Mathematical fluid mechanics

The aim of this advanced course is to give a self-contained introduction to the mathematical theory of incompressible fluids, starting from the derivation of the fundamental equations of motion and ending with an overview of current research in this very active field. No previous knowledge of fluid mechanics is required, all the prerequisites being covered by the PDE course in the first semester. The emphasis is put on the qualitative behavior of the solutions of the evolution problem, especially in domains without boundaries where the vorticity equation can be used to gain some insight into the dynamics. Flows with localized vorticity, such as isolated vortices in 2D or vortex filaments in 3D will receive a special attention.

Program :

• *Derivation of the fundamental equations.* Continuity equation, momentum and energy balance, Euler equations for perfect fluids. Internal friction, rate-of-strain tensor, Navier-Stokes equations for viscous fluids. Boundary conditions.

• *Perfect fluids.* Classical solutions of Euler equations in a smooth domain. Vorticity formulation, Yudovich's theory for weak solutions in $\mathbb{R}^2$. Flow past an obstacle, the d'Alembert paradox. Shear flows and stability criteria.

• *Viscous fluids without boundaries.* The Cauchy problem for the Navier-Stokes equations in $\mathbb{R}^2$ and $\mathbb{R}^3$. Critical function spaces, Fujita-Kato theory, and the Millenium Problem.

• *Boundary layers and inviscid limit.* The Navier-Stokes equations in a smooth two-dimensional domain. Prandtl's theory of boundary layers. Inviscid limit, the Kato criterion. The drag force around an obstacle, Stokes' formula. The plane and cylindrical Poiseuille flows.

• *Vortices and filaments.* Isolated vortices in inviscid planar fluids, the point vortex system. Oseen vortices and long-time asymptotics of two-dimensional viscous fluids. Vortex filaments, binormal flow, existing results and conjectures.

References :

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L. Landau and E. Lifshitz, *Fluid Mechanics*, Course of Theoretical Physics vol. **6**, Pergamon Press, London, 1959.

P.-G. Lemarié Rieusset, *The Navier-Stokes Problem in the 21st Century*, Chapman and Hall/CRC, 2016.

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Lecture notes : `https://www-fourier.univ-grenoble-alpes.fr/~gallay/cours.html`