In Memoriam: Geneviève Raugel

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Geneviève Raugel was born in 1951 in Strasbourg. She was the second child of Marguerite Huchler, a housewife, and Henri Michel Raugel, a school teacher. Strasbourg is the capital of Alsace and one of the major cities in Eastern France, at the meeting point of French and German cultural influences in the heart of Europe.

Geneviève spent her childhood and adolescence in Strasbourg. She attended the $Lyc\acute{e}e$ International des Pontonniers and, after her baccalaureate, the preparatory class at the $Lyc\acute{e}e$ Kléber. In 1972, she left Strasbourg to enter the $\acute{E}cole$ Normale Supérieure of Fontenay-aux-Roses, near Paris. This distinguished graduate school, founded in 1880, was still reserved for women at the time Geneviève was admitted. Like her, many female mathematicians have studied in this school, which eventually became co-educational in 1981.

In 1972, shortly after she began her studies in the Paris area, Geneviève met her future husband Gérard Laumon. They got married in 1980 and settled permanently in 1988 in Fontenay-aux-Roses, in a nice apartment in the city center located 500 m away from the former ÉNS.

After her studies at the ÉNS, Geneviève joined the CNRS as an associate researcher and was assigned to the IRMAR laboratory at the University of Rennes 1. There, she began her research activity under the direction of Michel Crouzeix. Her PhD thesis, defended in 1978, is devoted to the numerical analysis of partial differential equations and is entitled Résolution numérique de problèmes elliptiques dans des domaines avec coins (Numerical resolution of elliptic problems in domains with corners). Still under the direction of M. Crouzeix, she defended a thèse d'État (equivalent to habilitation) on the numerical approximation of nonlinear problems in 1984, one year after being promoted to a permanent research position at the CNRS. Two years later, she was appointed to the Centre de Mathématique de l'École Polytechnique, in Palaiseau, and in 1989 she joined the department of mathematics at Orsay, where she worked as a research director until she passes away in 2019.

The very first article of Geneviève Raugel [51] is concerned with the resolution by the finite element method of elliptic equations in domains with corners. With Christine Bernardi, she studied in [5] a finite element scheme for the Stokes problem, previously introduced by Michel Fortin, which is still used today under the name of "Bernardi-Fortin-Raugel element". Geneviève's interest in geometry and dynamics in infinite dimensions is already apparent in her work devoted to numerical analysis. During her thèse d'État, she studied the approximation of bifurcation problems involved in the buckling of a hyperelastic beam and showed in [18] that this problem can be considered as an imperfect

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bifurcation in the sense of Golubitsky and Schaeffer. In [11], she generalized a Morse lemma due to Martin Golubitsky and Jerrold Marsden.

The year 1985 was a turning point in the scientific life of Geneviève Raugel. During her first visit to Brown University, she attended a weekly seminar organized by Jack Hale. The numerous discussions she had with him played a decisive role in the choice of her future research subjects. This was the beginning of a long and fruitful collaboration on the dynamical properties of partial differential equations, which will only ended with Jack Hale's passing away in 2009.

Geneviève Raugel and Jack Hale were the first to study the continuity of attractors under singular perturbations of PDEs. In the article [19], they considered for example the perturbation generated by a numerical approximation, which is well in the spirit of Geneviève's early works. Other interesting cases were subsequently studied, in particular the thin domain approximation [25, 23, 26, 27, 30, 55] where a PDE defined on a very thin domain is approximated by the same PDE defined on a limiting domain of lower spatial dimension. Later works addressed fundamental questions in the study of dynamical systems generated by dissipative PDEs. The paper [32] shows that the globally bounded solutions of such a PDE are typically more regular than one would expect from the general Cauchy problem, even if the equation has no regularizing effect in finite time. The articles [33] and [34] also explore new techniques for the study of periodic solutions and their perturbations. This long collaboration - 17 co-authored papers and 5 long invitations for a total of more than two years - deeply influenced Geneviève's scientific activity.

Geneviève Raugel's first publications on fluid mechanics were devoted to flows in thin domains. In a series of works in collaboration with George Sell [60, 62, 63, 61], she obtained several results of global existence of strong solutions for the Navier-Stokes equation in three dimensions, based on the thin domains techniques and the well-understood two-dimensional case. The same ideas were used in [41, 43] with Jerrold Marsden and Tudor Ratiu to study the Euler equation on a thin spherical shell, and more recently with Dragos Iftimie for the Navier-Stokes equation with various boundary conditions [35, 36]. Together with Marius Paicu, Andrey Rekalo and Van-Sang Ngo, she explored in [46, 47, 49, 48, 45] more complex fluid models (hyperbolic Navier-Stokes, second grade fluids, anisotropic equations) but always with the original point of view of singular perturbations or hidden regularities.

Geneviève Raugel's contributions to the study of nonlinear PDEs considered as infinite dimensional dynamical systems are not limited to her collaboration with Jack Hale. After meeting with Klaus Kirchgässner, Geneviève became interested in the existence and stability of traveling fronts in reaction-diffusion systems [40, 59]. In collaboration with Thierry Gallay, she studied the same question for a class of hyperbolic equations with damping [14, 15, 17, 16, 13]. Following a research program initiated with Jack Hale - the generalization to PDEs of known results for differential equations in finite dimension - she strove to describe qualitatively the dynamics of generic PDEs. With Pavol Brunovský and Romain Joly, she was the first to establish the genericity of the Morse-Smale property for a damped wave equation or for a PDE without gradient structure [7, 37, 38, 8]. On the other hand, with Nicolas Burq and Wilhelm Schlag [9], she obtained a dichotomy between finite time blow-up and convergence to an equilibrium for a damped Klein-Gordon equation, thus improving on what was known by the classical methods of PDE analysis only.

Geneviève Raugel was deeply involved in the academic community. She was always very caring and generous with her time, especially towards young mathematicians. She directed 12 PhD theses very conscientiously, and also supervised many other young researchers.

Her supervision was not limited to mathematical discussions: she really introduced the newcomers to the world of research, supporting them during their first steps, and even helping them with more personal issues.

In terms of collective responsibilities, Geneviève served for many years in the central councils of the University of Paris-Sud (now Université Paris-Saclay). She was the codirector of the European structure federating the mathematical relations between France and Romania. She co-organized several important conferences, as well as a whole thematic semester on fluid mechanics, which was held in Lausanne in 2006. She was also co-editor-in-chief of the Journal of Dynamics and Differential Equations for 14 years, with George Sell and then Yingfei Yi. She carried out all these tasks with an exemplary dedication, even continuing to run JDDE from her hospital bed.

Being an open-minded and multi-skilled person, Geneviève Raugel had collaborators in various fields. The list of her research works is not limited to the bibliography below. On the one hand, she was very demanding about the quality of her results and the writing of her articles. She would not allow herself to publish a paper she felt imperfect, and consequently she left several works unfinished. On the other hand, she could spend hours in mathematical discussions without worrying about the outcome in short time, nor claiming anything once the problem was solved. So much so that several colleagues became regular collaborators and friends, without sharing a publication with her. Among them we can name Sylvain Crovisier, Constantine Dafermos, Bernold Fiedler, Konstantin Mischaikow, Peter Poláčik and Jürgen Scheurle.

Geneviève warmly welcomed her collaborators to Orsay, and her hospitality was proverbial. She also visited many foreign countries for conferences or long-term invitations, and she was well known in the worldwide community. She often traveled with her husband, Gérard Laumon, who is also a leading mathematician (he supervised the PhD thesis of two Fields Medal winners!). Following her invitations and those of her husband, Geneviève visited many new places, always grabbing the opportunity to meet people and to give her time for visits and lectures. This was for example the case during their last trip to Viet Nam at the end of August 2017, on the invitation of Ngô Báo Châu.

Geneviève Raugel was also open-minded and cultured during her spare time. She collected books on the history of Alsace, her native region, and its inhabitants. She had a passion for German literature, a language she mastered perfectly because of her origin, and she took advantage of her trips to Germany and Austria to enrich her book collection. She was also interested in Russian literature, which was facilitated by her knowledge of the Russian language. In addition to reading, her hobbies included music, with a predilection for baroque and religious pieces, her stamp collection, genealogy, and gardening. Together with Gérard, her husband, she loved hiking in the mountains, especially in Queyras, in the middle of the French Alps, or in Austria.

Geneviève used to spend most of her holidays and many weekends in the family house of Gérard, in Lyon, which they inherited in 2000. A lot of mathematical results were finalized there, and the house has welcomed many colleagues and friends over years. Geneviève loved to walk in the surrounding area, a very typical small district of Lyon, from which she knew the history, the streets and the monuments. Even when she was ill, between two stays at the hospital, she returned to rest in this house in Lyon. Today, Geneviève is buried in the Loyasse cemetery, close to the house where she and Gérard had such happy times.

All those who have known Geneviève Raugel as a colleague, supervisor, collaborator or friend will remember her as a rigorous mathematician and an eminently devoted, welcoming and generous person in every sense of this word. Her mathematical career has been







Left, with Gérard Laumon, her husband, and Ngô Báo Châu, in Viêt Nam (2003). Right, hiking in Queyras, in the french Alps.



With Jack Hale at Atlanta.





With collaborators and friends: Constantine Dafermos (2016) and Jerrold Marsden (2002).







With collaborators and friends: Yingfei Yi (2011), Pavol Brunovský and Bernold Fiedler (2008).

PhD thesis supervised by Geneviève Raugel

- Mostafa Abounouh 1993
- Vincent Moreau 1994
- Ionel Ciuperca 1995
- Jean Angles 1996
- Seifeddine Snoussi 1996
- Mohamed Ali Hamza 2001
- Bouthaina Abdelhedi 2005
- Romain Joly 2005
- Van-Sang Ngo 2009
- Basma Mourtada 2010
- Olivier Couland 2013
- Maxime Percie du Sert 2014

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