Dissipative structures in 2D flows?

GDT MathFluid 02/03/18

D’après Nguyen van yen, Farge, Schneider PRL 2011

Where is dissipated the energy of oceanic currents?

Large scale winds $\sim 1\ \mathrm{TW}$

↓

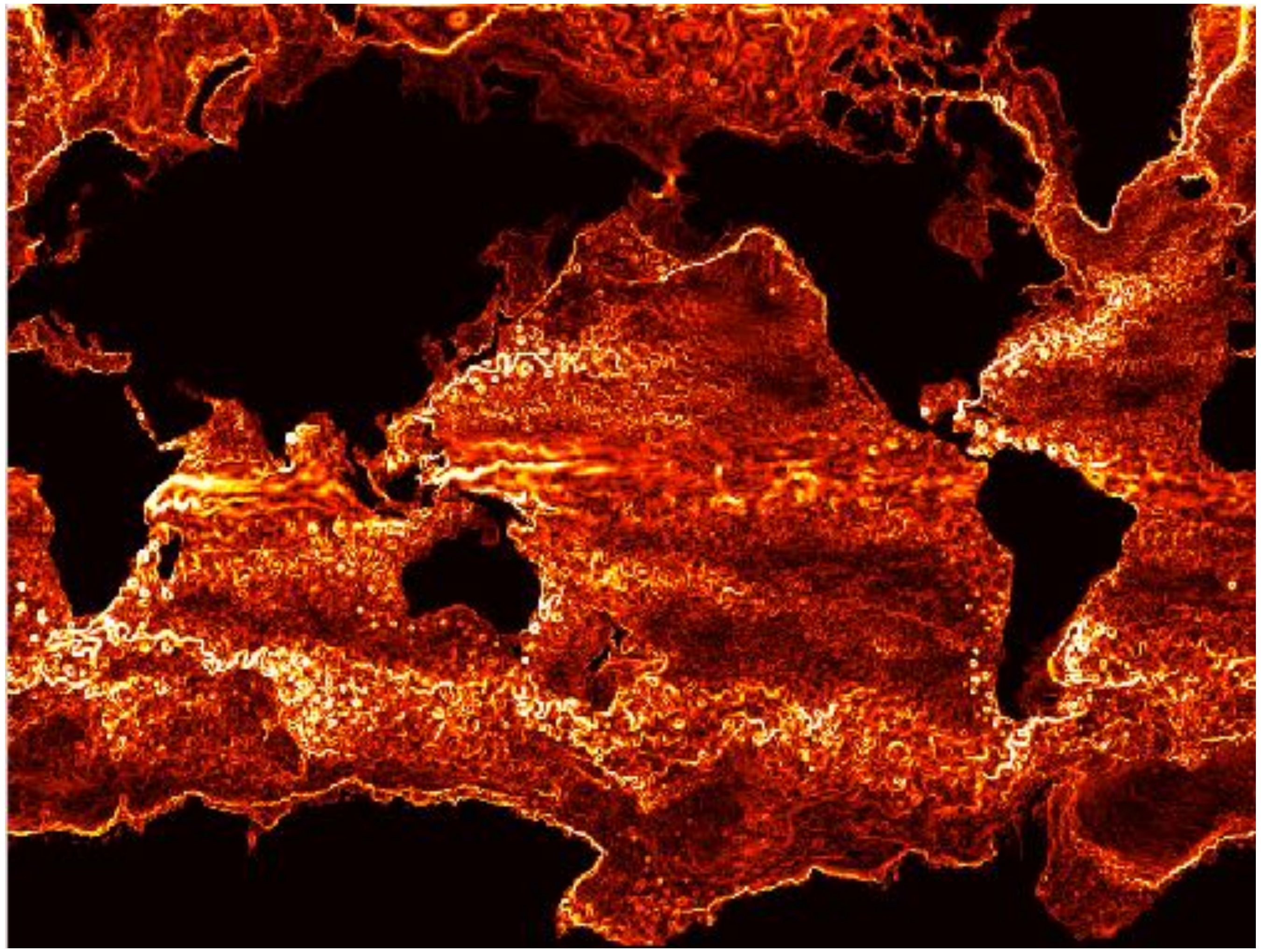
Instabilities

↓

Geostrophic turbulence

↓

Dissipation at boundaries.

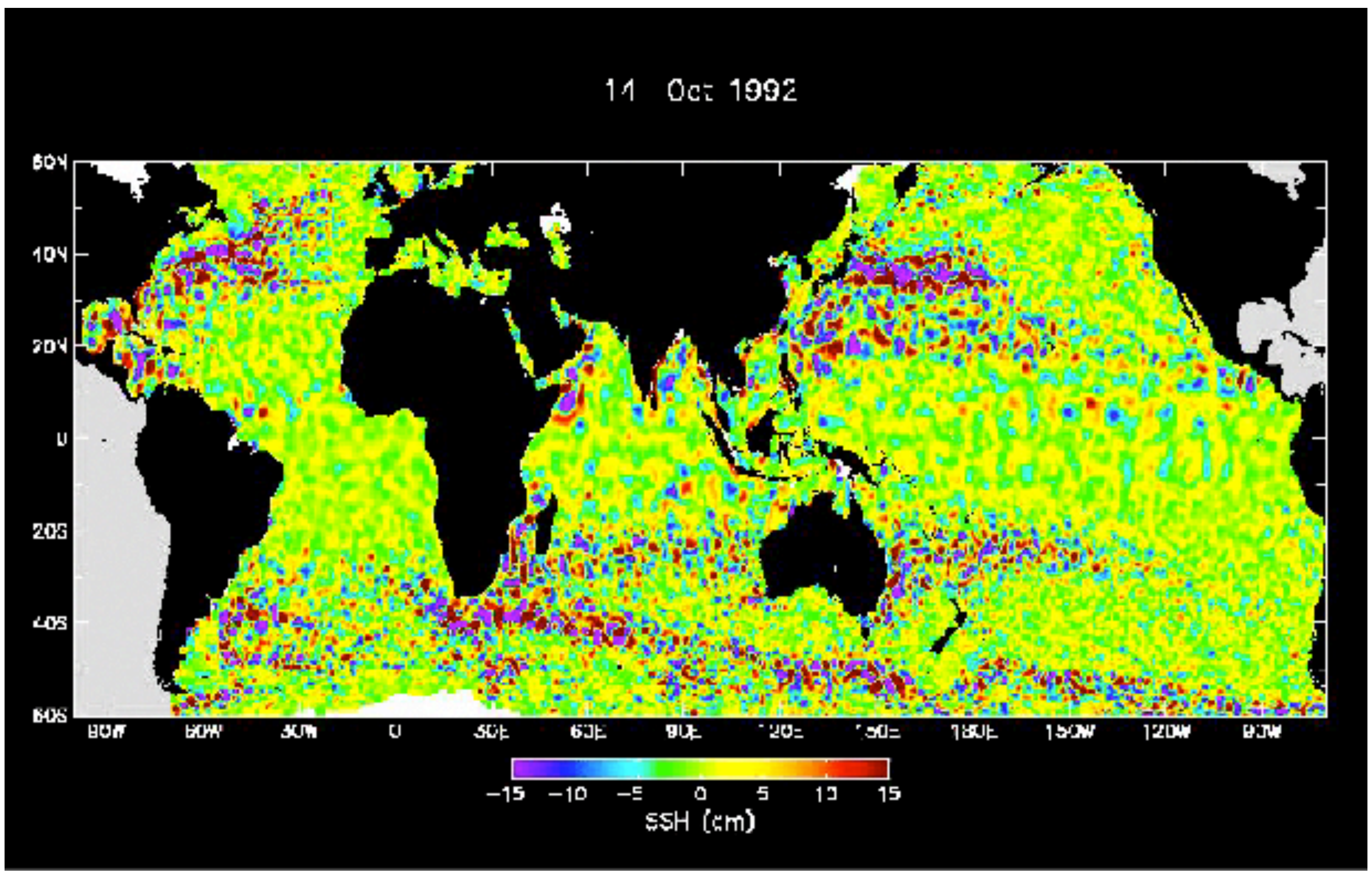


Ocean circulation kinetic energy: Reservoirs, sources, and sinks R Ferrari, C Wunsch 2009

LETTERS
PUBLISHED ONLINE: 22 AUGUST 2010 | DOI: 10.1038/NGEO943
nature geoscience

Significant sink of ocean-eddy energy near western boundaries

Xiaoming Zhai[1]*, Helen L. Johnson[2] and David P. Marshall[1]



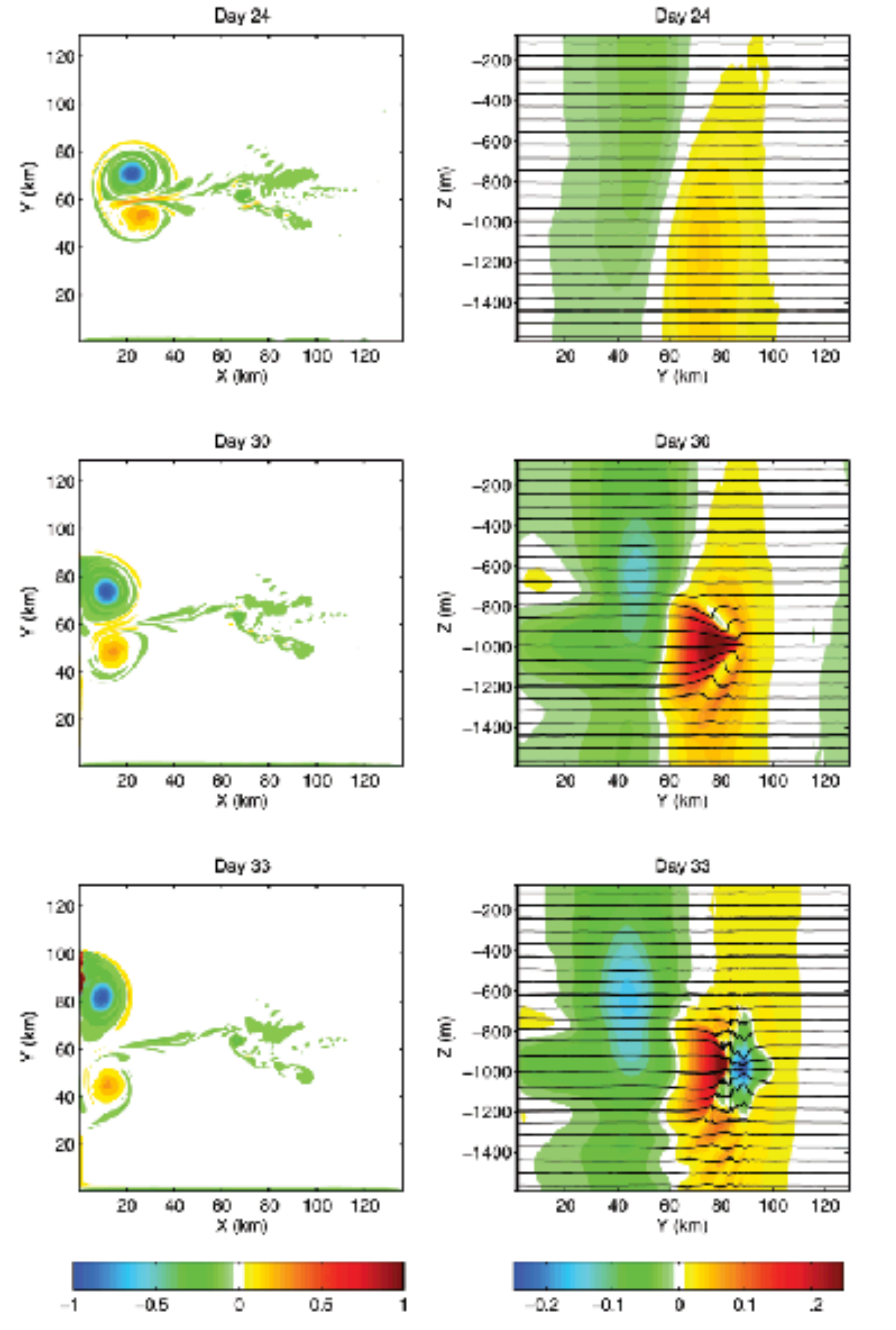
« We estimate a convergence of eddy energy near the western boundary of approximately **0.1–0.3TW**. This energy is most probably scattered into highwavenumber vertical modes, resulting in energy dissipation and diapycnal mixing »

http://jeffreyearly.com/science/qg-eddies-paper/

A dipole hitting a wall

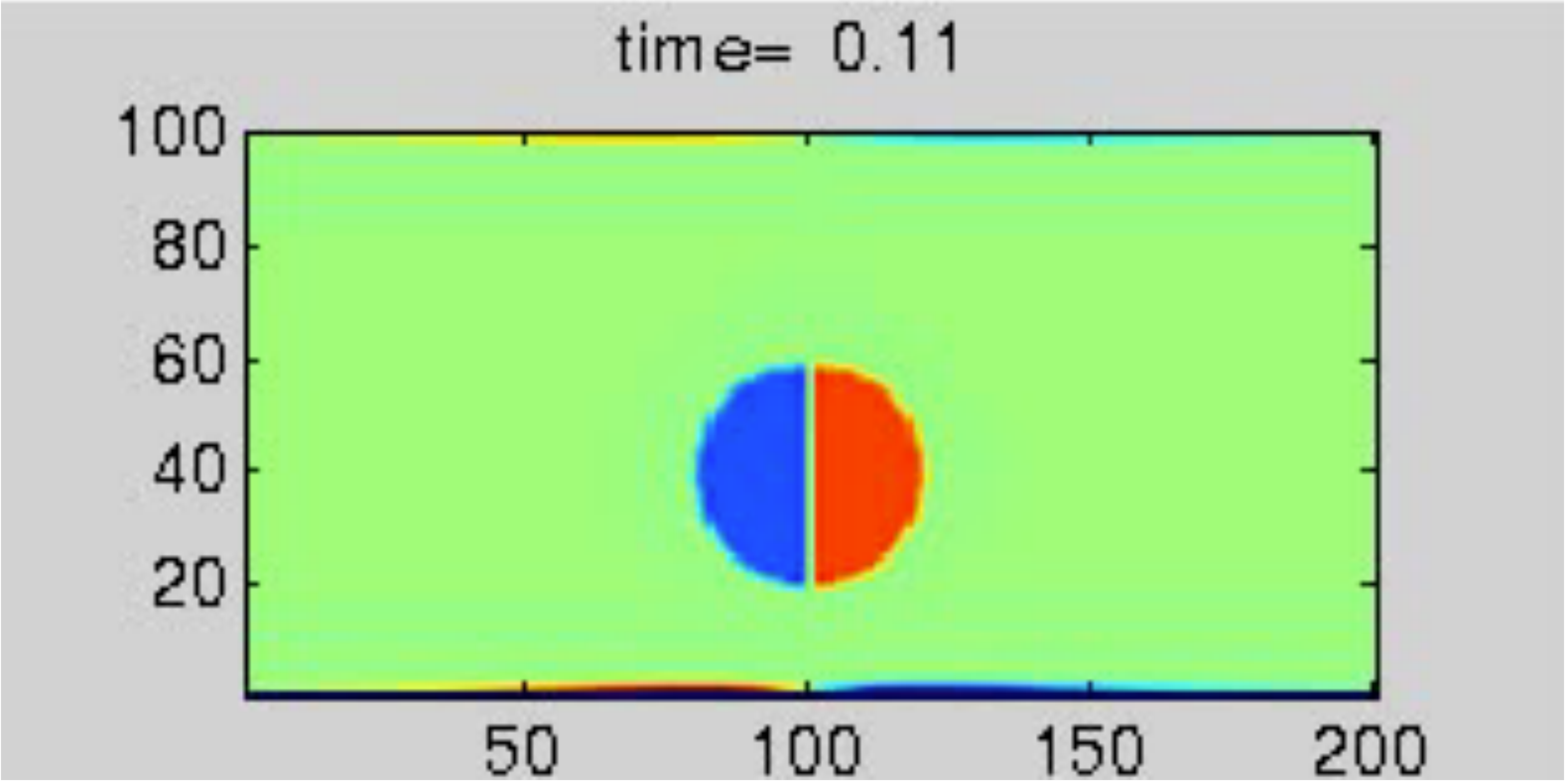
Model the interaction between a 'meddie' an a western boundary

Hydrostatic primitive equations on a f-plane, free slip



Submesoscale generation by boundaries, Dewar, Berloff and Hogg JMR 2011

What about 2d incompressible flows ?



Simulation from G. Roullet: http://stockage.univ-brest.fr/%7Eroullet/codes.html

First numerical study of this problem (with lab experiments):

Vortex dipole rebound from a wall Orlandi 1990 Physics of Fluids

This is the problem revisited by Nguyen Farge Schneider PRL 2011

Anomalous energy dissipation in incompressible 2D flow in the presence of a wall ?

$$\partial_t \omega + \mathbf{u} \cdot \nabla \omega = \nu \Delta \omega, \quad \nabla \cdot \mathbf{u} = 0,$$ **no slip condition at the wall**

Energy decay $$\frac{\mathrm{d}}{\mathrm{dt}} E = -\nu \int_D \omega^2$$

Energy decay, assuming constant enstrophy $$E(t_1) - E(t_0) \propto \nu$$

Energy decay, assuming Prandtl boundary layer $$E(t_1) - E(t_0) \propto \nu^{1/2}$$

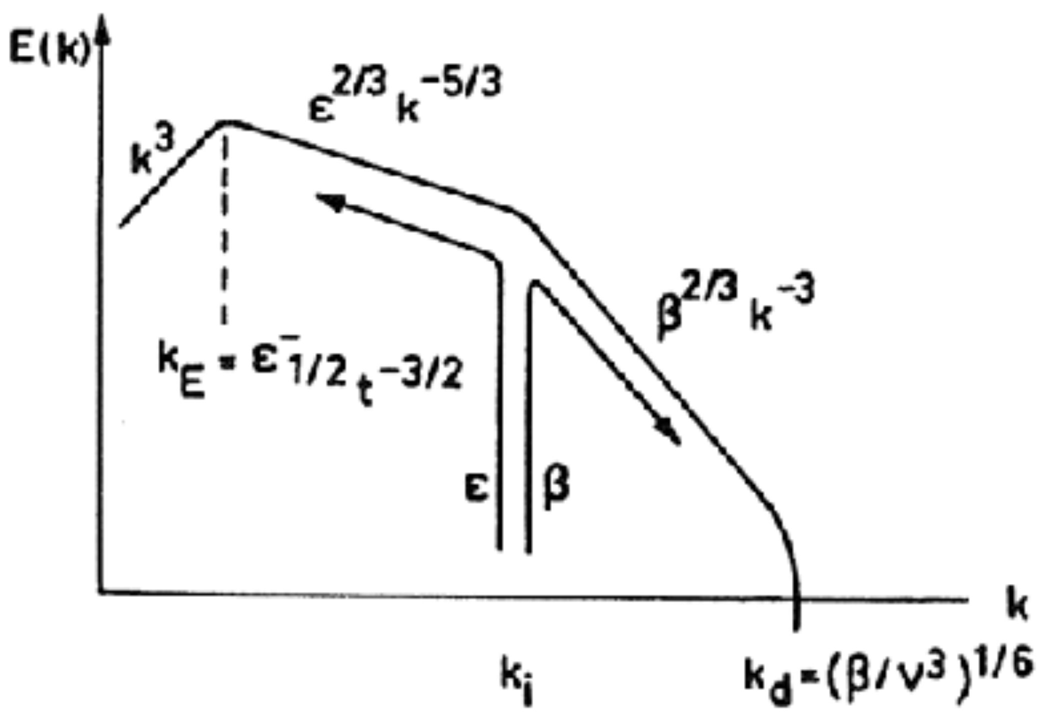
Dissipative boundary layer if $$\omega \propto 1/\nu \text{ over area} \propto \nu$$

Grid resolution and dissipative structures

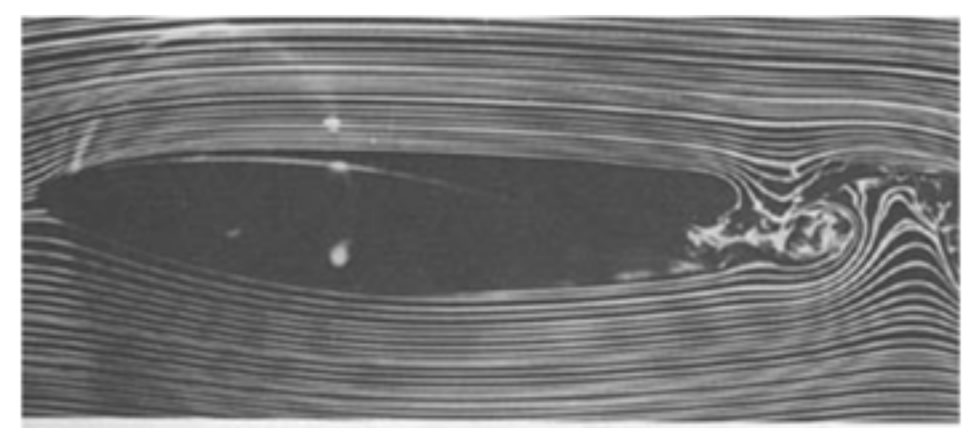
In simulations of 2d NS, common practice for mesh size is

$$\delta x \propto \nu^{1/2}$$

Without walls, this corresponds to the estimate of dissipation scale ending enstrophy cascade



With walls, this is the scaling of Prandtl layer thickness



One of the novelty in NFS is to consider a finer grid resolution $\delta x \propto \nu$

Dissipative structures in 2D flows with a wall: Kato theorem (84)

Kato, 1984, Remarks on zero viscosity limit for non stationary Navier-Stokes flows with boundary, MSRI Berkeley

In loose terms:

The solution of Navier-Stokes equation converges towards solution of Euler equation with same initial date if and only if the energy dissipation rate vanishes, and even if and only if the energy dissipation vanishes in a strip of width ν along the wall.

$$\frac{\mathrm{d}}{\mathrm{dt}} E = -\nu \int_D \omega^2$$

Important physical consequence on the generation of dissipative structures when there is boundary layer detachment

Numerical experiments

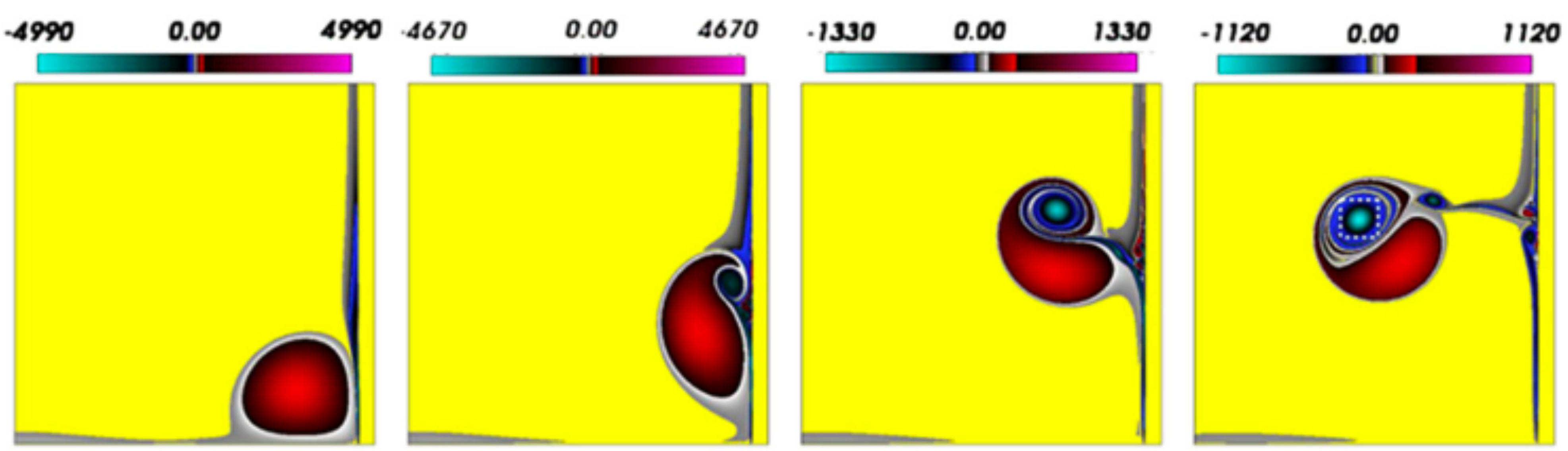
Penalisation method, pseudo-spectral code, boundaries enforced by mask χ_0

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\Delta\mathbf{u} - \frac{1}{\eta}\chi_0\mathbf{u} \\ \nabla\cdot\mathbf{u} = 0, \quad \mathbf{u}(0,\mathbf{x}) = \mathbf{v} \end{cases} \xrightarrow{\text{solution}} \mathbf{u}_{\mathrm{Re},\eta}$$

Angot et al 99

$$\left\| \mathbf{u}_{\mathrm{Re},\eta} - \mathbf{u}_{\mathrm{Re}} \right\| \leq C(\mathrm{Re})\eta^{\frac{1}{2}}$$

In practice, there is a third parameter given by mask width χ_0

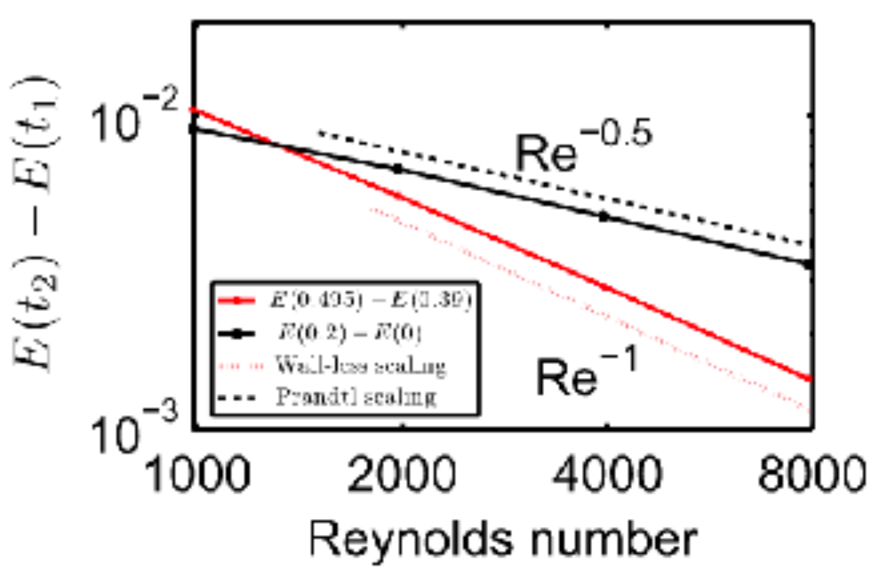


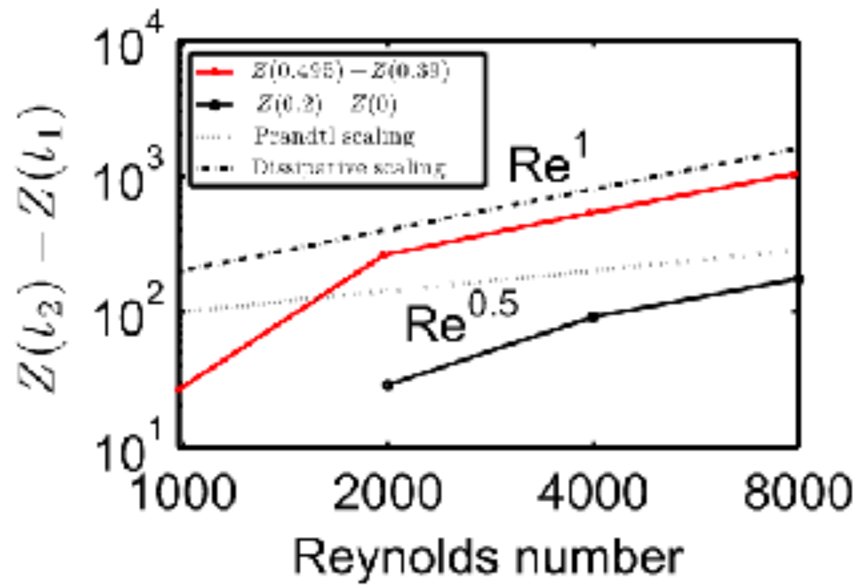
Results

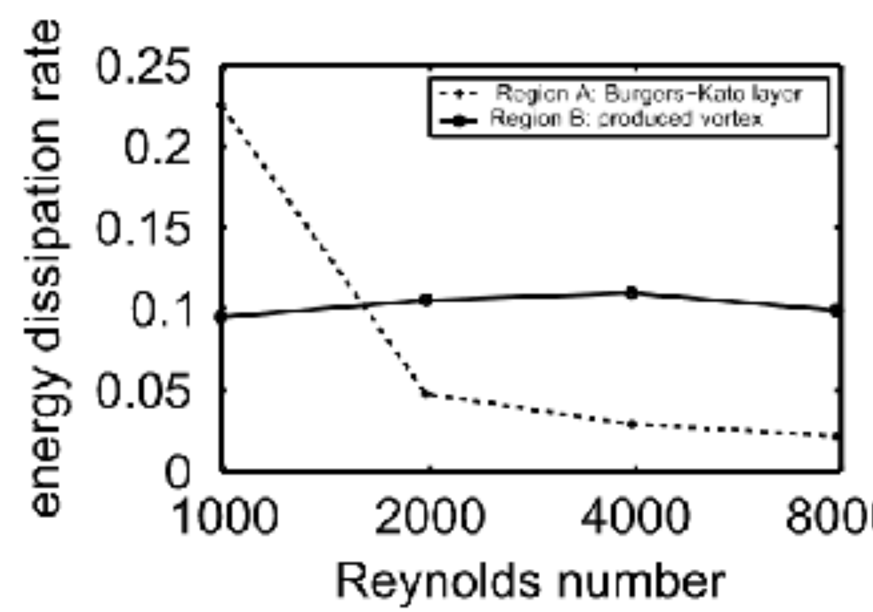
Energy decays before collision: wall-less scaling

Energy decays during collision before detachment: Prandtl scaling

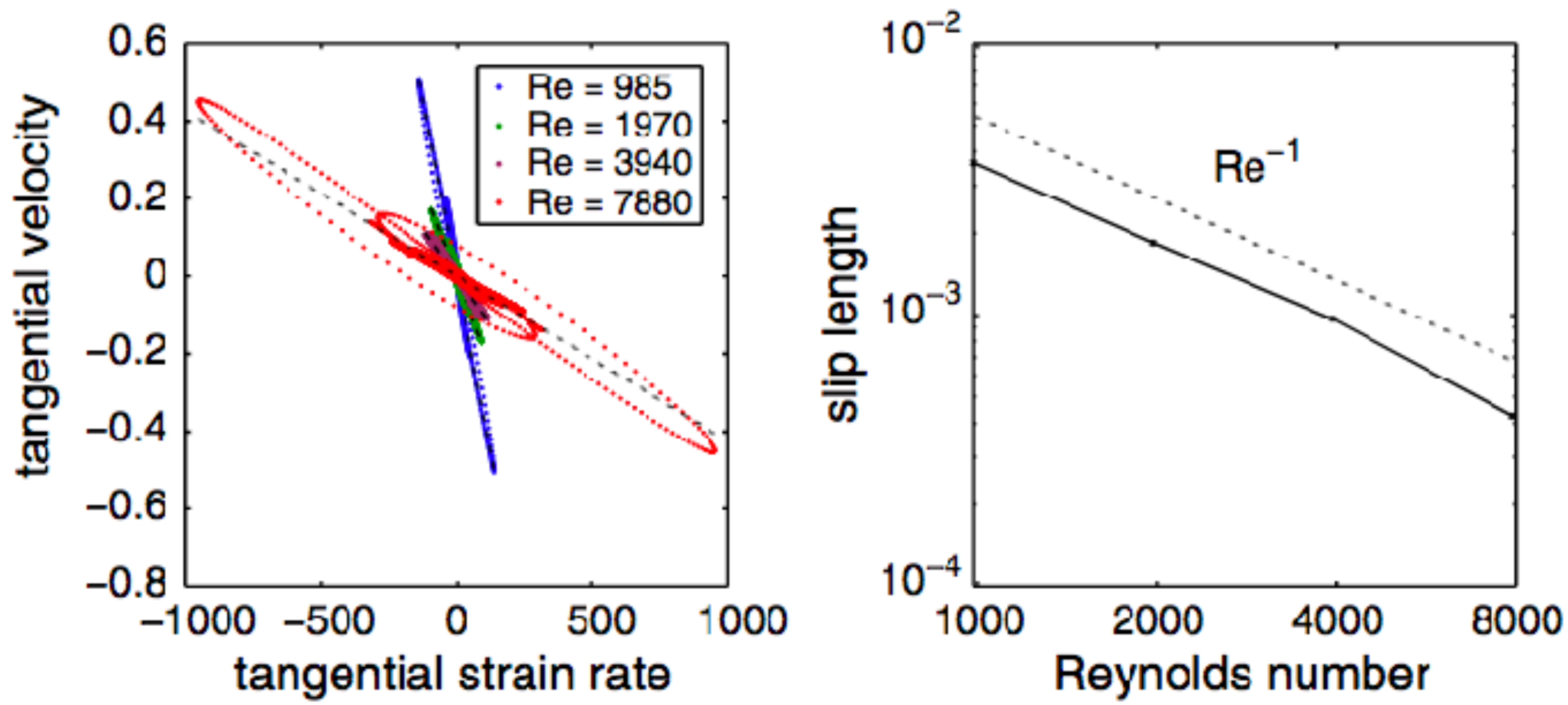
Energy decays as ~1 after detachment







Effective Navier condition at the boundary



$$\mathbf{u} \cdot \mathbf{n} = 0 \text{ on } \partial\Omega,$$

slip length dependence on Re

$$T\left(\frac{\partial \mathbf{u}}{\partial \mathbf{n}} + s_L^{-1}\mathbf{u}\right) = 0 \text{ on } \partial\Omega,$$

Other numerical experiments

Sutherland Macaskill Dritschel Phys Fluids 2013 compare the effect of No Slip, Free Slip and Partial Slip (Navier), using pseudo spectral method in x direction, compact differences on a Tchebichev grid in the other direction, with influence matrix method to enforce BC.

Sublinear growth of enstrophy with Re when the slip length is fixed and in no-slip case

Similar conclusion in Dissipation of coherent structures in confined two-dimensiona turbulence H. J. H. Clercx, and G. J. F. van Heijst 2017

The issue is not settled

A possible critic: effect of poor resolution? Reynolds not large enough?

Energy dissipation caused by boundary layer instability at vanishing viscosity

Nguyen van yen, Waidmann, Klein, Farge, Schneider arxiv 2017

Compare '2D Euler+Prandlt' and NS

Prandtl equations

$$\partial_t \omega_P + \nabla(\mathbf{u}_P \omega_P) = \partial^2_{y_P} \omega_P$$

$$\omega_P(x, y_P, 0) = 0$$

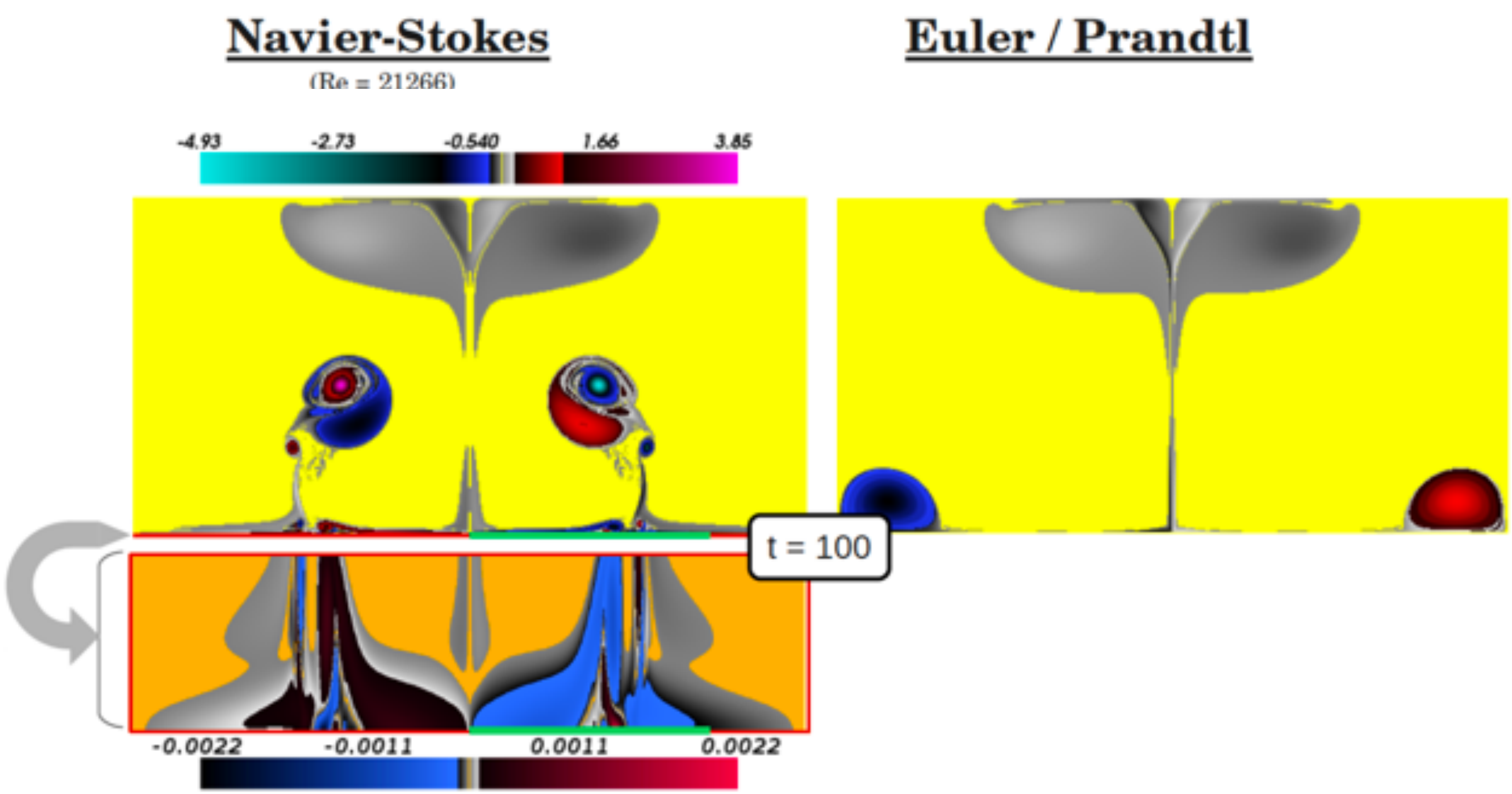
$$\psi_P(x, y_P, t) = \int_0^{y_P} \mathrm{d}y'_P \int_0^{y'_P} \mathrm{d}y''_P \omega_P(x, y''_P, t)$$

$$\partial_{y_P} \omega_P(x, 0, t) = -\partial_x p_E(x, 0, t),$$

No diffusion parallel to the wall

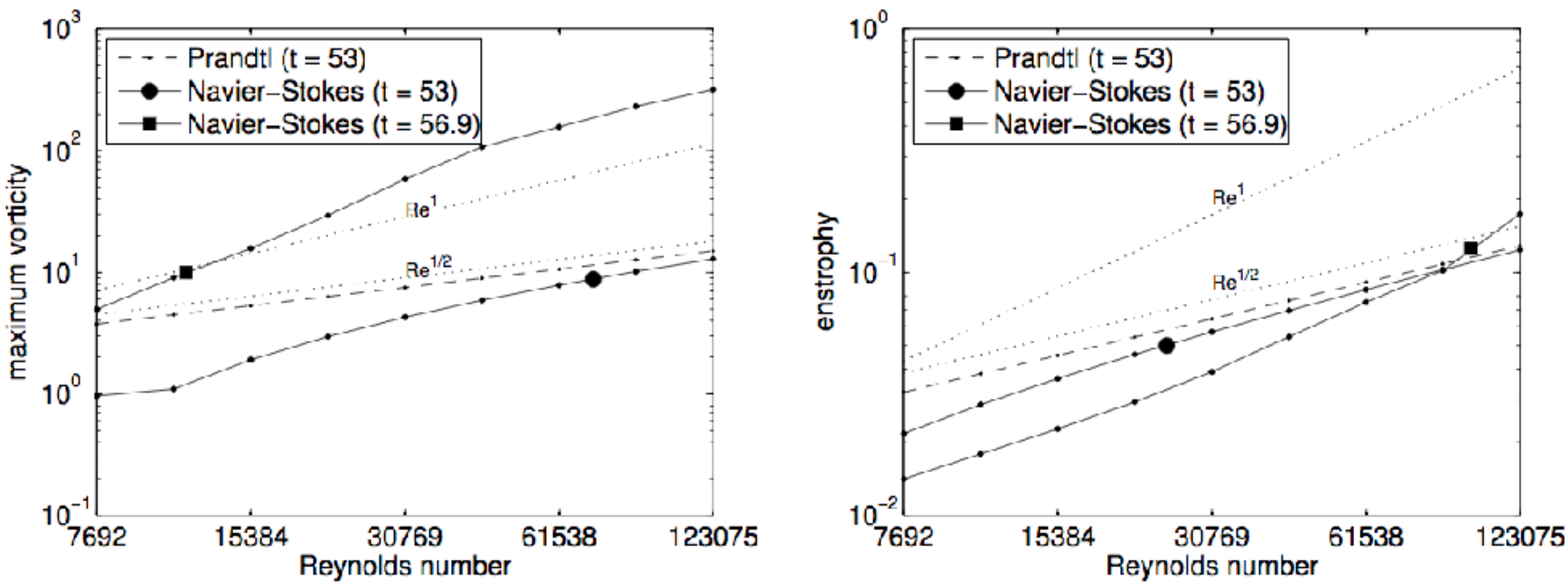
Prandtl solution blows up (parallel vorticity gradients)

Flow evolution



Evidence for dissipative structures

Vorticity maximum does not appear at the location of Prandtl singularity



Phenomenology: Tollmien–Schlichting-Rayleigh instability

Nguyen van yen, Waidmann, Klein, Farge, Schneider arxiv 2017

Linear dynamics around a prescribed velocity profile invariant along the wall direction (Orr Sommerfeld equation)

Condition for instability: recirculation at the boundary $u'_P(0) < 0$

Range of unstable modes scales as $\Delta k \sim Re^{1/2}$

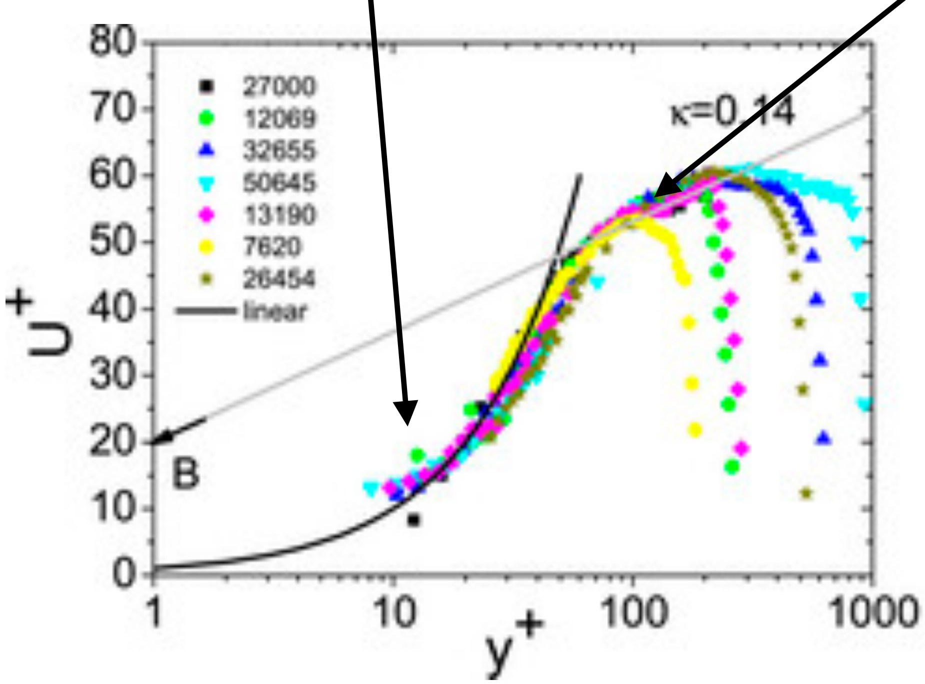
These modes are excited by the appearance of the singularity

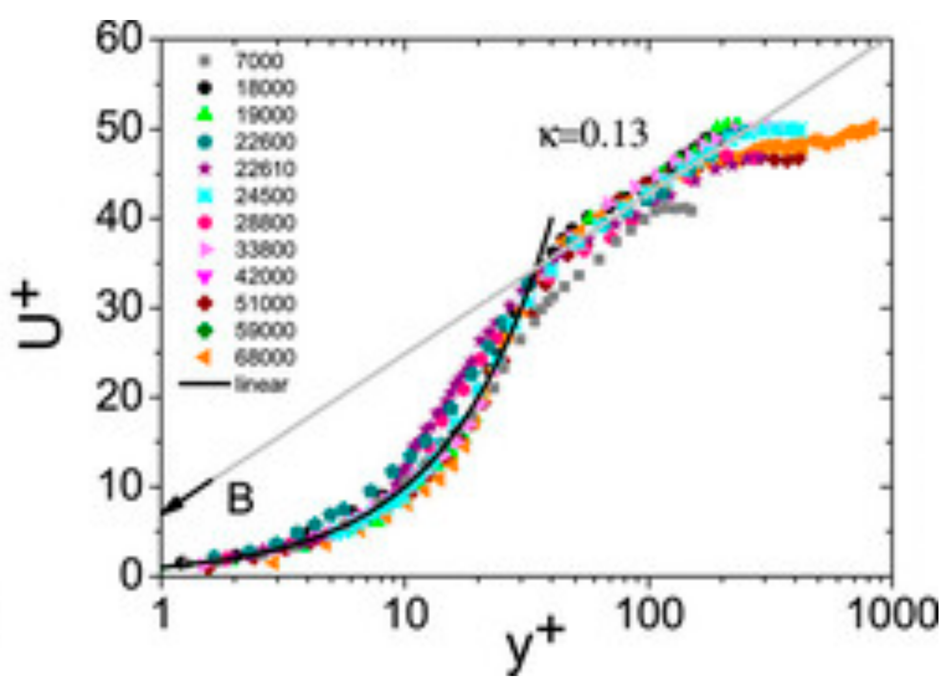
Assume all modes amplitude grow at the same rate until they reach the order of Prandtl vorticity $\omega_k \sim Re^{1/2}$

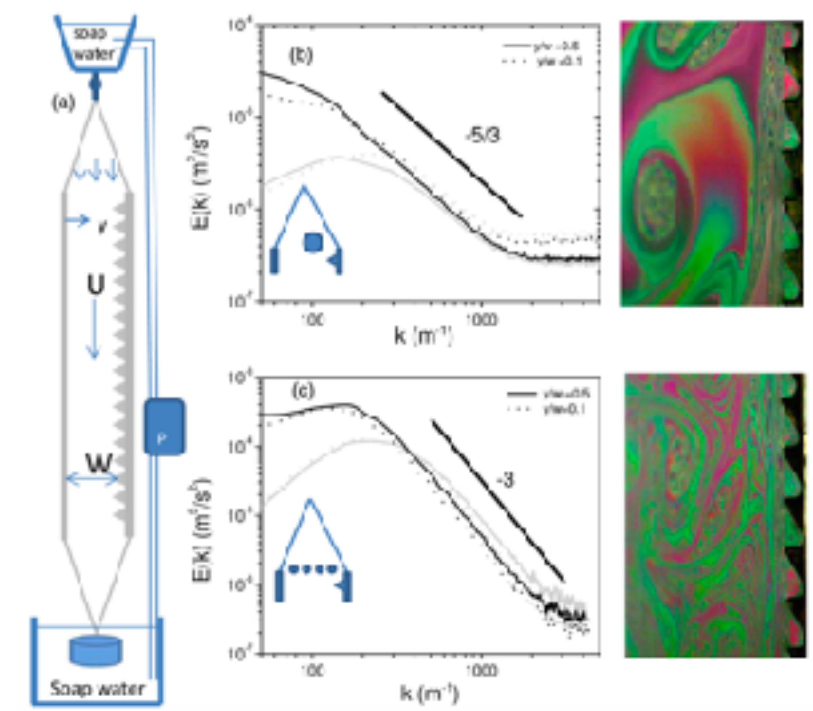
This corresponds to a wave packet with amplitude $\omega \sim Re$

Empirical law of the wall in 2d flows

Viscous layer: $\tau_0 \approx \nu \overline{u}/y$ **Inertial sublayer:** $\tau_0 \approx -\overline{u'v'} \approx \sqrt{\tau_0}\kappa y \partial_y \overline{u}$







Observation in soap films, Samanta et al 2014

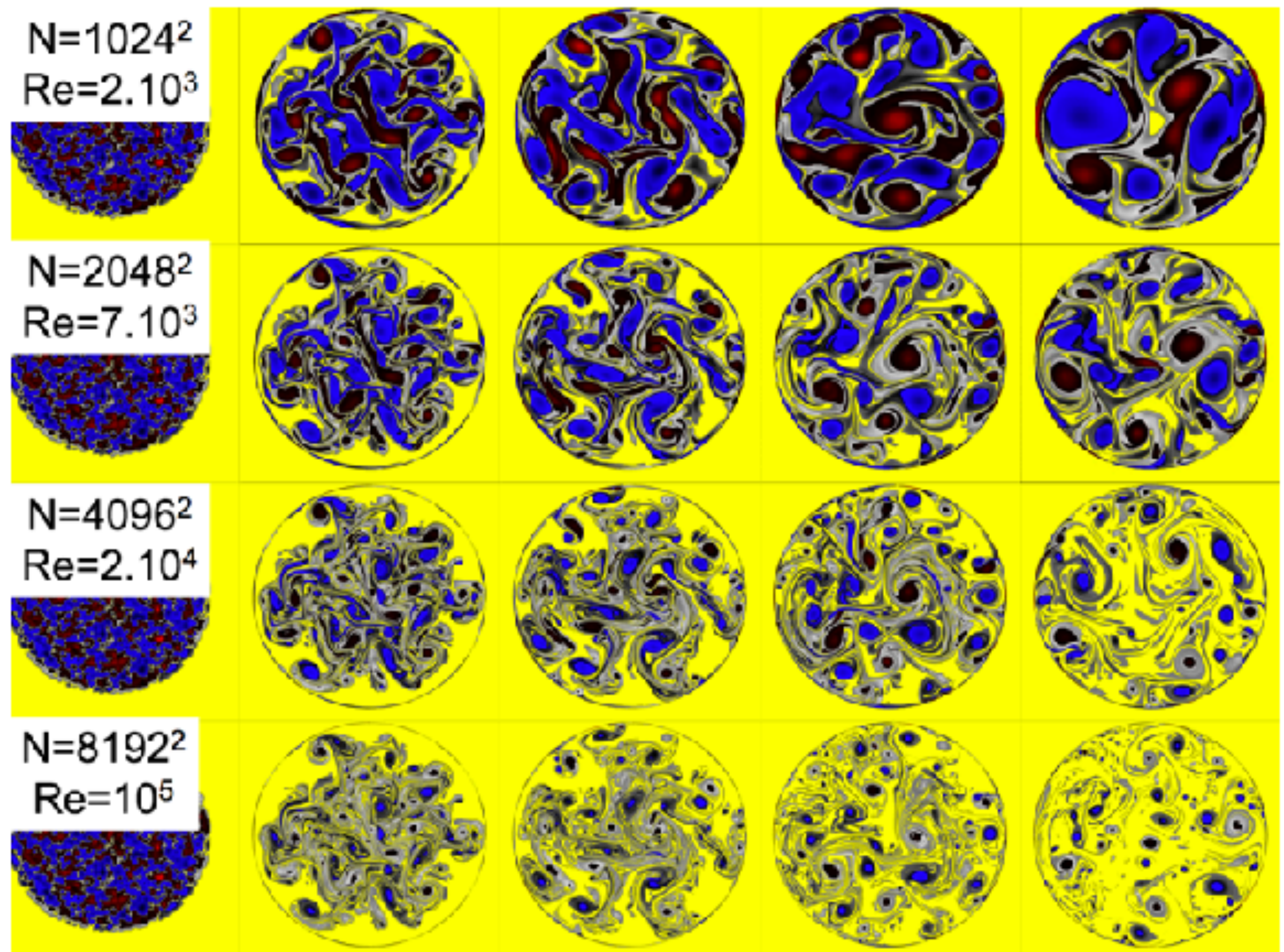
Transition at $\delta \approx \nu/\sqrt{\tau_0}$ $\overline{u} \approx \kappa\sqrt{\tau_0}\log(\delta\sqrt{\tau_0}/\nu)$

Statistical signature of dissipative structures?

Long time evolution of 2D flows with walls

Farge Schneider PRL 2005
NS + no slip (penalisation method)

(Previous work in a serie of papers by Clercx and *van Heijst*)

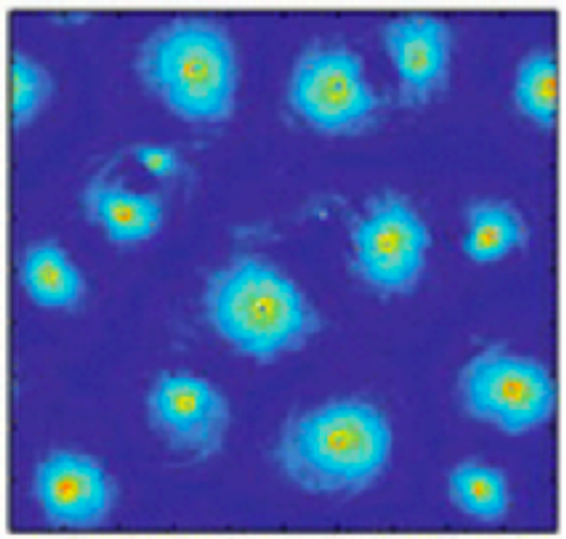


Roullet & McWilliams (ongoing work) implicite LES of 2d flows, no slip BC enforced by creating/destructing vorticity at the boundary.

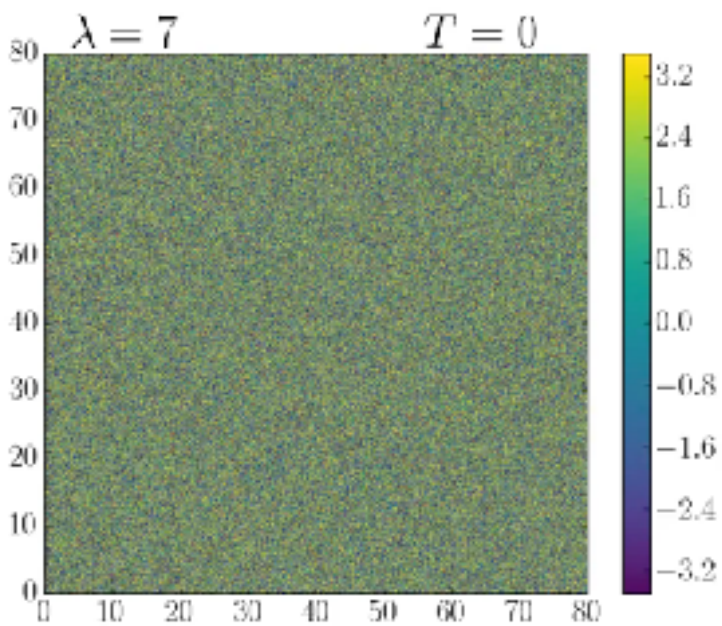
Vortex crystal states

Rotating moist convection

Held 2004 Emnauel 2011



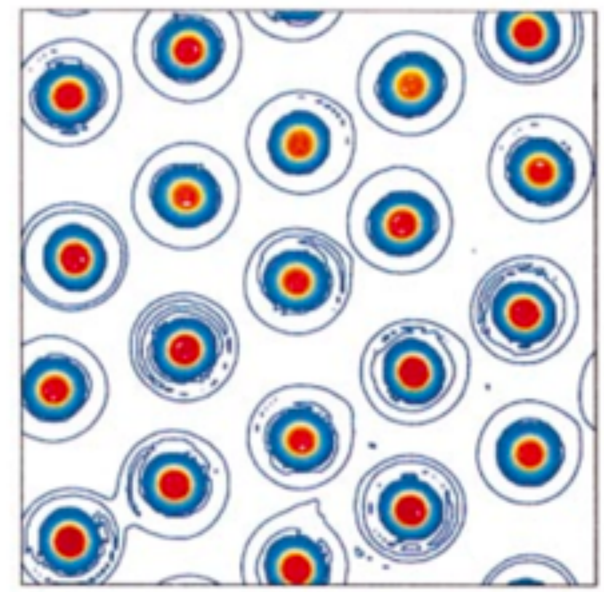
‘Active matter model’ James, Bos, Wilczek 18



$$\partial_t \boldsymbol{u} + \lambda \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p - (1+\Delta)^2 \boldsymbol{u} - \alpha \boldsymbol{u} - \beta \boldsymbol{u}^2 \boldsymbol{u}$$

Stratified quasi-geostrophic flows

Arbic & Flierl 2004



Plasma, atmospheres,…

Conclusion&Questions

Boundary layer detachment goes with the production of dissipative structures

Spatial scales of the order of the 1/Re

What are the consequences on self-organization in the domain bulk?

Consequences for geophysical flows ? Energy budget ? Gulf-Stream detachment?

Gulf Stream Separation in Numerical Ocean Models, Chassignet Marshall 2013

Misc: vortex methods on beta plane configurations?