AMS-EMS-SMF meeting – Low Dimensional Topology session
Grenoble, July 18–22

PROGRAM

	Monday 18	Tuesday 19	Wednesday 20
8:00–10:00 am			Le (8:00-8:50) Massuyeau (9:00-9:50)
10:30–12:30 am			Miller (10:30-11:20) Andersen (11:30-12:20)
1:30–3:30 pm	Powell (1:30-2:20) Kosanovic (2:30-2:55) Robert (3:00-3:25)	Piccirillo (1:30-2:20) Kjuchukova (2:30-2:55) Misev (3:00-3:25)	
4:00–6:00 pm	Zentner (4:00-4:50) Palmer-Anghel (5:00-5:25) Detcherry (5:30-5:55)	Lin (4:00-4:50) Cazassus (5:00-5:25) Ben Aribi (5:30-5:55)	

ABSTRACTS

Joergen Andersen (Southern Denmark Univ.)

Geometric Recursion

Geometric Recursion is a very general machinery for constructing mapping class group invariants objects associated to two dimensional surfaces. After presenting the general abstract definition we shall see how a number of constructions in low dimensional geometry and topology fits into this setting. These will include the Mirzakhani-McShane identities, mapping class group invariant functions and closed forms on Teichmüller space (including the Weil-Petterson symplectic form) and the Goldman symplectic form on moduli spaces of flat connections. We will then discuss how averaged of volume forms constructed by geometric recursion satisfies topological recursion and thereby argue that Geometric Recursion can in these cases also be seen as a tool for proving that various topological quantities satisfies Topological Recursion.

Fathi Ben Aribi (UC Louvain)

Fuglede–Kadison determinants over free groups and Lehmer's constants

The Mahler measure of a polynomial with integer coefficients is its geometric mean on the unit circle, and the famous Lehmer Problem studies whether these Mahler measures admit an accumulation point around 1. In 2019, Lück extended this question to Fuglede–Kadison determinants over torsion-free groups, which are determinants defined for infinite-dimensional equivariant operators, and which are hard to compute. Lehmer's constants of a group then measure the possible gap around 1 of Fuglede–Kadison determinants over that group. Beyond group theory and operator algebras, Fuglede–Kadison determinants appear in low-dimensional topology as building blocks of L^2-torsions, a class of topological invariants of 3-manifolds which detect strong information such as the hyperbolic volume.

In this talk, I will introduce these various concepts and present new values of Fuglede–Kadison determinants over free groups, obtained via combinatorics on trees. As a consequence, I will present new bounds on Lehmer's constants for a large class of groups, which partially answers Lück's question. Furthermore, I will present even finer bounds on Lehmer's constants for certain groups of hyperbolic 3-manifolds, obtained by studying L^2-torsions of Dehn fillings on the complement of the Whitehead link.

Guillem Cazassus (Univ. Oxford)

Hopf algebras, equivariant Lagrangian Floer homology, and cornered instanton theory

Let G be a compact Lie group acting on a symplectic manifold M in a Hamiltonian way. If L, L' is a pair of Lagrangians in M, we show that the Floer complex $CF(L, L')$ is an A_∞-module over the Morse complex $CM(G)$ (which has an A_∞-algebra structure involving the group multiplication). This permits to define several versions of equivariant Floer homology.

This should also imply that the Fukaya categoy $\mathcal{F}uk(M)$, in addition to its own A_∞ structure, is an A_∞ module over $CM(G)$. These two structures should be packaged into a single one: $CM(G)$ is an A_∞ bialgebra, and $\mathcal{F}uk(M)$ is a module over it. In fact, $CM(G)$ should have more structure, it should be a $\mathcal{H}opf_\infty$ algebra, a strong-homotopy structure (still unclear to us) that should induce the Hopf algebra structure on $H_*(G)$. In a certain sense, this refines a conjecture by Teleman.

Applied to some subsets of Huebschmann-Jeffreys extended moduli spaces introduced by Manolescu and Woodward, this construction should permit to define a cornered instanton theory analogous to Douglas-Lipshitz-Manolescus construction in Heegaard-Floer theory. This is work in progress, joint with Paul Kirk, Mike Miller-Eismeier and Wai-Kit Yeung.

Renaud Detcherry (Univ. Bourgogne-Franche Comté)

On the Heisenberg twisted homology representations of mapping class groups of surfaces

A big open question in the theory of mapping class groups of compact oriented surfaces is whether they are linear groups, i.e., whether they embed into $\mathrm{GL}_n(\mathbb{C})$. While mapping class groups of surfaces share all of the known special properties of finitely generated linear groups, it is mostly known that braid groups are linear by the work of Bigelow and Krammer on the faithfulness of Lawrence representations.

Recently, Blanchet, Palmer and Shaukat defined similar representations of the Torelli groups $\mathcal{T}(\Sigma_{g,1})$ of the surface $\Sigma_{g,1}$, using the action of $\mathrm{Mod}(\Sigma_{g,1})$ on some characteristic cover of $\mathrm{Conf}_n(\Sigma_{g,1})$, the space of configurations of n distinct points in $\Sigma_{g,1}$. Those representations take value in $\mathrm{GL}_n(\mathbb{Z}[H_g])$, where H_g is the so-called Heisenberg group.

We will explain how to extract from those representations some linear representations of $\mathcal{T}(\Sigma_{g,1})$, and we will try to argue why those representations might be faithful. In particular, we will state a faithfulness criterion for these representations in terms of twisted intersections of simple arcs on $\Sigma_{g,1}$. Finally, we will explain how one can relate those representations to Lawrence representations of braid groups. Joint work with Jules Martel.

Alexandra Kjuchukova (Univ. Notre Dame)

$\mathbb{CP}^2$*-slicing numbers of knots*

We say a knot K is H-slice in a closed smooth 4-manifold X^4 if K bounds a smooth null-homologous disk in $X \backslash \mathring{B}^4$. The $\mathbb{CP}^2$-slicing number of K is the smallest m such that K is H-slice in $\#^m\mathbb{CP}^2$. I will show how to extract a lower bound on the $\mathbb{CP}^2$-slicing number of a knot from its double branched cover, and I will compute some examples. Time permitting, I'll also give an upper bound on the topological counterpart of this quantity (defined analogously), and I will contrast the two. This talk is based on arXiv:2112.14596.

Danica Kosanović (ETH Zürich)

2*-knots and knotted families of arcs*

Knowing when one can embed a surface into a 4-manifold is a question of fundamental importance for 4-manifold theory. It gives rise to a field of 2-knot theory, which is in a certain sense even harder than classical knot theory. However, in certain situations – like in the setting when the embedded surfaces (disks) have a common light bulb (in the boundary) – one can completely classify them up to isotopy. I will explain how we do this using some quite general techniques from homotopy theory, which in turn lead to some surprising applications of higher homotopy groups of spaces of embeddings. Joint work with Peter Teichner.

Thang T. Q. Le (Georgia Tech)

Skein algebras and quantum Teichmüller spaces of surfaces

For a punctured surface (with boundary) the (stated) skein algebra is a quantization of the $SL_2(\mathbb{C})$-character variety. When the surface has no boundary but at least one puncture, Bonahon and Wong showed that there is an embedding, known as the quantumt trace map, of the skein algebra into the quantum enhanced (or holed) Teichmüller space. We extend the result to the case when the boundary is non-empty, which now involves also the decorated Teichmüller space. There is also a generalization, though with weaker results, for SL_n. This is a joint work with T. Yu.

Francesco Lin (Columbia Univ.)

Seiberg-Witten theory and hyperbolic three-manifolds

An outstanding problem in low-dimensional topology is to understand the relationship (if any) between Floer theoretic invariants and geometric invariants of hyperbolic three-manifolds. For example, a basic question is whether one can read off some Floer theoretic information of a three-manifold from quantities such as volume, injectivity radius, and the lengths of closed geodesics. In this talk I will discuss some partial results towards this goal obtained in joint work with M. Lipnowski.

Gwénaël Massuyeau (Univ. Bourgogne-Franche Comté)

On the non-triviality of the torsion subgroup of the abelianized Johnson kernel

The Johnson kernel is the subgroup of the mapping class group of a closed oriented surface that is generated by Dehn twists along separating curves. The rational abelianization of the Johnson kernel was calculated by Dimca, Hain and Papadima, then it was revisited by Morita, Sakasai and Suzuki. From this and using the theory of finite-type invariants of 3-manifolds, Nozaki, Sato and Suzuki showed that the torsion of the abelianized Johnson kernel is non-trivial.

In this talk, we will give a more elementary and more explicit proof of this non-triviality using only the action of the mapping class group of the surface on the Malcev Lie algebra of its fundamental group. The same infinitesimal methods will also allow us to diagrammatically reformulate the rational abelianization of the Johnson kernel, and consider the case of a surface with boundary as well.

Work in collaboration with Quentin Faes.

Maggie Miller (Stanford Univ.)

Knotted handlebodies

We construct 3-dimensional genus-g handlebodies H and H_0 in the 4-sphere so that H and H_0 have the same boundary and are homeomorphic rel boundary, but are not smoothly isotopic rel boundary (for all $g \geq 2$). In fact, H and H_0 are not even topologically isotopic rel boundary, even when their interiors are pushed into the 5-ball. This proves a conjecture of Budney and Gabai (who recently constructed smooth 3-balls in the 4-sphere with the same boundary that are not smoothly isotopic rel boundary) for $g \geq 2$ in a very strong sense. In this talk, Ill describe some useful facts about higher-dimensional knots that go into this construction and talk about remaining future directions. Joint work with Mark Hughes and Seungwon Kim.

Filip Misev (Univ. Regensburg)

Meridional rank of arborescent links with many twigs

The meridional rank of a link is the smallest number of meridians that generate the link group (that is, the fundamental group of the link complement). It is conjectured that the meridional rank equals the bridge number. I would like to discuss this conjecture and explain how Coxeter groups, arising as quotients of the link group, can be used to confirm it for a new class of links, the arborescent links with many twigs. Joint work with S. Baader, R. Blair, and A. Kjuchukova.

Cristina Palmer-Anghel (Univ. Geneva)

Coloured Jones and coloured Alexander invariants from two Lagrangians intersected in a symmetric power of a surface

The coloured Jones polynomials and the coloured Alexander polynomials are quantum invariants that come from the representation theory of the quantum group $U_q(sl(2))$. We construct a unified topological model for these two sequences of invariants. More precisely we prove that the N^{th} coloured Jones and N^{th} coloured Alexander invariants are different specializations of a *graded intersection* between *two explicit Lagrangians in a symmetric power of the punctured disc.*

In particular, the Jones polynomial and the Alexander polynomial are two specializations of the *same graded intersection in a configuration space.* Then, we notice that the intersection before specialization is (up to a quotient) an explicit interpolation between the Jones polynomial and Alexander polynomial.

Lisa Piccirillo (MIT)

4-manifolds with boundary and fundamental group $\mathbb{Z}$

In this talk I will discuss a classification of topological 4-manifolds with boundary and fundamental group $\mathbb{Z}$, under some mild assumptions on the boundary. We apply this classification to provide an algebraic classification of surfaces in simply-connected 4-manifolds with 3-sphere boundary, where the fundamental group on the surface complement is $\mathbb{Z}$. We also compare these homeomorphism classifications with the smooth setting, showing for example that everyHermitian form over the ring of integer Laurent polynomials arises as the equivariant intersection form of a pair of exotic smooth 4-manifolds with boundary and fundamental group $\mathbb{Z}$.

Mark Powell (Durham Univ.)

Counterexamples in 4-manifold topology

With an emphasis on examples, I will discuss the relationships between the equivalence relations on 4-manifolds of s-cobordism, h-cobordism, homotopy equivalence, and simple homotopy equivalence. Joint work with Daniel Kasprowski, Csaba Nagy, John Nicholson, and Arunima Ray.

Louis-Hadrien Robert (Univ. Luxembourg)

An $\mathfrak{sl}_2$-action on $\mathfrak{gl}_N$ link homology

I will explain how foams can be used to constuct $\mathfrak{gl}_N$ link homology. The case $N = 2$ corresponds to Khovanov homology. This description enables to endow the homology groups with an $\mathfrak{sl}_2$ action. This can be restricted to a p-DG structure when working in characteristic p. Joint work with You Qi, Joshua Sussan and Emmanuel Wagner.

Raphael Zentner (Univ. Regensburg)

$SL(2,\mathbb{C})$-character varieties of knots and maps of degree 1

We ask to what extend the $SL(2,\mathbb{C})$-character variety of the fundamental group of the complement of a knot in S^3 determines the knot. Our methods use results from group theory, classical 3-manifold topology, but also geometric input in two ways: The geometrisation theorem for 3-manifolds, and instanton gauge theory. In particular this is connected to $SU(2)$-character varieties of two-component links, a topic where much less is known than in the case of knots. This is joint work with Michel Boileau, Teruaki Kitano and Steven Sivek.