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Session Title
Primary and Secondary Mathematics Education: A Critical Look from a European and U.S. Perspective

Synopsis
In order to succeed in the information-based society of the twenty first century, students must have a solid command of core subjects – reading, science, history, and the arts – with mathematics at its center. A quality mathematics education must be an integral part of the learning experience. Almost every job today demands knowledge of mathematics that includes both conceptual understanding and basic skills. Although learning takes place throughout a lifetime, the acquisition of this knowledge is preferably achieved through school education. However, in recent years, primary and secondary mathematics education in Europe and the United States has suffered from severe deficiencies, which have been brought about by inadequate reforms, inappropriate responses to sociological changes, and poor didactical practices. What are the key issues framing this debate? How do we ensure that each student receives a quality mathematics education? This panel of experts will provide the context of the debate, then discuss the reforms, their origins, and their effects, before suggesting practical solutions to the critical problems facing mathematics education in Europe and the United States.

Panelists

Jean-Pierre Demailly,
Professor of Mathematics at Joseph Fourier University in Grenoble and Académie des Sciences de Paris, France
Wilfried Schmid,
Professor of Mathematics at Harvard University, U.S.A.

Address of Jean-Pierre Demailly

Dear Colleagues,

First, I wish to express my deep gratitude to the Organizing Committee for the invitation at the ICCM 2007 in Hangzhou, and especially for suggesting my participation to this panel on Mathematics Education. I would like to discuss the French situation from my standpoint, as a researcher in Mathematics, but also as an individual deeply involved in education matters since a few years – I am currently President of the GRIP (Interdisciplinary Group for the Study of School Programs), in charge of an experimental network of classes SLECC (« Savoir Lire Ecrire Compter Calculer »). It appears to me that the extremely worrying situation we face in France is by many aspects also encountered in other European countries, so I believe that these are general trends which are worth being analyzed at an international level.

As you all know, France is still a major player in Science, particularly in Mathematics : almost 25% of the talks presented last year at the International Congress in Madrid in 2006 were given by French Mathematicians. While not as strong in other scientific fields, France appears to be doing very well in Physics, Chemistry, Biology, Medecine (at least),

and is a leader in several prominent technological fields such as nuclear energy or aeronautics. However, I would say that this prominent position reflects the value of the French Educational System three or four decades ago - or more - given the long amount of time needed to train students and to develop successful research programs and successful curricula.
Actually, 40 years ago, France used to achieve high performance in primary and secondary education, with almost 100% population attending primary school, though only a small percentage of 20-30% of population went to high school. In those times, the general level of French schools was comparing favorably to that of most other countries, frequently by a large margin. The major turn had been taken in 1880 with Jules Ferry's law on « public instruction » granting free and compulsory schooling to everyone. The syllabusses proposed during the period 1880-1970 were generally of excellent quality - and even though everything was certainly not optimal - extremely good work had been done by Ferdinand Buisson and his successors. In particular, I would like to mention Buisson's monumental treatise « Dictionnaire de Pédagogie », whose general precepts and rules could still be useful today in primary education – would they not be considered a priori with disdain by a majority of modern educators. The Baccalauréat (final examination of high school in France) was of a high level, both in litterature and science.

A picture of the present situation in France

However, four decades after the end of this era, we are faced with a completely different picture.

- France performs poorly according to the PISA OECD 2006 study for science proficiency level (France lags behind Finland, Canada, Korea, China by a large margin, ...) - the U.K. is also quite weak. Germany got rather poor results in 2002 and appears to have somewhat improved in 2006. I want to stress that these international studies may not be extremely meaningful, given that they only measure basic skills and are probably not relevant to assess the performance of the few countries which provide richer content to pupils than the vast majority (as was the case of France long ago).
- in France, 65 % of population reaches the end of high school, but the current average level of Baccalauréat (final exam at K12 level) is extremely weak. Actually, this can be a matter of dispute, since the requirements are sometimes a strange combination of very advanced (too advanced ?) subjects in special areas, e.g. in biology, with rather inconsistent or ridiculous demands in other fundamental subjects : students are no longer supposed to be able to write correct sentences in correct academic French language, but they still should be able to grasp the profound nature of litterary texts according to their stylistic classification ; in mathematics, decomposing an integer into a product of prime numbers is no longer a requirement, but the exponential function is introduced (at K12 level) as the solution of a differential equation, for which existence and uniqueness is supposed to have been discussed ...
- Students appear to be discouraged from entering the science curriculum (except possibly for biology, medecine)
    - in the last decade, there has been of decay of more than 50 % students in mathematics in the first years of university (and an even bigger decrease in physics)
    - the industry suffer from a lack of properly trained engineers and technicians. While the level of studies in engineer schools may not be as bad as in universities, it is definitely weaker than it used to be 20 years ago – France is particular among Western countries for having its best engineer schools

being a branch of higher education completely separate from mainstream universities. (This is actually one important reason why French universities are not able to attract students who have the best qualifications. Another negative consequence is a large gap between research and industry in many areas.) However, any reasonable comparison which can be made with students of comparable age between France and China, even when looking at students of engineer schools of « reasonably good level », shows that Chinese students are nowadays performing substantially better on average : I was recently told that at INSA Lyon, the Chinese students who are studying there regularly get much higher scores in math and physics – even when their reason for pursuing studies in France is because they failed the harder entrance examinations in China – anyhow, they still have a much better background !

- teachers' background has become quite weak, especially in maths and physics : while students from the top elite schools are still doing rather well, this only concerns only a few dozens of exceptional students ; on the other hand, the average students enormously suffer from the current chaos - poorly trained young teachers will be a very serious concern in the future.
- There are too many students in disciplines which are perceived to be « easier » to learn (sports, arts, psychology, ...) or more atttractive professionally (medecine).
- The PhD programs in Mathematics are still of an excellent quality, but they have currently very few students (a sharp decline of 40-50% in the last four years) – again a consequence of the lack of properly trained students from previous levels. This means that French mathematicians are even more available than before to take care of Chinese students who would be willing to study in France, but within a rather short period, this could lead to a dramatic weakening of the PhD programs themselves - an extremely worrying perspective for French science, especially since the situation is more or less similar in several other european countries.

I am now going to describe the circumstances which can explain these severe problems. Of course there are a number of sociological factors, but it appears to me that the main factors are, by far, bad moves and bad reforms which have had an extremely negative impact on the quality of general education. Since the mistakes have been consistently repeated in almost all Western countries (possibly starting in United States and being often copied blindedly in Europe), I believe that it is important for Asian countries to analyze carefully these phenomena and to avoid reproducing similar mistakes. The United States have a notoriously bad average level in primary and secondary education, but they still manage to fill their PhD programs by importing large numbers of highly skilled students from Asia and Eastern Europe - I doubt France and Western Europe can afford to do the same !

General setup of the French educational system

First, I would like to briefly recall the general setup of the French educational system.

- Nursery school (ages 3 – 6) is available almost everywhere and heavily attended, although school is not compulsory before age 6.
- Primary education consists of grades 1 – 5 (ages 6 – 11).
- Secondary education consists of grades 6 – 12 (ages 11 – 18) and has two steps :
  « collège » grades 6 – 9 (ages 11 – 15),
  « lycée » grades 10 – 12 (ages 15 – 18) = high school in the US.

School is compulsory up to age 16.

- University : normal age for entering is 18 years.
  Europe has recently adopted a unified scheme for higher studies called the « Bologna system » or LMD (Licence-Master-Doctorate) in France :
  Licence = 3 years, Master = 2 years, Doctorate = 3+ years
  Each of the L,M,D cycles is to be completed by achieving a corresponding diplom.

Major reforms undertaken since 1970

Now, I will describe very schematically the main steps of the educational reforms which have taken place since 1970 – shortly after the so called May 1968 revolution in Paris, when students started large scale demonstrations in the streets during several weeks. These events gave the initial political signal for deploying the reforms, although they may not have influenced directly their contents - the ideas were already in the air (even among leading parties and fractions of the society who did not take part into the protests) ; I would say however that the reforms were applied hastily and without the necessary care to measure and analyze long term consequences.

- In the years 1969-72, important reforms of primary and secondary education took place[1]. « Active pedagogies » were introduced in primary education, while traditional methods for reading, writing and counting were changed. For reading, pupils started to be taught as if French were no longer an alphabetic language, turning actually French words into something similar to Chinese ideograms, global entities to be learnt just by memorizing the « photographic image ». Emphasis was put on socializing pupils rather than providing them specific knowledge in fundamental disciplines. In mathematics, the teaching of the 4 arithmetic operations was no longer recommended in grade 1, and pupils started to learn addition merely. At the same time, new maths were introduced at the lycée level (grades 10-12) : the emphasis was now on formal rigor, using the language of set theory and abstract linear algebra instead of traditional euclidean geometry, and introducing a highly formalized approach in analysis (epsilons and deltas). I was myself a pupil in that time, and my personal feeling is that these « new maths » at the lycée level were more or less OK, possibly slightly too ambitious for average pupils, and a bit lacking in certain directions, e.g. for developing geometric intuition. They were however still rather successful as long as pupils entered the lycée with proper training in arithmetic and elementary geometry, which was still the case of a majority of pupils in the science curriculum. Things turned into a disaster when « crazy » new maths were also introduced at lower levels, in primary education and even sometimes at kindergarten. Numbers were introduced in grade 1-2 as classes of equipotent sets ! My father, who was a math teacher, had to present negative numbers in grade 6 as equivalence classes of pairs of natural numbers with essentially « Grothendieck's method » for deriving a group from a monoid ! In grade 8, a line was defined as an abstract set together with a family of bijections to the the real numbers, transforming each other by affine changes of variables ! Of course all these matters were way too much for pupils, and the quality of training of the weaker ones started to suffer enormously.
- In 1975, a major change was introduced in secondary education, namely the « collège unique[2] » : pupils entering grade 6 were no longer differentiated between general schools and technical schools (with their more practical syllabusses). The fact is that primary school had become much less effective than earlier (see

1 The ministers in charge at that time were Edgar Faure, under presidency of Charles de Gaulle, and Olivier Guichard, under presidency of Georges Pompidou.

2 The minister in charge was René Haby, under presidency of Valéry Giscard d'Estaing.

above !) and large numbers of pupils were now failing badly in higher grades. Rather than accepting to accommodate weaker and weaker classes in the technical schools, a bold decision was taken : simply suppress these technical schools, in the hope that a re-unification would help to get pupils out of their difficulties through contact with the more skilled pupils – and with the general idea that more and more people should anyway enter university. Unfortunately, this turned out to be just wishful thinking, and is now recognized to have been essentially ineffective – not surprising since primary education was pursuing its own decline ! The only immediate result was a drastic decrease in the contents of « collège » syllabusses (grades 6-9), and it is not exaggeration to say that grade 9 is now probably less demanding than grade 5 used to be (if only pupils could still be seriously evaluated by teachers !)

- After these steps, it is clear that the 1969 programs of lycée (grades 10-12), especially new maths, were no longer sustainable for a large majority of pupils. In 1989, starting the second term of the President of Republic François Mitterrand, a new reform was on its way, which I would describe as based on politics and abstract ideology rather than anything else. Slogans were « 80 % of population should enter university » (Japan being taken as the model to follow). Mathematics, especially new maths, were accused implicitly or explicitly of being anti-democratic, just as any other formalized corpus of knowledge. In 1985, the minister of Education Jean-Pierre Chevènement signed new programs established by his administration, in which :

  - major parts of elementary arithmetic disappear from primary and secondary programs (even for the science sections) : prime numbers, lcd, gcd are no longer taught explicitly !

  - one can observe a tendancy to reduce the mathematical approach to rote learning of rudimentary facts[3].

  Slightly later, a devastating paradigm was introduced : children should rediscover everything by themselves through experimentation, and professors should be there only to guide their steps. This was the start of a kind of « cultural revolution » in French education. The actual reforms were adopted around 1991-1992[4], and resulted in a new drastic depletion of curricula. The 10th grade which used to be differentiated between arts, science and economy started to be undifferentiated, and only one science curriculum was established, instead of at least 3 formerly : math-physics, maths-physics-biology, maths-technology. Even more severely, the new ideologies had tremendously bad effects on teaching practices and on the general organization of syllabusses. Mathematics and Physics were no longer taught in a coherent way, with such an emphasis on experimentation (at the expense of more theoretical parts) that any fruitful interaction between math and physics became impracticable. In reality, almost all disciplines, including litterature, science, history, geography, saw their programs completely « reshaped » (and depleted, as far as fundamental knowledge is concerned...). The reform was accompanied with the introduction of IUFM (University Institutes for Training of Teachers), where the main concern was apparently more to indoctrinate future teachers with the new pedagogical theories and paradigms rather than introducing them with a solid knowledge of their discipline, along with efficient and well tested teaching practices.

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3 *Translation from the explanations accompanying the new French 10th grade syllabus in 1985* « This is why the theoretical part of the course should be brief : its contents should be limited to essential concepts and results. The design of such a course cannot be viewed as an ordered sequence of concepts and theorems ; all newly introduced concepts should be presented by starting from concrete and sufficiently rich situations ».

4 under the ruling of the minister of education Lionel Jospin (who became later Prime Minister in 1997).

It still took a few years for these reforms to show their effects, especially at university, and also the administration in charge was - and probably still is - biased enough, to the point of not willing to analyze and report accurately about the damages – e.g. by declaring « obsolete » important items which could have been used to evaluate the situation. In any case, here is a factual comparison of the introduction of the 4 arithmetic operations during the period 1920-1970, versus the most recent programs for primary schools (2002).

Subject	Scheduled in grade X			Official recommendations
	1920 - 1970	in 2003	delay	Legend — roman : end of primary curriculum in 2002 — *italics* : maximal requirements in 6th grade in 1995. — [....] : our comments.
Operations on whole integers				
Addition of 2 digit numbers	1	2	1 year	« at the end of grade 3, only the technique of addition can be required » (curriculum 2002.)
Subtraction of 2 digit integers	1	4	> 2 yrs	
Multiplication/division by 2 & 5	1	4	> 2 yrs	
Multiplication by 2 digit number	3	6	> 3 yrs	« compute the product of two integers (3 digits by 2 digits) with paper and pencil. » In 2001, 46,2% of 6th grade pupils could not multiply 64 ×39 canceled from 6th grade national tests since 2002 !!!
Division of integer by 2 digit integer	3	5*	> 2 yrs	[* but with restriction] « dividend < 10 000 »
Division of arbitrary integers	4	never	?	
Operations on decimal numbers				
Multiplication of decimal numbers	4	6	2 yrs	[Canceled from primary curriculum in 1995]
Division of decimal by integer	4	6	2 yrs	[Canceled from primary curriculum in 2002] *« cannot be required at the end of primary school »*
Division of two decimal numbers	5	never	?	[Canceled from primary curriculum in 1980]

Results of such a policy (French national tests circa 2000) :
— in 2001, 46.2 % of 6th grade pupils could not multiply 64 ×39 ;
canceled from 6th grade national tests since 2002 !
— in 2002, 62.7 % of 7th grade pupils cannot multiply 9.74 × 3.5 ; 74.2 % cannot divide 178.8 by 8.

As the reader will see, between 1970 and 2007, the delay in acquiring the same basic calculation skills is of at least 2 years - without taking into account the fact that this delay is probably accompanied by a decline in performance and understanding, even with the 2 year difference in age and all other conditions equal.

What can be done and what has been done ?

Up to now, it has been very hard in France to reach a general agreement on the analysis of difficulties. Education is a highly political subject ; many people who have been involved in the process of the previous reforms are still in charge of responsibilities (and are of course not willing to recognize their past failures – possibly even not able to realize !). However the situation has recently moved a little bit, and I will try to summarize some of the events which have taken place and are of interest to us.

- France is centralized, so one « way of attack » is just to convince the administration in Paris that things are going wrong, hoping then for some positive decisions to be taken at the national level. This becomes unfortunately easier and easier – especially after the events at autumn 2005 where thousands of teenagers were involved in riots in the French suburbs, and where the lack of proper education and professional perspective was the most obvious reason for their anger. This approach may of course be less effective in more decentralized countries.
- In 2003, a group of university colleagues of mine and of secondary/primary teachers joined to form the GRIP (French acronym for Interdisciplinary Group for the Study of School Programs - I am the current president). We started to write critical reports and counter-proposals for educational matters. GRIP now has more than 60 members (and hundreds of people following its activities).
- The GRIP activity quickly focused on evaluating and modifying the existing syllabusses and curricula - in our view, the main issue involved in the crisis of education.
- The main points of the GRIP proposals are to (re-)introduce a systematic approach to teaching basic knowledge : always start from concrete elementary facts, and proceed through progressive generalization to more elaborate questions ; rely as much as possible on individual work of pupils, through exercises of many different types ; create links between disciplines which can bring a rich interplay.

The goals of the GRIP is to link theory and practice, by assessing the proposed curricula through experimentation in class, especially at the most fundamental levels of primary school. The GRIP also actively seeks for international cooperation, since the main issues are not limited to France or Europe – it is clear from just looking at the international conferences that some « bad ideologies » still propagate through official channels ; fact is that it is now almost impossible in Western countries to test certain solutions that we would like to implement, e.g. in secondary education, simply because pupils that would be properly trained to experiment them do not exist any more in sufficient numbers. Also, many « trendy pedagogical theories » introduce implicit assumptions that prevent their promoters to even consider certain possibilities. (For these reasons, national or international studies which « prove impossible or impractical » to teach such and such items at a given level, such as e.g. division in grade 1 as was written recently by a so-called « expert » in France, have to be taken with a grain of salt – after all, a very large majority of French 6-7 year old pupils had been able to divide a number in the range 1-100 by 2 and 5 from 1880 to 1970 ...).

The SLECC program

In 2005, the GRIP entered into negotiations with the Primary School Department of the Ministry of Education and obtained permission and support for running a National network of « experimental » primary schools (5 year program from grade 1 to grade 5). The French acronym is SLECC (Savoir Lire Ecrire Compter Calculer = Learning how to Read Write Count Calculate). SLECC started with approximately 15 classes in 2005, and runs about 60 classes throughout France since September 2007.

The project is funded by the Ministry of Education, and has obtained two specific positions attributed to teachers taking care of the network. The classes are randomly distributed according to the availability of teachers willing to enter the experiment ; there is a priori no reason to believe that there could be a sociological bias in the distribution of children attending SLECC.

The SLECC program includes a drastic reform of mathematics education for primary school, but viewing it from just a mathematical perspective would greatly reduce its scope and value. In fact, its sphere of activity covers all other disciplines, in particular their relation to mathematics : links with scientific activities are certainly involved, but the main concern is to develop links between language and mathematics from the very beginning of learning, i.e. in France from the last year of nursery school (5-6 year pupils).

The SLECC program also includes some classes in secondary education – up to now, their activity is more to remediate to the deficit in fundamental skills rather than trying to promote more ambitious programs, an impossible task right now.

Main principles of SLECC pedagogy

The foundation of the SLECC curriculum is the exact opposite of the general trend in pedagogy introduced at the end of the 60's which has led to sacrifice the contents of the primary school curriculum under the fallacious principle that a pupil learns better if there is less to learn. Actually, the reason for the failure of this principle is quite simple : the elimination of certain links in the logical chain of knowledge leaves the remaining notions more difficult or even impossible to learn !

The main principles of the SLECC pedagogy are :

- Simultaneous teaching of reading and writing : the « phonics » method called « writing-reading » (this actually disqualifies from scratch the decades-long debate of *« whole language reading »,* simply because there cannot be any *« whole language writing »* !
- Simultaneous teaching of counting and calculating - more precisely the simultaneous teaching of the 4 arithmetic operations along with numbering. After all, 34 means 3 times 10 plus 4, so multiplication and addition are certainly involved already when numbering. For similar reasons, subtraction and division should not be detached either.
- A strong interplay between various disciplines, e.g. language and math. When doing elementary mathematics, pupils have to write down statements and conclusions as complete sentences (they should not just fill holes in prepared sheets !). This helps them in formulating problems and getting acquainted to elaborate explanations properly. There are many other examples : math and geography do have interests in common ; when learning to exploit a map, it is important to understand scales and the relation with conversions of units. Of course, other rich interconnections are to be considered between observation in science, measurements and elementary calculations (proportionality, etc), and again language capabilities to describe a situation or phenomenon.
- No fake opposition between memorizing and understanding, which are necessary to each other. When needed, multiplication tables have to be known fluently.
- Strong insistence on pencil and paper algorithms of the four arithmetic operations, including decimal number operations. Actually, this is a very good preparation for algebra in later grades, since the algorithms of arithmetic operations are similar to a large extent to the algorithms for multiplying or dividing polynomials. Calculators are discouraged at primary school level (and beginning of secondary level), at least as long as pupils have not reached a routine expertise in handling arithmetic operations on paper.
- Operations on pure numbers and concrete numbers are taught simultaneously. It turns out that our brain seems to handle much better physical quantities than abstract numbers, especially when the intuitive meaning of operations is involved. This is of course also an excellent preparation for the introduction of elementary physics and dimensional analysis. SLECC textbooks insist on writing formulas such as

3 m + 4 m = 7 m (even very early in the 1st grade curriculum),
and later on,
6.2 kg + 250 g = 6200 g + 250 g = 6450 g = 6.45 kg ;
units should always appear in the operations whenever appropriate[5] !

- Concrete manipulation in geometry : paper, scissors, ruler, compasses, angles ... Even the proof of the formula for the area of a disc (i.e., deriving that area = πR^2 from the definition of π = perimeter/diameter = P/2R) can be done in 4th grade through cutting a paper disc in 16 sectors, and re-assembling them in an approximate parallelogram ; this of course assumes that the concept of area has already been taught in a systematic way, from rectangles to parallelograms and then from parallelograms to triangles. The case of rectangles can itself be started at an earlier stage by counting squares in case the edges are integers, then by extending to the general case of decimal numbers through changes of the units of length and area.

As we see, the pupils are introduced to mathematical proofs at a very early level (in a form suitable for their age – we do not mean by this a « completely formalized proof »). This can be done even in very early grades, e.g. by observing on the multiplication table that 6 x 8 = 7 x 7 - 1 (and similar cases as well), and proving it by displacing a row into a column in an arrangement of wooden squares. Later on, at the beginning of secondary education, such proofs or justifications can be given in a systematic way – e.g. for Pythagoras' theorem which is the cornerstone of Euclidean geometry. We certainly recommend the (re)-introduction of elementary geometry (in dimension 2 and possibly also dimension 3) at the beginning of secondary education, starting with the so called « equality cases » for triangles, for which a long tradition in France has shown beyond any doubt that they constitute a sane and very accessible basis for introducing proofs and reasoning in geometry - if needed to convince mathematicians, this approach could even be formalized into a rigorous and simple mathematical theory according to modern standards, using only the axiom of euclidean distance, together with certain properties of real numbers such as the existence of square roots ...

Main observations from the SLECC « experiment »

- The use of a systematic methodology creates confidence, pupils feel safer because they receive precise instruction and have rules to learn, the use of which empowers them to solve problems.
- Pupils are busy with regular and well planned activities, very early they have to learn to work by themselves – as a consequence discipline is easier to maintain in classes !
- Pupils who have difficulties realize better that they have problems than when the main issue of school is to « socialize », so that children and their parents accept better when a pupil has to repeat a level (and such a repetition then becomes much more useful and meaningful than with classes in which the goals are not clear enough).
- Even more important, pupils taught under these conditions seem to outperform pupils of « standard classes », even when they come from areas which have social problems and are compared to pupils of more favored regions.

Reactions from society and institution

As is easy to imagine, the combination of politics and ideology has started what can be described as a « school war » between supporters of the still dominant pedagogical

5 Of course, there is nothing much original here, such a use of units had been standard in France along the decades 1880-1970, so one can feel confident that this can become really effective with pupils, at least when these matters are properly taught ...

theories and their opponents. However, faced to what is now perceived to be a « school disaster », the ministry of education has started to consult the associations which were previously leading the opposition to current programs – and the ministry of education now supports SLECC openly.

Of course, a lot still has to be done. Especially there is a very severe lack of adequate training of a majority of teachers, especially after three or four decades of insane teaching practices. We have to face strong ideological opposition of certain teacher trade unions and of former « pedagogical gurus » ... Returning to a situation where French education performs in a reasonable manner appears to be a daunting task. We hope that GRIP and SLECC will contribute in a useful way.

References

References / reports / textbooks (mostly in French) can be found on our web sites

http://grip.ujf-grenoble.fr/

http://www.slecc.fr/

Results of PISA study 2006

Percentage of students at each proficiency level on the science scale

	Proficiency levels													
	Below Level 1 (below 334.94 score points)		Level 1 (from 334.94 to 409.54 score points)		Level 2 from 409.54 to 484.14 score points		Level 3 (from 484.14 to 558.73 score points		Level 4 from 558.73 to 633.33 score points)		Level 5 [from 633.33 to 707.93 score points		Level 6 (above 707.93 score points)	
	%	S.E.	%	S.E.	%	S.E.	%	S.E.	%	S.E.	%	S.E.	%	S.E.
OECD														
Australia	3,0	(0,3)	9,8	0,5)	20,2	(0,6)	27,7	(0,5)	24,6	0,5)	11,8	(0,5)	**2,8**	(0,3)
Austria	4,3	(0,9)	12,0	1,0)	21,8	(1,0)	28,3	(1,0)	23,6	1,1)	8,8	(0,7)	**1,2**	(0,2)
Belgium	4,8	(0,7)	12,2	0,6)	20,8	(0,8)	27,6	(0,8)	24,5	0,8)	9,1	(0,5)	1,0	(0,2)
Canada	**2,2**	(0,3)	7,8	0,5)	19,1	(0,6)	28,8	(0,6)	27,7	0,6)	12,0	(0,5)	**2,4**	(0,2)
Czech Republic	3,5	(0,6)	12,1	0,8)	23,4	(1,2)	27,8	(1,1)	21,7	0,9)	9,8	(0,9)	1,8	(0,3)
Denmark	4,3	(0,6)	14,1	0,8)	26,0	(1,1)	29,3	(1,0)	19,5	0,9)	6,1	(0,7)	0,7	(0,2)
Fnland	0,5	(0,1)	3,6	0,4)	13,6	(0,7)	29,1	(1,1)	32,2	0,9)	17,0	(0,7)	3,9	(0,3)
France	6,6	(0,7)	14,5	1,0)	22,8	(1,1)	27,2	(1,1)	20,9	1,0)	**7,2**	(0,6)	0,8	(0,2)
Germany	4,1	(0,7)	11,3	1,0)	21,4	(1,1)	27,9	(1,1)	23,6	0,9)	10,0	(0,6)	1,8	(0,2)
Greece	**7,2**	(0,9)	16,9	0,9)	28,9	(1,2)	29,4	(1,0)	14,2	0,8)	**3,2**	(0,3)	**0,2**	(0,1)
Hungary	**2,7**	(0,3)	12,3	0,8)	26,0	(1,2)	31,1	(1,1)	21,0	0,9)	**6,2**	(0,6)	0,6	(0,2)
Iceland	5,8	(0,5)	14,7	0,8)	25,9	(0,7)	28,3	(0,9)	19,0	0,7)	5,6	(0,5)	0,7	(0,2)
Ireland	3,5	(0,5)	12,0	0,8)	24,0	(0,9)	29,7	(1,0)	21,4	0,9)	8,3	(0,6)	1,1	(0,2)
Italy	7,3	(0,5)	18,0	0,6)	27,6	(0,8)	27,4	(0,6)	15,1	0,6)	**4,2**	(0,3)	0,4	(0,1)
Japan	**3,2**	(0,4)	8,9	0,7)	18,5	(0,9)	27,5	(0,9)	27,0	1,1)	12,4	(0,6)	**2,6**	(0,3)
Korea	**2,5**	(0,5)	8,7	0,8)	21,2	(1,0)	31,8	(1,2)	25,5	0,9)	**9,2**	(0,8)	1,1	(0,3)
Luxembourg	6,5	(0,4)	15,6	0,7)	25,4	(0,7)	28,6	(0,9)	18,1	0,7)	5,4	(0,3)	0,5	(0,1)
Mexico	18,2	(1,2)	32,8	0,9)	30,8	(1,0)	14,8	(0,7)	**3,2**	0,3)	0,3	(0,1)	0,0	a
Netherlands	**2,3**	(0,4)	10,7	0,9)	21,1	(1,0)	26,9	(0,9)	25,8	1,0)	11,5	(0,8)	1,7	(0,2)
New Zealand	4,0	(0,4)	9,7	0,6)	19,7	(0,8)	25,1	(0,7)	23,9	0,8)	13,6	(0,7)	4,0	(0,4)
Norway	5,9	(0,8)	15,2	0,8)	27,3	(0,8)	28,5	(1,0)	17,1	0,7)	5,5	(0,4)	0,6	(0,1)
Poland	**3,2**	(0,4)	13,8	0,6)	27,5	(0,9)	29,4	(1,0)	19,3	0,8)	6,1	(0,4)	0,7	(0,1)
Portugal	5,8	(0,8)	18,7	1,0)	28,8	(0,9)	28,8	(1,2)	14,7	0,9)	3,0	(0,4)	0,1	(0,1)
Slovak Republic	**5,2**	(0,6)	15,0	0,9)	28,0	(1,0)	28,1	(1,0)	17,9	1,0)	**5,2**	(0,5)	0,6	(0,1)
Spain	4,7	(0,4)	14,9	0,7)	27,4	(0,8)	30,2	(0,7)	17,9	0,8)	4,5	(0,4)	0,3	(0,1)
Sweden	3,8	(0,4)	12,6	0,6)	25,2	(0,9)	29,5	(0,9)	21,1	0,9)	6,8	(0,5)	1,1	(0,2)
Switzerland	4,5	(0,5)	11,6	0,6)	21,8	(0,9)	28,2	(0,8)	23,5	1,1)	9,1	(0,8)	1,4	(0,3)
Turkey	12,9	(0,8)	33,7	1,3)	31,3	(1,4)	15,1	(1,1)	**6,2**	1,2)	0,9	(0,3)	0,0	a
United Kingdom	4,8	(0,5)	11,9	0,6)	21,8	(0,7)	25,9	(0,7)	21,8	0,6)	10,9	(0,5)	**2,9**	(0,3)
United States	7,6	(0,9)	16,8	0,9)	24,2	(0,9)	24,0	(0,8)	18,3	1,0)	7,5	(0,6)	1,5	(0,2)
OECD total	6,9	(0,3)	16,3	0,3)	24,2	(0,4)	25,1	(0,3)	18,7	0,3)	7,4	(0,2)	1,4	(0,1)
OECD average	**5,2**	(0,1)	14,1	0,1)	24,0	(0,2)	27,4	(0,2)	20,3	0,2)	7,7	(0,1)	1,3	(0,0)
Partners														
Argentina	28,3	(2,3)	27,9	1,4)	25,6	(1,3)	13,6	(1,3)	4,1	0,6)	0,4	(0,1)	0,0	a
Azerbaijan	19,4	(1,5)	53,1	1,6)	22,4	(1,4)	4,7	(0,9)	0,4	0,2)	0,0	a	a	a
Brazil	27,9	(1,0)	33,1	1,0)	23,8	(0,9)	11,3	(0,9)	3,4	0,4)	0,5	(0,2)	0,0	(0,0)
Bulgaria	18,3	(1,7)	24,3	1,3)	25,2	(1,2)	18,8	(1,1)	10,3	1,1)	**2,6**	(0,5)	0,4	(0,2)
Chile	13,1	(1,1)	26,7	1,5)	29,9	(1,2)	20,1	(1,4)	8,4	1,0)	1,8	(0,3)	0,1	(0,1)
Colombia	26,2	(1,7)	34,0	1,6)	27,2	(1,5)	10,6	(1,0)	1,9	0,3)	**0,2**	(0,1)	0,0	a
Croatia	3,0	(0,4)	14,0	0,7)	29,3	(0,9)	31,0	(1,0)	17,7	0,9)	4,6	(0,4)	0,5	(0,1)
Estonia	1,0	(0,2)	6,7	0,6)	21,0	(0,9)	33,7	(1,0)	26,2	0,9)	10,1	(0,7)	1,4	(0,3)
Hong Kong-China	1,7	(0,4)	7,0	0,7)	16,9	(0,8)	28,7	(0,9)	29,7	1,0)	13,9	(0,8)	**2,1**	(0,3)
Indonesia	20,3	(1,7)	41,3	2,2)	27,5	(1,5)	9,5	(2,0)	1,4	0,5)	0,0	a	a	a
Israel	14,9	(1,2)	21,2	1,0)	24,0	(0,9)	20,8	(1,0)	13,8	0,8)	4,4	(0,5)	0,8	(0,2)
Jordan	16,2	(0,9)	28,2	0,9)	30,8	(0,8)	18,7	(0,8)	5,6	0,7)	0,6	(0,2)	0,0	a
Kyrgyzstan	58,2	(1,6)	28,2	1,1)	10,0	(0,8)	**2,9**	(0,4)	0,7	0,2)	0,0	a	a	a
Latvia	3,6	(0,5)	13,8	1,0)	29,0	(1,2)	32,9	(0,9)	16,6	1,0)	3,8	(0,4)	0,3	(0,1)
Liechtenstein	**2,6**	(1,0)	10,3	2,1)	21,0	(2,8)	28,7	(2,6)	25,2	2,5)	10,0	(1,8)	**2,2**	(0,8)
Lithuania	4,3	(0,4)	16,0	0,8)	27,4	(0,9)	29,8	(0,9)	17,5	0,8)	4,5	(0,6)	0,4	(0,2)
Macao-China	1,4	(0,2)	8,9	0,5)	26,0	(1,0)	35,7	(1,1)	22,8	0,7)	5,0	(0,3)	0,3	(0,1)
Montenegro	17,3	(0,8)	33,0	1,2)	31,0	(0,9)	14,9	(0,7)	3,6	0,4)	0,3	(0,1)	0,0	a
Qatar	47,6	(0,6)	31,5	0,6)	13,9	(0,5)	5,0	(0,4)	1,6	0,1)	0,3	(0,1)	0,0	(0,0)
Romania	16,0	(1,5)	30,9	1,6)	31,8	(1,6)	16,6	(1,2)	**4,2**	0,8)	0,5	(0,1)	0,0	a
Russian Federation	**5,2**	(0,7)	17,0	1,1)	30,2	(0,9)	28,3	(1,3)	15,1	1,1)	3,7	(0,5)	0,5	(0,1)
Serbia	11,9	(0,9)	26,6	1,2)	32,3	(1,3)	21,8	(1,2)	6,6	0,6)	0,8	(0,2)	0,0	a
Slovenia	**2,8**	(0,3)	11,1	0,7)	23,1	(0,7)	27,6	(1,1)	22,5	1,1)	10,7	(0,6)	**2,2**	(0,3)
Chinese Taipei	1,9	(0,3)	9,7	0,8)	18,6	(0,9)	27,3	(0,8)	27,9	1,0)	12,9	(0,8)	1,7	(0,2)
Thailand	12,6	(0,8)	33,5	1,0)	33,2	(0,9)	16,3	(0,8)	4,0	0,4)	0,4	(0,1)	0,0	a
Tunisia	27,7	(1,1)	35,1	0,9)	25,0	(1,0)	10,2	(1,0)	1,9	0,4)	0,1	(0,1)	0,0	a
Uruguay	16,7	(1,2)	25,4	(1,1)	29,8	(1,5)	19,7	(1,1)	6,9	(0,5)	1,3	(0,2)	0,1	(0,1)

France is substantially below OECD average, already very weak compared to better performing countries in Asia ![6]

6 The reference given here to the PISA study is in no way an endorsement of its general value for assessing the results of students in a general and arbitrary context. We think on the contrary that it is of very limited interest when the point comes to ensuring that students have deeper knowledge and understanding. We certainly do not recommend such evaluations as a general practice in a working educational system.