# Gravity waves, compensating subsidence and detrainment around cumulus clouds

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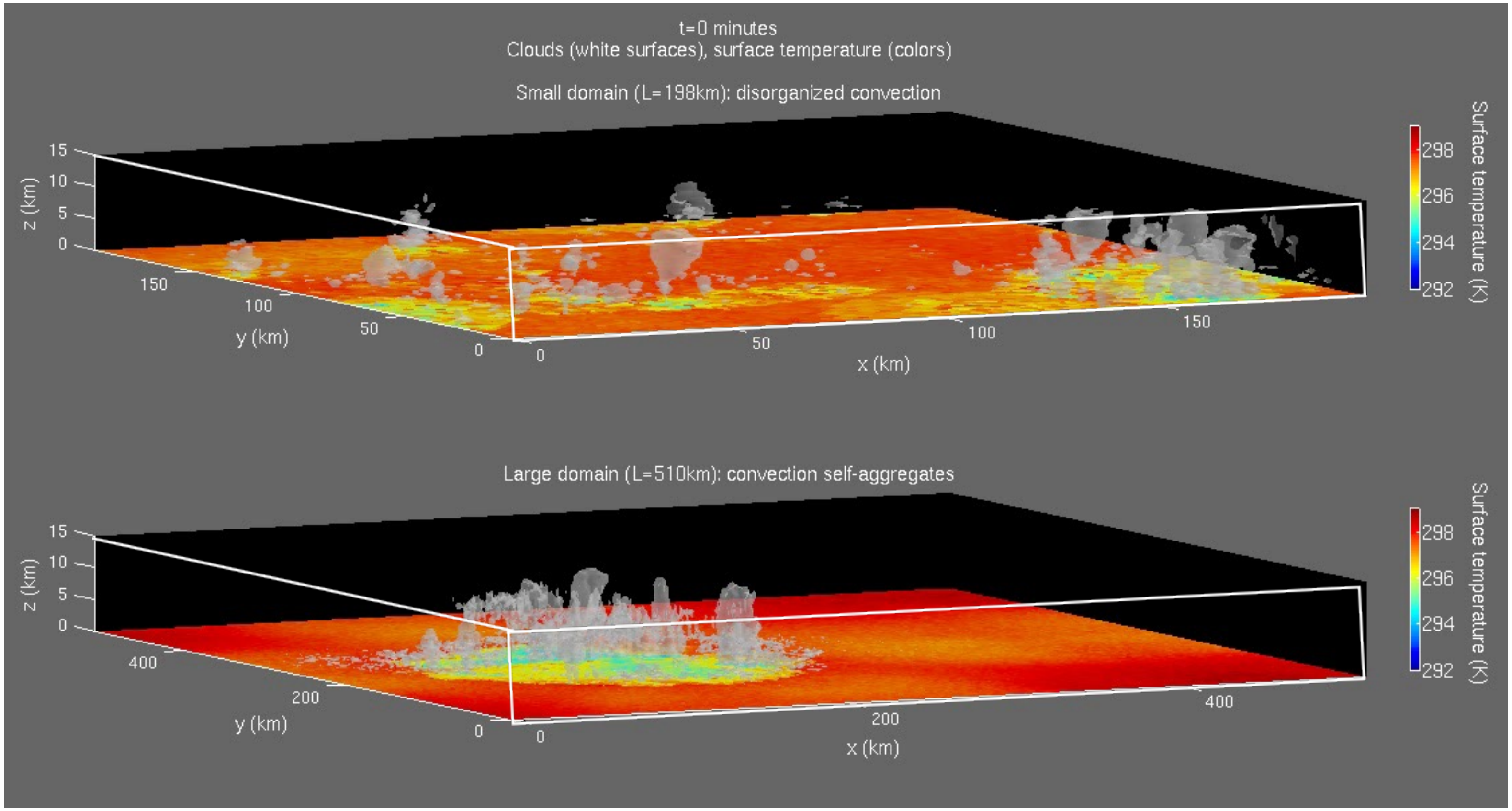
# Initial motivation

*Clouds over near-surface temperature in cloud-resolving model SAM*
*(sea-surface temperature uniform; doubly periodic; no large-scale forcing)*

*[Khairoutdinov & Randall, JAS 2003]*



*« pop corn »*
*convection*

*Self-aggregates*

Self Aggregation = Instability of disorganized Radiative-Convective Equilibrium “pop corn” state

*[Muller et al ARFM 2022]*

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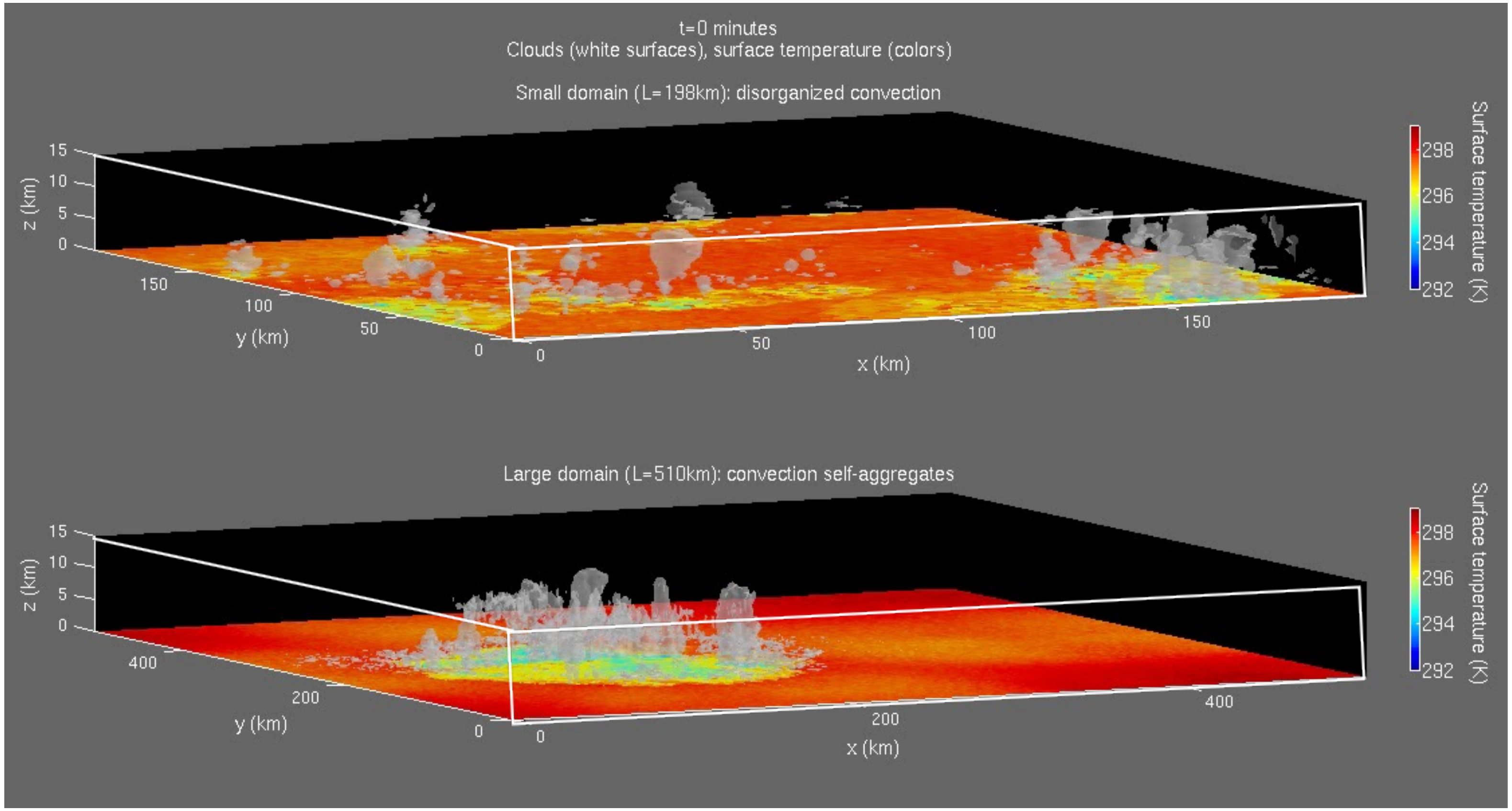

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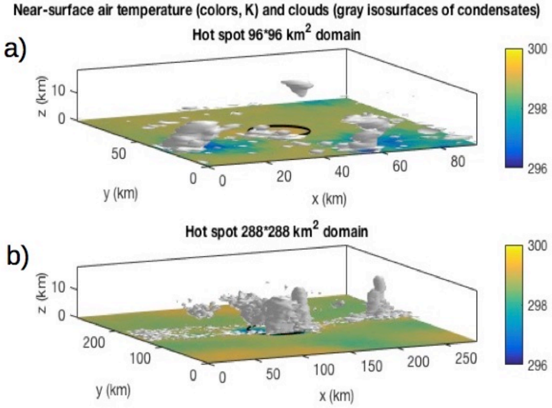

Impact of hot spot (Shamekh et al 2021)

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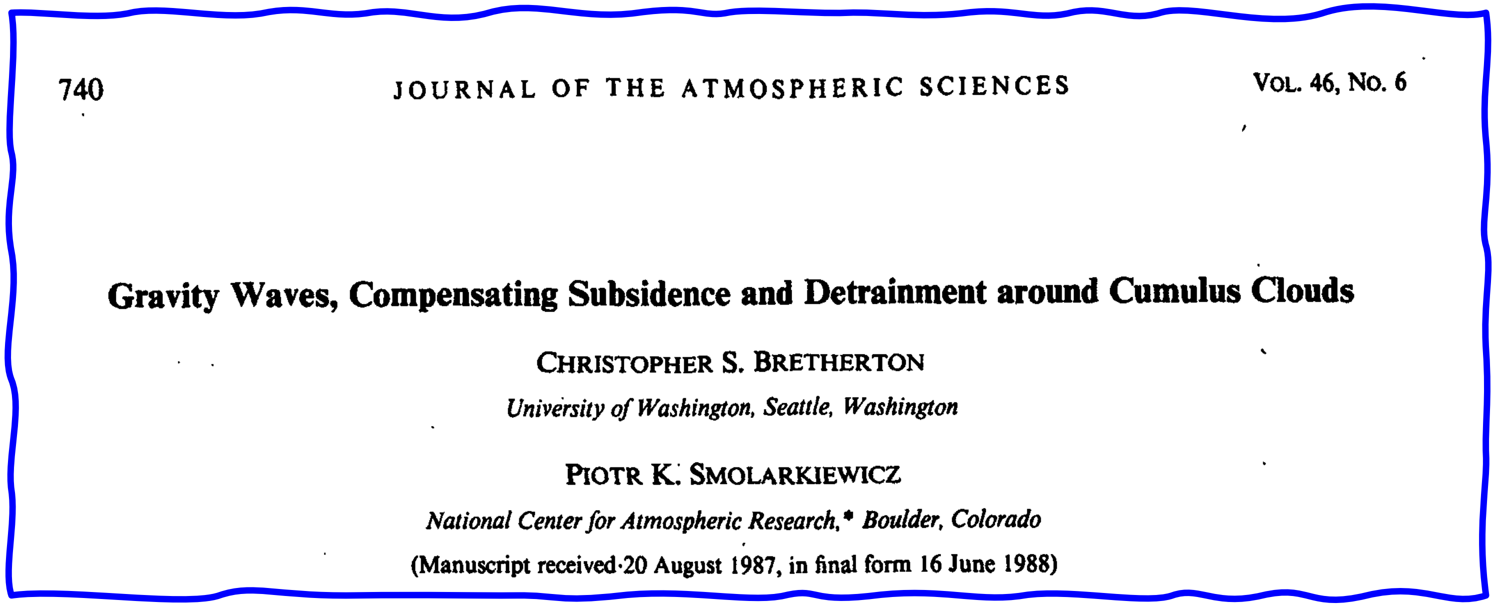
## 1. Context: Moist thermodynamics and stability

*(Dry adiabat, moist adiabat, potential temperature, equivalent potential temperature, conditional instability)*

## 2. The paper



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**Gravity Waves, Compensating Subsidence and Detrainment around Cumulus Clouds**

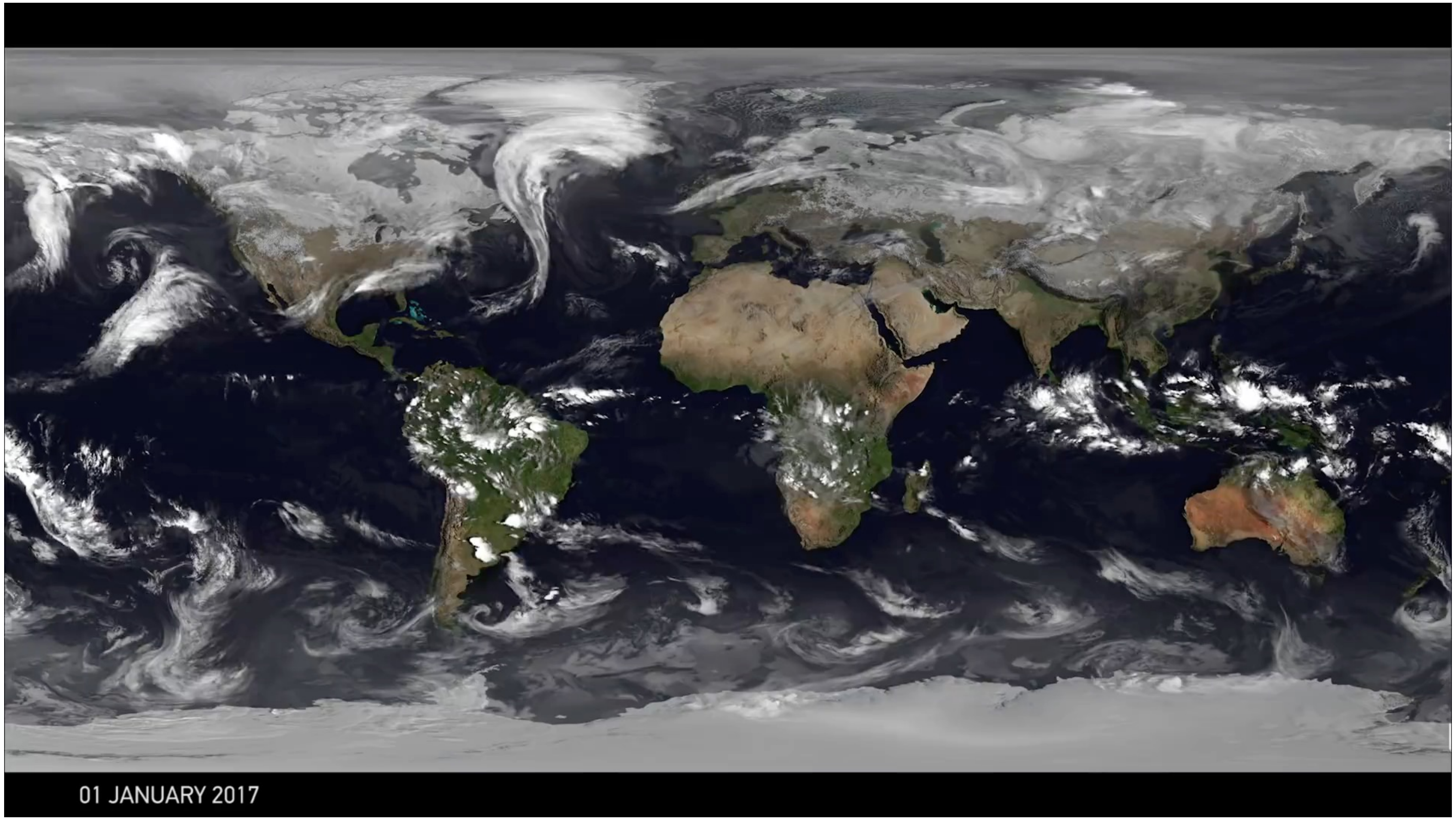
CHRISTOPHER S. BRETHERTON
*University of Washington, Seattle, Washington*

PIOTR K. SMOLARKIEWICZ
*National Center for Atmospheric Research,* Boulder, Colorado*

(Manuscript received 20 August 1987, in final form 16 June 1988)

# Context: Moist thermodynamics and stability

*Brightness temperature from satellite (white ⇔ cold cloud tops)*



Large extratropical
storm systems
⇒ **High clouds**

Subtropics
⇒ **No high clouds**

ITCZ = Intertropical
convergence zone
⇒ **Tropical**
**Convective clouds**

*« A year of weather »*

# Context: Moist thermodynamics and stability

*Courtesy : Octave Tessiot*

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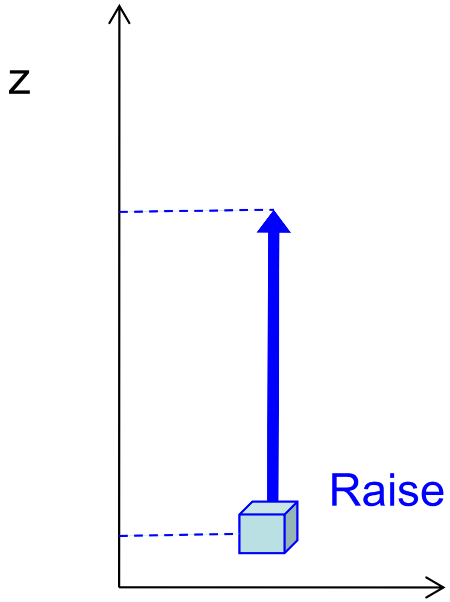
# Context: Moist thermodynamics and stability

## Dry convection

T decreases with height.
But p as well.

Density = ρ(T,p).
How determine stability? The parcel method



Raise parcel adiabatically. Comes back to initial position?

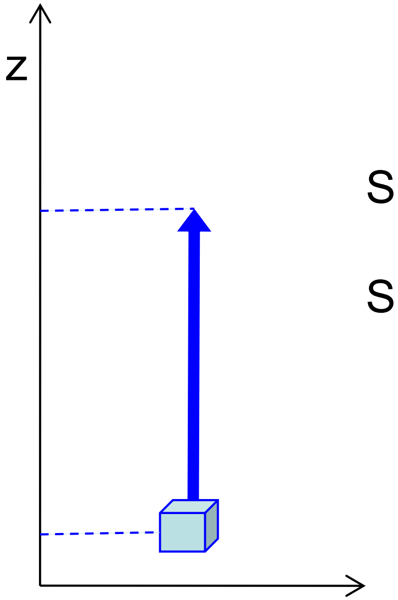
# Context: Moist thermodynamics and stability

## Dry convection

Potential temperature $\theta = T\,(p_0 / p)^{R/cp}$ conserved under adiabatic displacements :

*Adiabatic displacement*

1st law thermodynamics: d(internal energy) = Q (heat added) – W (work done by parcel)



$$c_v\,dT = -\,p\,d(1/\rho)$$

Since $p = \rho R T$, $\quad c_v\,dT = -\,p\,d(R\,T / p) = -\,R\,dT + R\,T\,dp / p$

Since $c_v + R = c_p$, $\quad c_p\,dT / T = R\,dp / p$

$$\Rightarrow d \ln T - R / c_p\, d \ln p = d\ \ln (T / p^{R/cp}) = 0$$

$$\Rightarrow\ T / p^{R/cp} = \text{constant}$$

Hence $\theta = T\,(p_0 / p)^{R/cp}$ potential temperature is conserved under adiabatic displacement

(R=gaz constant of dry air; $c_p$=specific heat capacity at constant pressure; $R/c_p \sim 0.286$ for air)

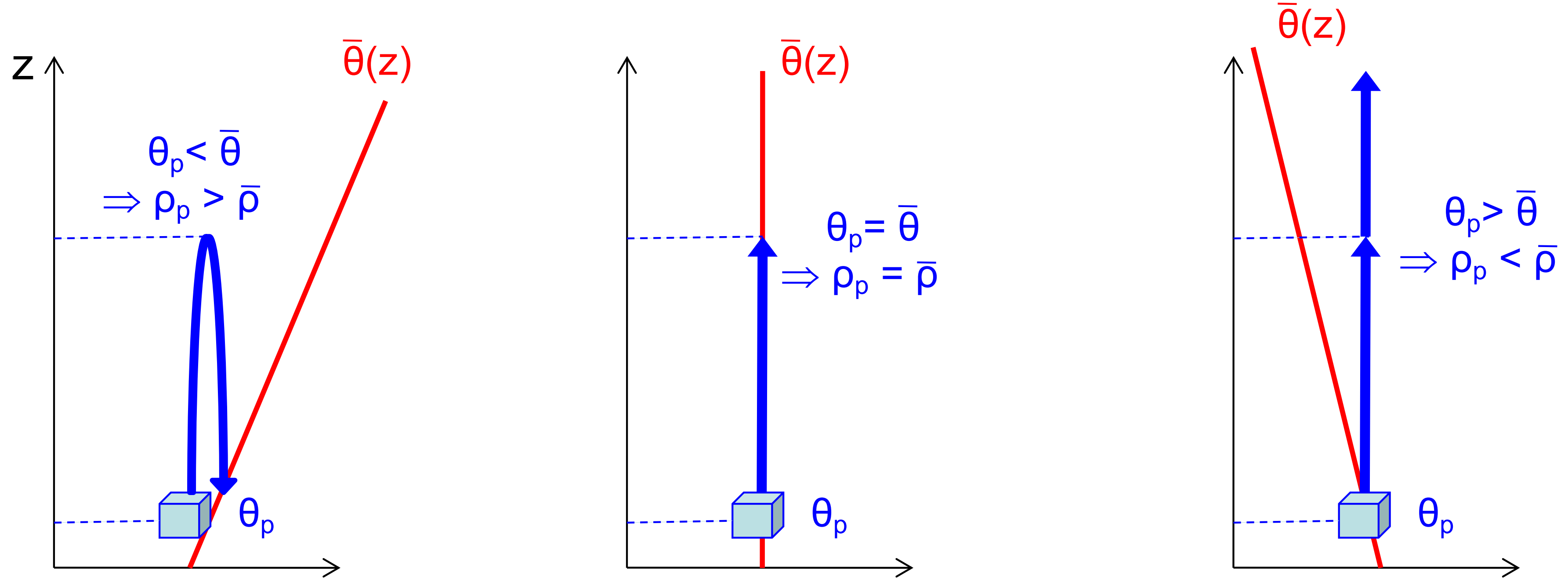
# Context: Moist thermodynamics and stability

**When is an atmosphere unstable to dry convection?**

When potential temperature $\theta = T\,(p_0 / p)^{R/cp}$ decreases with height !

The parcel method:

Small vertical displacement of a fluid parcel adiabatic (=> $\theta$ = constant).
During movement, pressure of parcel = pressure of environment.



STABLE

NEUTRAL

UNSTABLE

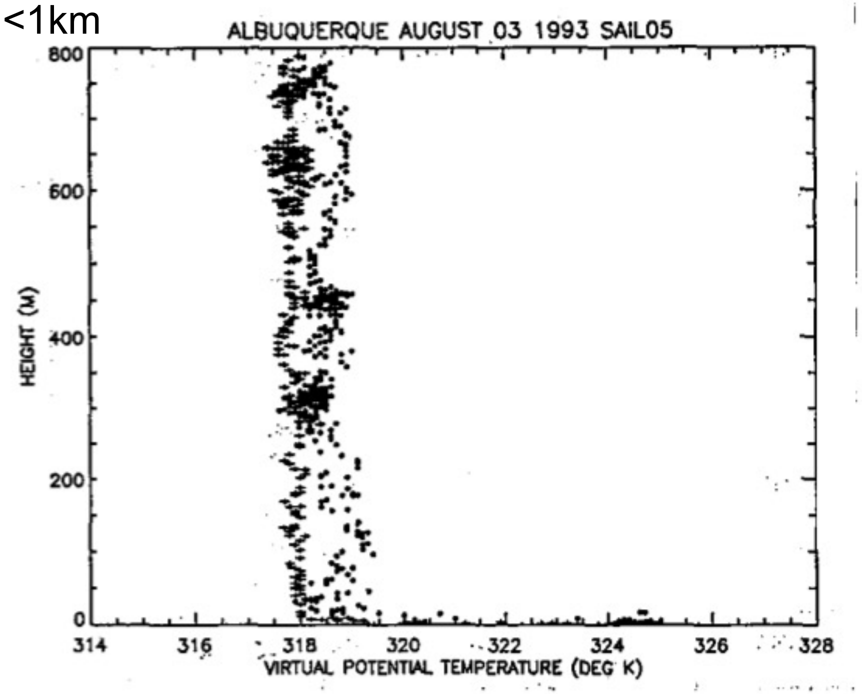
# Context: Moist thermodynamics and stability

Convective adjustment time scales is very fast (minutes for dry convection) compared to destabilizing factors (surface warming, atmospheric radiative cooling…)

**=> The observed state is very close to convective neutrality**

Dry convective boundary layer over daytime desert

<1km



*[Renno and Williams, 1995]*

But above a thin boundary layer, not true anymore that θ = constant. Why?…

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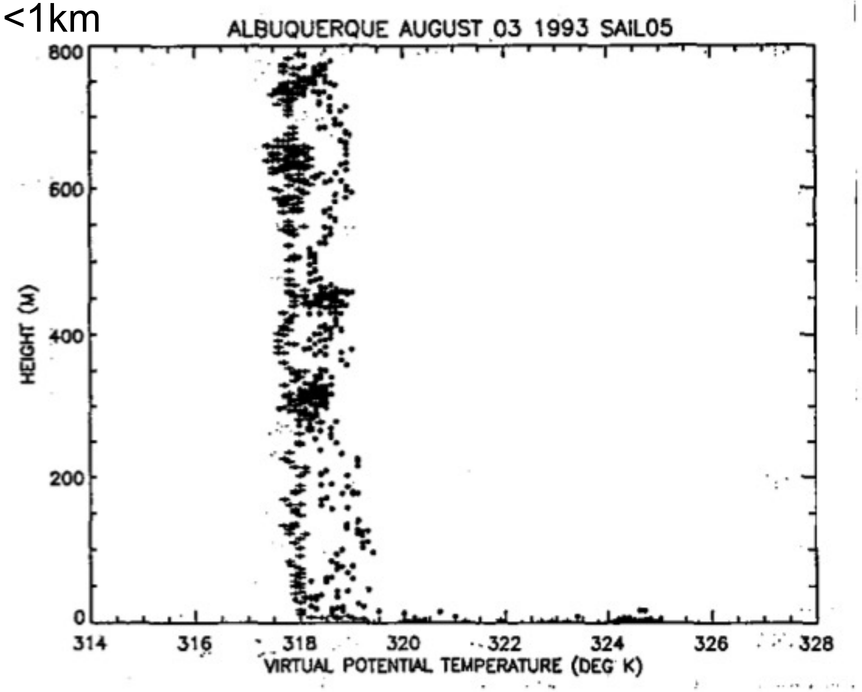

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But above a thin boundary layer, not true anymore that θ = constant. Why?…

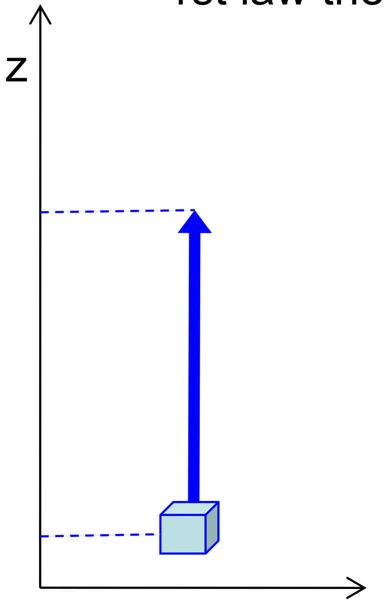
Most atmospheric convection involves phase change of water

Significant latent heat with phase changes of water = **Moist Convection**

# Context: Moist thermodynamics and stability

## When is an atmosphere unstable to moist convection ?

Equivalent potential temperature $\theta_e = T\,(p_0 / p)^{R/cp}\ e^{Lv\,qv / (cp\,T)}$ is conserved under adiabatic displacements :



1st law thermodynamics if air saturated ($q_v$=$q_s$) :

d(internal energy) = Q (latent heat) – W (work done by parcel)

$$c_v\,dT = -\ L_v\,dq_s - \ p\,d(1/\rho)$$

$$\Rightarrow d \ln T - R / c_p\, d \ln p = d\ \ln (T / p^{R/cp}) = \ -L_v / (c_p\,T)\,dq_s$$

$$\Rightarrow d\ \ln (T / p^{R/cp}) \sim\ -L_v / c_p\, d(q_s/T)$$

$$\Rightarrow\ T / p^{R/cp}\ \ e^{Lv\,qs / (cp\,T)}\ \sim \text{constant}$$

Note: Air saturated => $q_v$=$q_s$
Air unsaturated => $q_v$ conserved

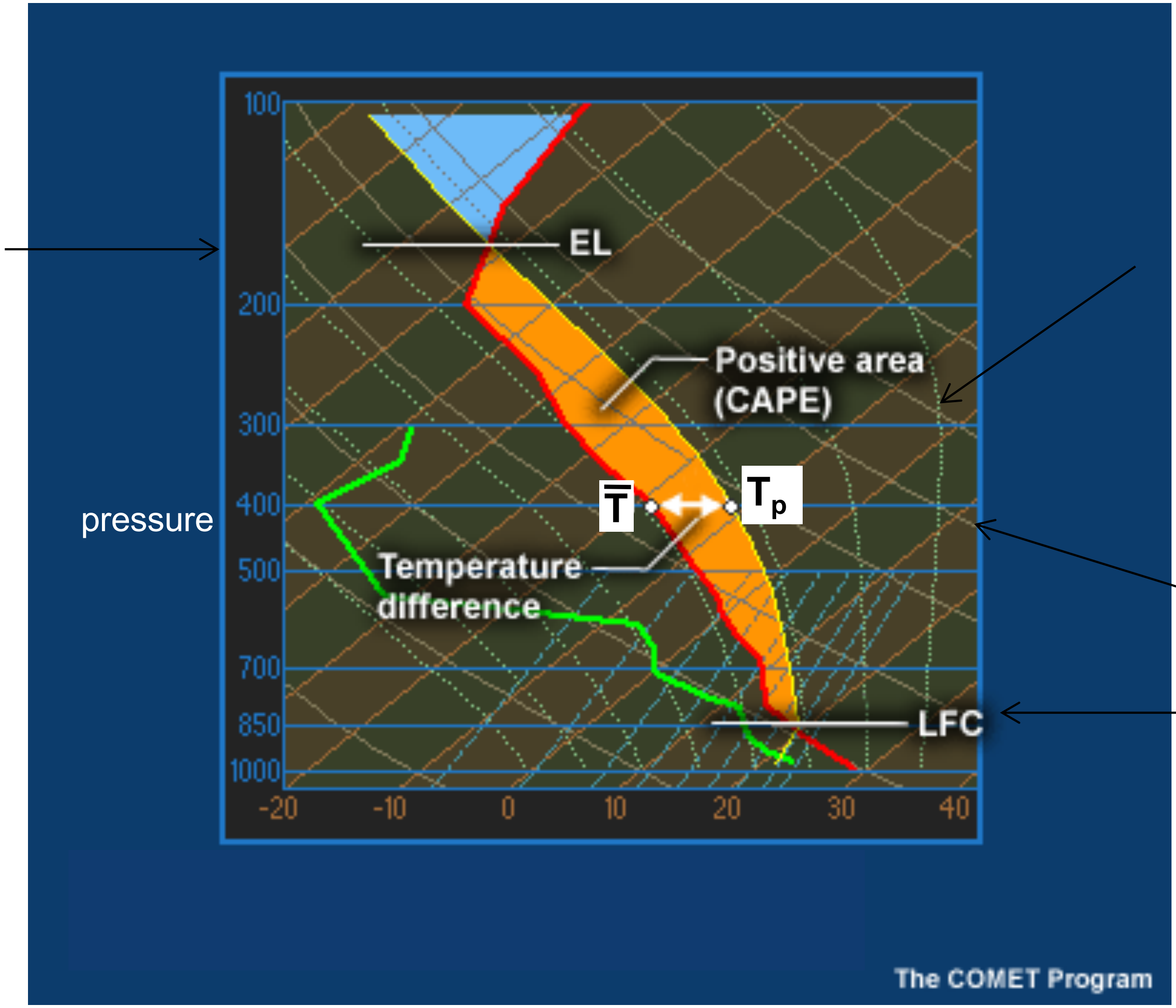
Hence

$\theta_e = T\,(p_0 / p)^{R/cp}\ e^{Lv\,qv / (cp\,T)}$ equivalent potential temperature is approximately conserved

# Context: Moist thermodynamics and stability

## When is an atmosphere unstable to moist convection ?

Skew T diagram (isoT slanted), atmospheric T in red



EL equilibrium level

Moist adiabat $\theta_e$= cstt

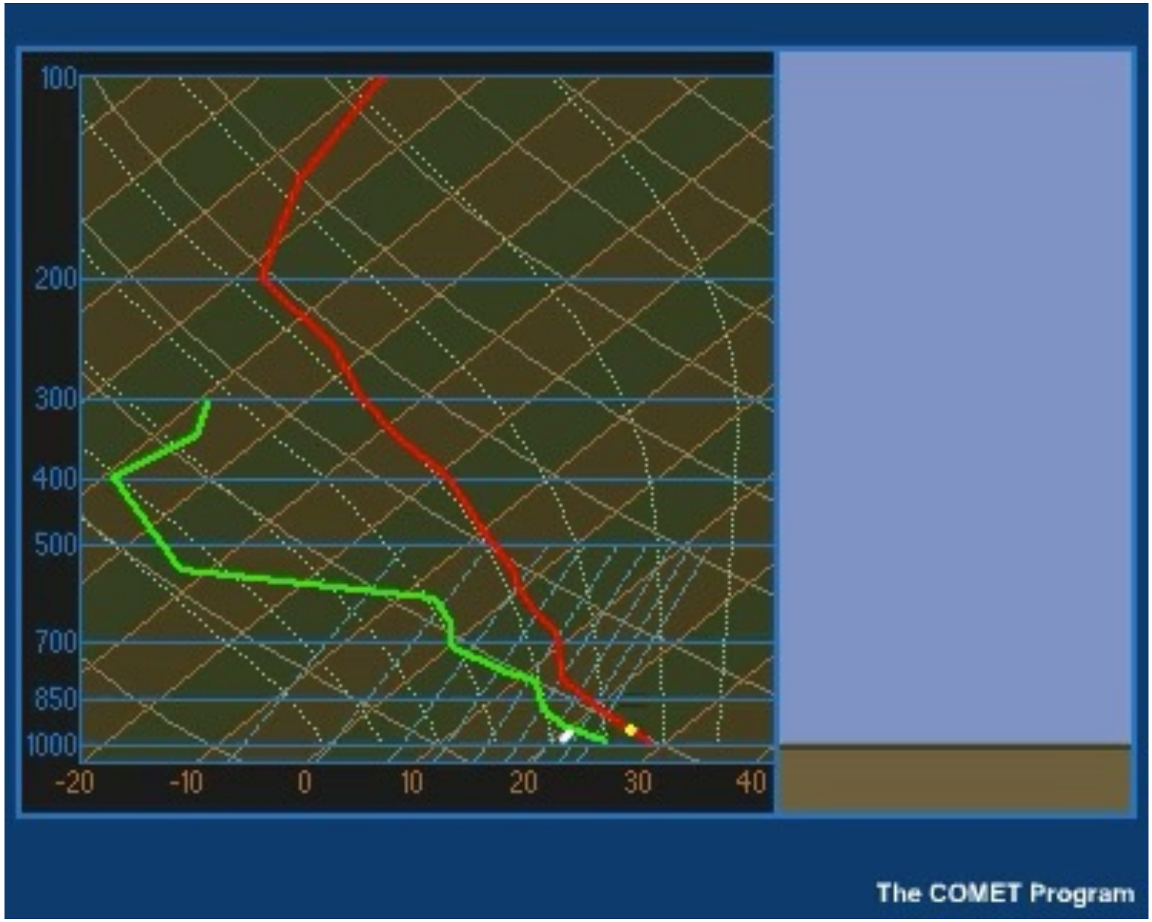
Dry adiabat $\theta$ = cstt

LFC level of free convection (= LCL lifted condensation level for simplicity)

CAPE: convective available potential energy

# Context: Moist thermodynamics and stability

Parcel = yellow dot



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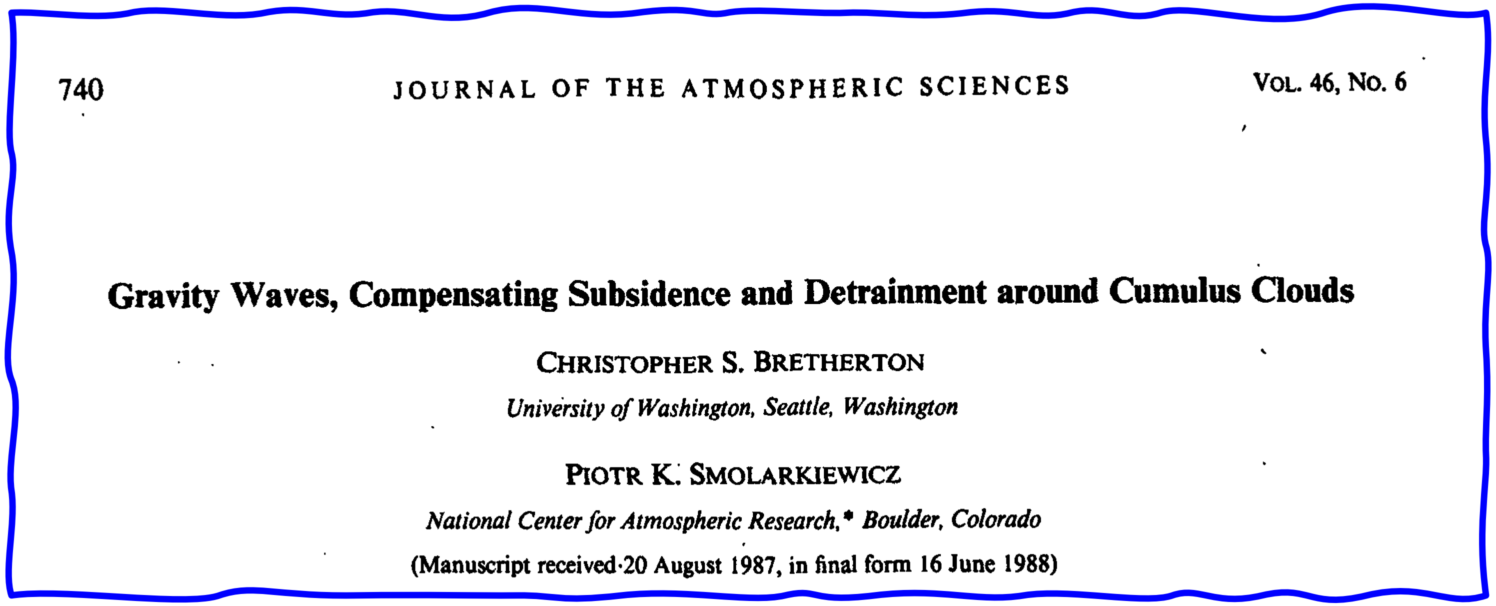

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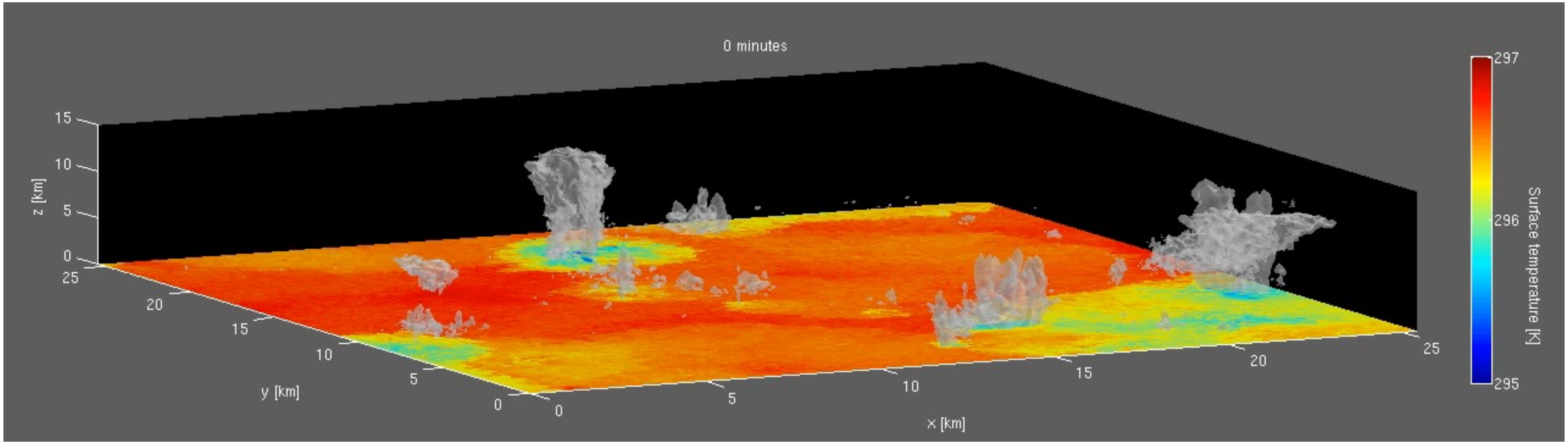
**Question addressed:** Redistribution of heat and moisture by convective clouds in a conditionally unstable atmosphere = “moist convective adjustment”

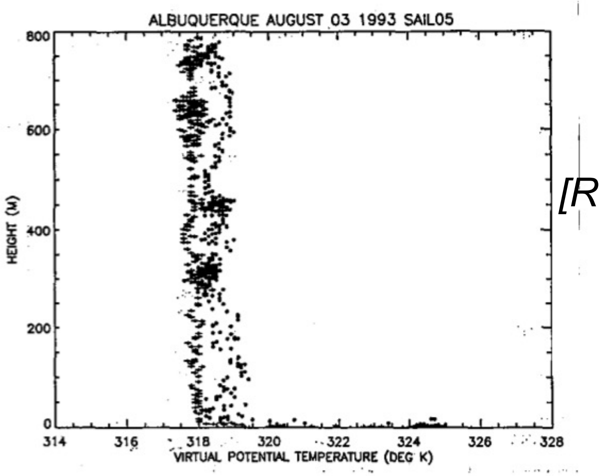
Clouds over near-surface temperature (Cloud resolving model SAM *Khairoutdinov & Randall 2003*)



T in cloud follows moist adiabat.
How does the cloud-free environmental (CFE) adjust?

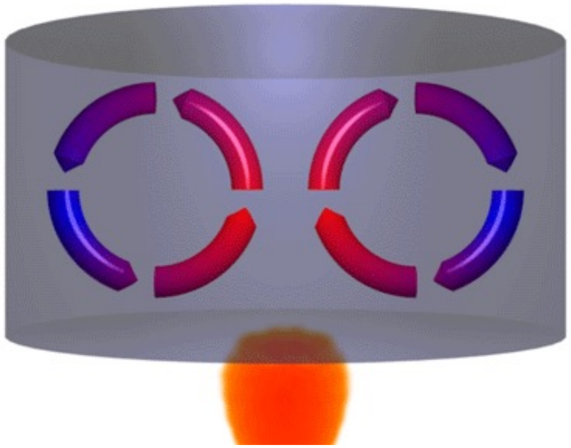
# **One possible mechanism of convective adjustment:** Mixing of all (water, θ…) invariants of dry boundary layer convection

Dry convective boundary layer over daytime desert

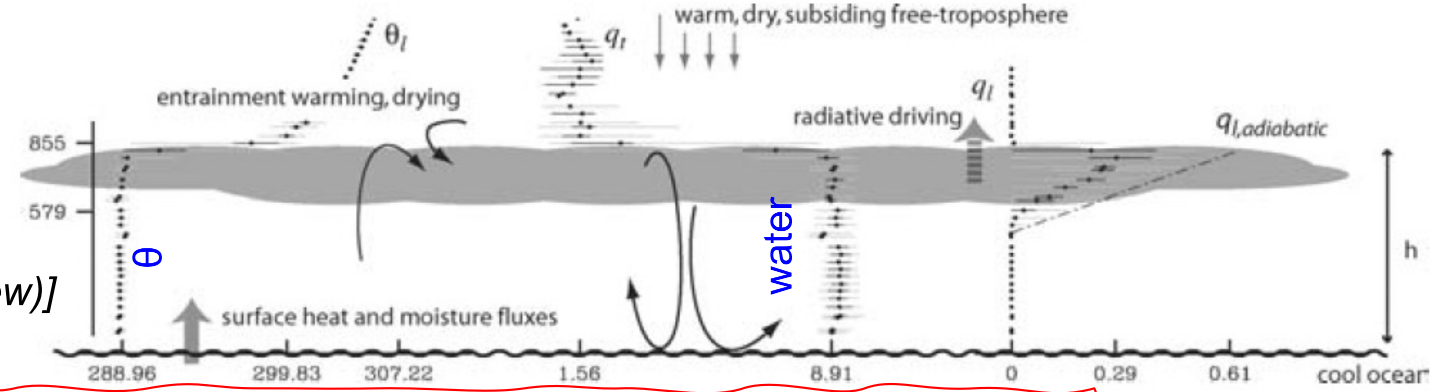


*[Renno and Williams, 1995]*

Convective mixing



Schematic boundary layer with aircraft data



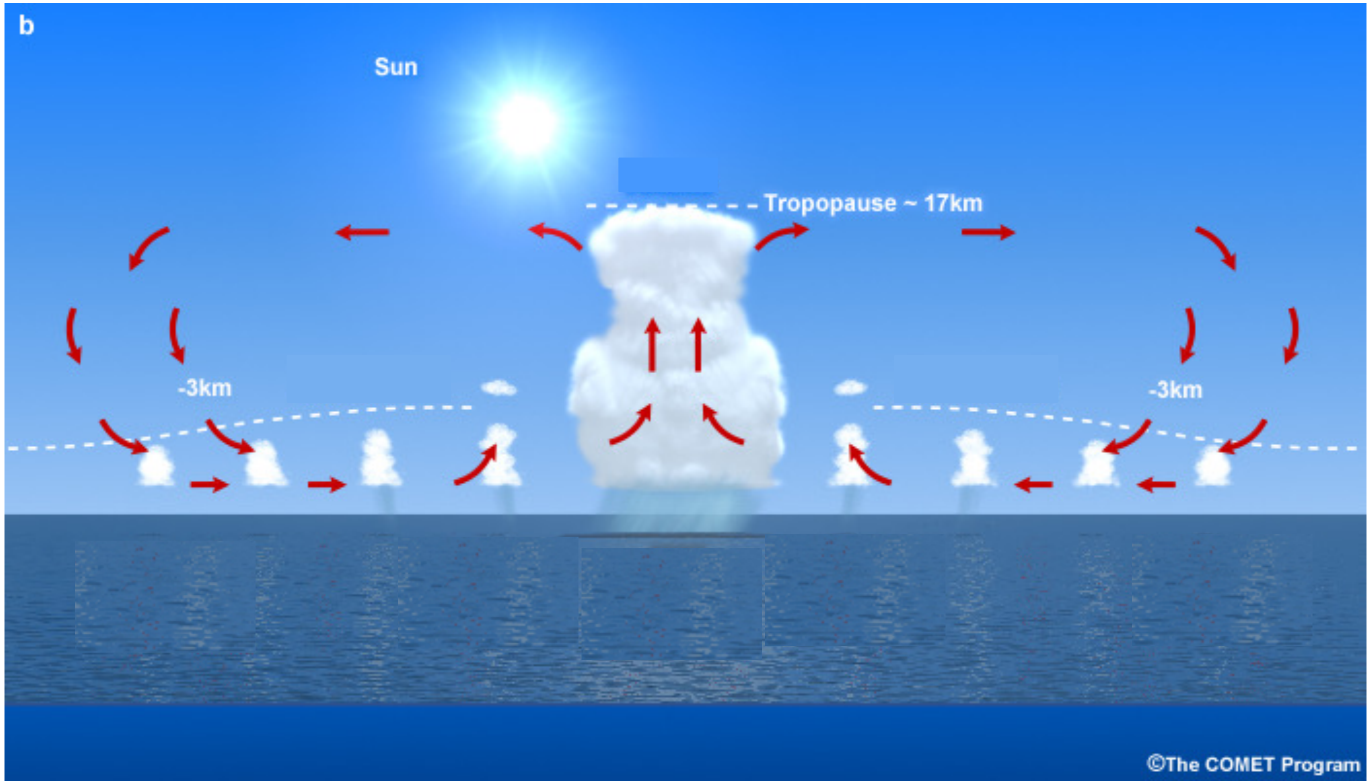
*[Stevens 2005 (review)]*

⇒ Not the case for moist convection

⇒ E.g. water field NOT homogeneous

# Proposed mechanism of convective adjustment:

## Compensating subsidence between clouds



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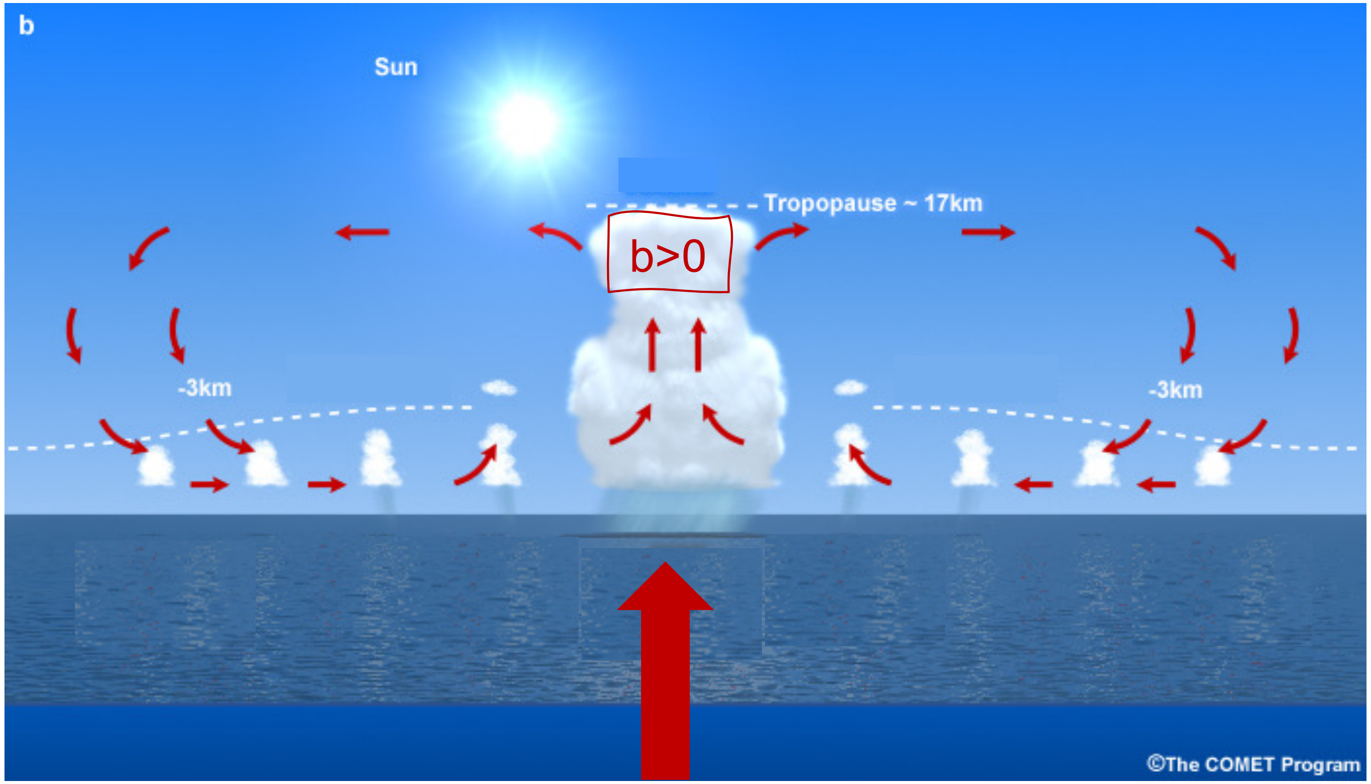

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**B source of convection**

⇒ Brings convective zone on moist adiabat

# Proposed mechanism of convective adjustment:

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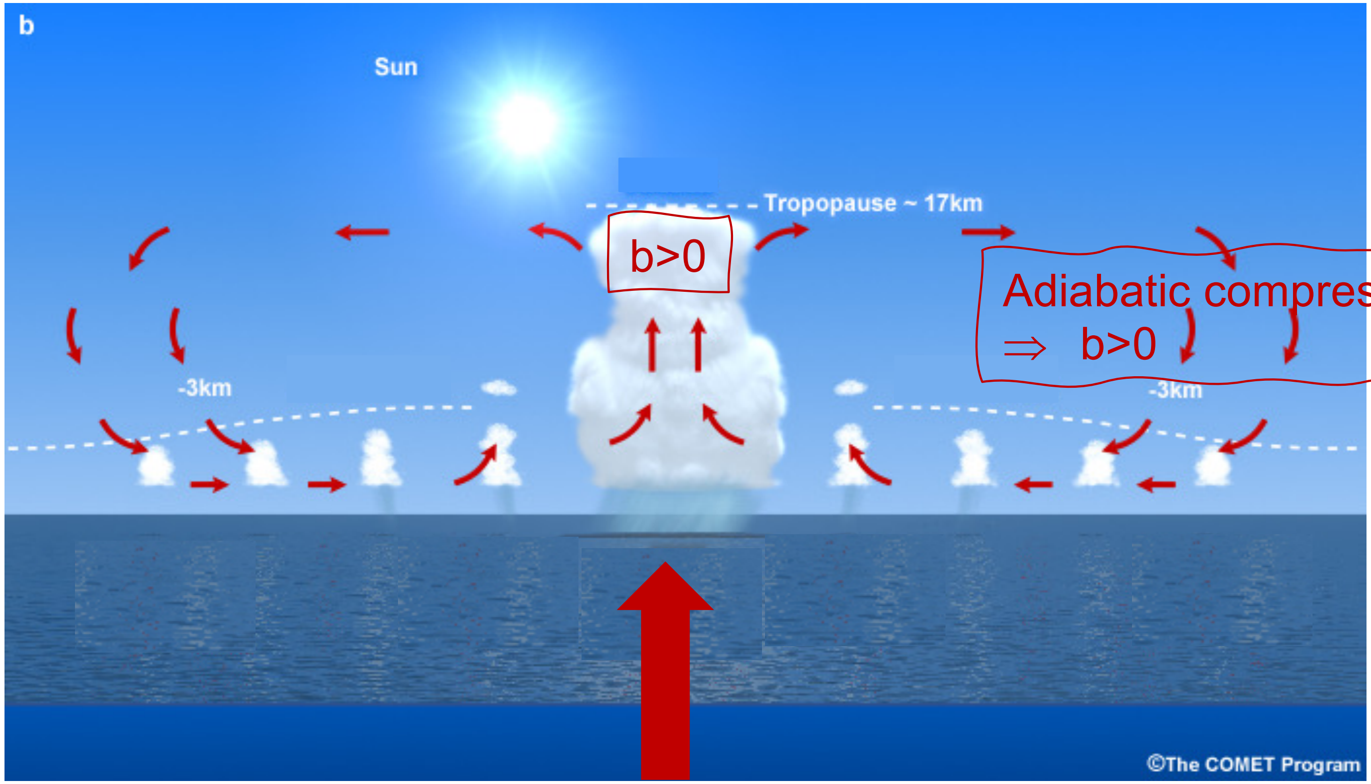

b>0

Adiabatic compression
⇒ b>0

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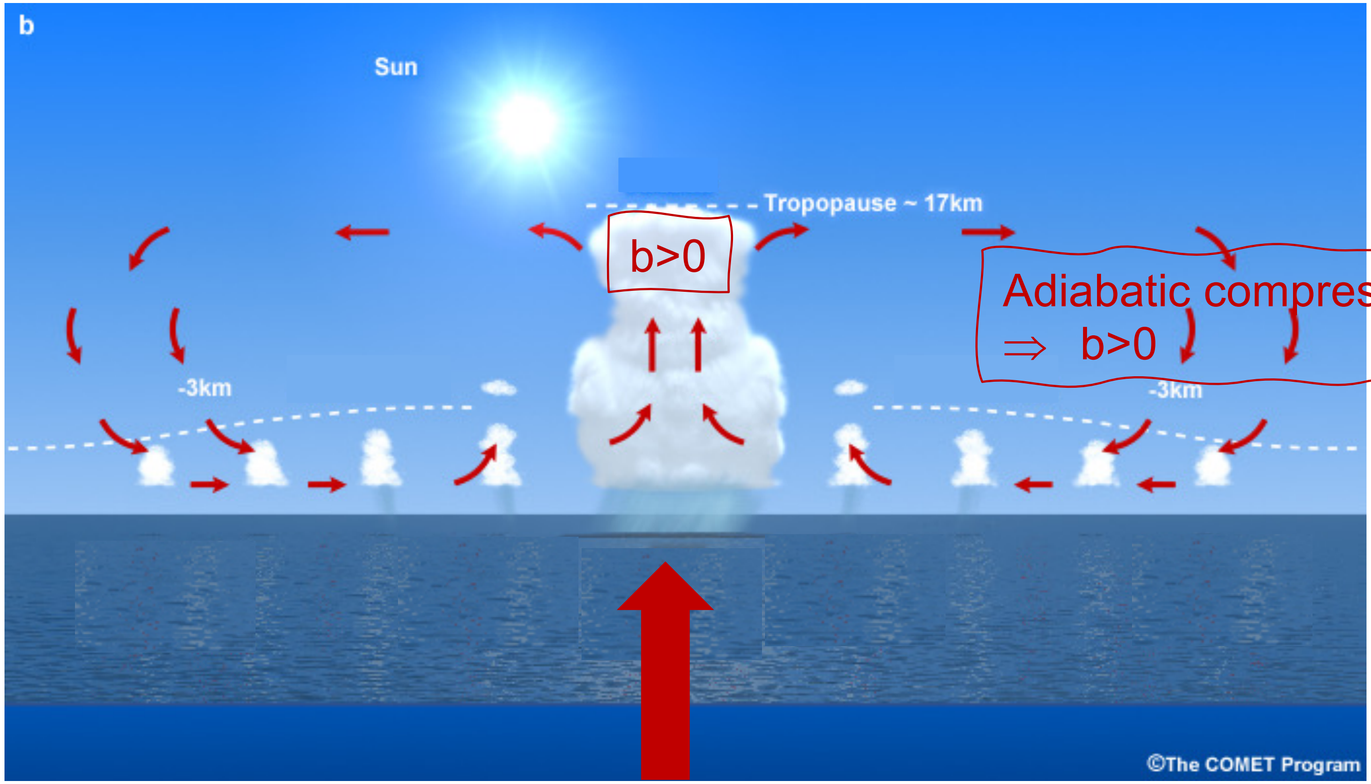
**B source of convection**

⇒ Brings convective zone on moist adiabat

# Proposed mechanism of convective adjustment:

## Compensating subsidence between clouds



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b>0

Adiabatic compression
⇒ b>0

⇒ All T profile moist adiabatic

**B source of convection**

⇒ Brings convective zone on moist adiabat

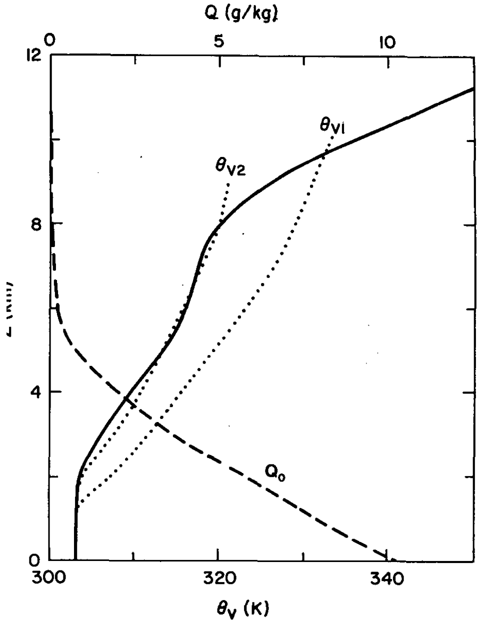
# Whole paper:

numerical experiments
+ linear theory

$\Rightarrow$ to validate this mechanism

# Numerical experiment:

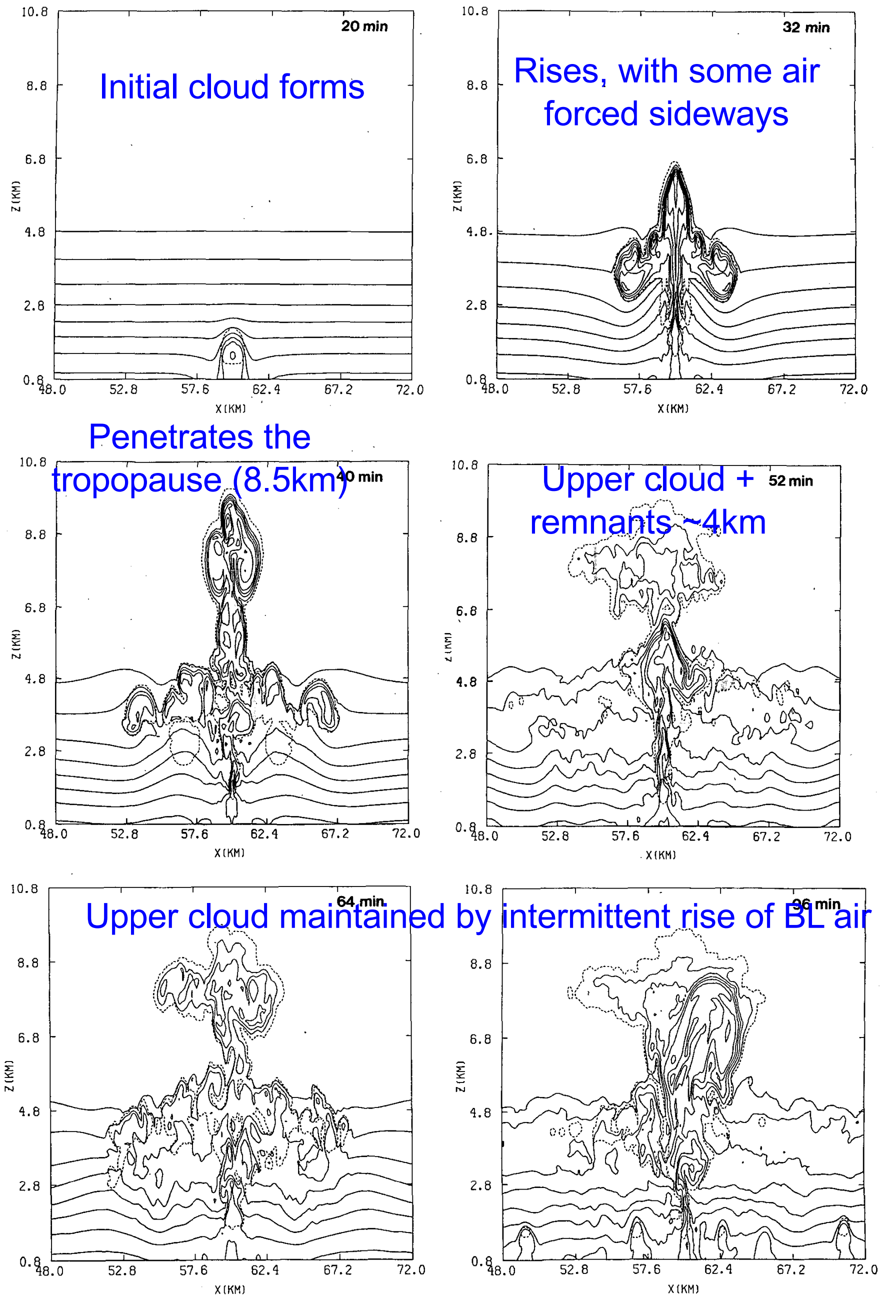
Initial sounding θ and total water mixing ratio Q (g water / kg dry air)



Expt E1:
Convection initiated by imposing 4 min Gaussian surface heat flux
H = 600 exp [-(x-60)^2] W/m^2

ulation. In experiment E1, convection was initiated by imposing for 4 min a small localized Gaussian surface heat flux $H = 600 \exp[-(x - 60)^2]$ W m$^{-2}$. This created a small "hot spot" with a maximum buoyancy of 1 K concentrated in the lowest two grid points. The

Time evolution of Q contours



Initial cloud forms

Rises, with some air forced sideways

Penetrates the tropopause (8.5km)

Upper cloud + remnants ~4km

Upper cloud maintained by intermittent rise of BL air

FIG. 2 (*Continued*)

# Numerical experiment:

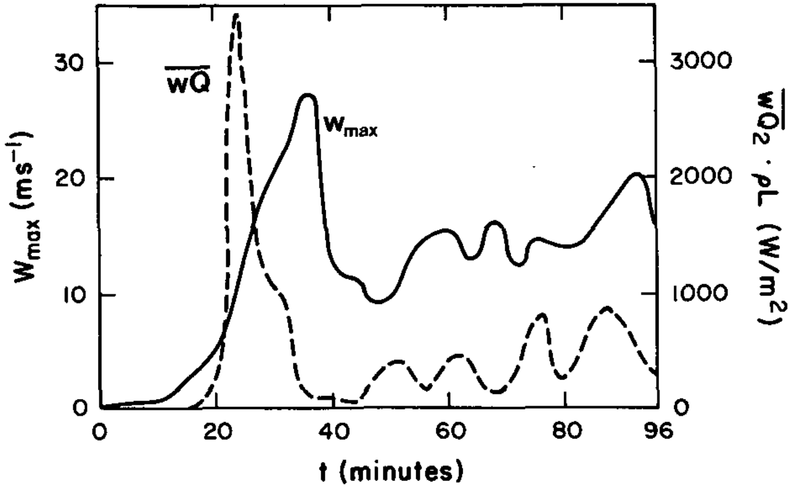
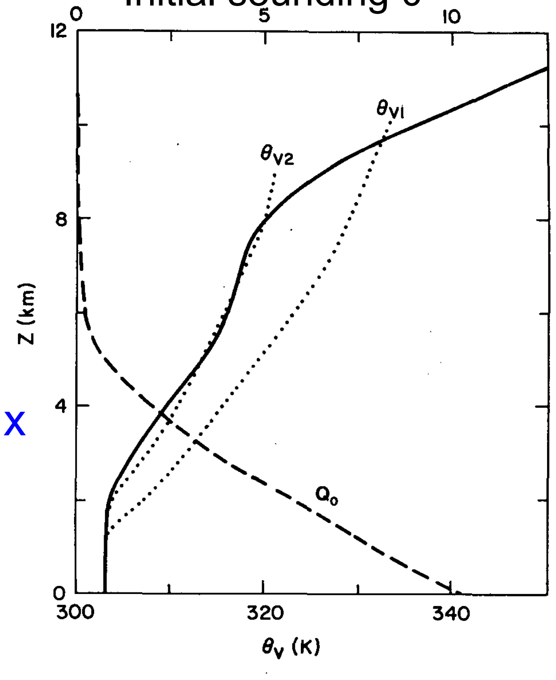


FIG. 3. Maximum vertical velocity in the inner model $w_{max}$ (solid) and $\rho L\overline{wQ}$ (dashed) vs time.

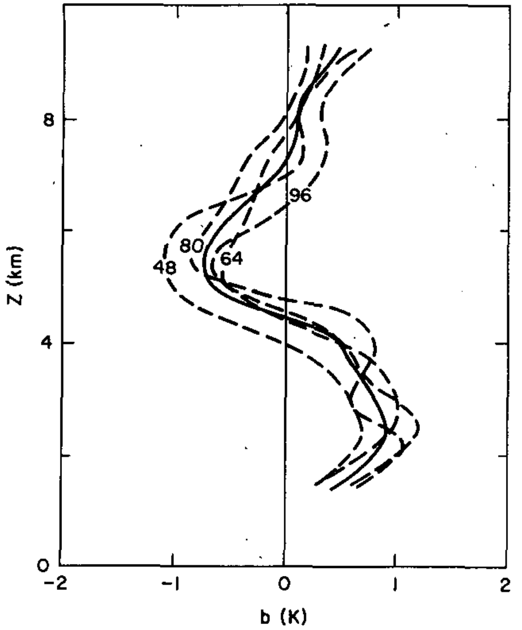
⇒ Quasi steady energy release in the cloud (wmax~cstt)

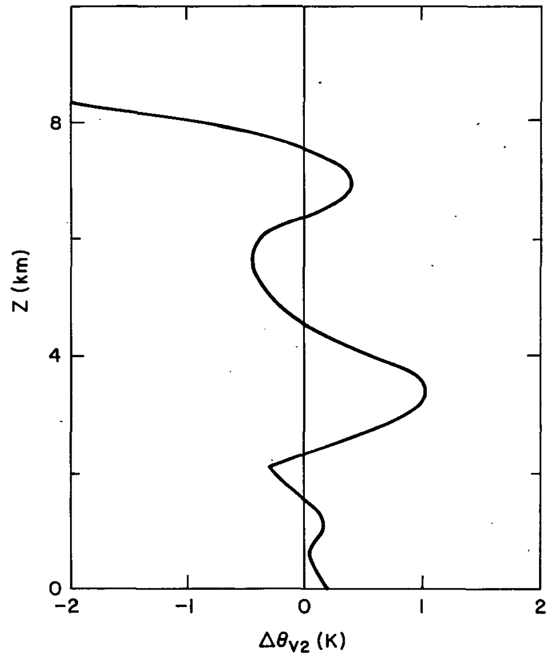
⇒ Initiated by moisture flux (wQ)

Initial sounding θ



Buoyancy anomaly in cloud
–quasi steady
–moist adiabatic

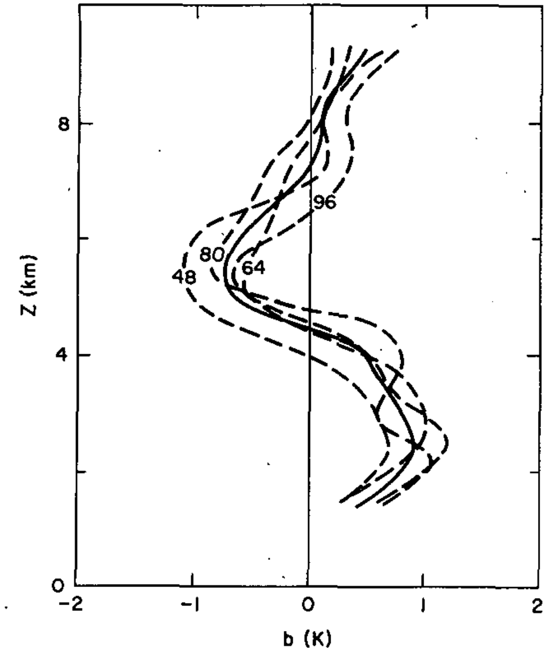




Buoyancy of moist adiabat θv2

*(conditional instability only released over a number of hours because most unstable air near the surface (no mixed layer) so pressure too weak for strong convergence and slow release of PE)*

# Numerical experiment:



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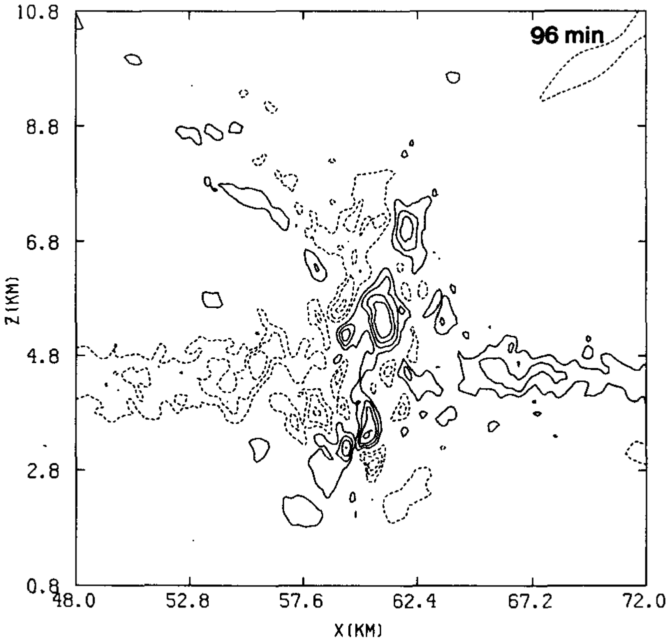
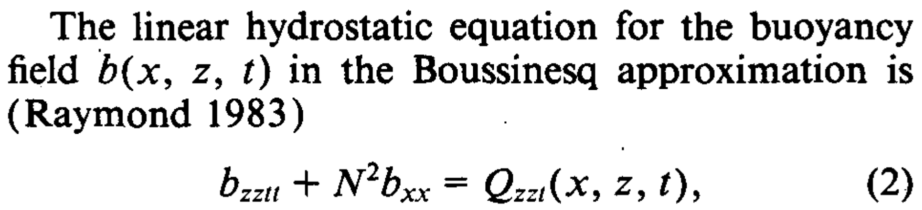


U velocity

⇒ outflow 3-5km

# Linear theory:

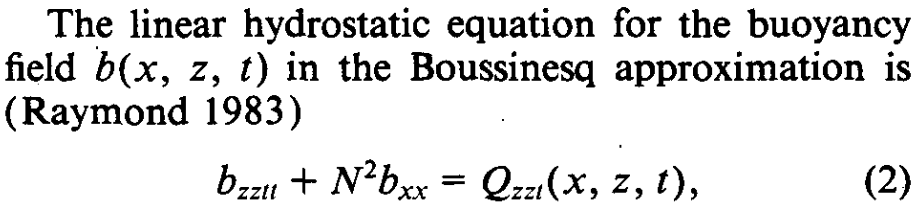


The linear hydrostatic equation for the buoyancy field $b(x, z, t)$ in the Boussinesq approximation is (Raymond 1983)

$$b_{zztt} + N^2 b_{xx} = Q_{zzt}(x, z, t), \qquad (2)$$

Maybe some background on Boussinesq…

# Linear theory:



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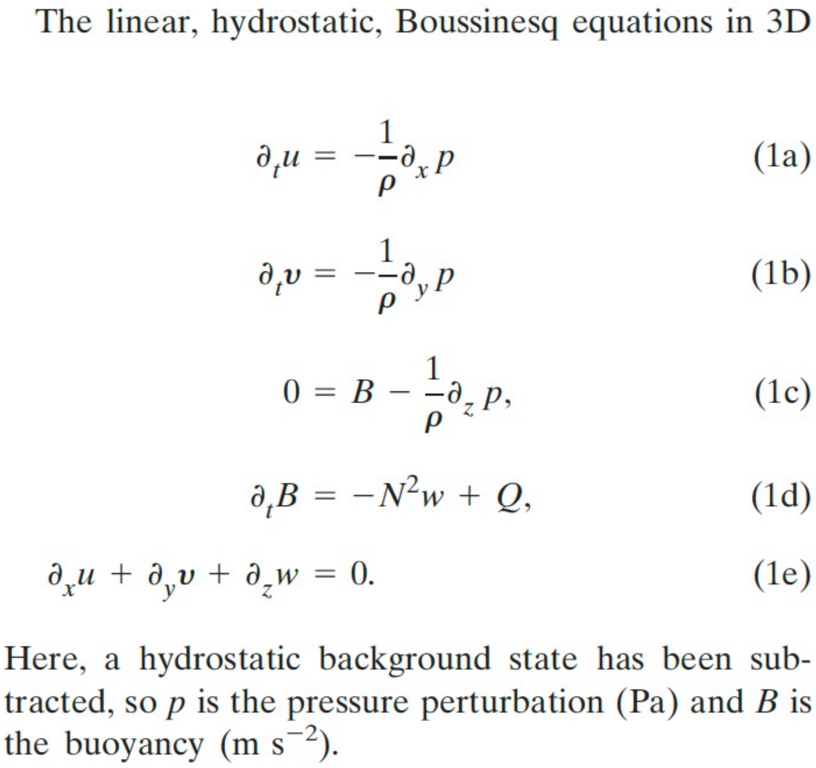


The linear, hydrostatic, Boussinesq equations in 3D

$$\partial_t u = -\frac{1}{\rho}\partial_x p \tag{1a}$$

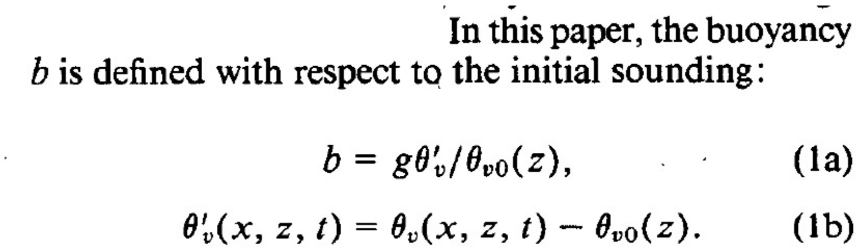
$$\partial_t v = -\frac{1}{\rho}\partial_y p \tag{1b}$$

$$0 = B - \frac{1}{\rho}\partial_z p, \tag{1c}$$

$$\partial_t B = -N^2 w + Q, \tag{1d}$$

$$\partial_x u + \partial_y v + \partial_z w = 0. \tag{1e}$$

Here, a hydrostatic background state has been subtracted, so $p$ is the pressure perturbation (Pa) and $B$ is the buoyancy (m $\mathrm{s}^{-2}$).

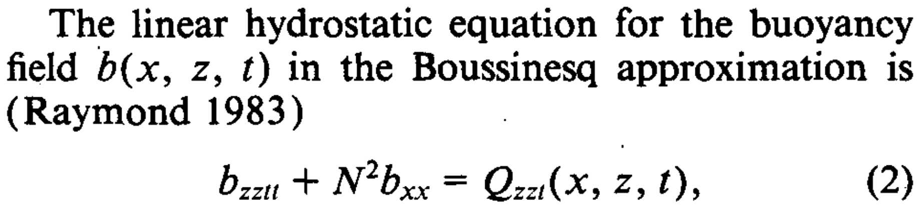


In this paper, the buoyancy $b$ is defined with respect to the initial sounding:

$$b = g\theta'_v/\theta_{v0}(z), \tag{1a}$$

$$\theta'_v(x, z, t) = \theta_v(x, z, t) - \theta_{v0}(z). \tag{1b}$$

# Linear theory:



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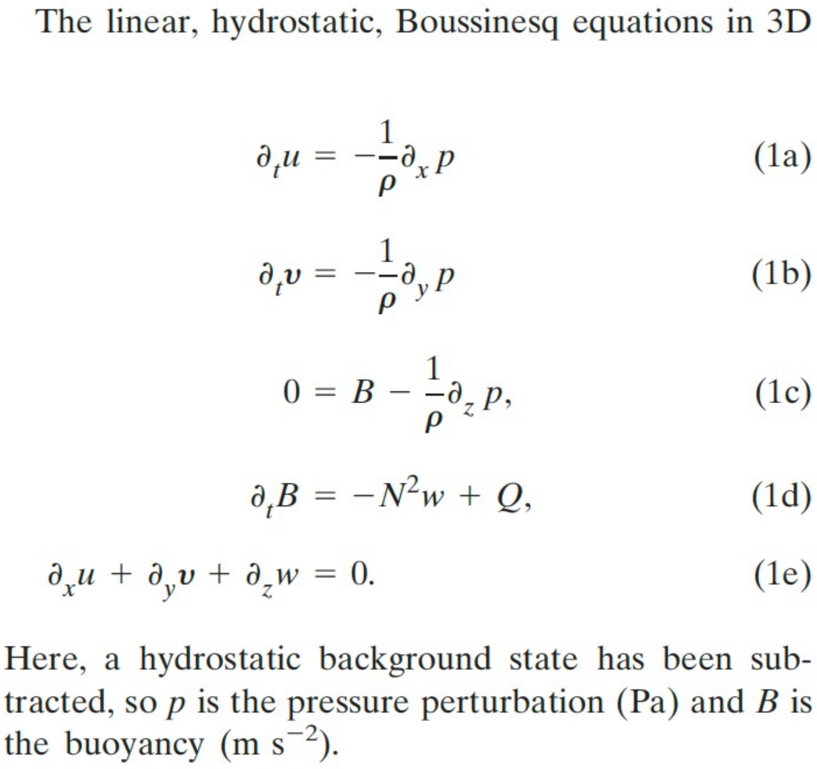

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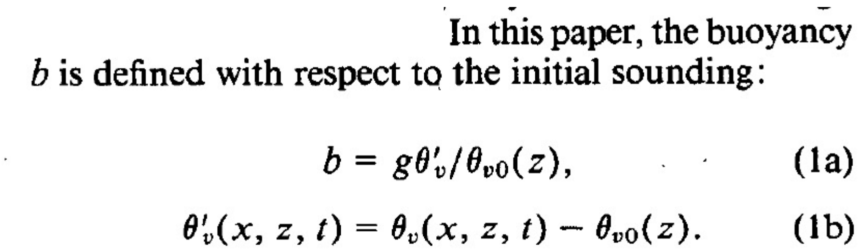

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# Linear theory:

The linear hydrostatic equation for the buoyancy field $b(x, z, t)$ in the Boussinesq approximation is (Raymond 1983)

$$b_{zztt} + N^2 b_{xx} = Q_{zzt}(x, z, t), \qquad (2)$$

$$b(x, z, 0) = 0. \quad \text{(initially at rest)}$$

The cloud is assumed to be a steady buoyancy source for $t > 0$: $Q(x, z, t) = Q_0 \delta(x) \sin(mz) H(t)$.

Separating z dependence

⇒ Forced 1D wave equation with solution

$$b(x, z, t) = \beta \sin(mz) H(c_m t - |x|), \qquad (3)$$

where $c_m = N/m$ and $\beta = Q_0/c_m$. There is a discontinuous transition between the undisturbed atmosphere ($|x| > c_m t$) and the "adjusted" atmosphere ($|x| < c_m t$) in which the buoyancy is the same as the cloud buoyancy. This spreading disturbance can be interpreted as a superposition of gravity waves due to the cloud buoyancy source.

# Linear theory:

Polarization relations yield

$$u(x, z, t) = -(\beta/N)\cos(mz)H(c_m t - |x|)\,\mathrm{sgn}(x), \tag{4a}$$

$$w(x, z, t) = -(\beta/mN)\sin(mz)\{\delta(c_m t - x) + \delta(c_m t + x) - 2\delta(x)\}. \tag{4b}$$

Here sgn($y$) is the sign function, 1 for $y > 0$, 0 for $y = 0$ and $-1$ for $y < 0$.

Figure 6 graphically depicts the adjustment process. The transition lines $|x| = c_m t$ are shown as thin verti-

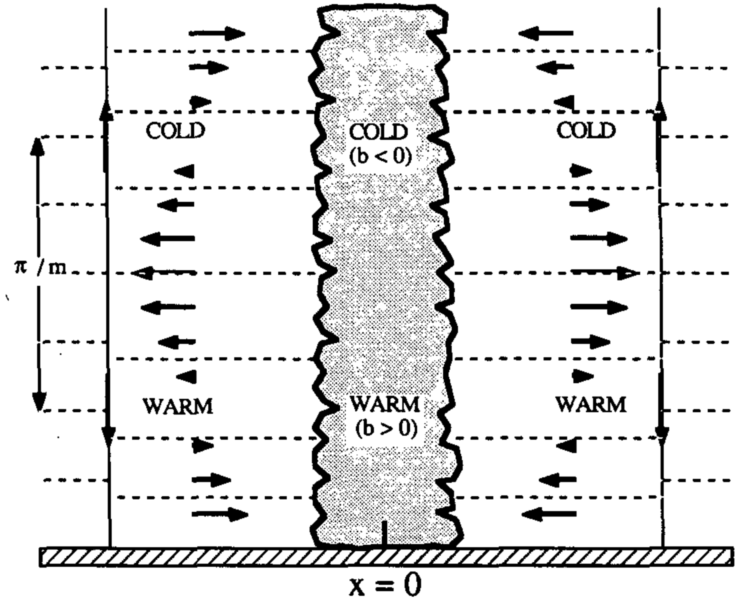
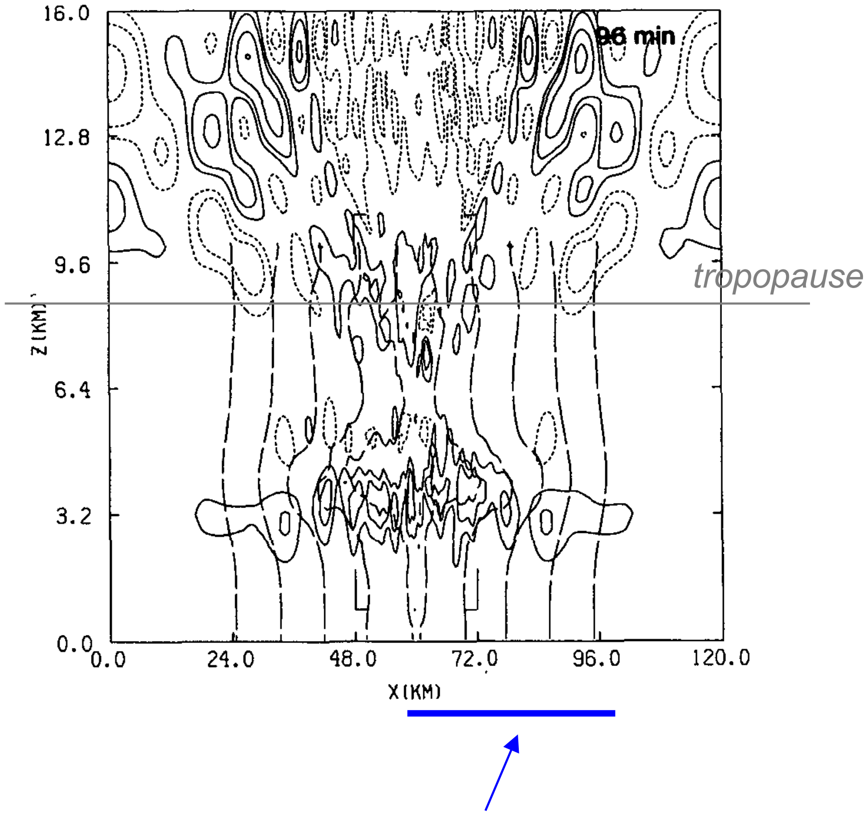


FIG. 6. The response to an idealized cloud maintaining a sinusoidal buoyancy perturbation in an atmosphere of uniform stratification.

At fronts x=ct, air displaced vertically to match buoyancy in cloud

Cloud buoyancy decreases => w decreases => outflow

Agrees with numerical results:
Buoyancy field (solid:b>0)
(Long dashed contours = tracer ξ)



within~25km of cloud => $b_{CFE}=b_{cloud}$

(dt~60min from cloud mature
25km => $c_{num}$ = 7m/s

$c_{theor}$=N/m ~ 10 m/s CQFD :)

# Linear theory:

- Relax steady forcing

The linear hydrostatic equation for the buoyancy field $b(x, z, t)$ in the Boussinesq approximation is (Raymond 1983)

$$b_{zztt} + N^2 b_{xx} = \cancelto{0}{Q_{zzt}(x, z, t)}, \qquad (2)$$

~~$b(x, z, 0) = 0$. (initially at rest)~~

$b(x, z, 0^+) = B_0 \sin(mz)\delta(x)$, where $t = 0^+$ refers to a time just after $t = 0$. We solve (2) subject to this initial condition to obtain

$$b(x, z, t) = -(B_0/2)\sin(mz) \times [\delta(x - c_m t) + \delta(x + c_m t)], \qquad (5)$$

$$u(x, z, t) = (B_0/2N)\cos(mz) \times [\delta(x - c_m t) - \delta(x + c_m t)]. \qquad (6)$$

*(similar: e.g. still outflow where b decreases, …)*

- Relax 2D

$Q(x, y, z, t) = Q_0\delta(x)\delta(y)\sin(mz)H(t)$ => (2) becomes

$$b_{zztt} + N^2(b_{rr} + r^{-1}b_r) = Q_{zzt}. \qquad (9)$$

In the Appendix, we find the solution to (9) such that $b$ is identically zero before $t = 0$:

$$b(r, z, t) = \frac{Q_0}{2\pi c^2 t}\frac{H(ct - r)}{(1 - [r/ct]^2)^{1/2}}\sin(mz), \qquad (10)$$

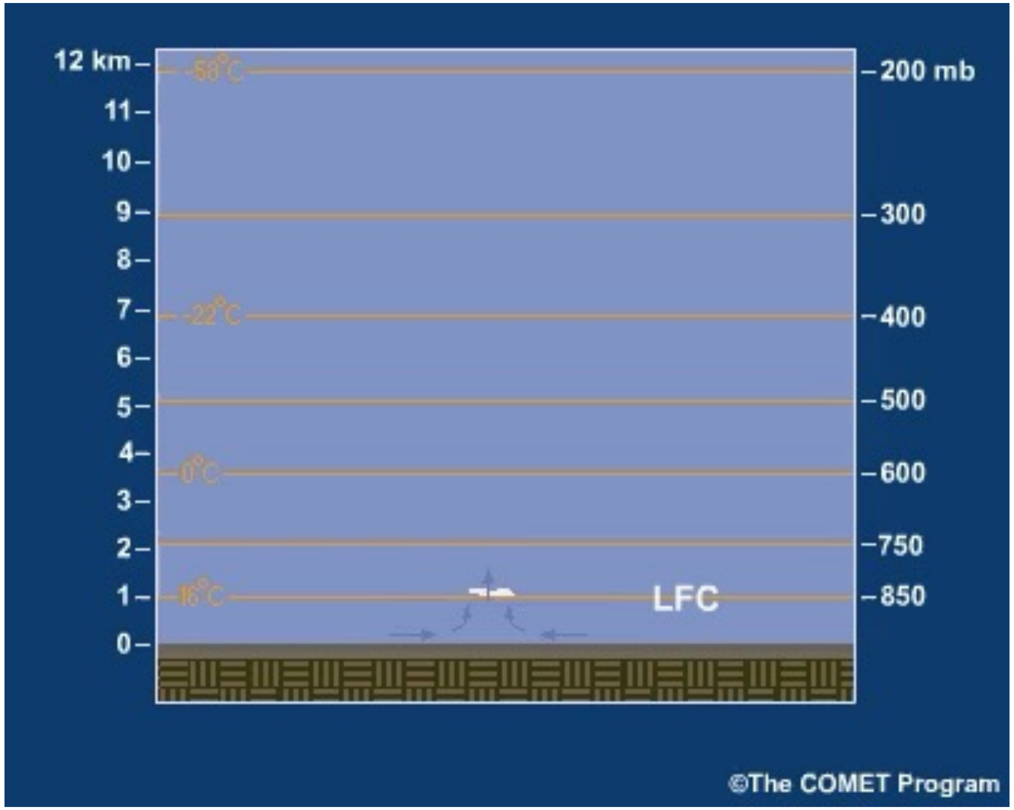
$$u(r, z, t) = \frac{mQ_0}{2\pi r N^2(1 - [r/ct]^2)^{1/2}}\cos(mz). \qquad (11)$$

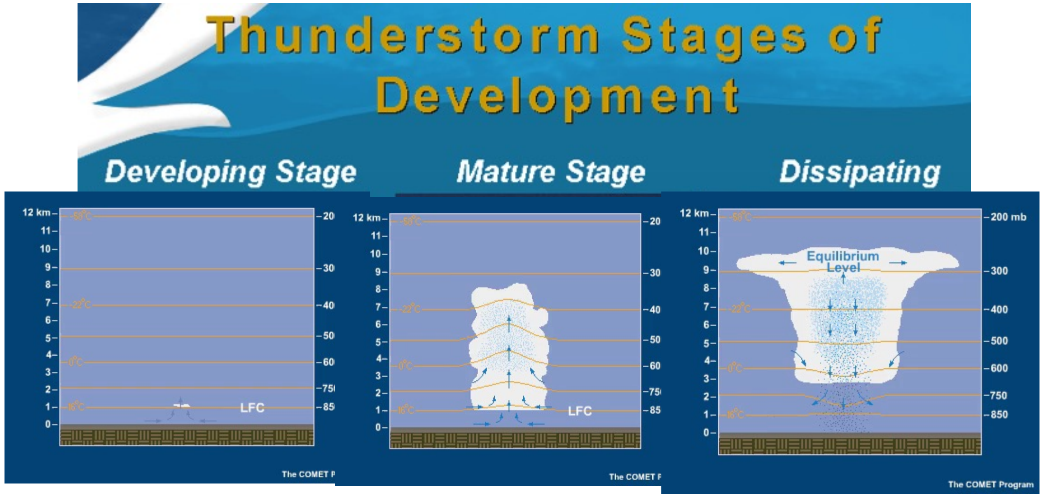
Once again, there is outflow in the region $r < ct$ in the region where the heating is decreasing with height. However, heating produces much larger displacements and hence much larger buoyancies near the wave front $r = ct$ than near the heat source.

*Rk: b increases as r -> ct?!?*
*i.e. larger buoyancy near the wave front…*

# Add precip => Downdrafts

Strong updrafts develop in the cumulus cloud => mature, deep cumulonimbus cloud.
Associated with heavy rain, lightning and thunder.





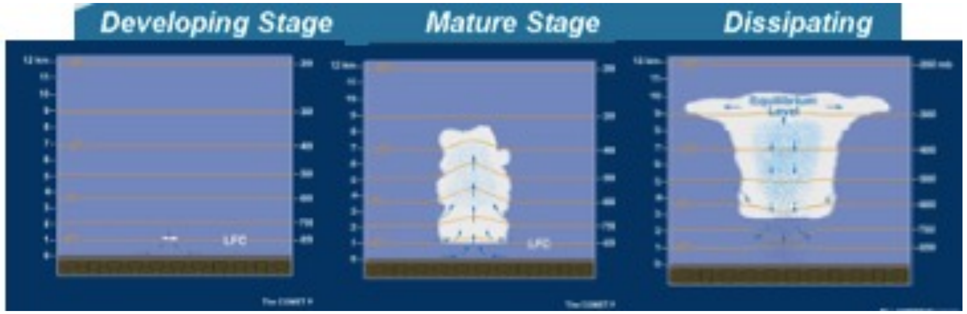
Evaporative driven cold pools

For more: see « atmospheric thermodynamics » by Bohren and Albrecht

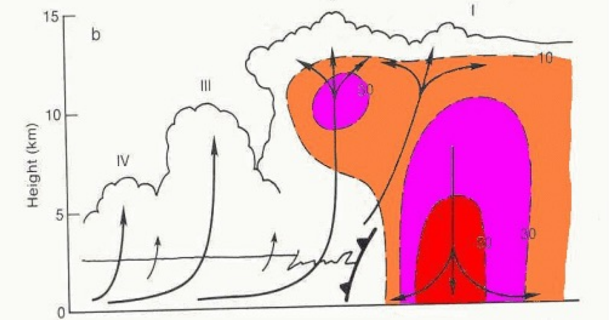
# Rk: Beyond single cloud

Note that thunderstorms can be :

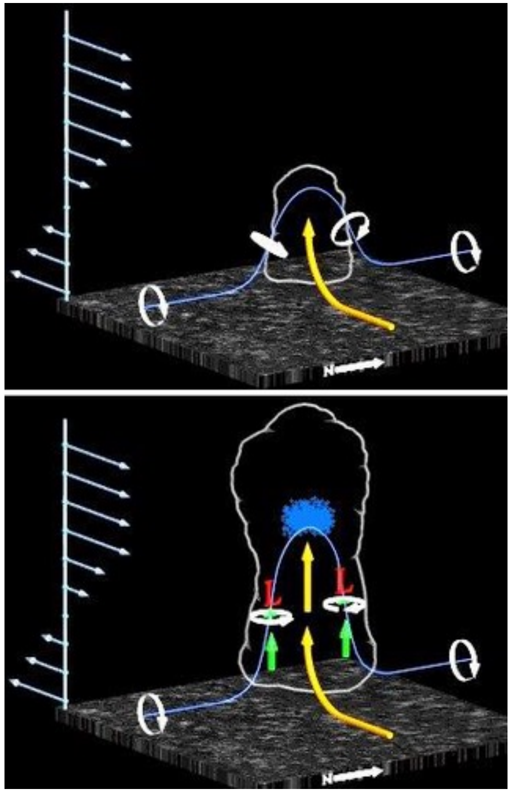
single-cell (typically with weak wind shear)

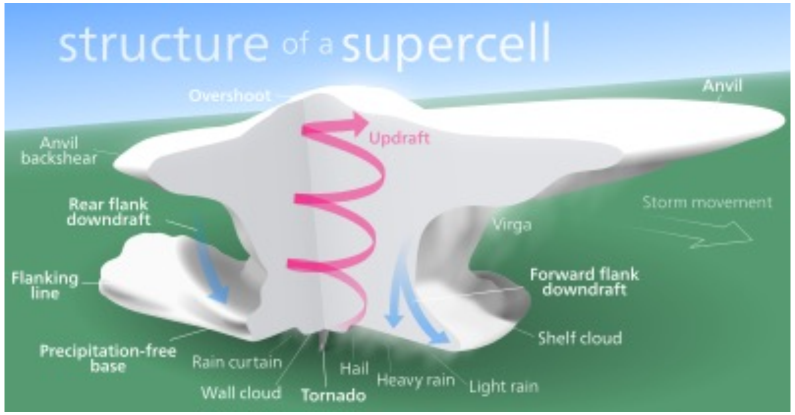


multi-cell (composed of multiple cells, each being at a different stage in the life cycle of a thunderstorm.



or supercell, characterized by the presence of a deep, rotating updraft





Typically occur in a significant vertically-sheared environment

*[See Houze book: Cloud Dynamics; Muller – Cloud chapter, Les Houches Summer School Lecture Notes]*

# Some thoughts: (1) reference if interested to learn more about atmospheric dynamics

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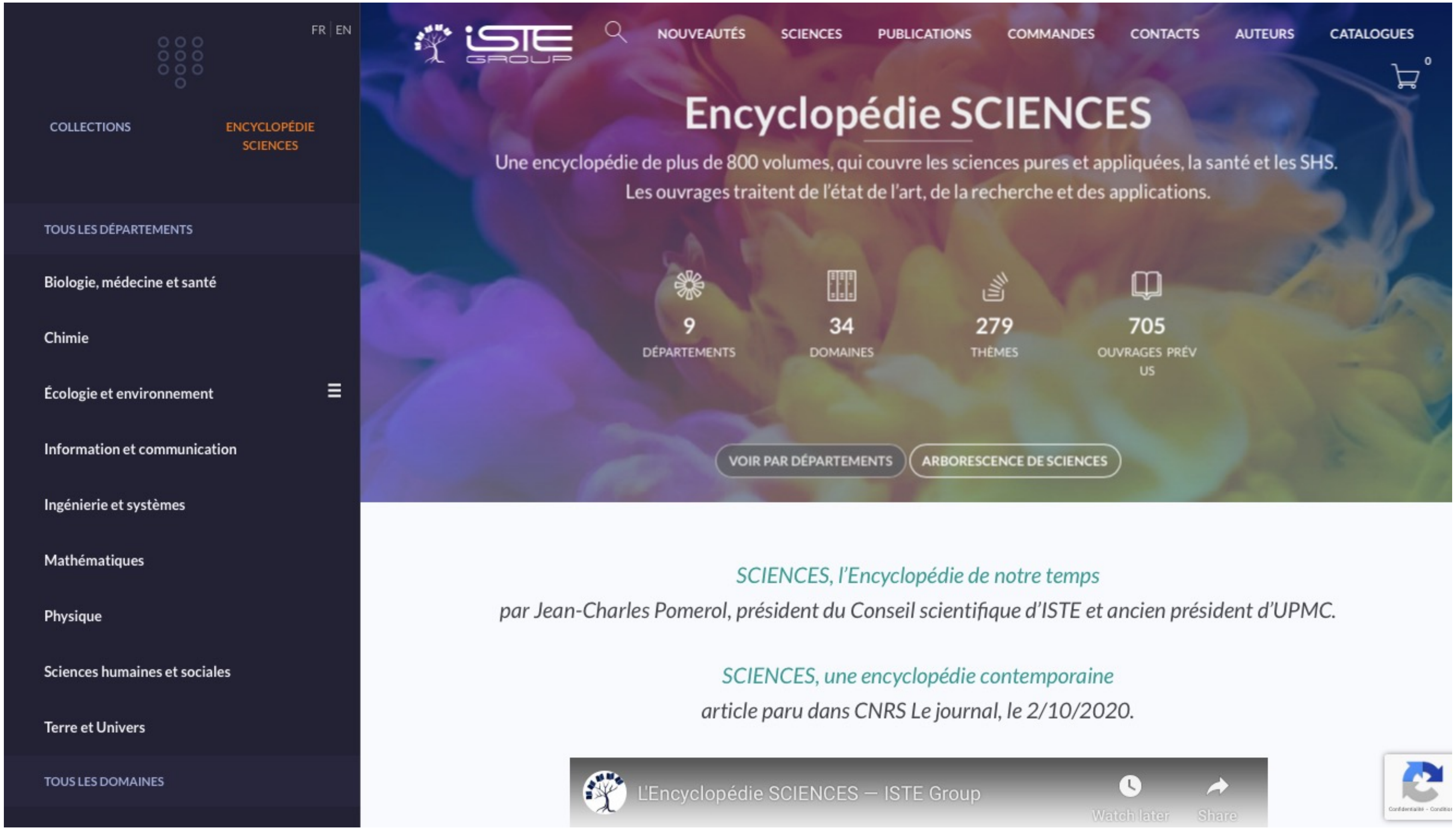
« Sciences » : Title given to an encyclopedia in preparation (some volumes available) This enormous endeavour will involve 20 000 authors, and will present the state-of-the-art in sciences and technologies.

Website : https://www.istegroup.com/fr/sciences/



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*SCIENCES, l'Encyclopédie de notre temps*

*par Jean-Charles Pomerol, président du Conseil scientifique d'ISTE et ancien président d'UPMC.*

*SCIENCES, une encyclopédie contemporaine*

*article paru dans CNRS Le journal, le 2/10/2020.*

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# Some thoughts: (1) reference if interested to learn more about atmospheric dynamics

**Atmospheric Dynamics**

A. **Large-scale Atmospheric Dynamics**

B. **Small-scale Atmospheric Dynamics**

C. **Variability, from intraseasonal to multidecadal and climate time scales**

# Some thoughts: (1) reference if interested to learn more about atmospheric dynamics (2) How far does the “convective adjustment” go?

2D: adjustment goes to infty

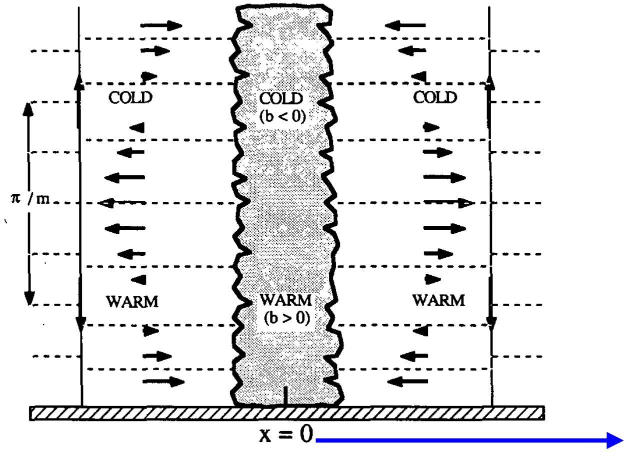
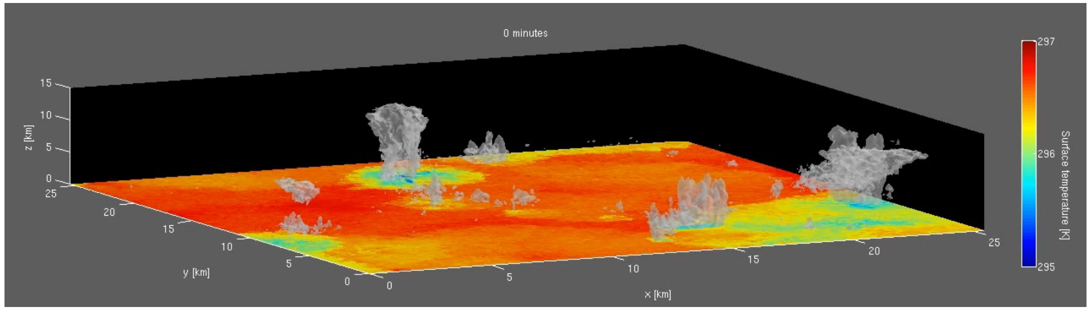
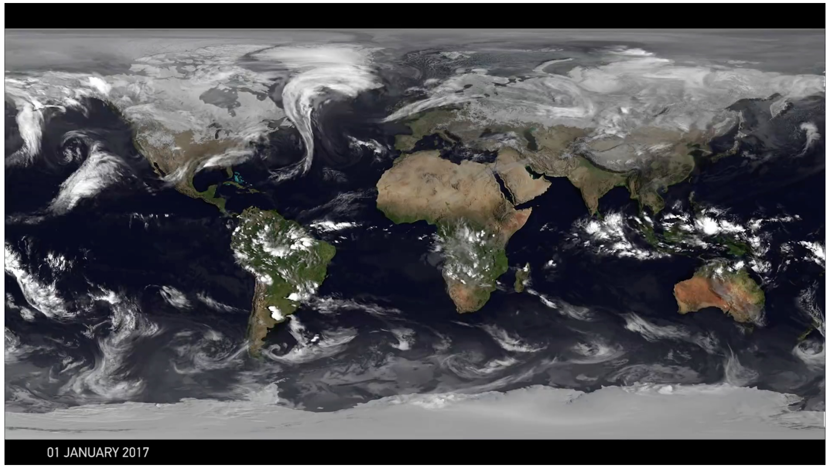


FIG. 6. The response to an idealized cloud maintaining a sinusoidal buoyancy perturbation in an atmosphere of uniform stratification.

CRM without Coriolis: frictional scale (surface friction or wave energy leak to stratosphere?)



In real tropics? Beta effect? (f ~ β y)



$$\partial_t u - \beta v = -\frac{1}{\rho}\partial_x p$$

$$\partial_t v + \beta u = -\frac{1}{\rho}\partial_y p$$

$$0 = B - \frac{1}{\rho}\partial_z p,$$

$$\partial_t B = -N^2 w + Q,$$

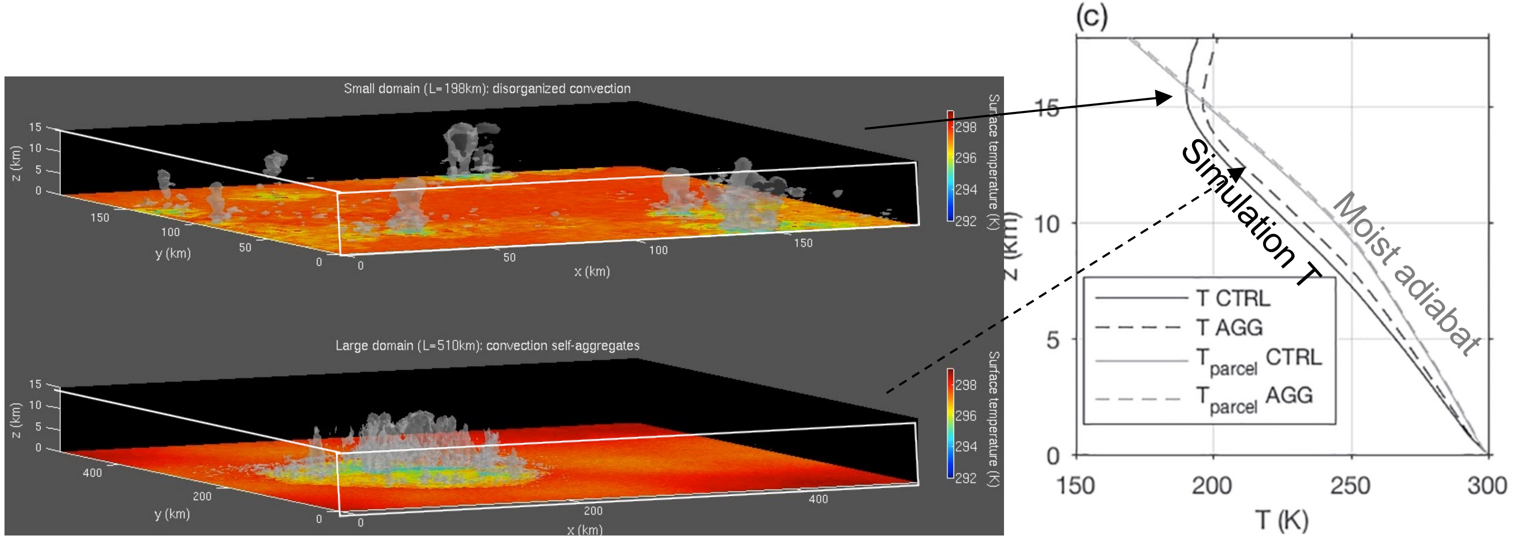
$$\partial_x u + \partial_y v + \partial_z w = 0.$$

# Some thoughts: (1) reference if interested to learn more about atmospheric dynamics
# (2) How far does the “convective adjustment” go?
# (3) Is convection undilute?

Not simply parcel going up undilute!

Entrainment of environmental air (turbulent entrainment at the edge of clouds)

Depends on convective organization in space (more clustered, less dilution by entrainment)



*Da Silva et al 2021*

# Some thoughts: (4) dry vs moist convection

## How does moist convection respond to radiation?

**Impact of radiative cooling in simplified model** of moist convection (with water vapor q):

$$\nabla \cdot \boldsymbol{u} = 0,$$ …………..Mass conservation

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \nu \nabla^2 \boldsymbol{u} - \beta T \boldsymbol{g},$$ …..Momentum conservation

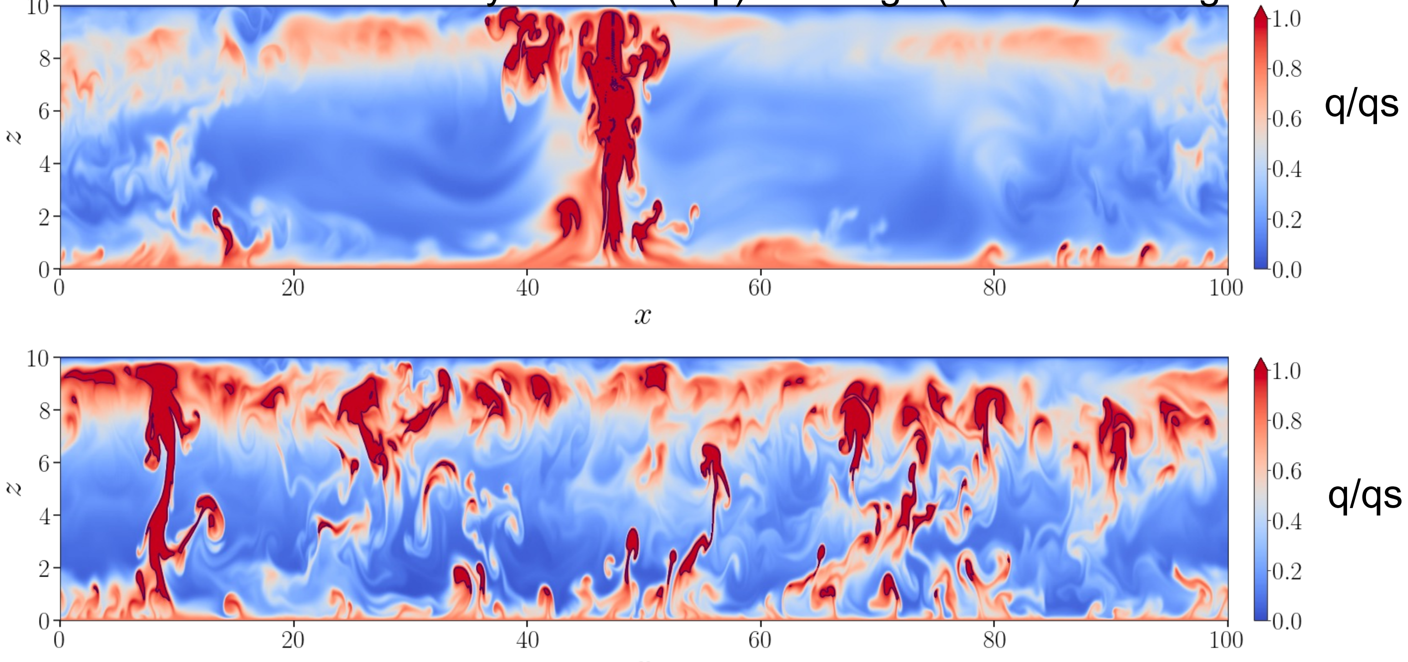
Radiative cooling

$$\partial_t T + \boldsymbol{u} \cdot \nabla T + \Gamma w = \kappa \nabla^2 T + L_v \tau^{-1} (q - q_s)_+ \boxed{- R}$$ ………..Energy conservation

$$\partial_t q + \boldsymbol{u} \cdot \nabla q = \kappa_q \nabla^2 q \boxed{- \tau^{-1} (q - q_s)_+}.$$ ………..Water conservation

Simple condensation scheme (fast (tau) condensation upon saturation)

Relative humidity at small (top) and large (bottom) cooling R



⇒ **Simple model reproduces results** from complex cloud-resolving model

*[Agasthya Muller 2023;*
*Agasthya Muller 2024]*

# Gravity waves, compensating subsidence and detrainment around cumulus clouds

**_Thank you for your attention!_**

Caroline Muller