Nonlinear Ekman Dynamics

Linear solution: Ekman (1905)

Start from Boussinesq

$$D_t \boldsymbol{u} + \boldsymbol{f} \times \boldsymbol{u} = -\nabla\phi + \hat{\boldsymbol{z}} b + \nabla \cdot \boldsymbol{\tau}/\rho_0$$
$$\nabla \cdot \boldsymbol{u} = 0$$

Assume: steady state, geostrophy, hydrostasy and incompressibility

$$\boldsymbol{f} \times \boldsymbol{u} = -\nabla\phi + \partial_z \boldsymbol{\tau}/\rho_0 \longleftarrow \boldsymbol{\tau}/\rho_0 = A\partial_z \boldsymbol{u}$$
$$\partial_z \phi = b \longrightarrow \partial_z \phi = 0$$
$$\nabla \cdot \boldsymbol{u} = 0$$

Linear solution

Separate into interior and boundary solutions

$$\boldsymbol{u} = \boldsymbol{u}_g + \boldsymbol{u}_E \quad ; \quad \phi = \phi_g + \phi_E$$

$$\boldsymbol{f} \times (\boldsymbol{u}_g + \boldsymbol{u}_E) = -\nabla(\phi_g + \cancel{\phi_E}) + A\partial_{zz}(\cancel{\boldsymbol{u}_g} + \boldsymbol{u}_E)$$

$$\partial_z \phi_g = 0 \quad \longrightarrow \quad \partial_z \phi_E = 0 \quad \longrightarrow \quad \phi_E = 0$$

Linear solution

$$\boldsymbol{f} \times (\boldsymbol{u}_g + \boldsymbol{u}_E) = -\nabla \phi_g + A\partial_{zz}\boldsymbol{u}_E$$

Geostrophic component

$$\boldsymbol{f} \times \boldsymbol{u}_g = -\nabla \phi_g$$

Equations for the boundary layer

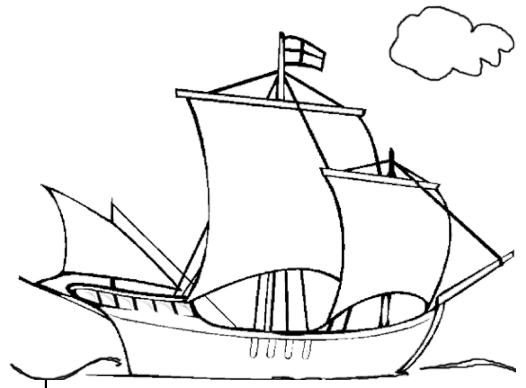
$$\boldsymbol{f} \times \boldsymbol{u}_E = A\partial_{zz}\boldsymbol{u}_E$$

$$\frac{\partial^4 u_E}{\partial z^4} = -\left(\frac{f}{A}\right)^2 u_E$$

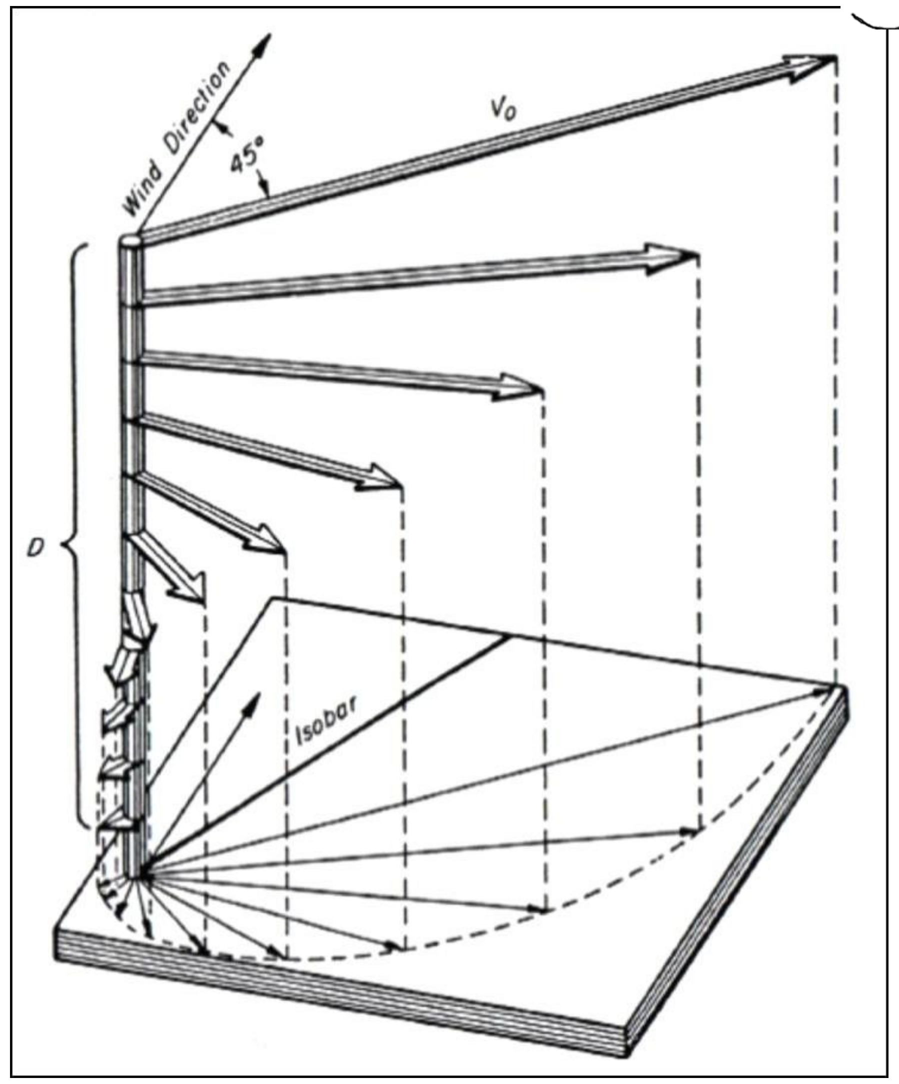
$$u_E = \mathbf{Re}\left(\underbrace{\mathrm{C_1 e}^{(1+\mathrm{i})z/\delta_\mathrm{E}} + \mathrm{C_2 e}^{(1-\mathrm{i})z/\delta_\mathrm{E}}}_{\text{Surface}} + \underbrace{\mathrm{C_3 e}^{-(1+\mathrm{i})z/\delta_\mathrm{E}} + \mathrm{C_4 e}^{-(1-\mathrm{i})z/\delta_\mathrm{E}}}_{\text{Bottom}}\right), \qquad \delta_E = \sqrt{2A/f}$$

Linear solution

$$u_E = \mathbf{Re}\left(C_1 e^{(1+i)z/\delta_E} + C_2 e^{(1-i)z/\delta_E} + C_3 e^{-(1+i)z/\delta_E} + C_4 e^{-(1-i)z/\delta_E}\right)$$



Surface boundary condition and solution



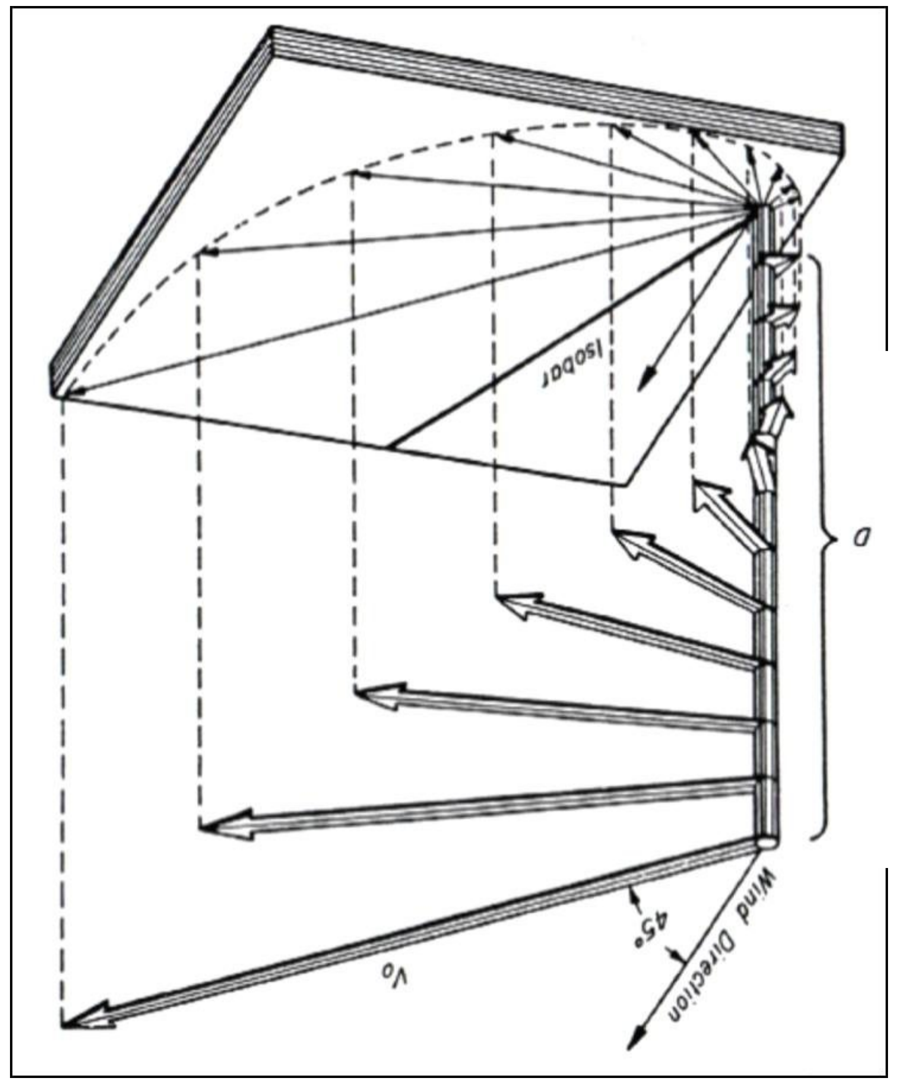
$$A\partial_z \boldsymbol{u}_E(z=0) = \boldsymbol{\tau}_a/\rho_0 \quad ; \quad \boldsymbol{u}_E(z\to -\infty) = 0$$

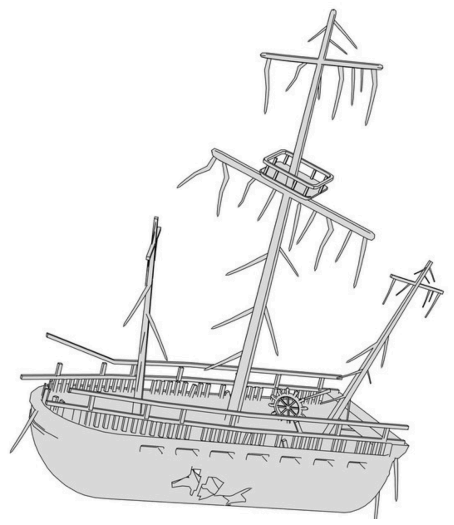
Surface

$$u_E = \frac{\sqrt{2}}{f\delta_E} e^{z/\delta_E}\left(\tau^x \cos(z/\delta_E - \pi/4) - \tau^y \sin(z/\delta_E - \pi/4)\right)$$

$$v_E = \frac{\sqrt{2}}{f\delta_E} e^{z/\delta_E}\left(\tau^x \sin(z/\delta_E - \pi/4) + \tau^y \cos(z/\delta_E - \pi/4)\right)$$

Bottom boundary condition and solution





$$\boldsymbol{u}(z=-H) = 0 \quad ; \quad \boldsymbol{u}_E(z\to +\infty) = 0$$

Bottom

$$u_E = -e^{-(z+H)/\delta_E}\left(u_g \cos((z+H)/\delta_E) + v_g \sin((z+H)/\delta_E)\right)$$

$$v_E = e^{-(z+H)/\delta_E}\left(u_g \sin((z+H)/\delta_E) - v_g \cos((z+H)/\delta_E)\right)$$

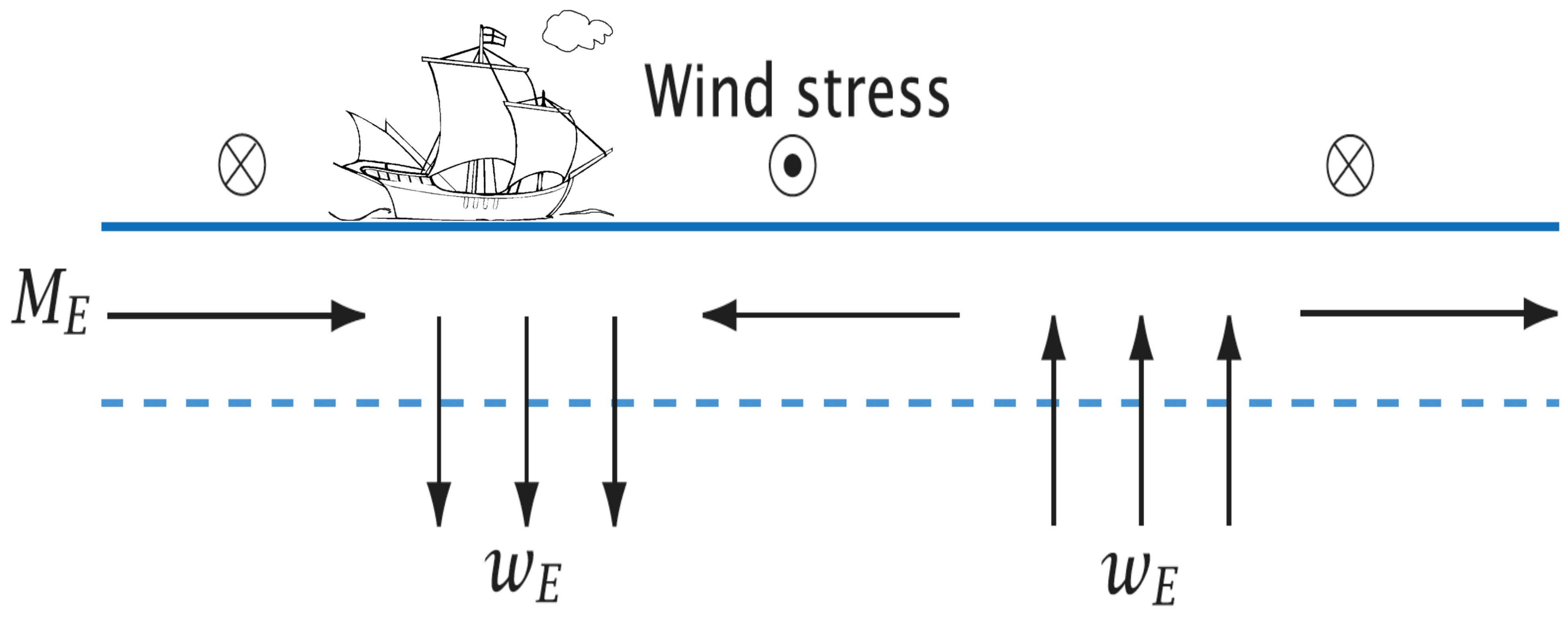
Ekman Transport and Pumping

Top layer Ekman Transport

$$\int_{-h_E}^{0} (\boldsymbol{f} \times \boldsymbol{u}_E = \partial_z \boldsymbol{\tau}/\rho_0)\, dz \quad \text{with} \quad \int_{-h_E}^{0} (\nabla \cdot \boldsymbol{u}_E = -\partial_z w_E)\, d.$$

$$\boldsymbol{f} \times \boldsymbol{U}_E = \boldsymbol{\tau_a}/\rho_0 \quad \text{with} \quad \nabla \cdot \boldsymbol{U}_E = -w_E$$

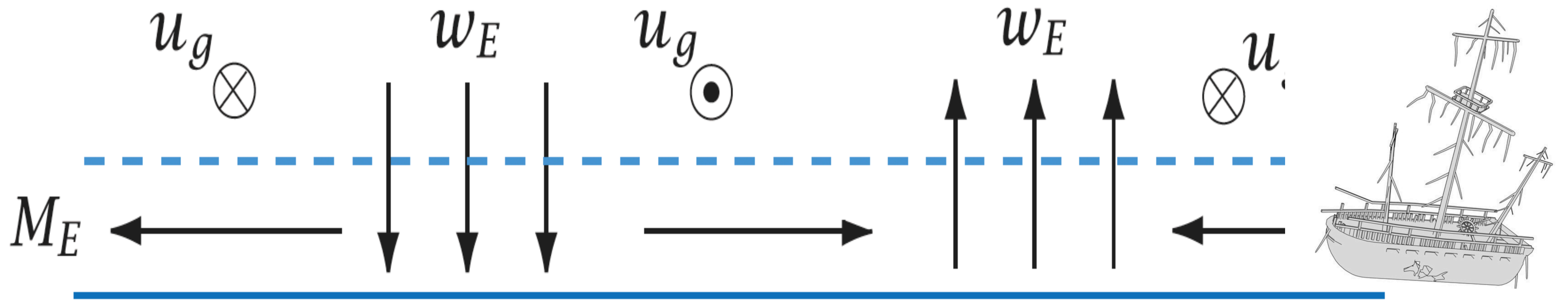
$$w_E = -\nabla \cdot \boldsymbol{U}_E = \nabla \cdot \left(\frac{\boldsymbol{\tau_a} \times \hat{\boldsymbol{z}}}{\rho_0 f} \right) = \hat{\boldsymbol{z}} \cdot \nabla \times \left(\frac{\boldsymbol{\tau_a}}{\rho_0 f} \right)$$



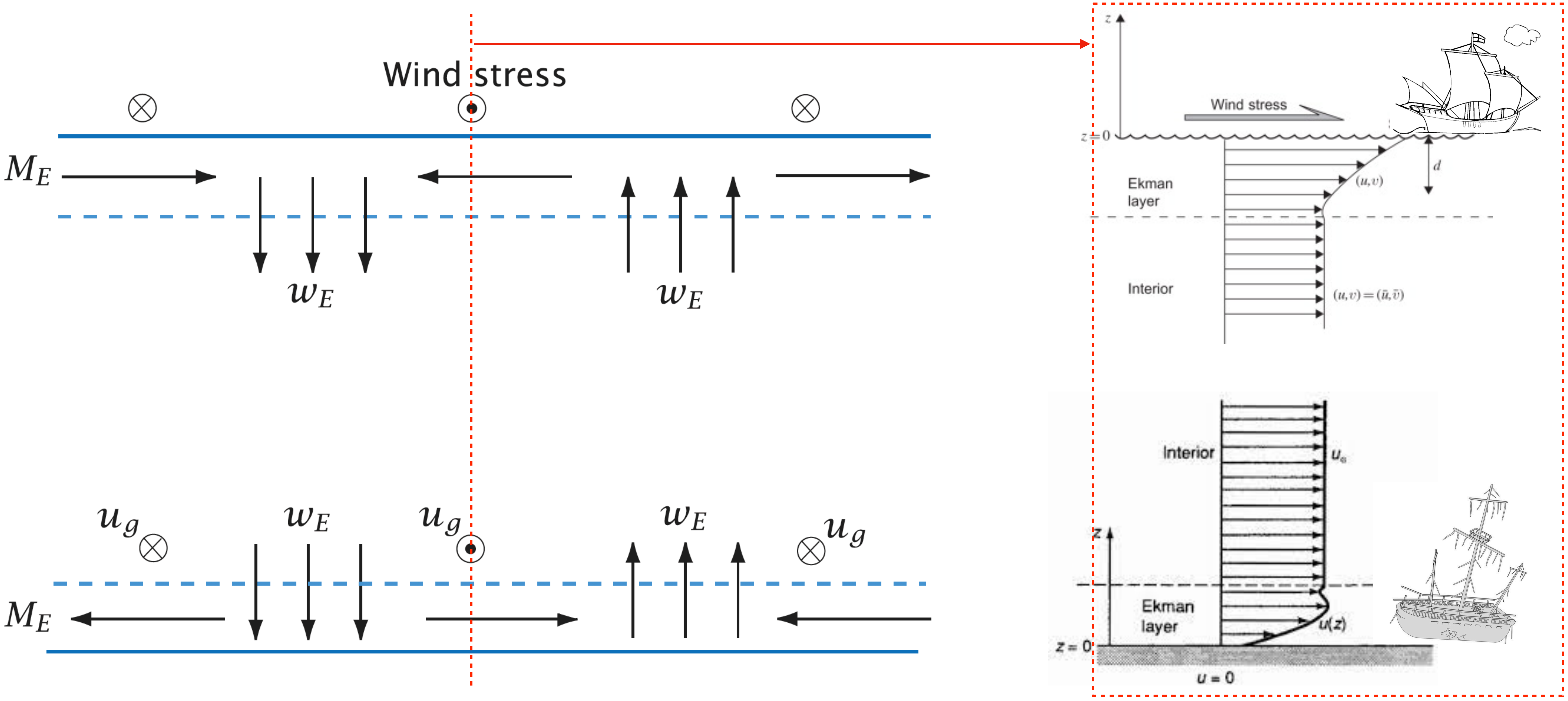
Bottom layer Ekman Transport

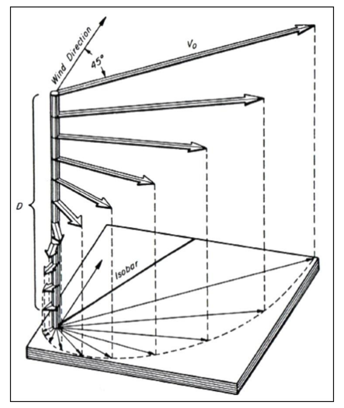
$$\boldsymbol{f} \times \boldsymbol{U}_E = \boldsymbol{\tau}_{\text{Bottom}}/\rho_0$$

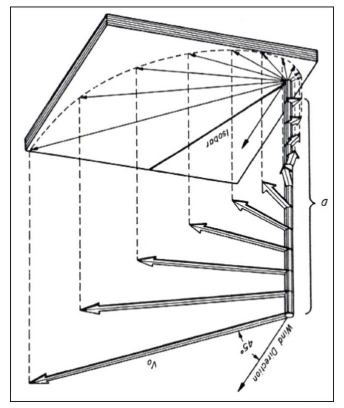
$$w_E = \frac{\hat{\boldsymbol{z}} \cdot \nabla \times \delta_E \boldsymbol{u}_g}{2\rho_0 f} = \frac{\delta_E \zeta_g}{2\rho_0 f}$$



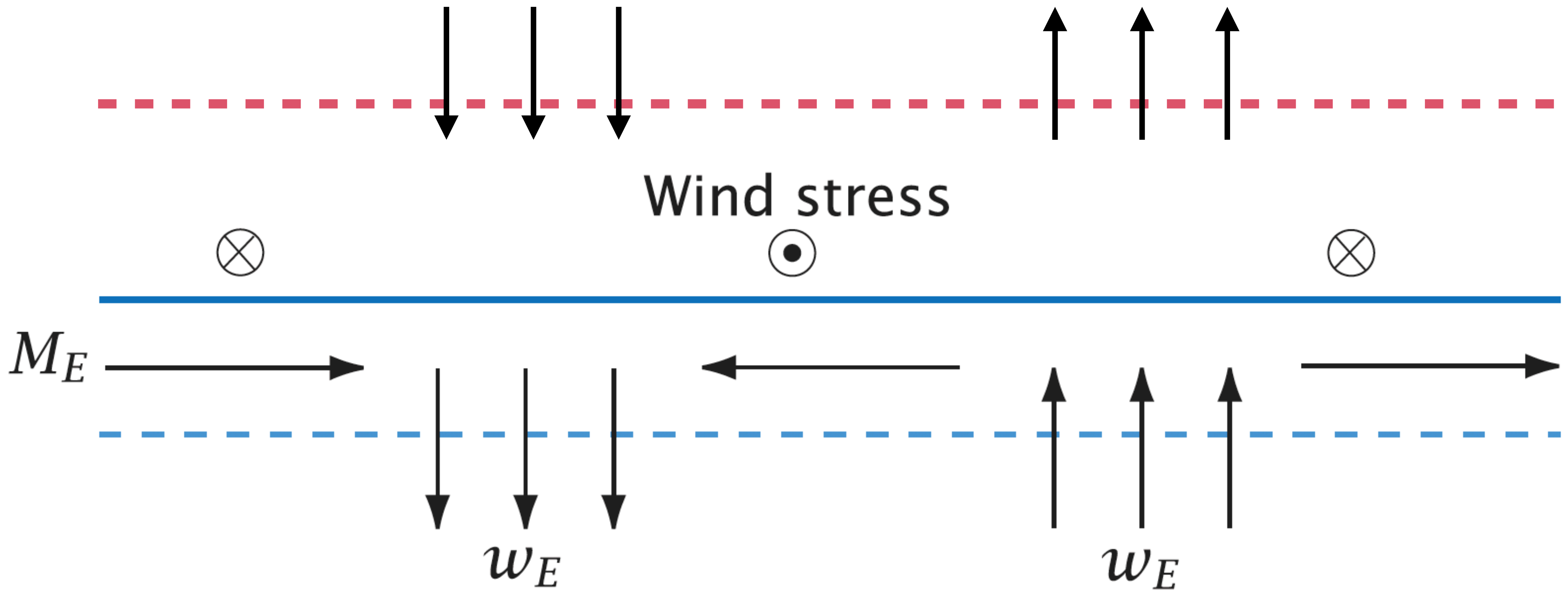
Ekman Transport and Pumping

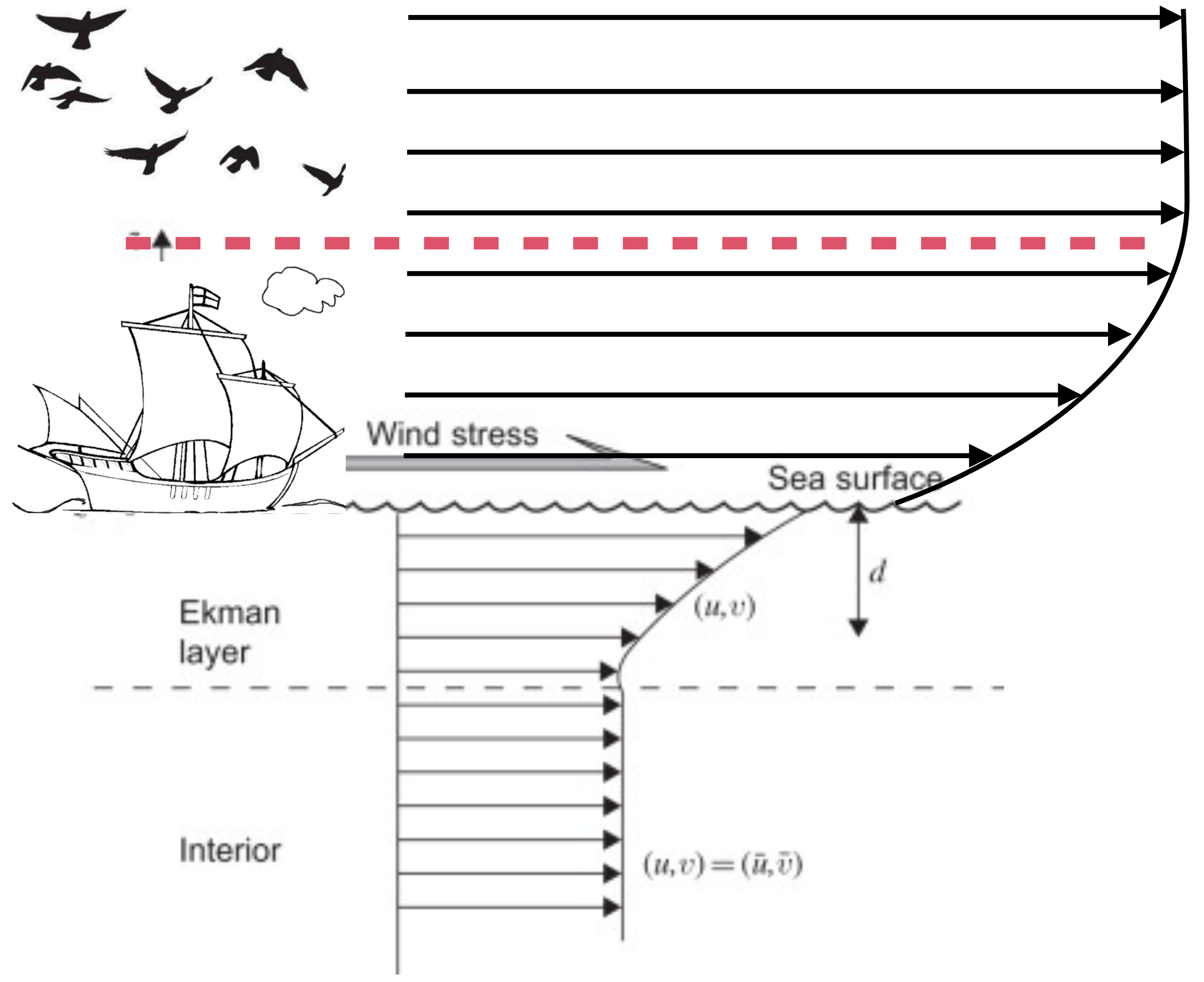


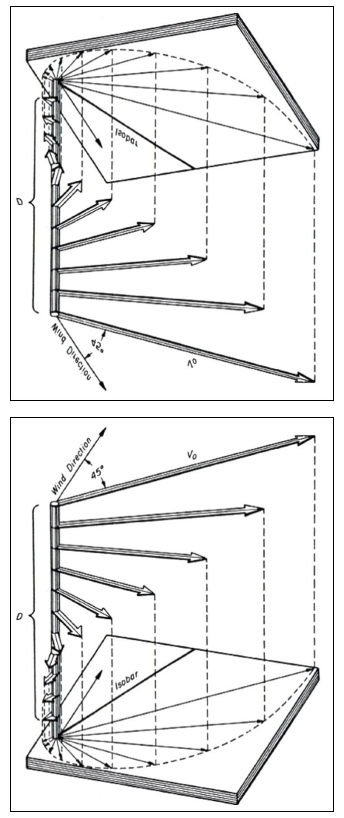


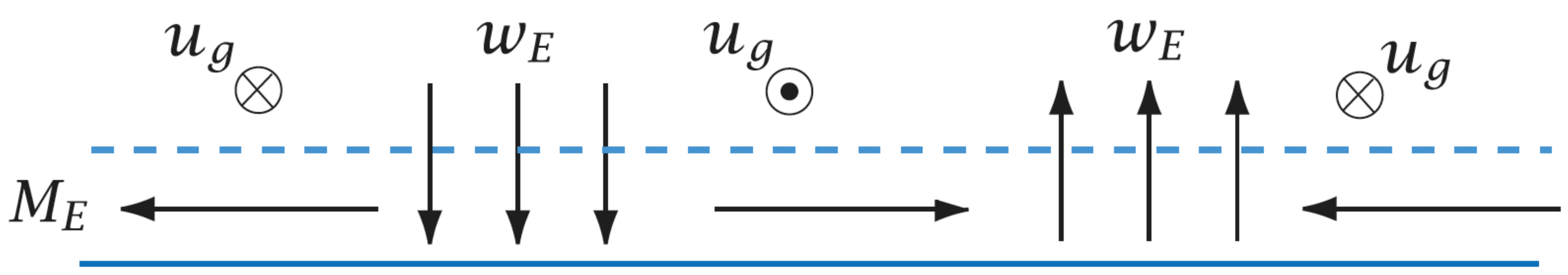


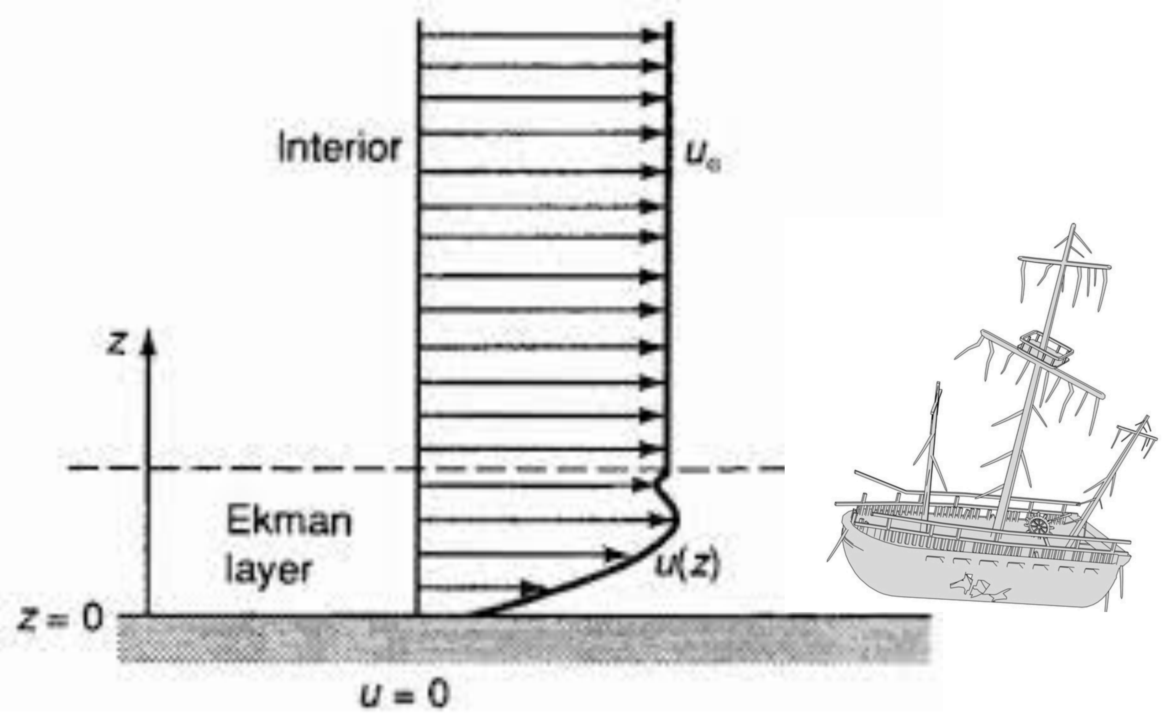
Adding the atmospheric boundary layer

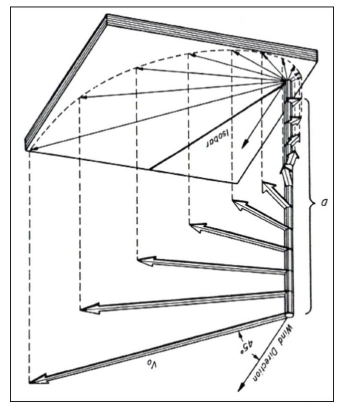












Nonlinear boundary layer

Nonlinear boundary layer

Steady state equations

$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} + \boldsymbol{f} \times \boldsymbol{u} = -\nabla \phi + \partial_z \boldsymbol{\tau}/\rho_0$$

Separate into interior and boundary solutions similarly to Ekman

$$\boldsymbol{u} = \boldsymbol{u}_g + \boldsymbol{u}_E$$

$$(\boldsymbol{u}_g + \boldsymbol{u}_E) \cdot \nabla(\boldsymbol{u}_g + \boldsymbol{u}_E) + \boldsymbol{f} \times (\boldsymbol{u}_g + \boldsymbol{u}_E) = -\nabla(\phi_g) + \partial_z \boldsymbol{\tau}/\rho_0$$

Define the geostrophic component

$$\boldsymbol{u}_g \cdot \nabla \boldsymbol{u}_g + \boldsymbol{f} \times \boldsymbol{u}_g = -\nabla(\phi_g)$$ ← **Interior, balanced, ...**

Equations for the boundary layer

$$\boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_E + \boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_g + \boldsymbol{u}_g \cdot \nabla \boldsymbol{u}_E + \boldsymbol{f} \times \boldsymbol{u}_E = \partial_z \boldsymbol{\tau}/\rho_0$$

Nonlinear boundary layer

$$\underbrace{\boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_E}_{} + \underbrace{\boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_g}_{} + \underbrace{\boldsymbol{u}_g \cdot \nabla \boldsymbol{u}_E}_{} + \underbrace{\boldsymbol{f} \times \boldsymbol{u}_E}_{} = \partial_z \boldsymbol{\tau} / \rho_0$$

Self-advection

(Higher higher order)

Stern(1965)

Higher order

Niiler(1969)

Hart(2000)

W&T(2016)

Ekman(1905)

Stern (1965):

Interaction of a uniform wind stress with a geostrophic vortex

$$\boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_g \; + \; \boldsymbol{f} \times \boldsymbol{u}_E = \partial_z \boldsymbol{\tau} / \rho_0$$

$$(\boldsymbol{f} + \boldsymbol{\zeta}_g) \times \boldsymbol{u}_E = -\nabla B + \partial_z \boldsymbol{\tau} / \rho_0$$

Transport and Pumping

$$(\boldsymbol{f} + \boldsymbol{\zeta}_g) \times \boldsymbol{U}_E = -\nabla \int_{-h_E}^{0} B + \boldsymbol{\tau_a} / \rho_0$$

$$w_E = -\nabla \cdot \boldsymbol{U}_E = \nabla \cdot \left(\frac{\boldsymbol{\tau_a} \times \hat{\boldsymbol{z}}}{\rho_0 (f + \zeta_g)} \right)$$

But notice that

$$\boldsymbol{U}_E = \frac{\boldsymbol{\tau_a} \times \hat{\boldsymbol{z}}}{\rho_0 (f + \zeta_g)} + \nabla \times \boldsymbol{A}$$

Stern (1965):

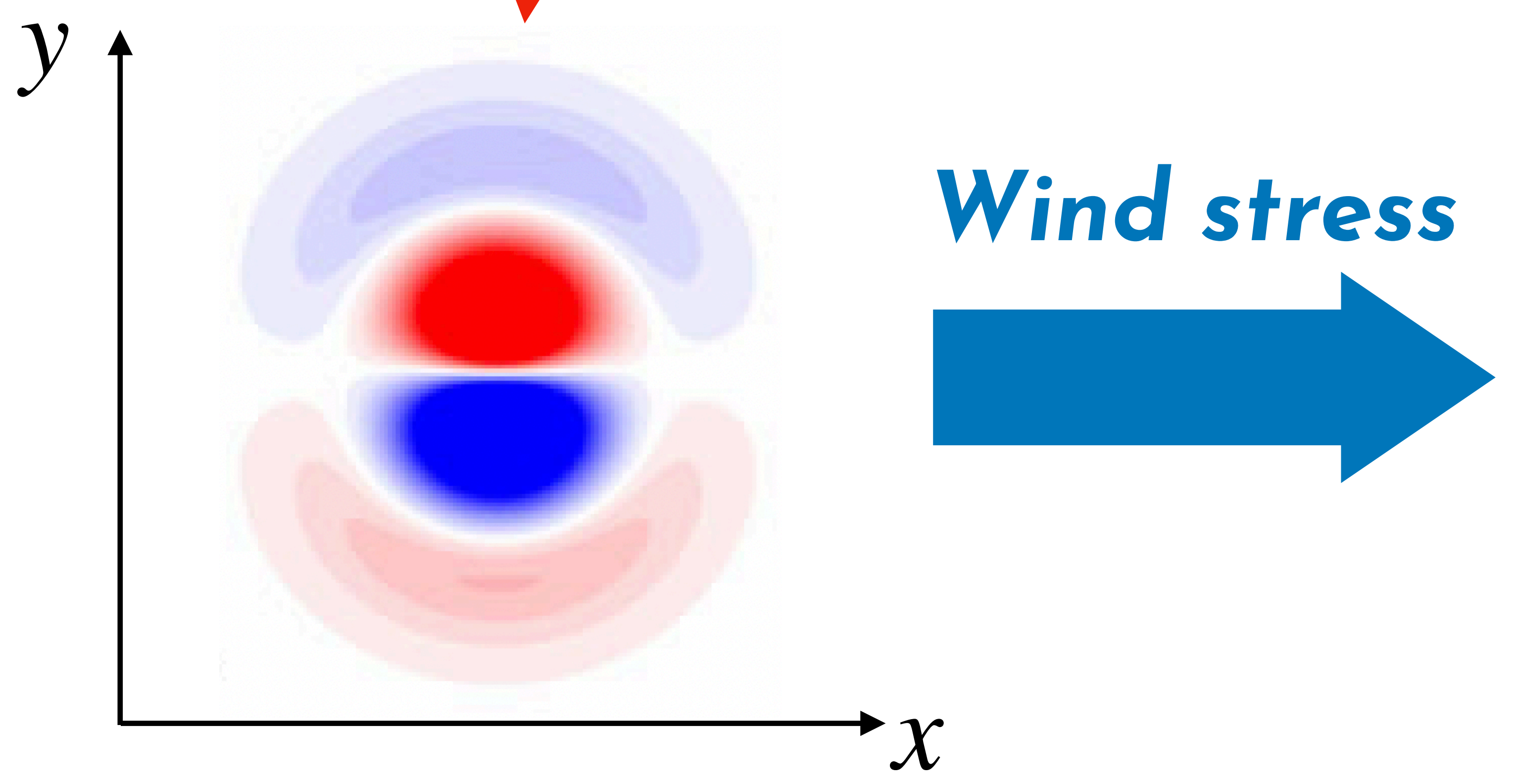
$$w_E = \hat{\boldsymbol{z}} \cdot \nabla \times \left(\frac{\boldsymbol{\tau_a}}{\rho_0 (f + \zeta_g)} \right) = \hat{\boldsymbol{z}} \cdot \nabla \times \left(\frac{\boldsymbol{\tau_a}}{\rho_0 f (1 + \varepsilon)} \right)$$

Rossby Number: $\varepsilon = \frac{U}{fL} = \frac{\zeta_g}{f}$

If f constant:

$$w_E = \frac{\hat{\boldsymbol{z}} \cdot \nabla \times \boldsymbol{\tau_a}}{\rho_0 (f + \zeta_g)} + \frac{1}{\rho_0 (f + \zeta_g)^2} \left(\tau_a^x \partial_y \zeta_g - \tau_a^y \partial_x \zeta_g \right)$$

$= 0$ if τ_a is constant



Assuming uniform wind stress over a nondivergent circular eddy (Gaussian streamfunction)

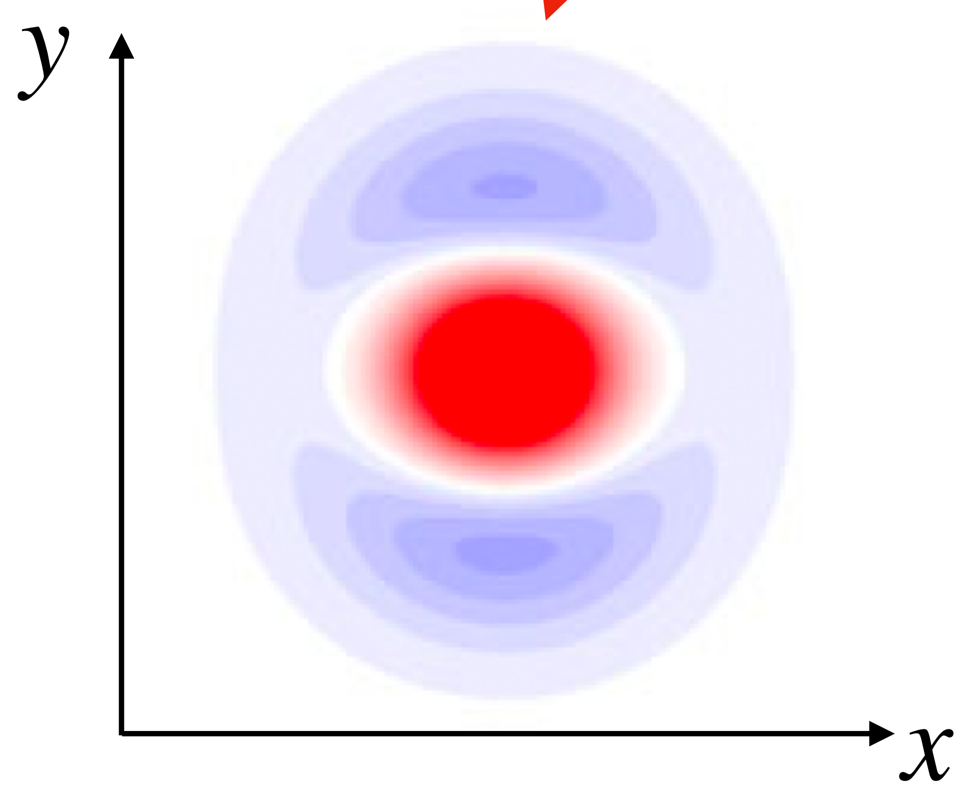
Stern (1965):

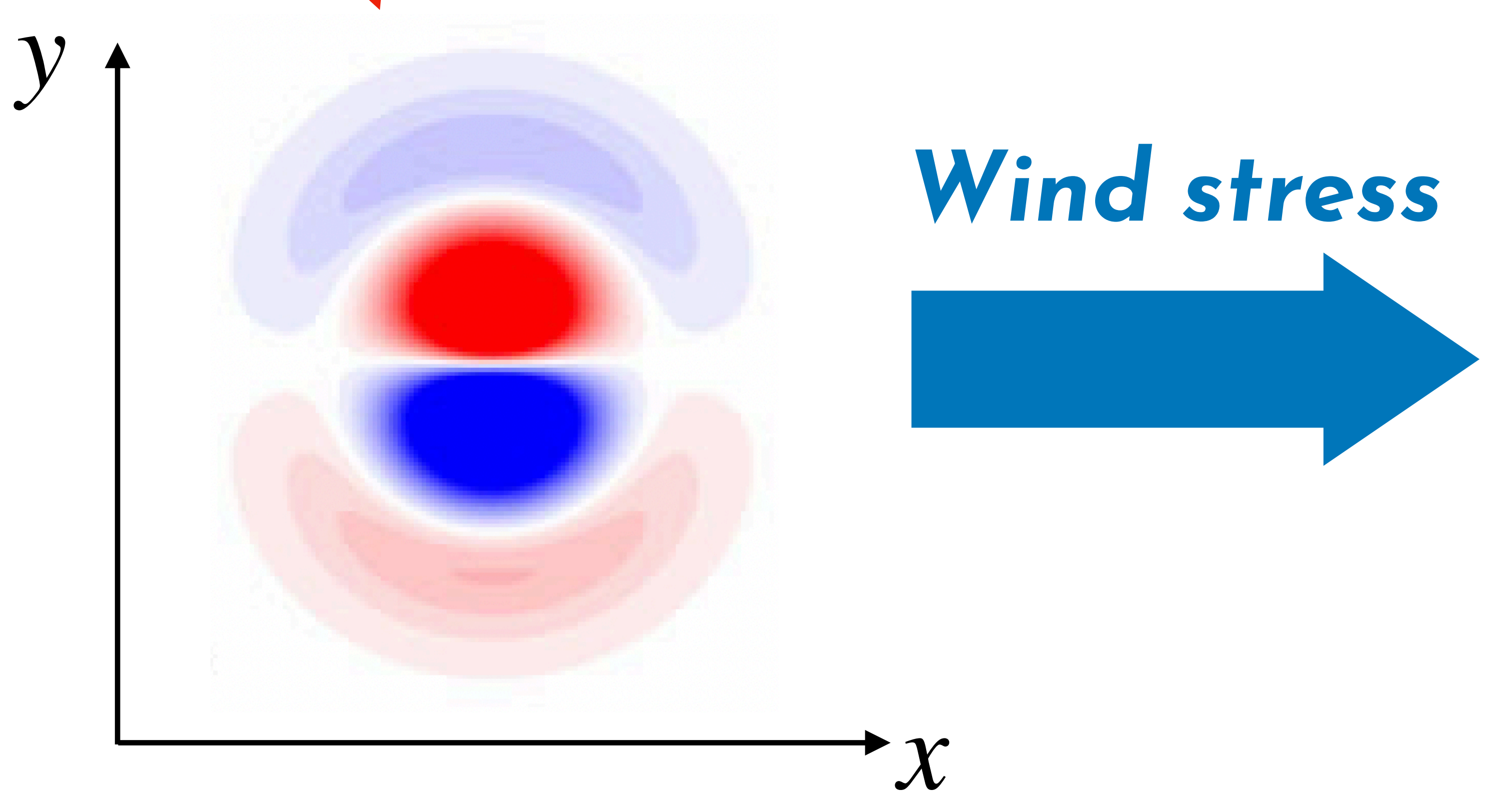
$$w_E = \nabla \cdot \left(\frac{\boldsymbol{\tau_a} \times \hat{\boldsymbol{z}}}{\rho_0 (f + \zeta_g)} \right)$$

If f constant: $$w_E = \frac{\nabla \times \boldsymbol{\tau_a}}{\rho_0 (f + \zeta_g)} + \frac{1}{\rho_0 (f + \zeta_g)^2} \left(\tau_a^x \partial_y \zeta_g - \tau_a^y \partial_x \zeta_g \right)$$

Using a velocity dependent stress :

$$\boldsymbol{\tau}_a = \rho_a C_d (\boldsymbol{u}_a - \boldsymbol{u}) |\boldsymbol{u}_a - \boldsymbol{u}|$$





Assuming uniform wind stress over a nondivergent circular eddy (Gaussian streamfunction)

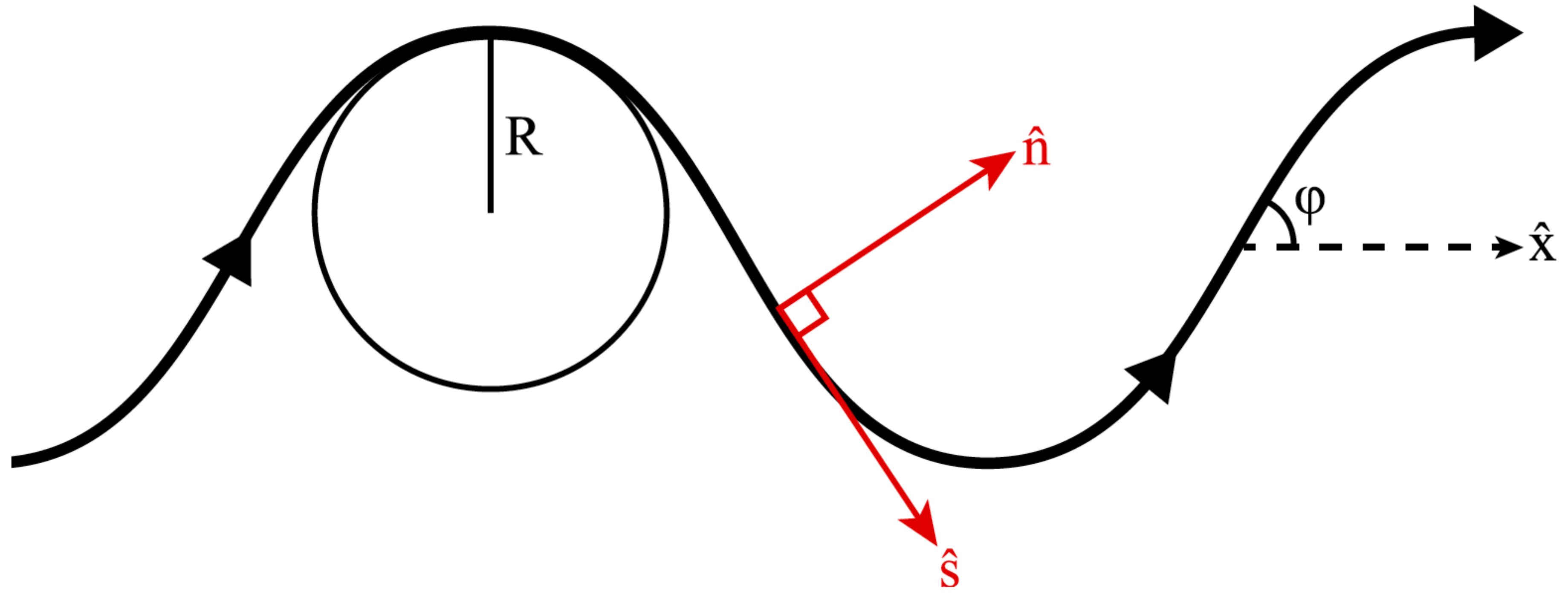
Wenegrat and Thomas (2017):

Ekman Transport in Balanced Currents with Curvature

$$\boldsymbol{u}_g \cdot \nabla \boldsymbol{u}_E + \boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_g + \boldsymbol{f} \times \boldsymbol{u}_E = \partial_z \boldsymbol{\tau}/\rho_0$$

Balanced natural coordinate system

$$\begin{cases} \mathbf{u} = (\overline{u} + u_e)\hat{\mathbf{s}} + v_e\hat{\mathbf{n}} + w_e\hat{\mathbf{z}} \\ \Omega \equiv \overline{u}k \\ \zeta \equiv -\partial\overline{u}/\partial n + \Omega \\ k \equiv (\partial\hat{\mathbf{s}}/\partial s)\cdot\hat{\mathbf{n}} = 1/R \end{cases}$$



$$\varepsilon\overline{u}\frac{\partial v_e}{\partial s} + (1+\varepsilon 2\Omega)u_e = \frac{\partial \tau_n}{\partial z}$$

$$\varepsilon\overline{u}\frac{\partial u_e}{\partial s} - (1+\varepsilon\zeta)v_e = \frac{\partial \tau_s}{\partial z}$$

$$(M_s, M_n) = \left(\int u_e\,dz, \int v_e\,dz\right)$$

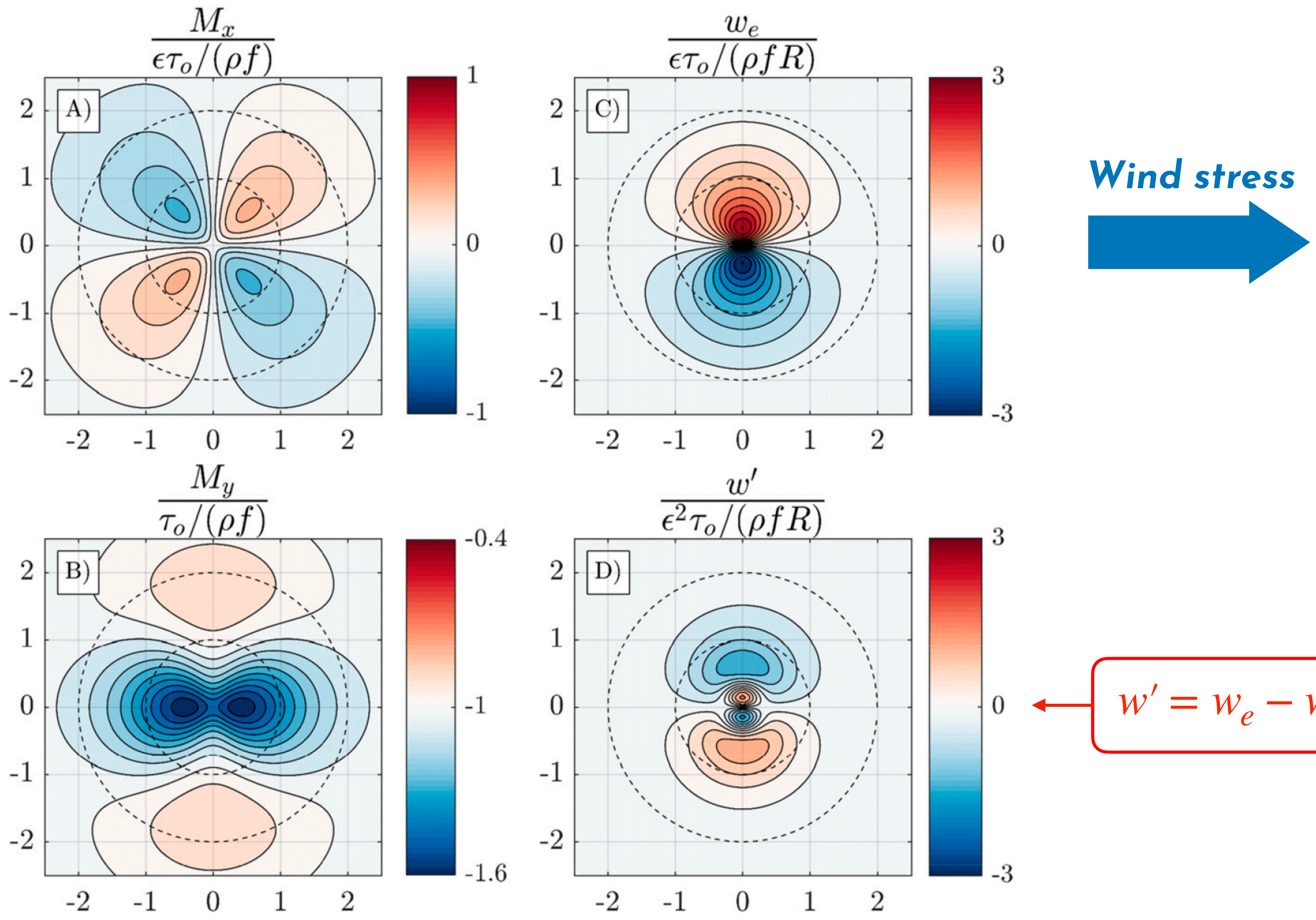
Transport equations

$$\begin{cases} \varepsilon\overline{u}\dfrac{\partial M_n}{\partial s} + (1+\varepsilon 2\Omega)M_s = \tau_n \\ \varepsilon\overline{u}\dfrac{\partial M_s}{\partial s} - (1+\varepsilon\zeta)M_n = \tau_s \end{cases}$$

Solution for a mesoscale vortex $\epsilon = 0.003$

Response to a uniform wind stress over a horizontally nondivergent circular eddy (Gaussian streamfunction)

Solution for a submesoscale anticyclone $\epsilon = 0.3$



$w' = w_e - w_{\text{Stern}}$

What about the tendency and self-advection?

Geostrophic currents influence Ekman pumping

Above: Stern (1965), Niiler(1969), Hart(2000), Wenegrat and Thomas (2016)

Geostrophic currents influence near-inertial oscillations

NI energy quickly imprinted on mesoscale eddies by refraction, from cyclones to anticyclones
-> wave energy exits surface layer (Rocha et al. 2018; Asselin and Young 2020).

Ekman–Near-Inertial interactions?

Part 1:

Interaction of Nonlinear Ekman Pumping, Near-Inertial Oscillations, and Geostrophic Turbulence

(With Yanxu Chen and David Straub)

Using a "slab layer" :

Consider that boundary layer correction is embedded near the top of the surface layer of a shallow water model

Interaction of Nonlinear Ekman Pumping, Near-Inertial Oscillations, and Geostrophic Turbulence

(With Yanxu Chen and David Straub)

2 slab models :

S1:

$$\frac{\partial}{\partial t}\mathbf{U}_s + (\mathbf{U}_s \cdot \nabla)\mathbf{u}_1 + (\mathbf{u}_1 \cdot \nabla)\mathbf{U}_s + f\hat{\mathbf{z}} \times \mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

S2:

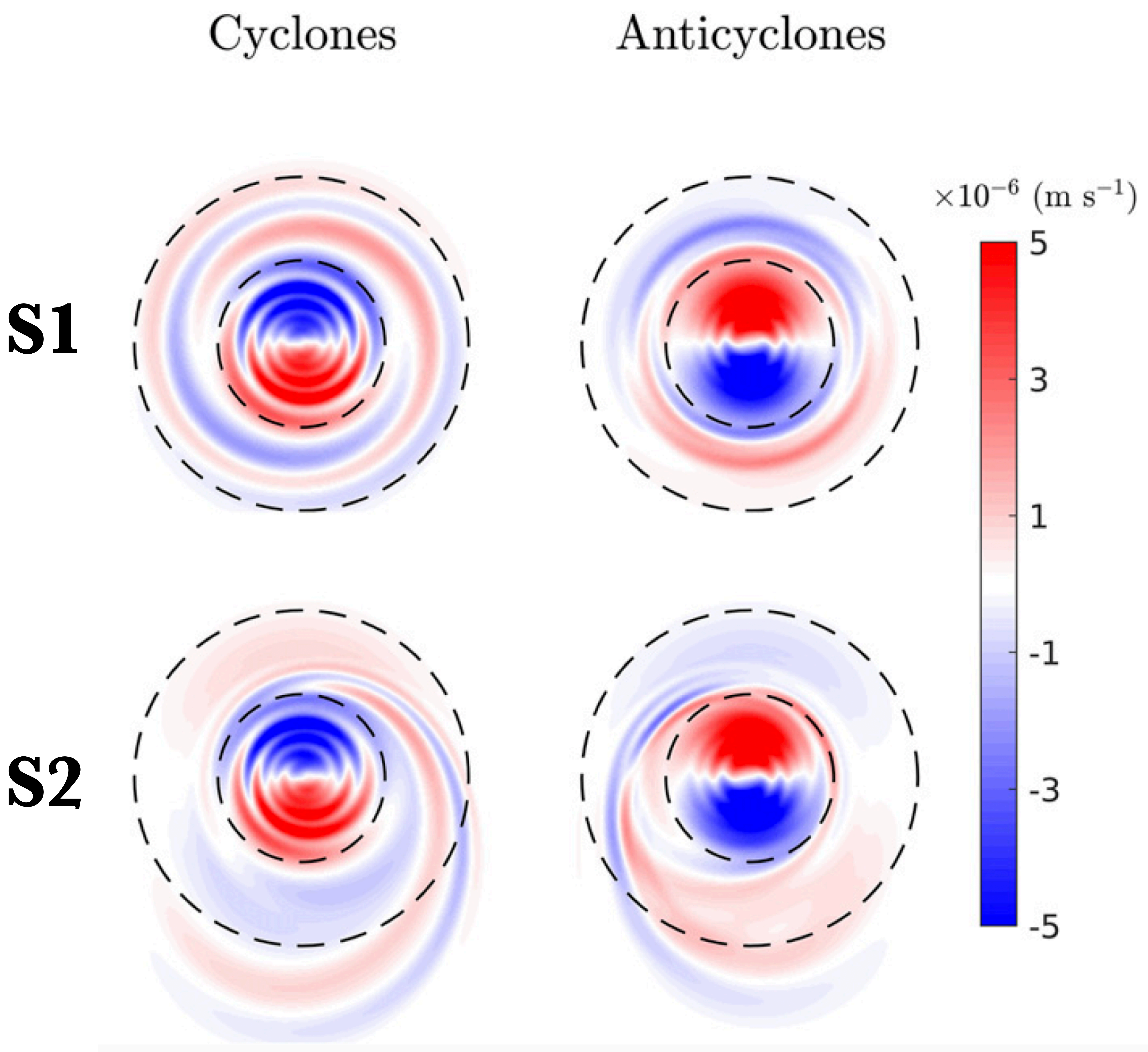
$$\frac{\partial}{\partial t}\mathbf{U}_s + \frac{1}{H_s}(\mathbf{U}_s \cdot \nabla)\mathbf{U}_s + (\mathbf{U}_s \cdot \nabla)\mathbf{u}_1 + (\mathbf{u}_1 \cdot \nabla)\mathbf{U}_s + f\hat{\mathbf{z}} \times \mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

Tendency — Self-advection — Stern — Higher order — Ekman

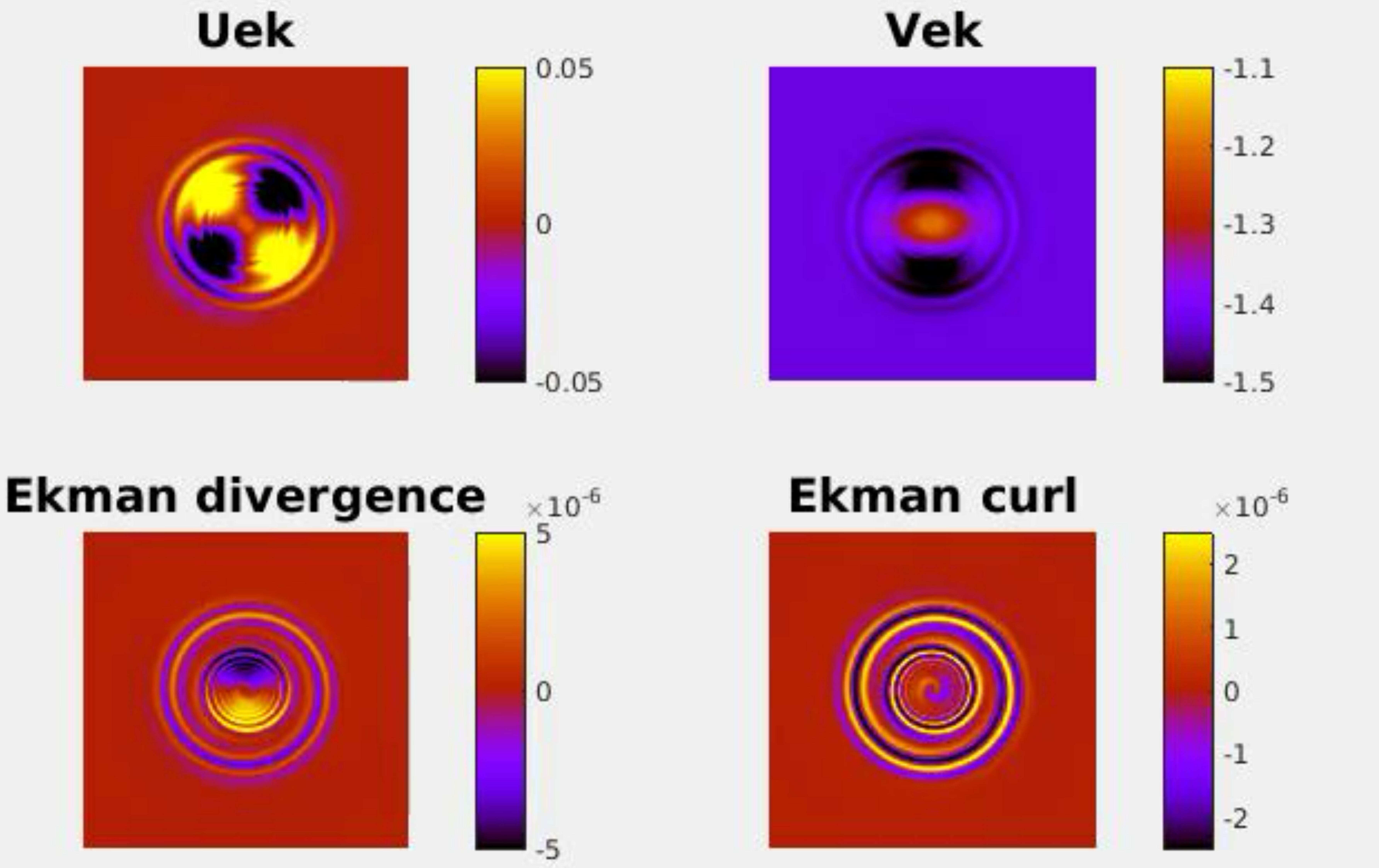
Stand-alone slab model

Response to a uniform wind stress blowing over a horizontally nondivergent circular eddy

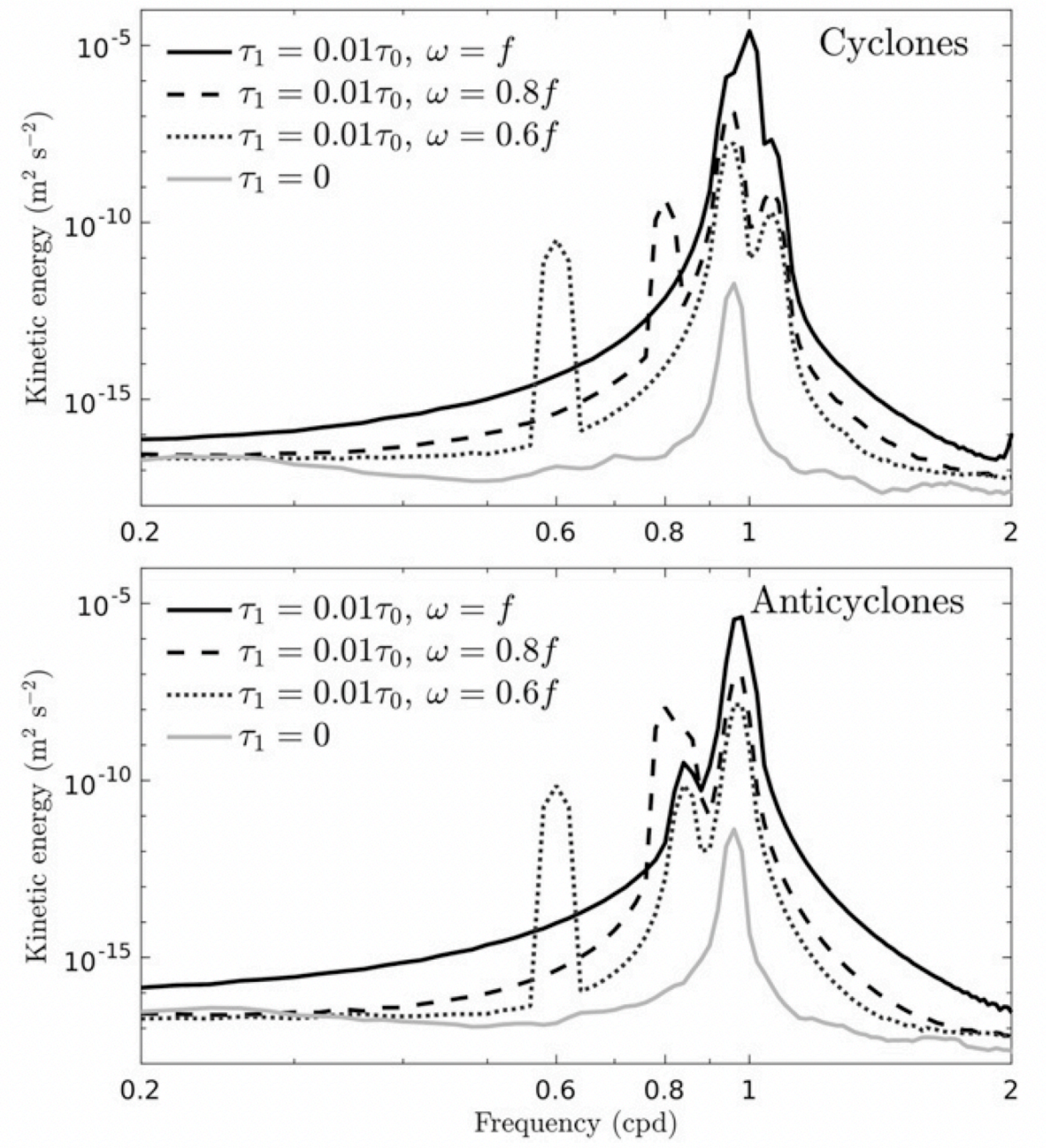
- *fast time scale transients*
- *transients are evident even when forcing is ramped up over several inertial periods*



Stand-alone S1 slab model for a cyclone



Stand-alone S1 slab model using variable wind $\tau = [\tau_0 + \tau_1(t)]$



Coupled model

Slab

S1: $$\frac{\partial}{\partial t}\mathbf{U}_s + \qquad + (\mathbf{U}_s\cdot\nabla)\mathbf{u}_1 + (\mathbf{u}_1\cdot\nabla)\mathbf{U}_s + f\hat{\mathbf{z}}\times\mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

S2: $$\frac{\partial}{\partial t}\mathbf{U}_s + \frac{1}{H_s}(\mathbf{U}_s\cdot\nabla)\mathbf{U}_s + (\mathbf{U}_s\cdot\nabla)\mathbf{u}_1 + (\mathbf{u}_1\cdot\nabla)\mathbf{U}_s + f\hat{\mathbf{z}}\times\mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

Shallow Water

$$\frac{\partial}{\partial t}\mathbf{u}_1 + (\mathbf{u}_1\cdot\nabla)\mathbf{u}_1 + f\hat{\mathbf{z}}\times\mathbf{u}_1 = -\nabla\phi_1 + \mathbf{D}_1 + \delta_{\mathrm{BF}}\frac{\boldsymbol{\tau}}{\rho_0 d_1},$$

$$\frac{\partial}{\partial t}\mathbf{u}_2 + (\mathbf{u}_2\cdot\nabla)\mathbf{u}_2 + f\hat{\mathbf{z}}\times\mathbf{u}_2 = -\nabla\phi_2 + \mathbf{D}_2,$$

$$\frac{\partial}{\partial t}d_1 + \nabla\cdot(\mathbf{u}_1 d_1) = (\delta_{\mathrm{BF}} - 1)\nabla\cdot\mathbf{U}_s,$$

$$\frac{\partial}{\partial t}d_2 + \nabla\cdot(\mathbf{u}_2 d_2) = 0,$$

Coupled model

Slab

S1: $$\frac{\partial}{\partial t}\mathbf{U}_s + \quad + (\mathbf{U}_s \cdot \nabla)\mathbf{u}_1 + (\mathbf{u}_1 \cdot \nabla)\mathbf{U}_s + f\hat{\mathbf{z}} \times \mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

S2: $$\frac{\partial}{\partial t}\mathbf{U}_s + \frac{1}{H_s}(\mathbf{U}_s \cdot \nabla)\mathbf{U}_s + (\mathbf{U}_s \cdot \nabla)\mathbf{u}_1 + (\mathbf{u}_1 \cdot \nabla)\mathbf{U}_s + f\hat{\mathbf{z}} \times \mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

Shallow Water

$$\frac{\partial}{\partial t}\mathbf{u}_1 + (\mathbf{u}_1 \cdot \nabla)\mathbf{u}_1 + f\hat{\mathbf{z}} \times \mathbf{u}_1 = -\nabla\phi_1 + \mathbf{D}_1 + \boxed{\delta_{\mathrm{BF}} \frac{\boldsymbol{\tau}}{\rho_0 d_1}},$$

$$\frac{\partial}{\partial t}\mathbf{u}_2 + (\mathbf{u}_2 \cdot \nabla)\mathbf{u}_2 + f\hat{\mathbf{z}} \times \mathbf{u}_2 = -\nabla\phi_2 + \mathbf{D}_2,$$

$$\frac{\partial}{\partial t}d_1 + \nabla \cdot (\mathbf{u}_1 d_1) = \boxed{(\delta_{\mathrm{BF}} - 1)\nabla \cdot \mathbf{U}_s},$$

$$\frac{\partial}{\partial t}d_2 + \nabla \cdot (\mathbf{u}_2 d_2) = 0,$$

Coupled model Forcing/Dissipation

Wind Forcing

$$\boldsymbol{\tau} = [\tau_0 + \tau_1(t)] \cos\left(\frac{2\pi y}{L}\right) \hat{\mathbf{x}}$$

$$\tau_1(t) = \sum_{n=1}^{30\,000} A_n \sin(\omega_n t + \varphi_n)$$

A_n chosen to correspond to an Ornstein-Uhlenbeck process with damping time scale of 5 days

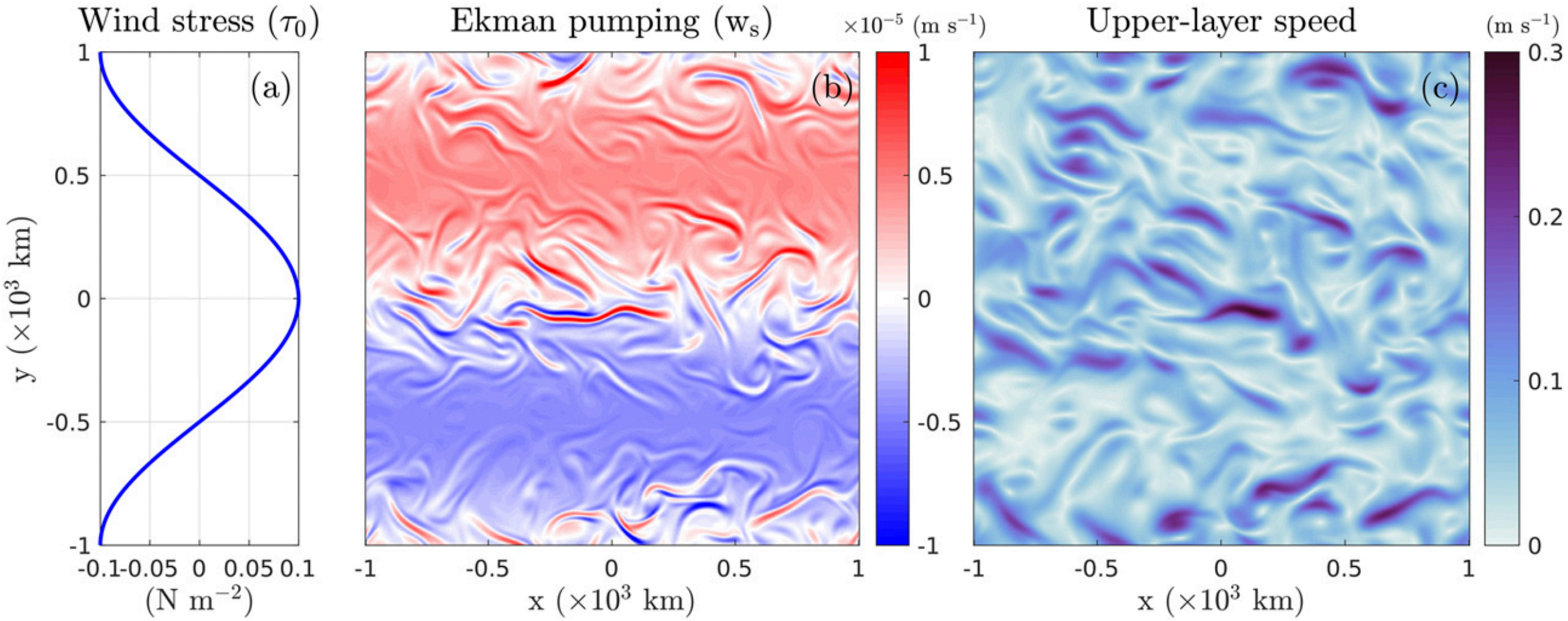
Dissipation

$$\mathbf{D}_s = -A_{\mathrm{bh}} \nabla^4 \mathbf{U}_s$$

$$\mathbf{D}_1 = r_{\mathrm{invLap}} \nabla^{-2} \mathbf{u}_1 - A_{\mathrm{bh}} \nabla^4 \mathbf{u}_1$$

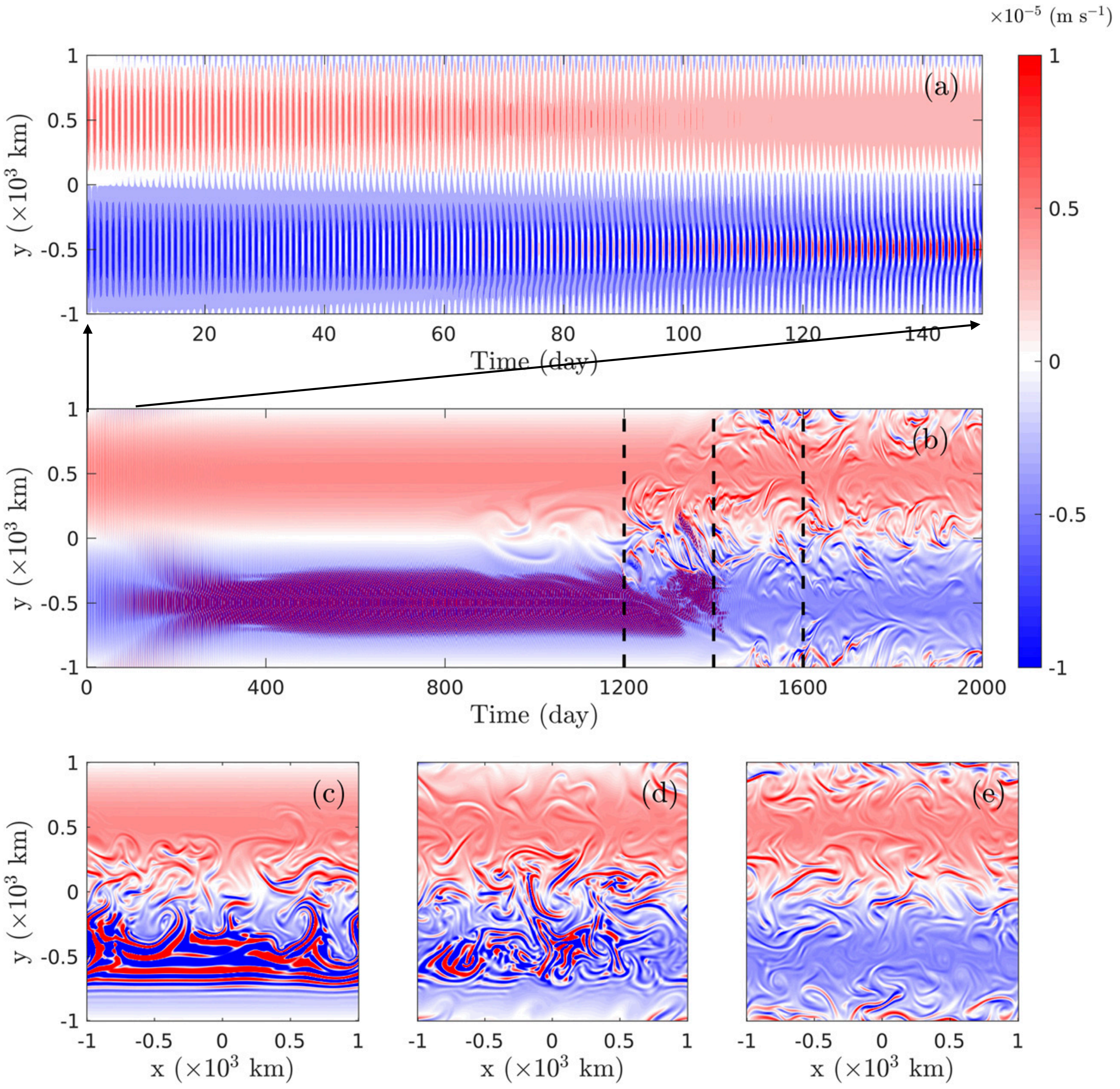
$$\mathbf{D}_2 = r_{\mathrm{invLap}} \nabla^{-2} \mathbf{u}_2 - A_{\mathrm{bh}} \nabla^4 \mathbf{u}_2 - r_{\mathrm{drag}} \mathbf{u}_2$$

S1 coupled model results with steady forcing $(\tau_1(t) = 0)$



Spinup under steady forcing using the S2 model

(pumping velocity)



Spinup instability using the S2 model

S2: $$\frac{\partial}{\partial t}\mathbf{U}_s + \frac{1}{H_s}(\mathbf{U}_s\cdot\nabla)\mathbf{U}_s + (\mathbf{U}_s\cdot\nabla)\mathbf{u}_1 + (\mathbf{u}_1\cdot\nabla)\mathbf{U}_s + f\hat{\mathbf{z}}\times\mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

$$\mathbf{U}_s \approx V_{\mathrm{Ek}}\hat{\mathbf{y}} + \mathbf{U}' \qquad V_{\mathrm{Ek}} = -\tau^x/(\rho_0 f) \qquad w_{\mathrm{Ek}} = (\partial/\partial y)V_{\mathrm{Ek}}$$

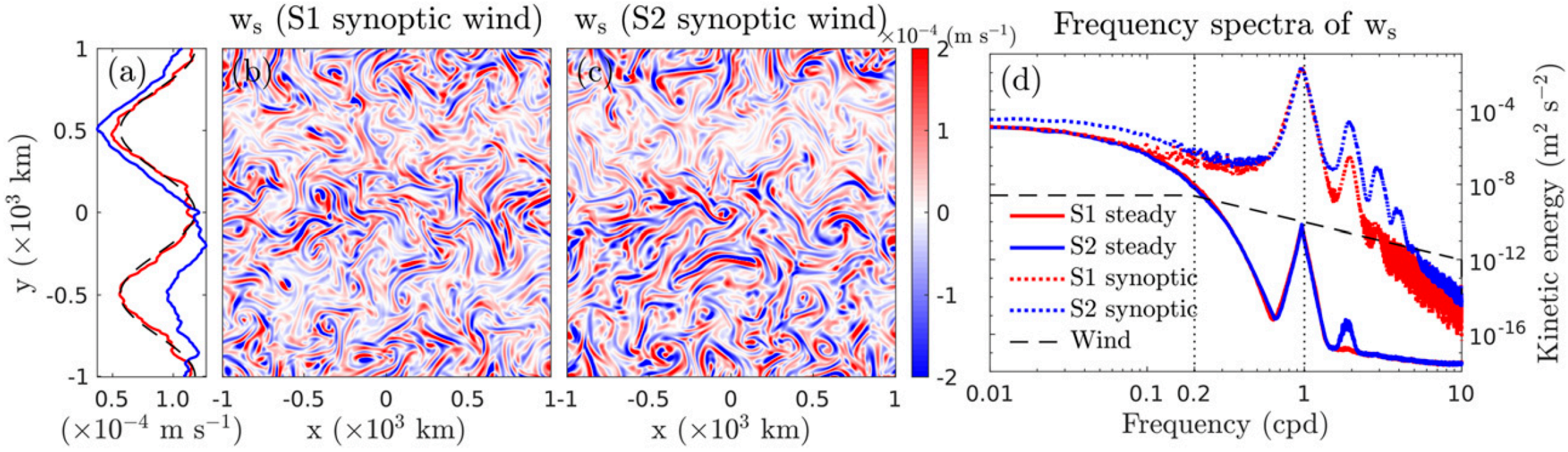
$\mathbf{u}_1$ small compared to $\mathbf{U}'/H_s$

$$\left\{ \begin{aligned} &\frac{\partial}{\partial t}U' + \frac{V_{\mathrm{Ek}}}{H_s}\frac{\partial}{\partial y}U' = fV', \\ &\frac{\partial}{\partial t}V' + \frac{V_{\mathrm{Ek}}}{H_s}\frac{\partial}{\partial y}V' = -fU' - \frac{w_{\mathrm{Ek}}}{H_s}V' \end{aligned} \right\}$$

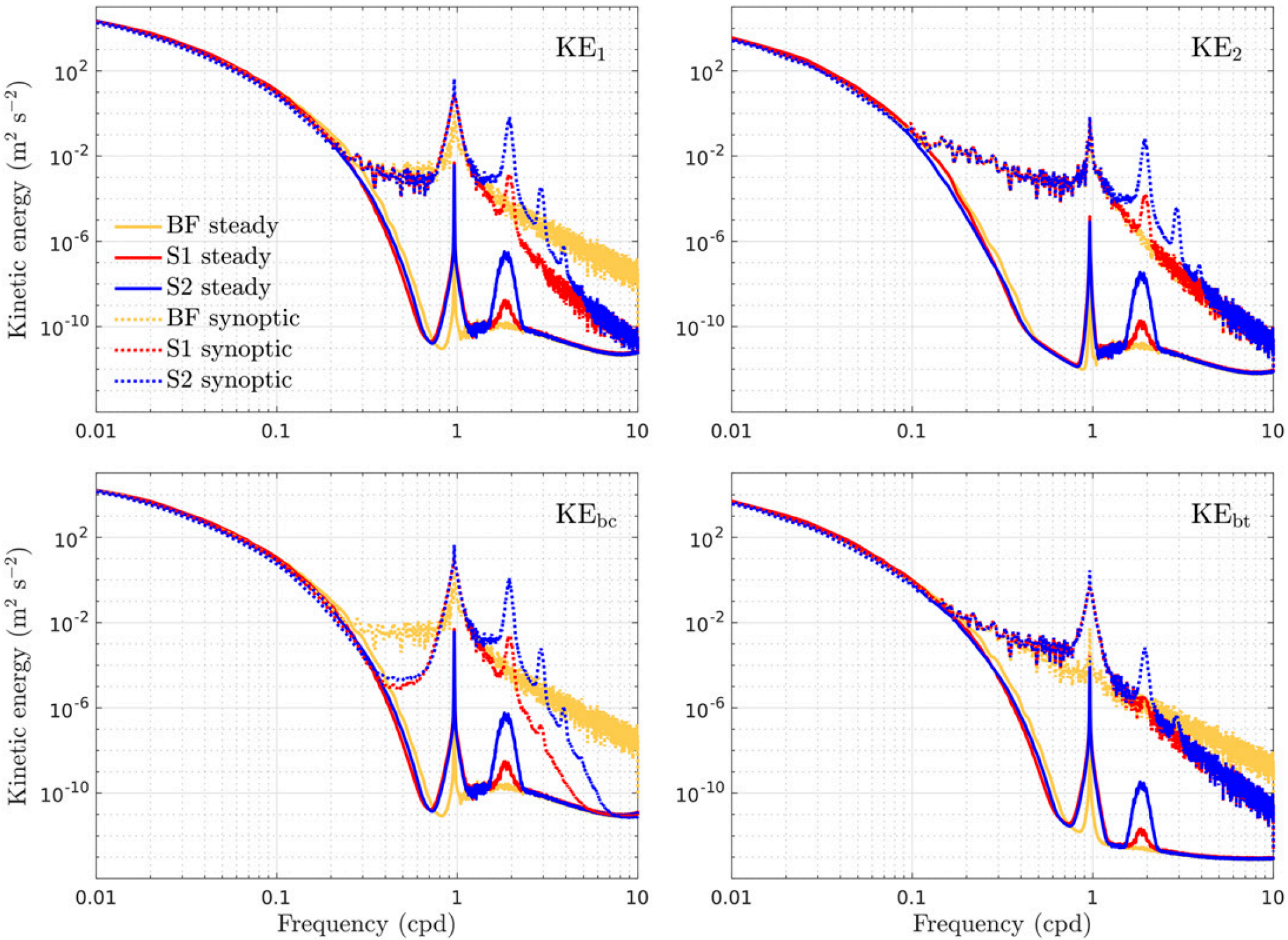
$$V'_{tt} + \sigma V'_t + f^2 V' = 0$$

$$\sigma = w_{\mathrm{Ek}}/H_s$$

Coupled model results with synoptic wind



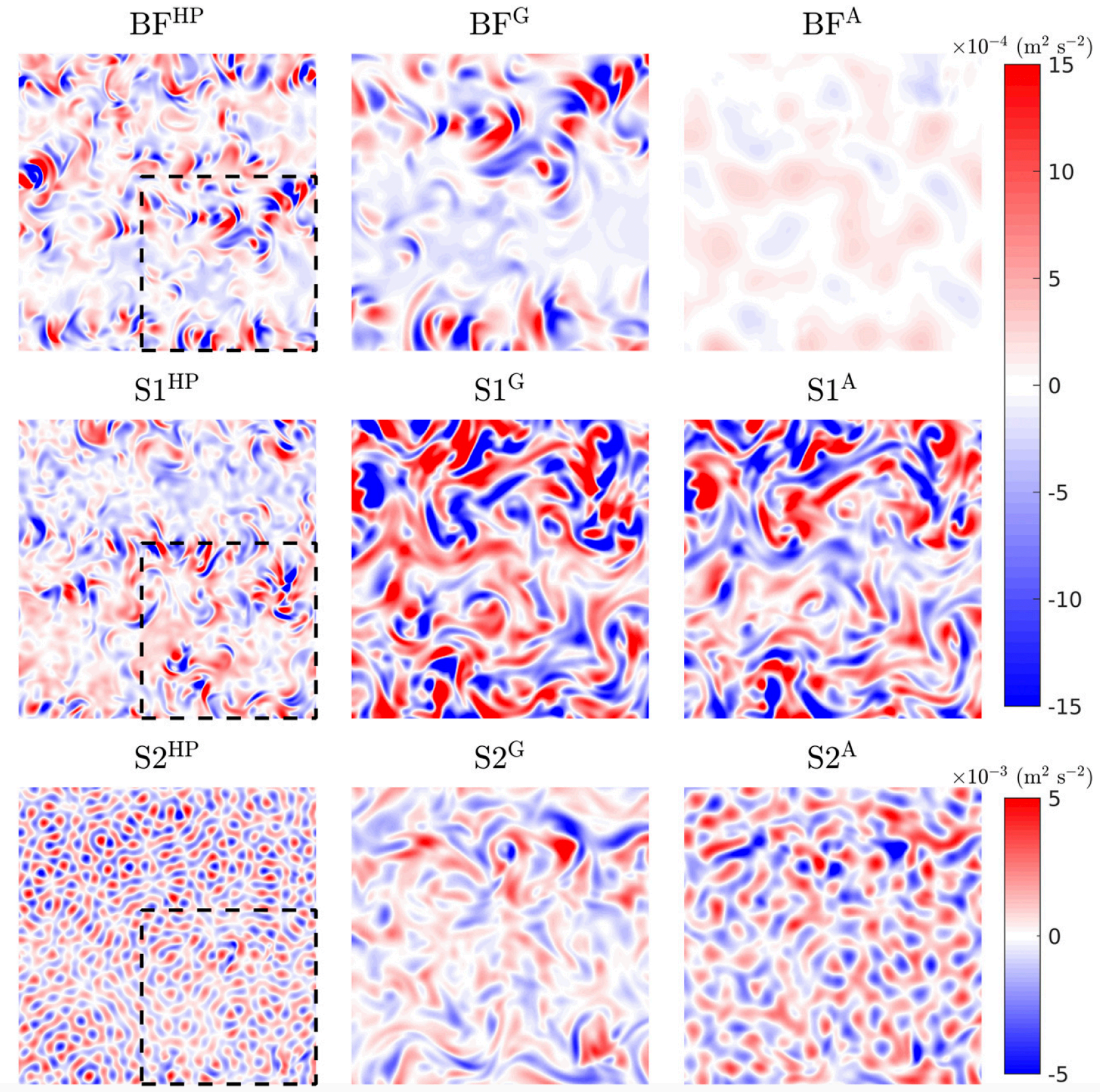
Interior response



Projections onto surface pressure

How does the high-frequency signals project onto surface pressure?

$$\nabla^2\psi - \frac{f^2}{gH_{\mathrm{eff}}}\psi = \zeta_{\mathrm{BC}} - \frac{f}{H_{\mathrm{eff}}}\eta$$

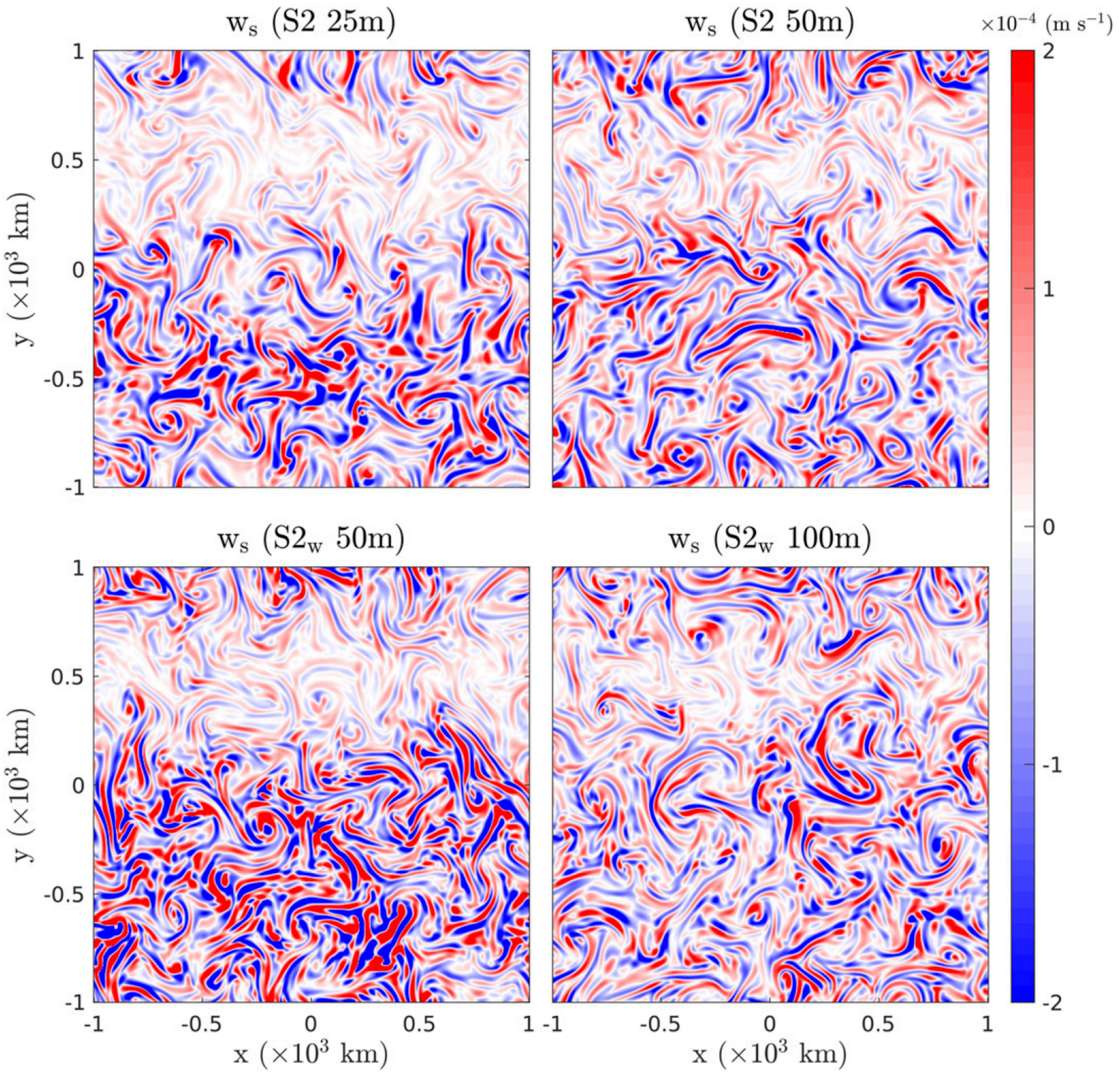


Vertical advection terms

S2:

$$\frac{\partial}{\partial t}\mathbf{U}_s + \frac{1}{H_s}(\mathbf{U}_s \cdot \nabla)\mathbf{U}_s + (\mathbf{U}_s \cdot \nabla)\mathbf{u}_1 + (\mathbf{u}_1 \cdot \nabla)\mathbf{U}_s + f\hat{\mathbf{z}} \times \mathbf{U}_s = \frac{\boldsymbol{\tau}}{\rho_0} + \mathbf{D}_s$$

$$\int w_s \frac{\partial}{\partial z}\mathbf{u}_s \, dz = \frac{1}{2H_s}(\nabla \cdot \mathbf{U}_s)\mathbf{U}_s$$

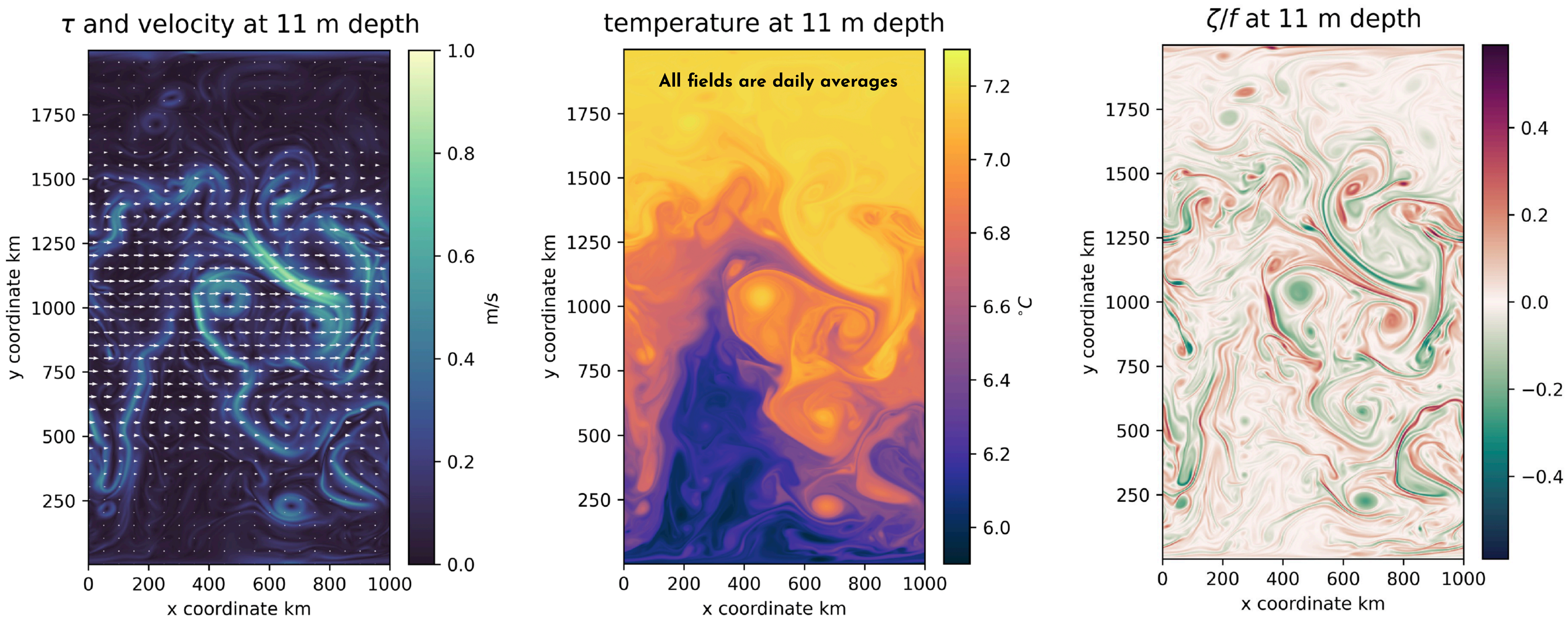


Ongoing work diagnosing non-linear pumping from GCM results

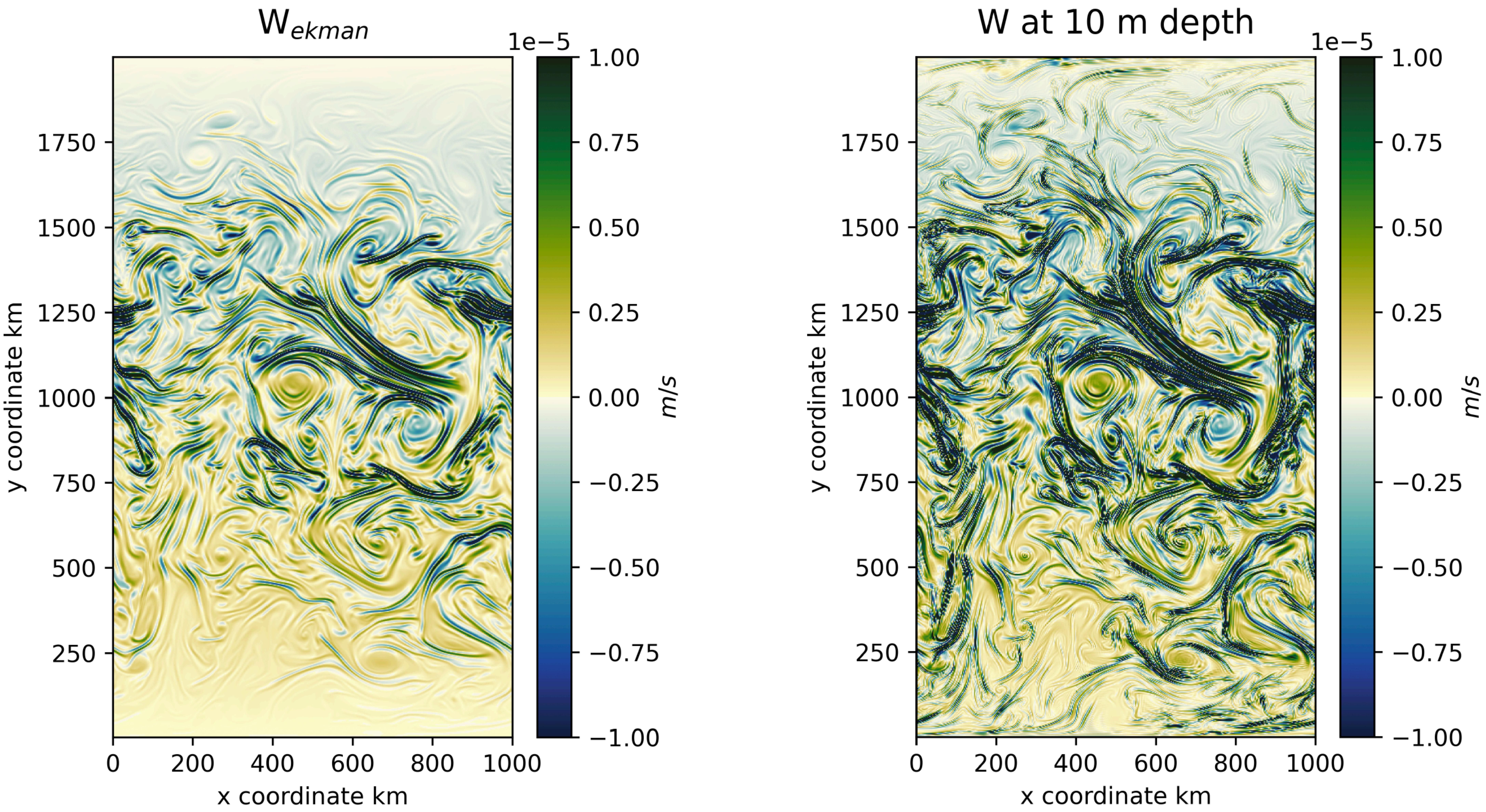
(Jan Klaus Rieck and David Straub)

… going back to S1 : $$\boldsymbol{u}_g \cdot \nabla \boldsymbol{U}_E + \boldsymbol{U}_E \cdot \nabla \boldsymbol{u}_g + \boldsymbol{f} \times \boldsymbol{U}_E = \frac{\boldsymbol{\tau}}{\rho_0} + A_h \nabla^2 \boldsymbol{U}_E$$

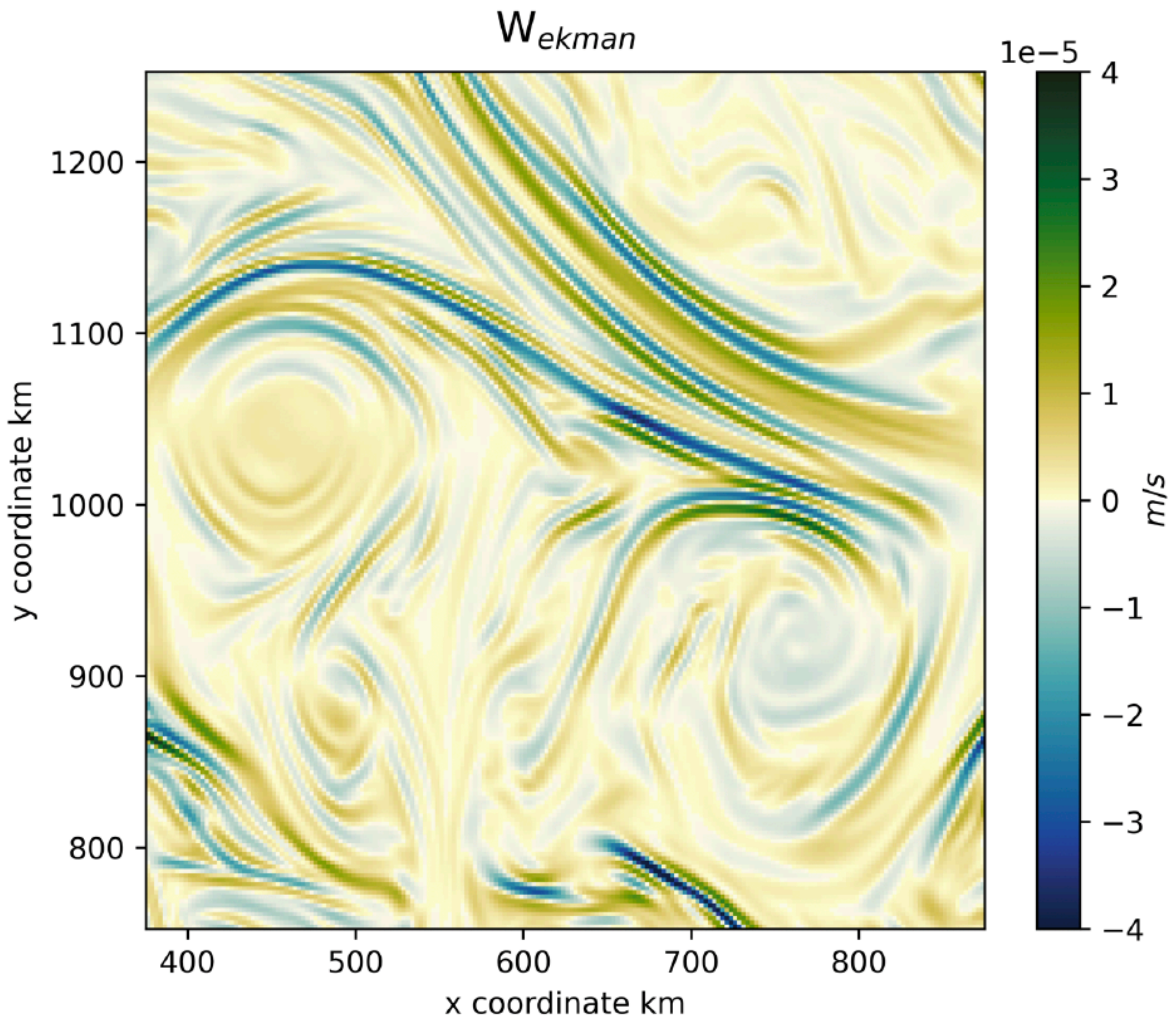
$$\text{Given } \boldsymbol{u}_g\text{, this is linear} \Rightarrow A\boldsymbol{x} = \boldsymbol{b}$$

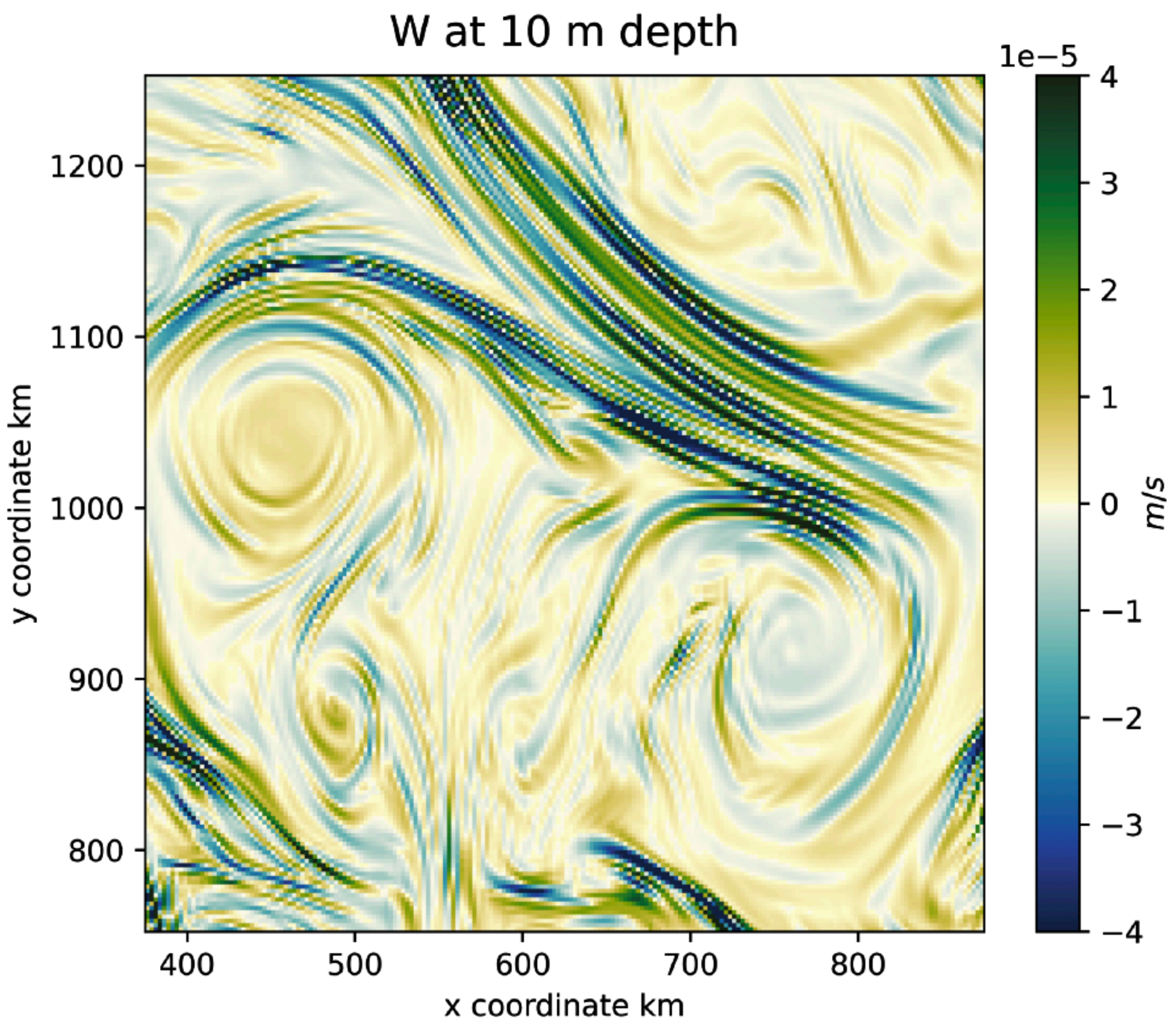


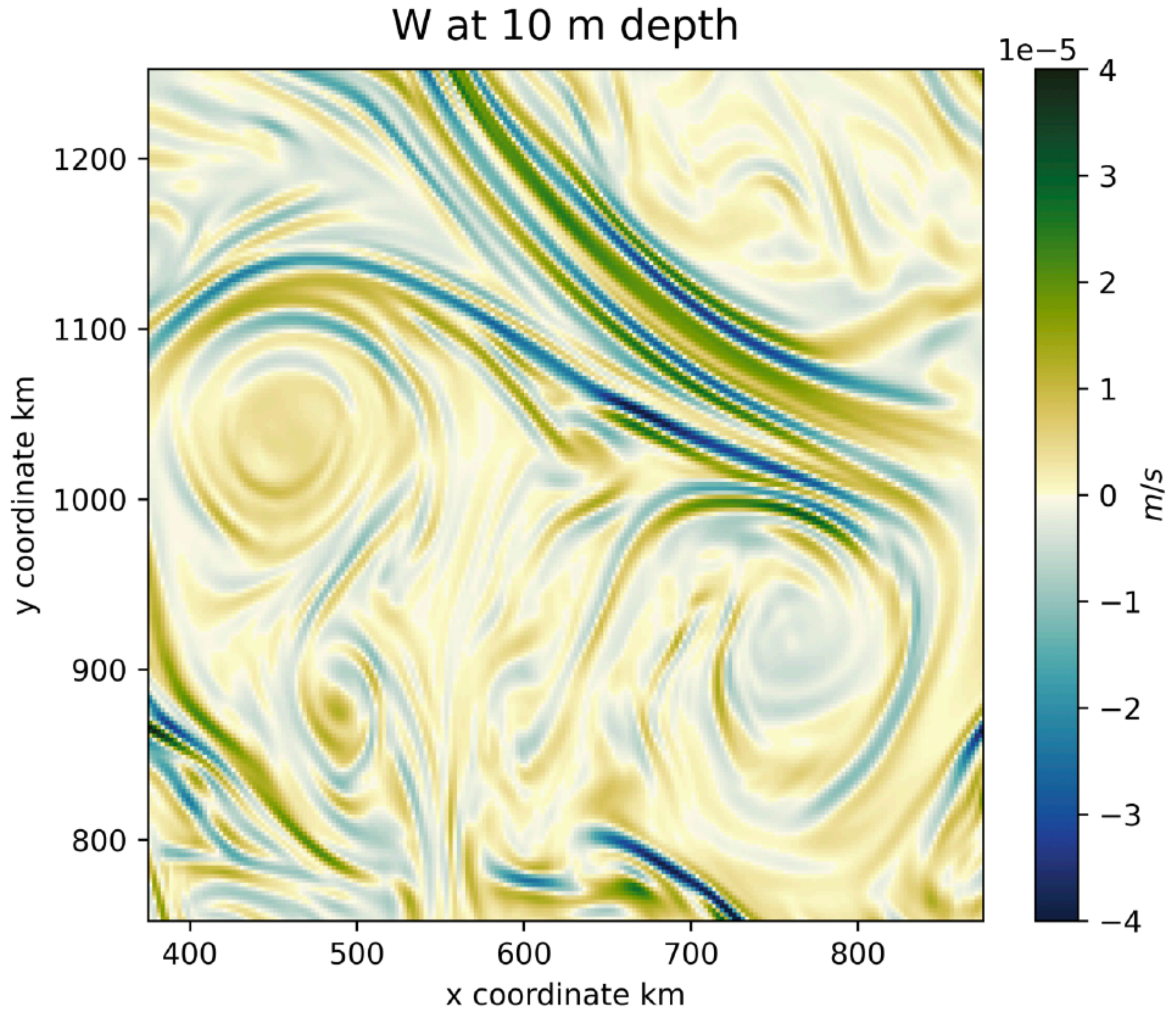
Diagnosing non-linear pumping from GCM results



Diagnosing non-linear pumping from GCM results







NL Ekman solver interpolates ug,vg, tau to an A grid and uses a vorticity-Bernoulli form of the equations. The interpolation may serve to reduce the effective resolution. Also, the viscous term may be too big.

GCM

Smoothed version of model w-field

Diagnosing non-linear pumping from GCM results

Linear Ekman Pumping (ocean-dependent stress)

$\boldsymbol{\tau}_a = \rho_a C_d (\boldsymbol{u}_a - \boldsymbol{u}) |\boldsymbol{u}_a - \boldsymbol{u}|$

NL Ekman Pumping

Calculated for the same u_g but using atm-only stress

Part 1: Future work

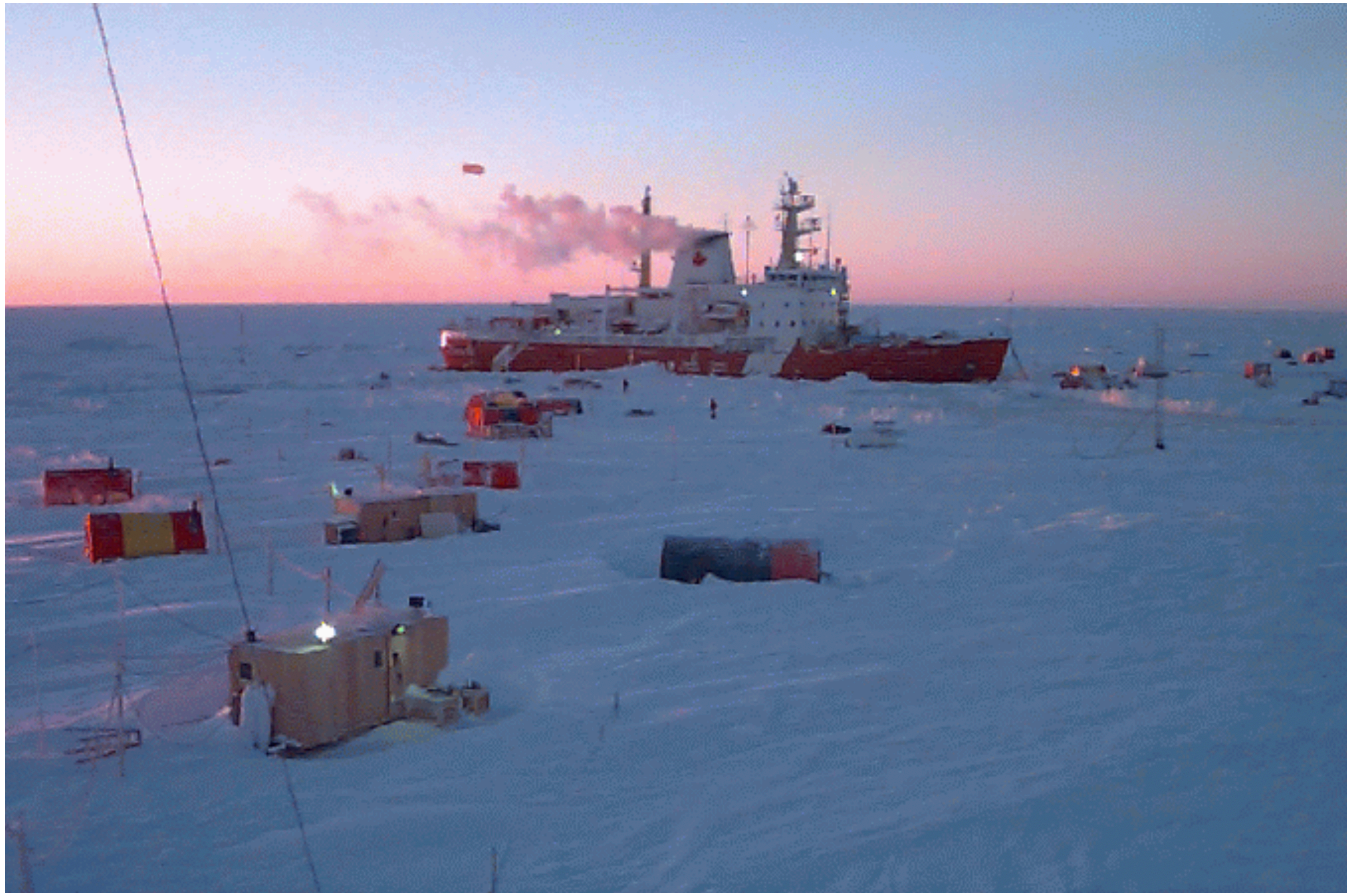
Goal: to ‘disentangle’ submesoscale w into Ekman, waves, …

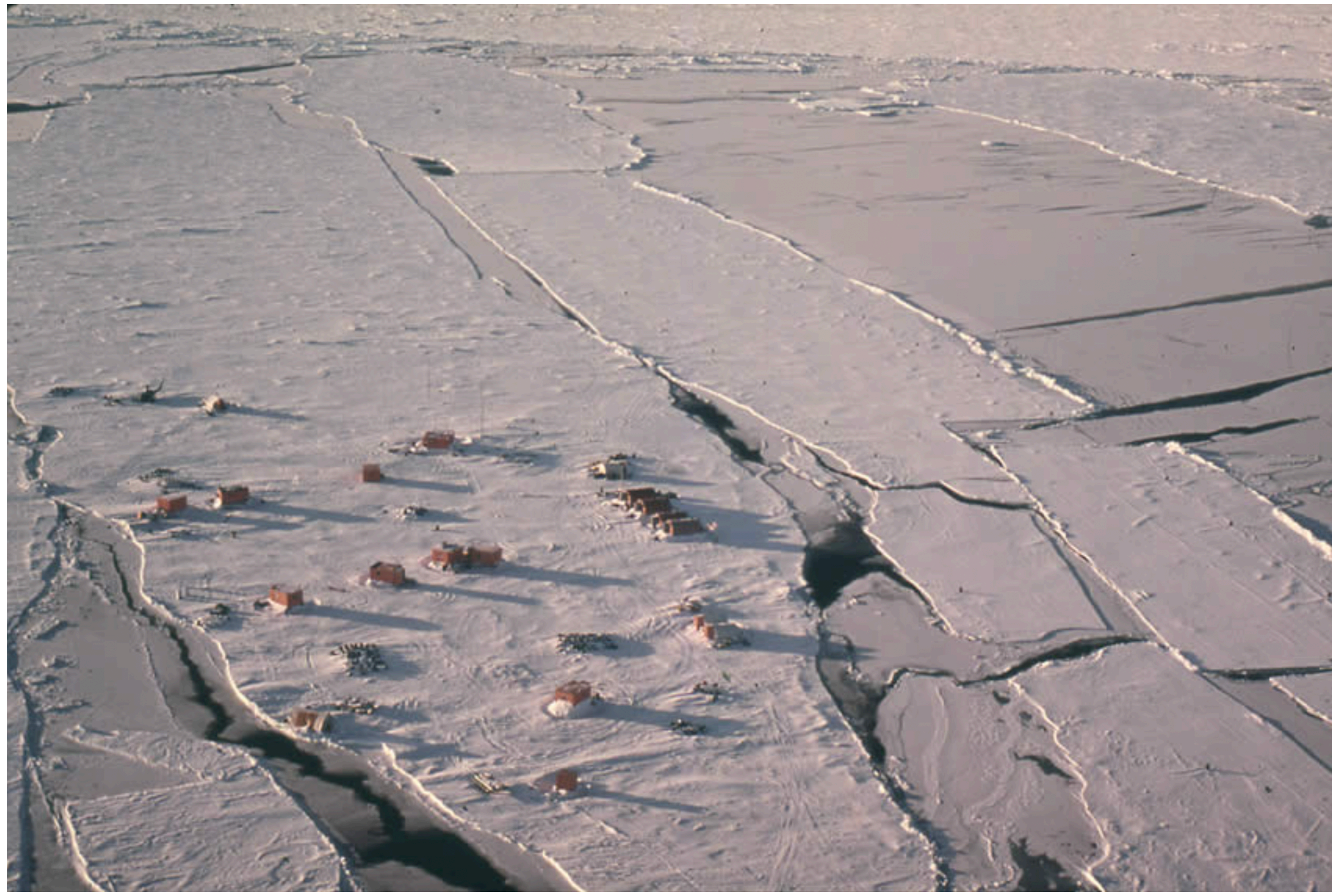
Ideally doing so from altimetry/scatterometer data

Various refinements: e.g. adding info from tendency and self-advection

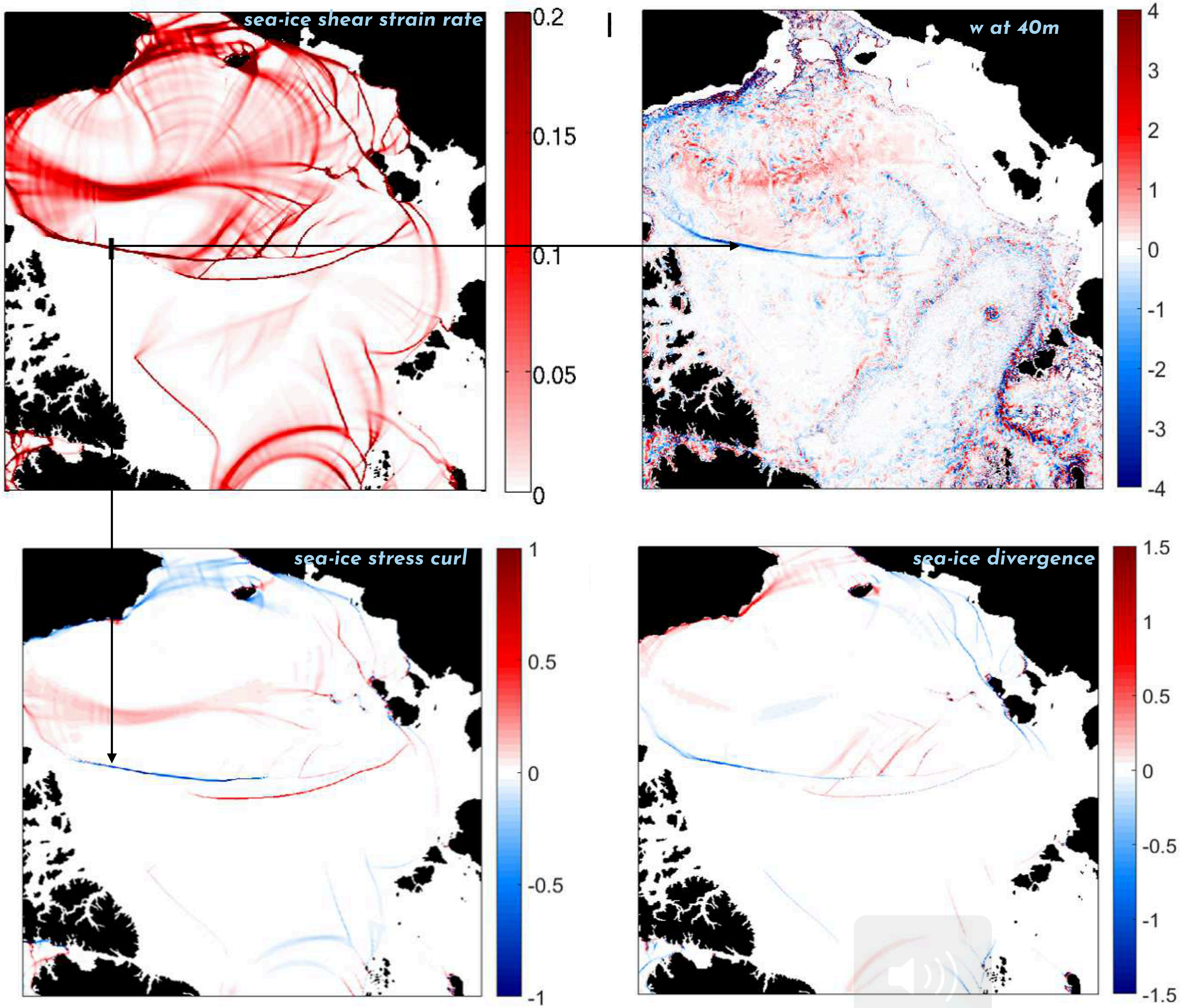
Part 2: Linear Kinematic Features in sea ice

(with P. Bourgault, K. Duquette, D. Straub, B. Tremblay)

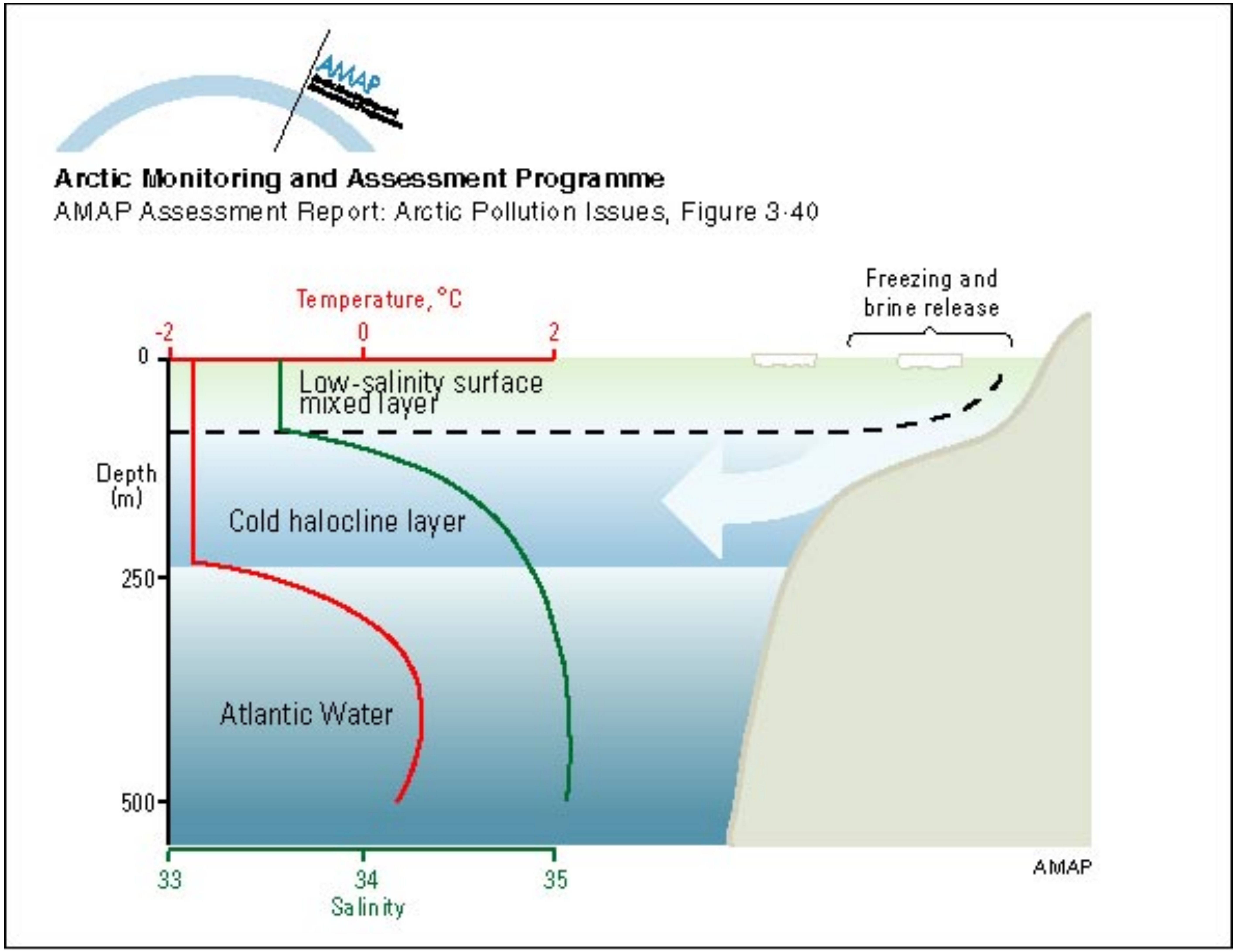




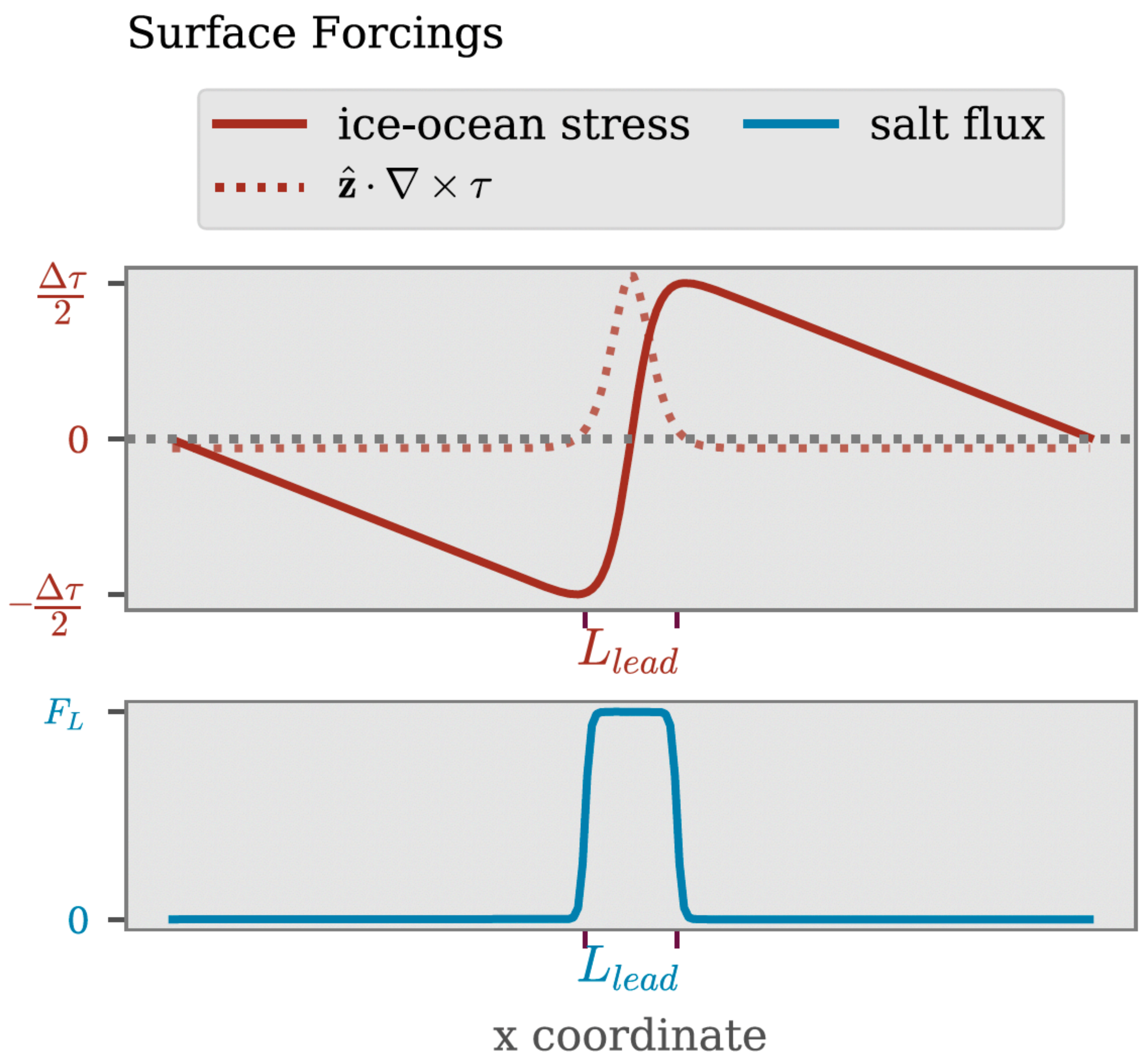
Vertical ocean heat fluxes beneath Linear Kinematic Features in the Arctic Ocean

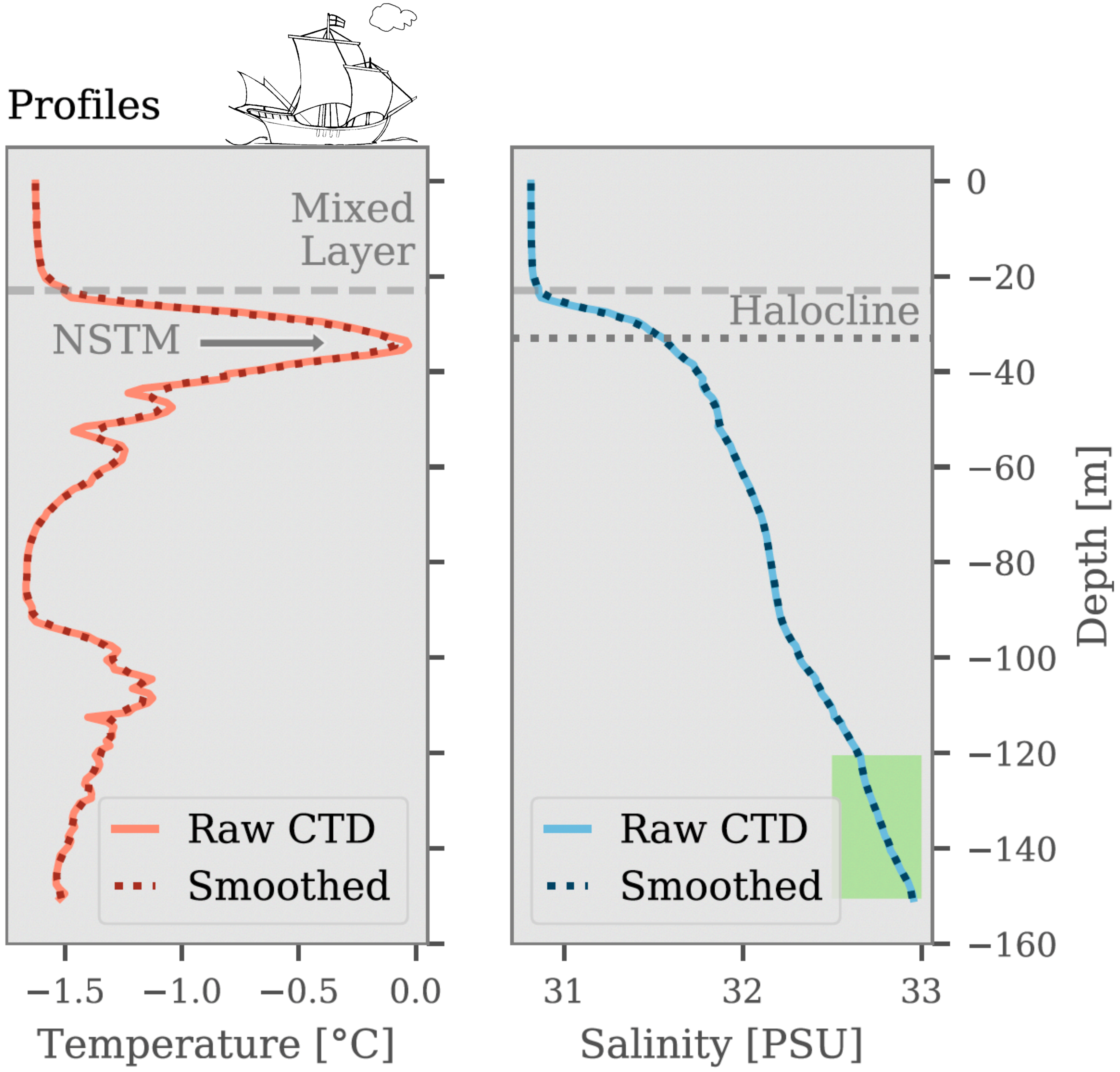


Vertical ocean heat fluxes beneath Linear Kinematic Features in the Arctic Ocean

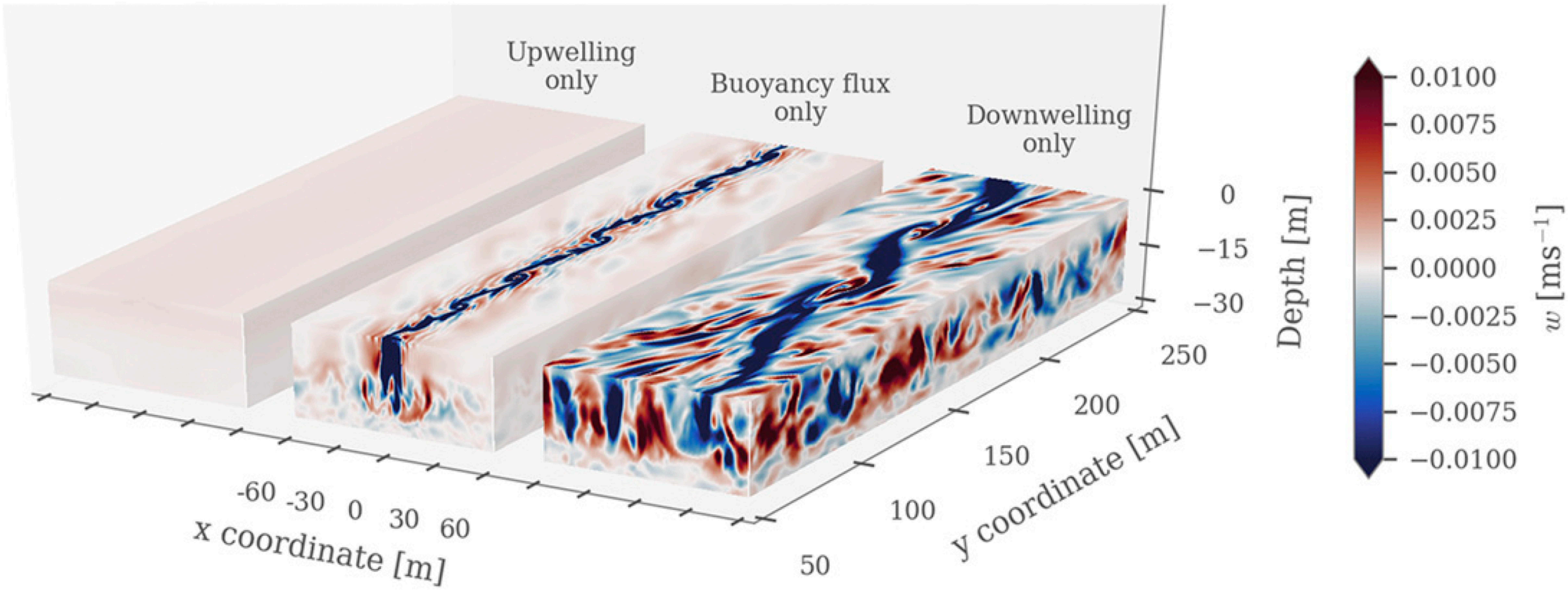


Model Setup: 3D LES

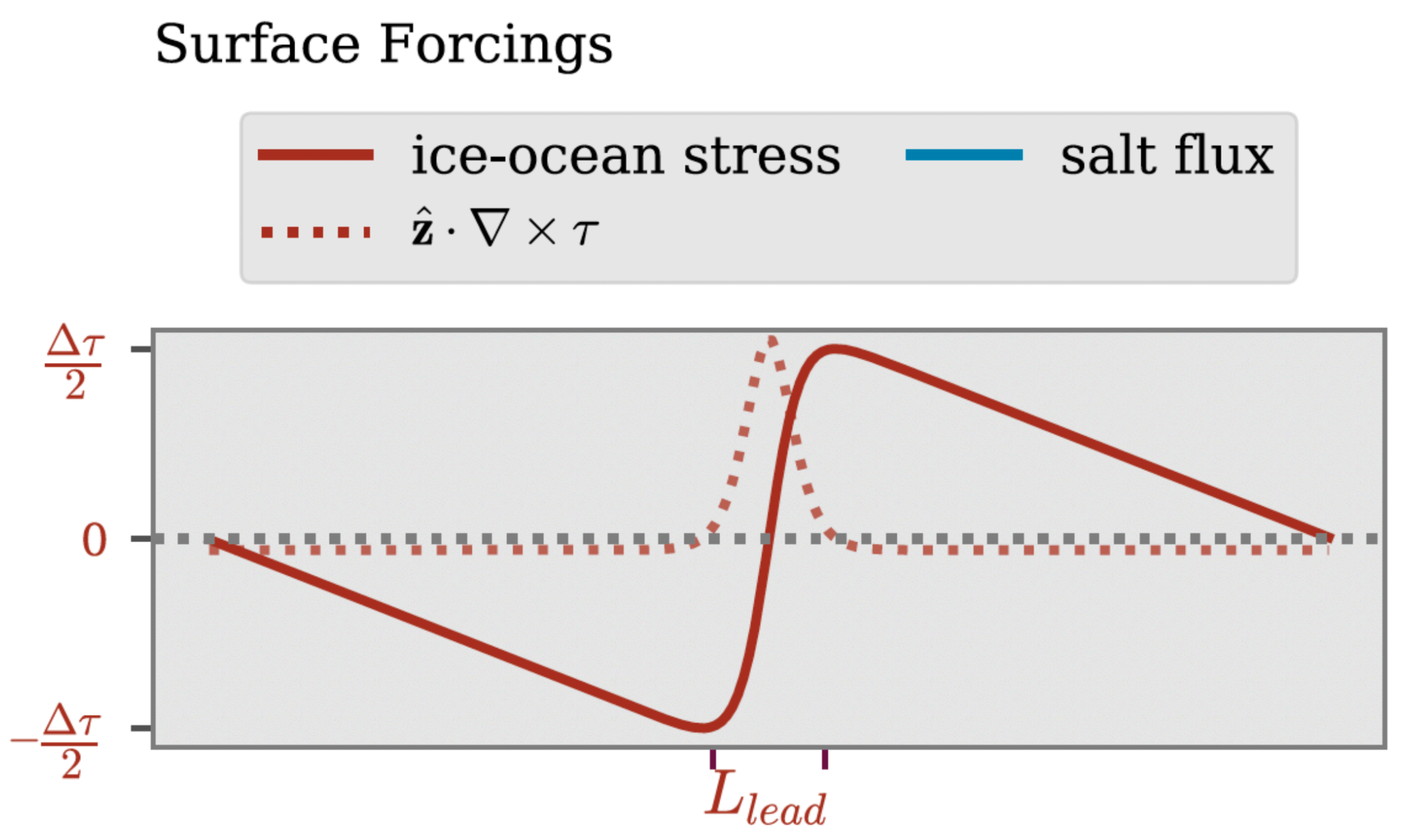


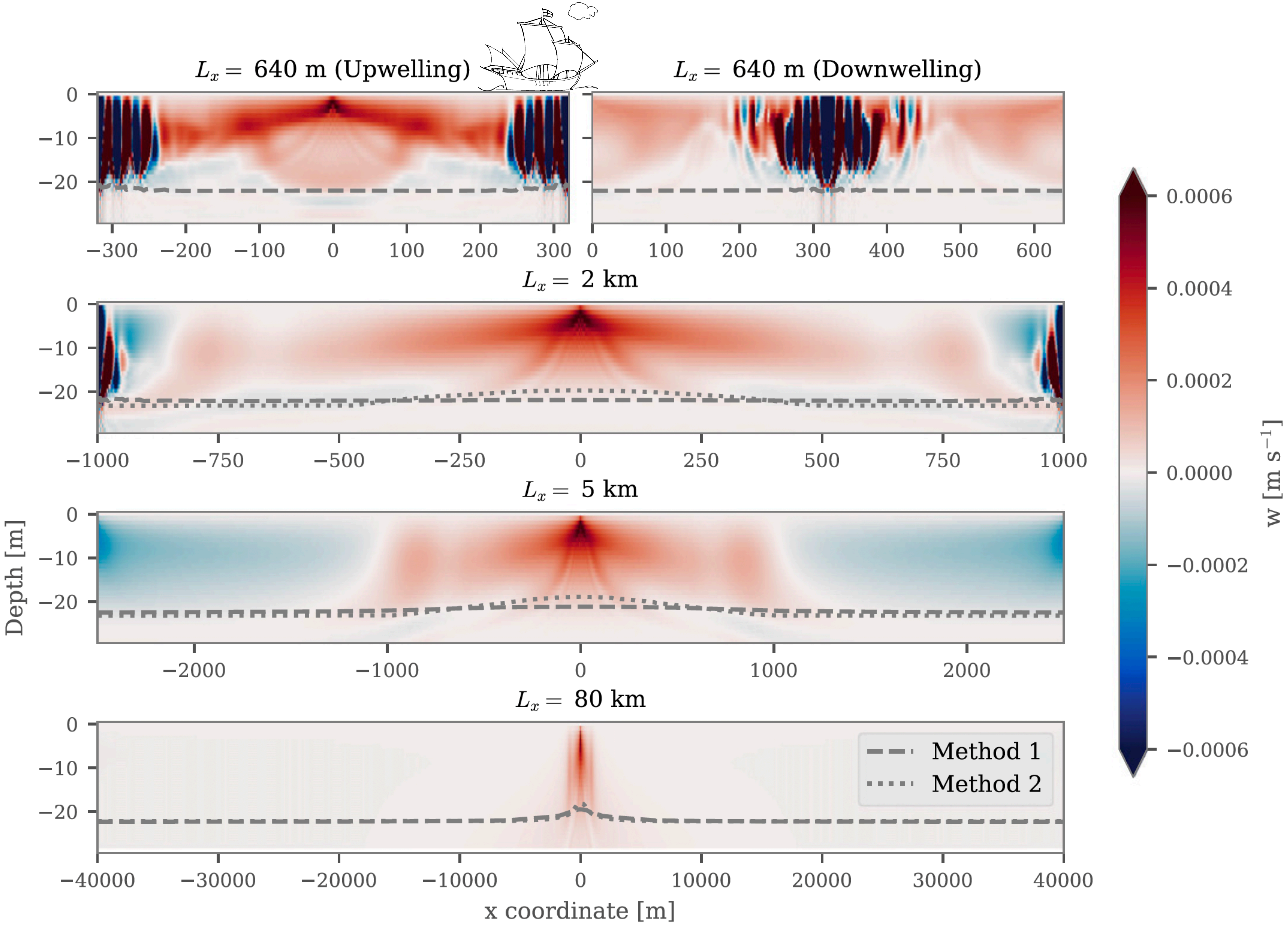


Vertical Velocity

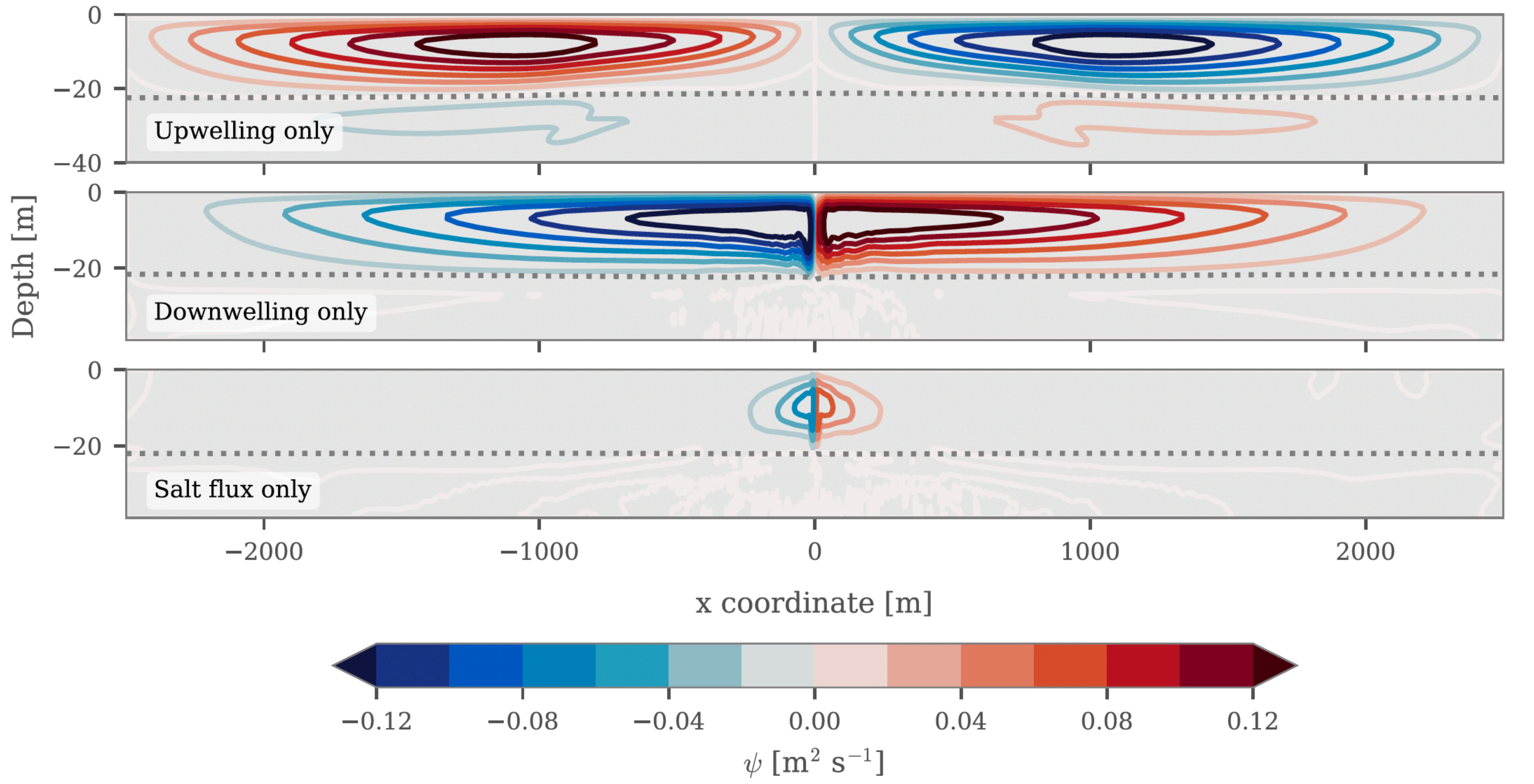


Sensitivity on the size of the periodic domain

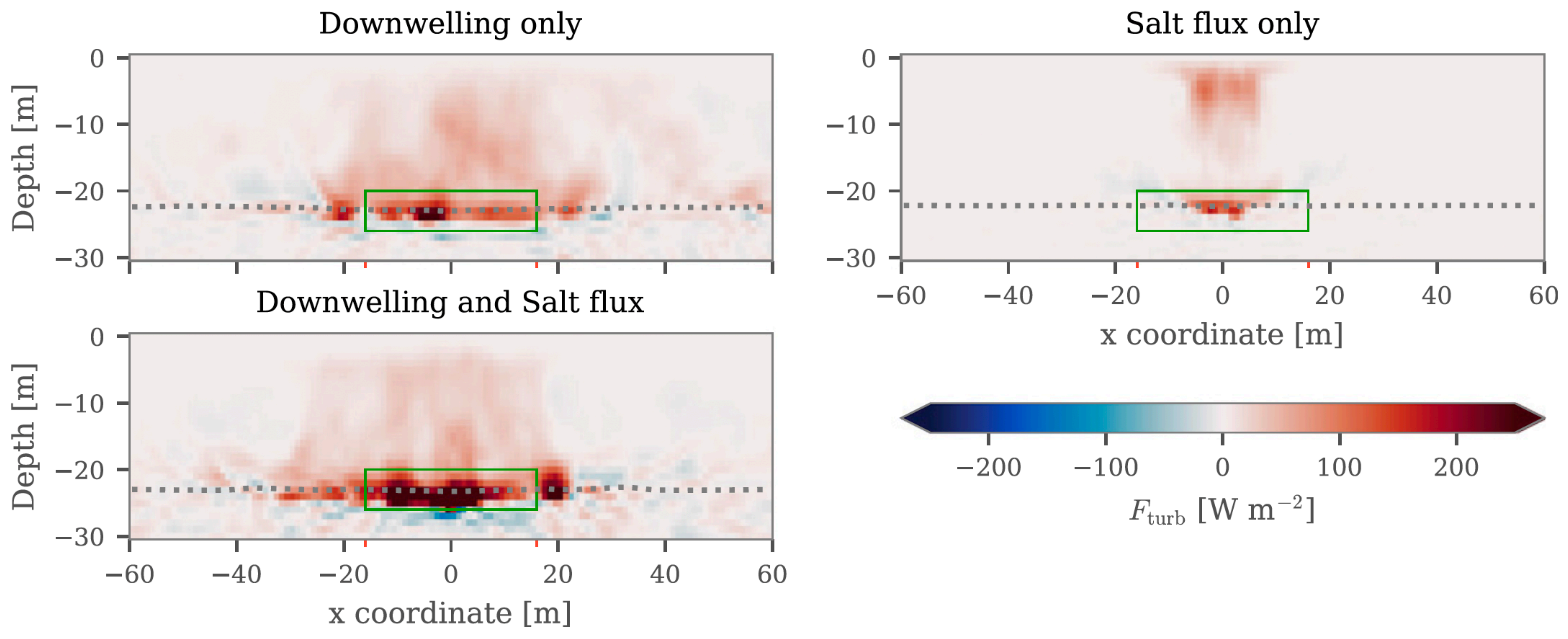




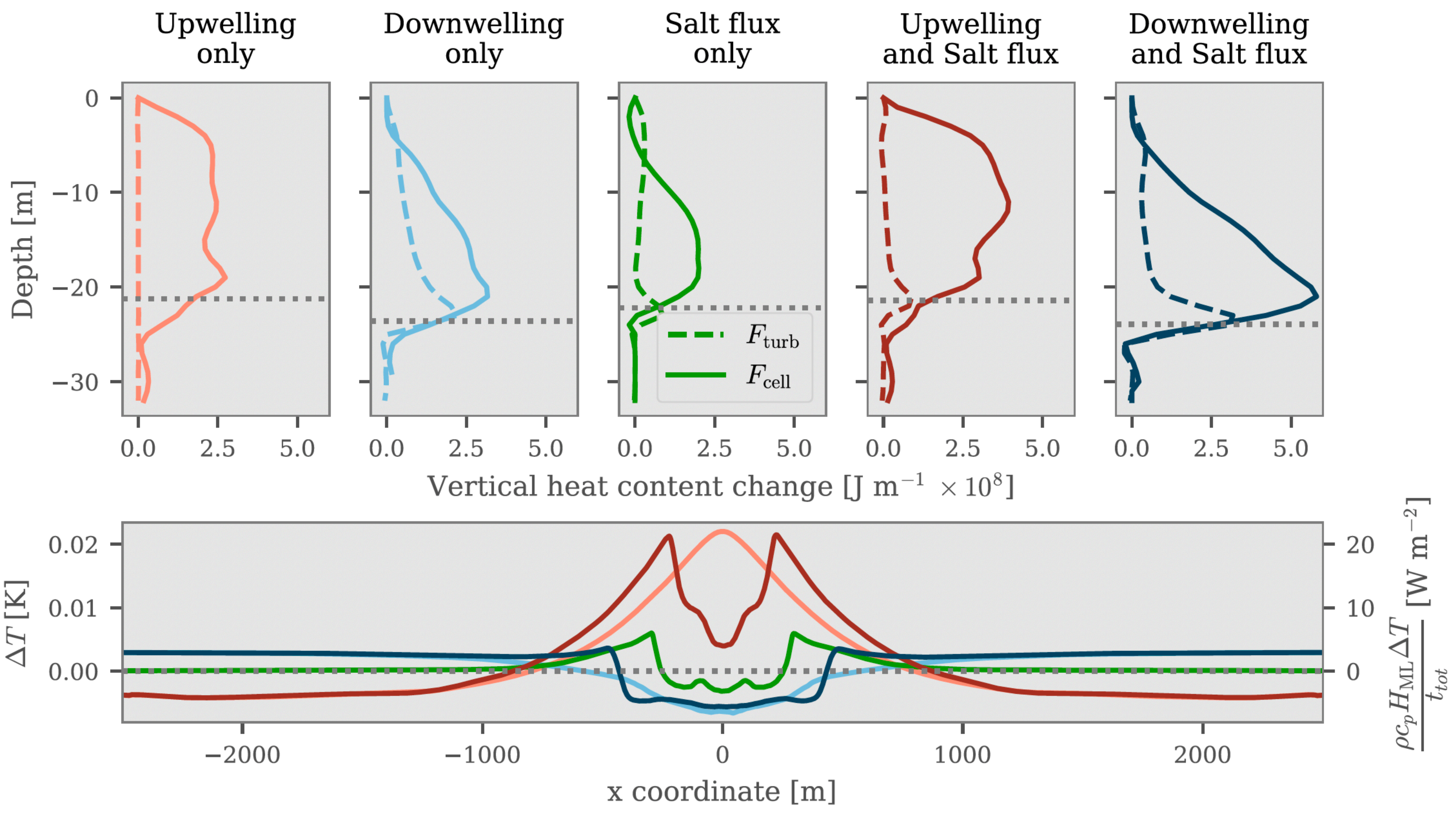
y-averaged circulation



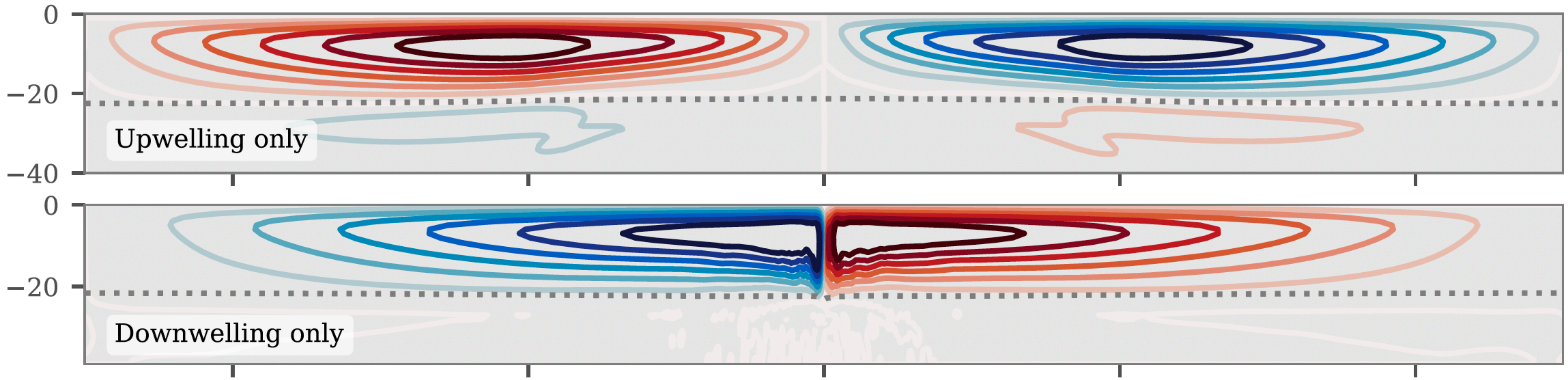
Deviations



Vertical heat content change



What explains the Upwelling/Downwelling asymmetry ?



1st possibility, Stern: $w_E = \nabla \cdot \left(\frac{\boldsymbol{\tau_a} \times \hat{\mathbf{z}}}{\rho_0 (f + \zeta_g)} \right) = \nabla \cdot \left(\frac{\boldsymbol{\tau_a} \times \hat{\mathbf{z}}}{\rho_0 f (1 + \varepsilon)} \right)$

$$\varepsilon = \frac{U}{fL} = \frac{\zeta}{f}$$

2nd possibility,

Revisiting Stern (1965) using the self-advection term

$$\boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_E \; + \; \boldsymbol{f} \times \boldsymbol{u}_E = \partial_z \boldsymbol{\tau} / \rho_0$$

$$(\boldsymbol{f} + \boldsymbol{\zeta}_E) \times \boldsymbol{u}_E = -\nabla B + \partial_z \boldsymbol{\tau} / \rho_0$$

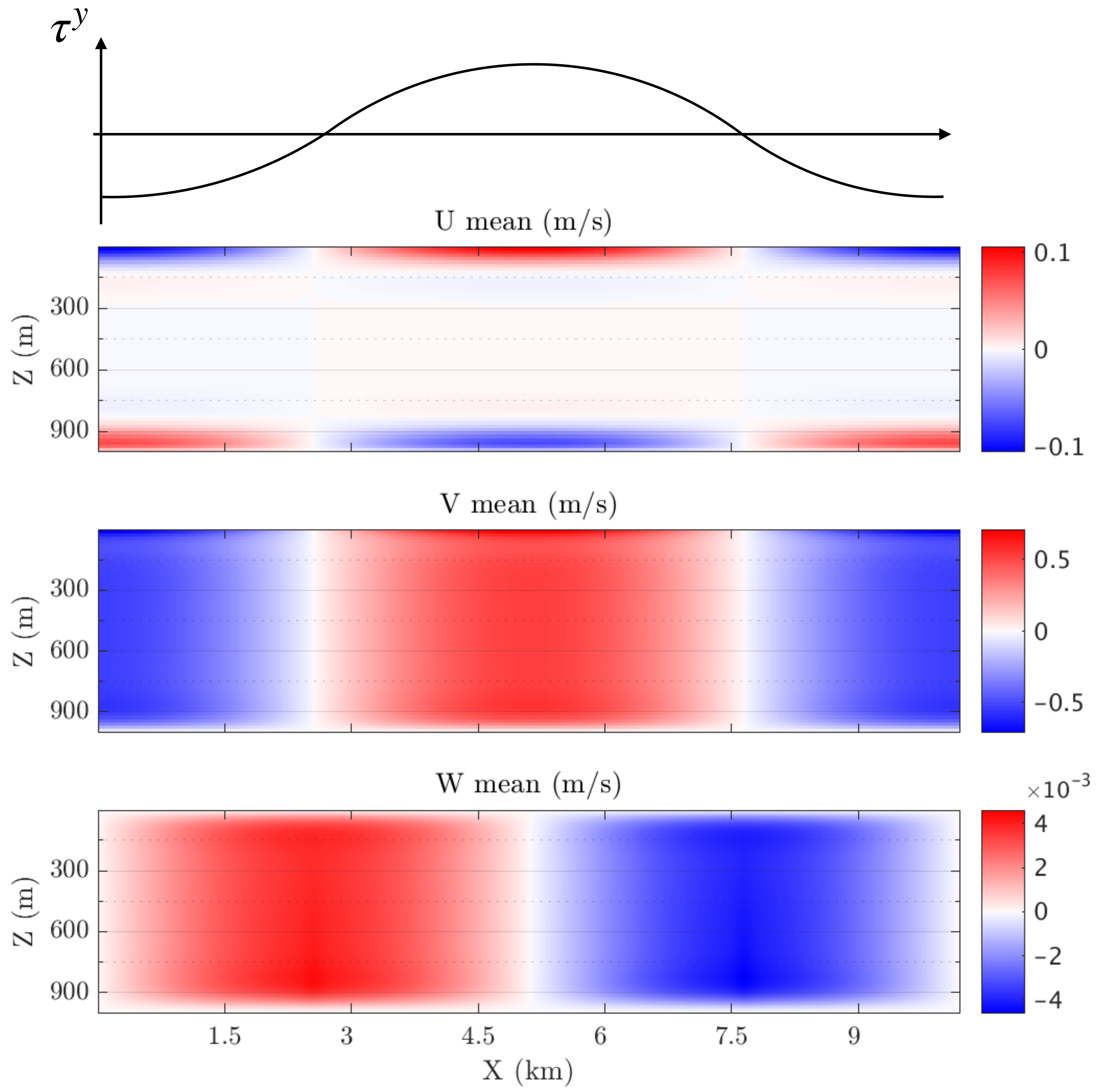
Transport and Pumping

$$(\boldsymbol{f} + \boldsymbol{\zeta}_E) \times \boldsymbol{U}_E = -\nabla \int_{-h_E}^{0} B + \boldsymbol{\tau_a} / \rho_0$$

$$w_E = \hat{\boldsymbol{z}} \cdot \nabla \times \left(\frac{\boldsymbol{\tau_a}}{\rho_0 (f + \zeta_E)} \right) = \hat{\boldsymbol{z}} \cdot \nabla \times \left(\frac{\boldsymbol{\tau_a}}{\rho_0 f (1 + \varepsilon_E)} \right)$$

$$\varepsilon_E = \frac{\mathrm{U_E}}{fL} = \frac{1}{fL} \left(\frac{\tau}{\rho_0 f \delta_E} \right)$$

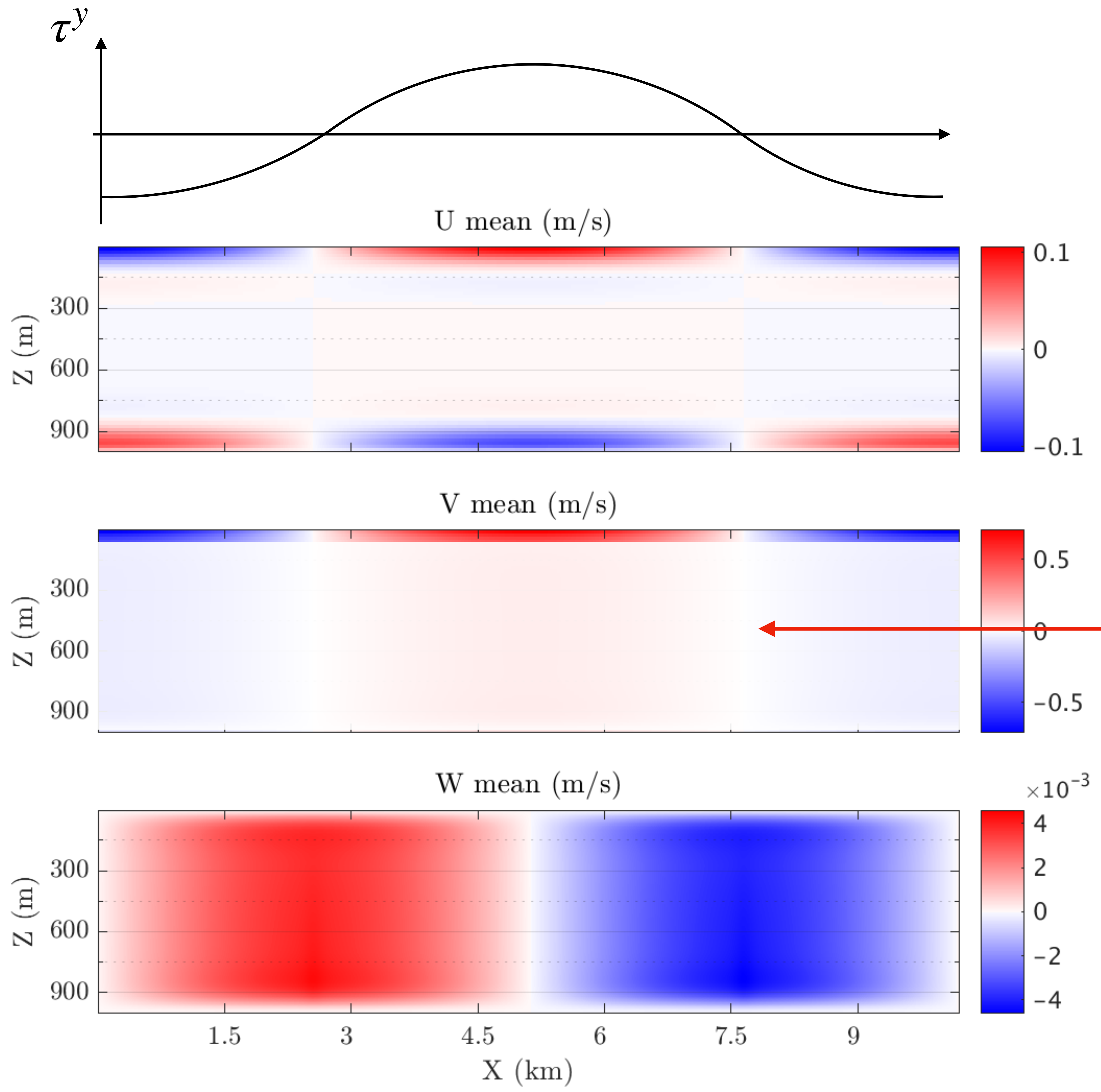
Modified model Setup with 2D LES



Body force in y direction in order to impose $v_g = 0$

$$\delta v(x,y,z,t) = \begin{cases} \frac{\tau}{\rho_0 dz} & \text{at the surface} \\ -\frac{\tau}{\rho_0 (H-dz)} & \text{under the surface} \end{cases}$$

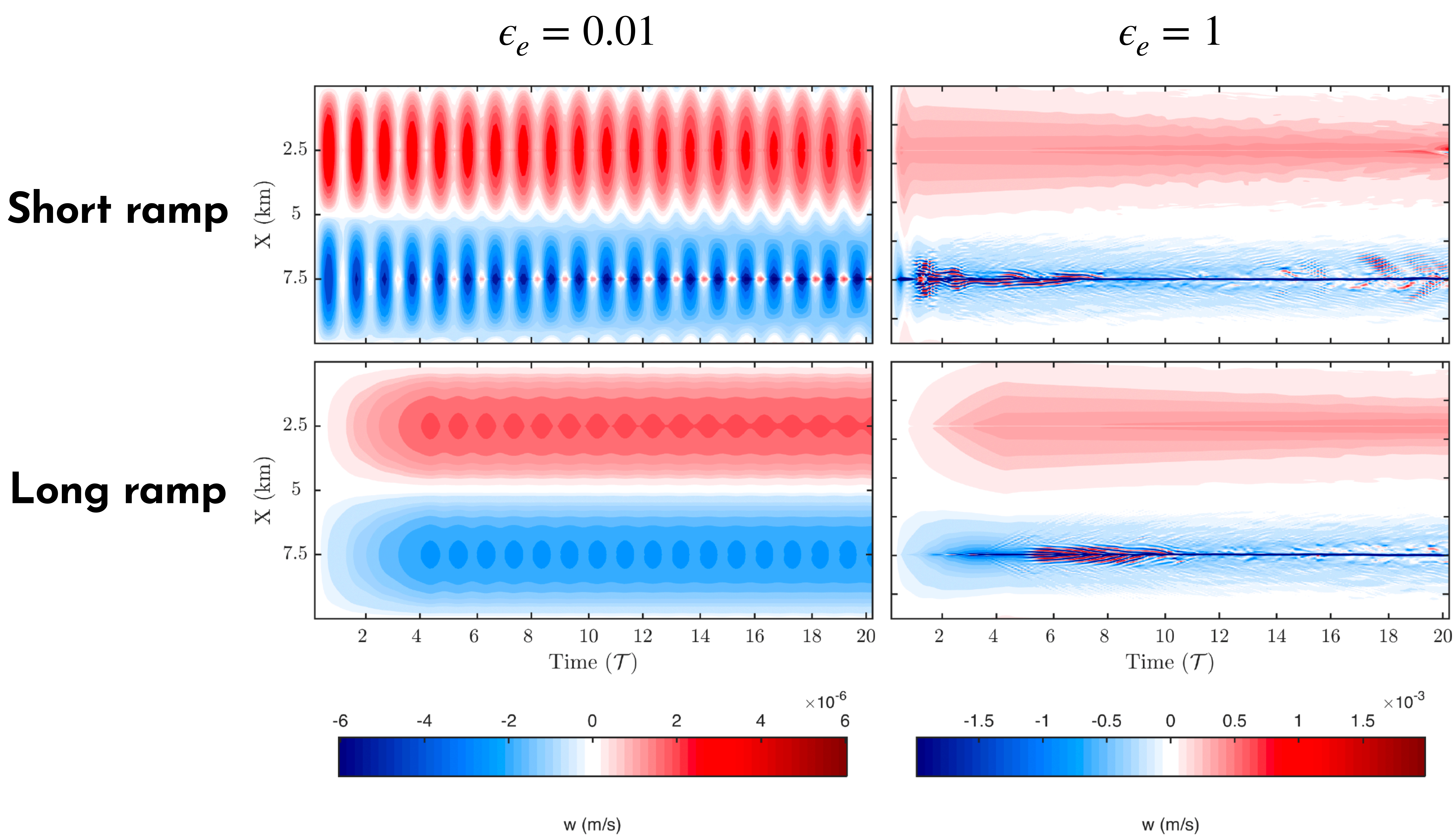
Modified model Setup with 2D LES



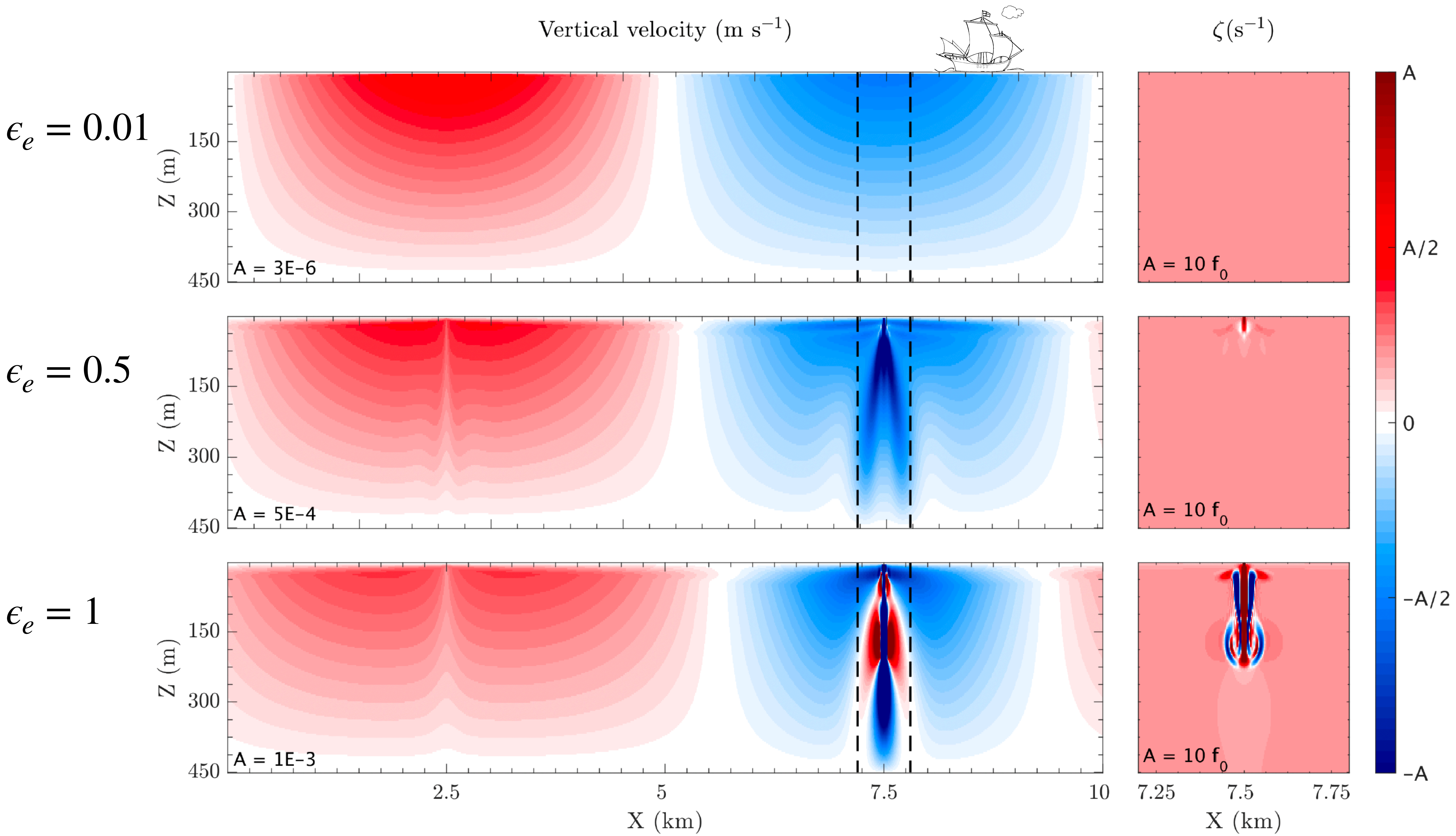
Body force in y direction in order to control v_g

$$\delta v(x,y,z,t) = \begin{cases} \frac{\tau}{\rho_0 dz} & \text{at the surface} \\ -\frac{\tau}{\rho_0 (H-dz)} & \text{under the surface} \end{cases}$$

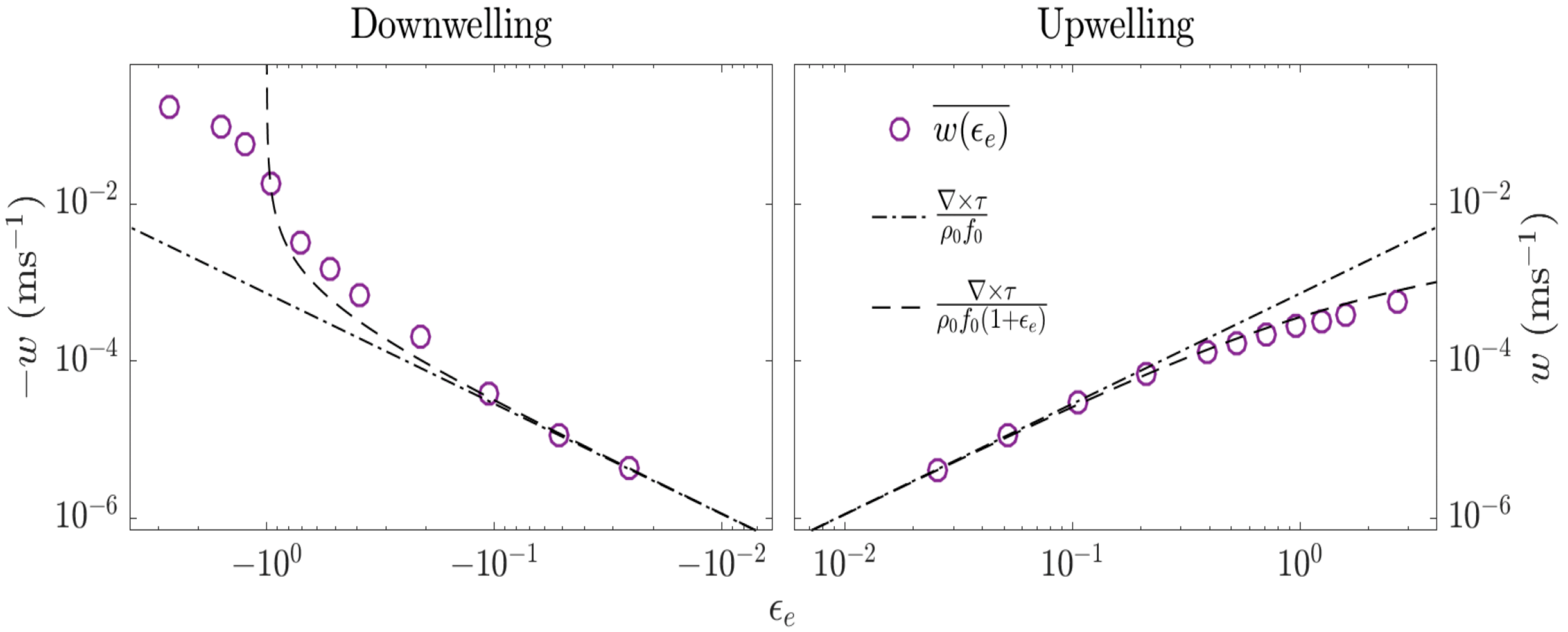
Spinup using the body force $(v_g = 0)$



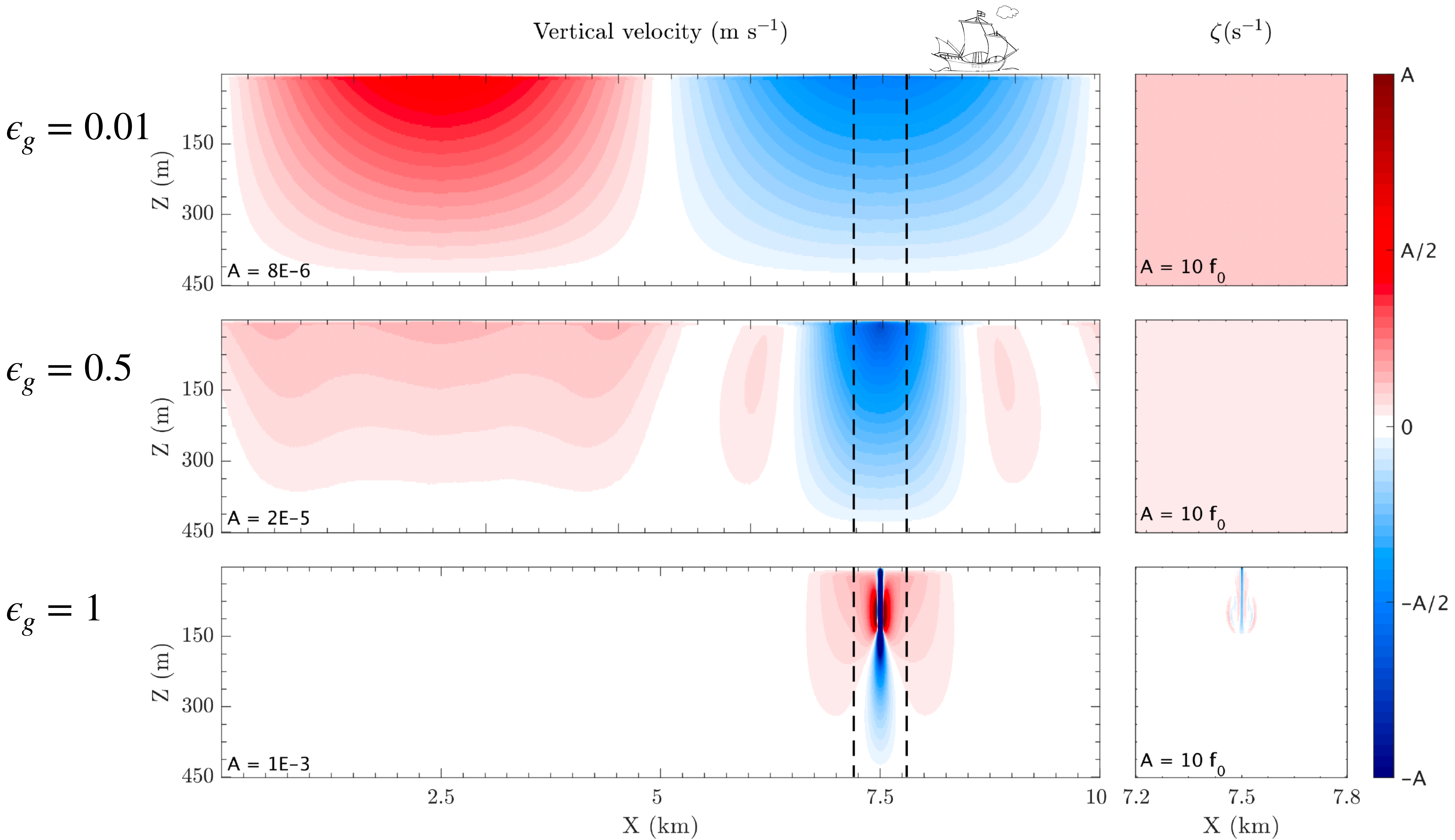
Pumping asymmetry using body force $(v_g = 0)$



Pumping asymmetry using the body force $(v_g = 0)$



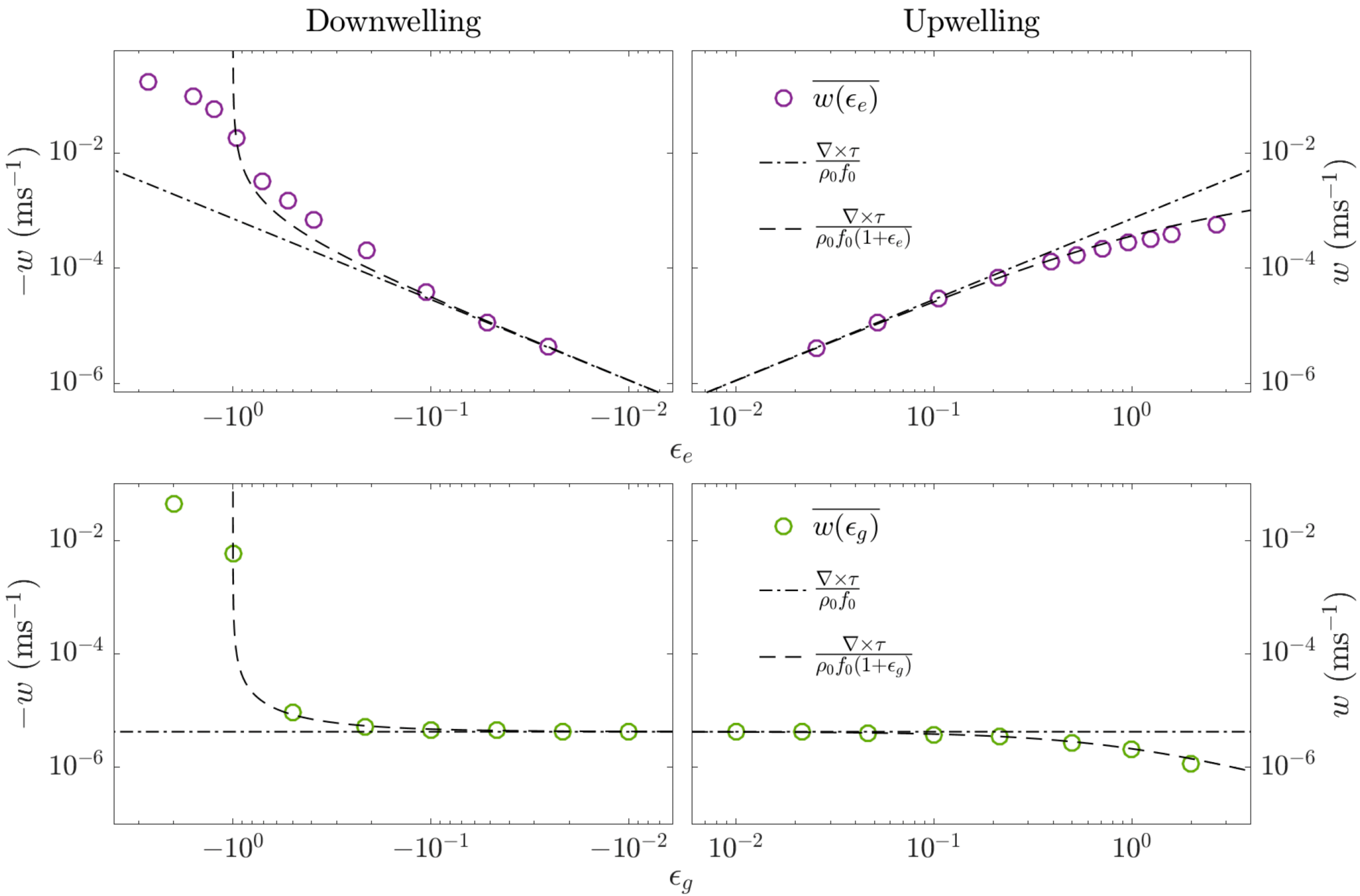
Comparison with Stern: Remove body force, varying $\epsilon_g = \zeta_g/f$ while fixing $\epsilon_e = 0.01$



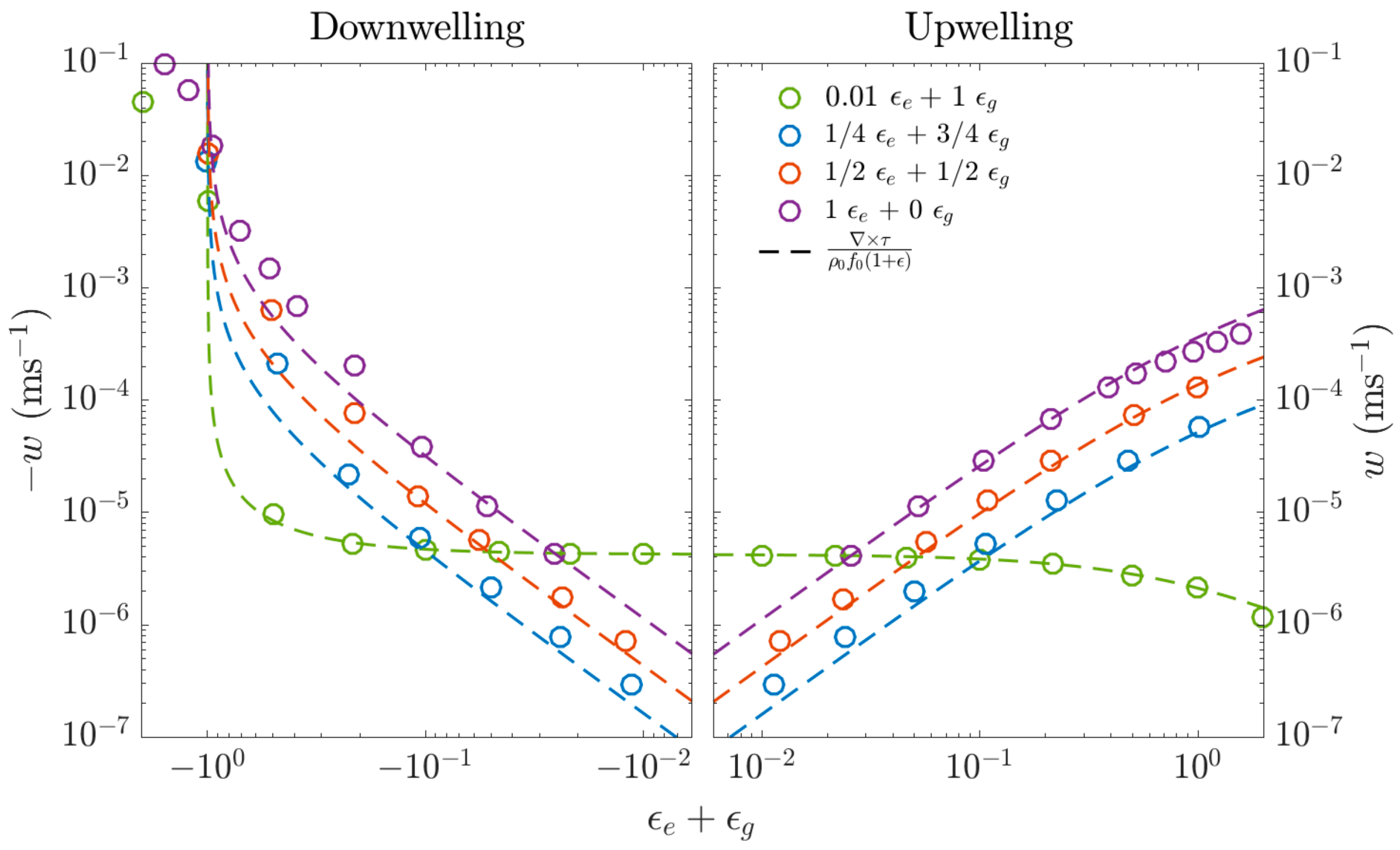
Pumping asymmetry comparison: self-advection vs Stern

self-advection

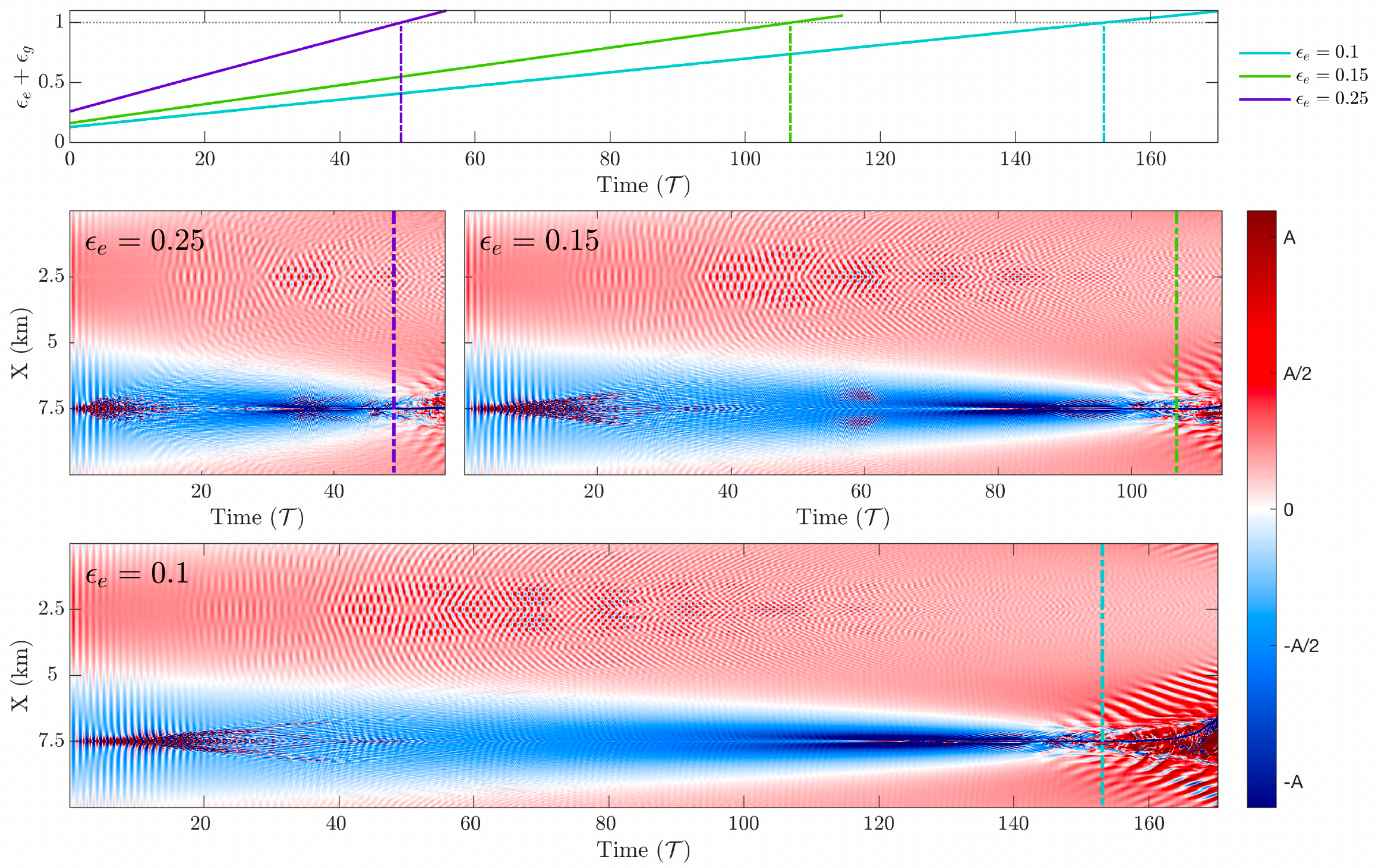
Stern



Addition of both regimes



Addition of both regimes: Spinup (no ramp)



Mechanistic model

Consider the j component of the vorticity :

$$\hat{\boldsymbol{j}} \cdot \nabla \times [D_t \boldsymbol{u} + \boldsymbol{f} \times \boldsymbol{u} = -\nabla \phi + \hat{\boldsymbol{z}} b + A \nabla^2 \boldsymbol{u}]$$

System of equations at steady state :

$$\nabla^2 \omega = \frac{1}{A}(J(\psi, \omega) - f \partial_z v)$$

$$\nabla^2 \partial_z v = \frac{1}{A}(J(\psi, \partial_z v) + f \partial_{zz} \psi)$$

$$\nabla^2 \psi = \omega$$

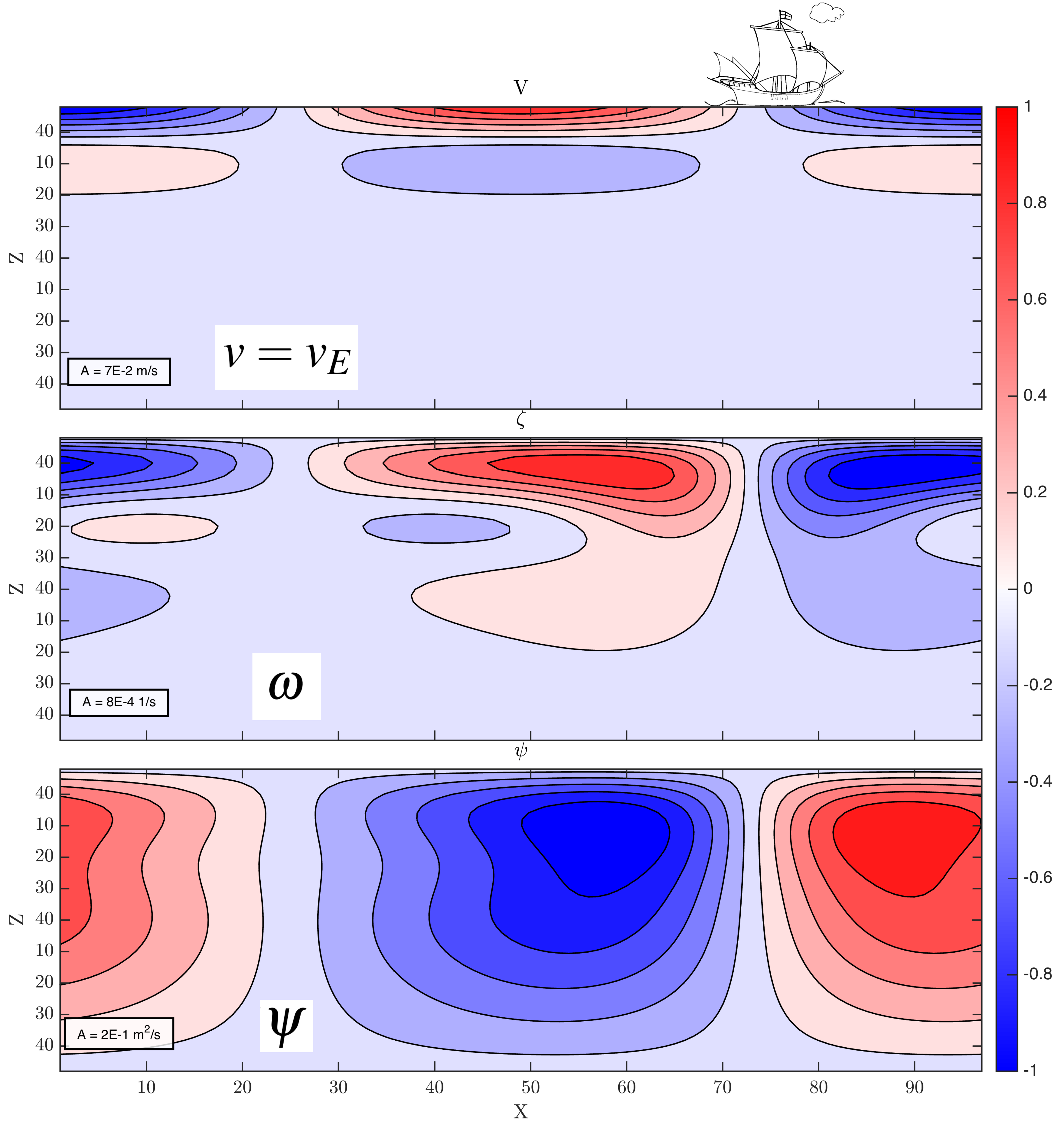
$$u = \partial_z \psi \quad ; \quad w = -\partial_x \psi$$

A mechanistic model

Assume v is given by (symmetric) linear Ekman solution :

$$v = v_E = \frac{\sqrt{2}\tau^y}{f\delta_E} e^{z/\delta_E} \cos(z/\delta_E - \pi/4)$$

$$\nabla^2\omega = \frac{1}{A}(J(\psi,\omega) - f\partial_z v_E)$$
$$\nabla^2\psi = \omega$$



Conclusion Part 1 & 2

$$\partial_t \boldsymbol{u}_E + \boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_E + \boldsymbol{u}_E \cdot \nabla \boldsymbol{u}_g + \boldsymbol{u}_g \cdot \nabla \boldsymbol{u}_E + \boldsymbol{f} \times \boldsymbol{u}_E = \partial_z \boldsymbol{\tau} / \rho_0$$

Near-Inertial Oscillations

Fun ‼

Instability

u_g is small

ϵ_e approaches 1

First order

ϵ_g approaches 1

Higher order

Curvature of u_g

Ekman(1905)