Problems suggested by J.-P. Demailly.

PROBLEM 1: *Kawamata-Viehweg vanishing theorem for compact Kähler manifolds.*
Let L a line bundle over a compact Kähler manifold (X, ω) of complex dimension n.

Definition. *L is a numerically effective line bundle, nef for short, if for every $\epsilon > 0$, there is a smooth metric h_ϵ on L such that, its hermitian curvature $i\Theta_{h_\epsilon}$ satisfies $i\Theta_{h_\epsilon} \geq -\epsilon\omega$. Let L a nef line bundle on X. One defines the numerical dimension of L to be*

$$\nu(L) = \max\{k \in N\,;\; c_1(L)^k \neq 0 \text{ in } H^{2k}(X, \mathbb{R})\}$$

where K_X denotes the canonical bundle of X.

If $D = \sum \alpha_j D_j \geq 0$ is an effective $\mathbb{Q}$-divisor, we define the multiplier ideal sheaf $\mathcal{I}(D)$ to be equal to $\mathcal{I}(\varphi)$ (for a precise definition of the multiplier ideal sheaf $\mathcal{I}(\varphi)$, we refer to the quoted article of J.-P. Demailly) where $\varphi = \sum \alpha_j \log |g_j|$ is the locally defined plurisubharmonic function defined by generators g_j of $\mathcal{O}(D_j)$.

Let (X, ω) be a Kähler manifold and F a line bundle over X such that some positive multiple mF can be written $mF = L \otimes D$, where L is nef and D is an effective divisor.

Is it true that $H^q(X, K_X \otimes F \otimes \mathcal{I}(m^{-1}D)) = 0$ for all $q > n - \nu(L)$? For projective manifolds, this is a consequence of the Kawamata-Viehweg vanishing theorem and a trivial slicing argument by hyperplane sections (see e.g. Theorem 6.12 in J.-P. Demailly, Lecture Notes in Math. n° 1646).

PROBLEM 2: *Towards a Mori theory for Kähler varieties.*
Assume that the Kähler manifold (X, ω) is such that its canonical bundle K_X is not nef. Does there exist a rational curve C in X such that $-K_X \cdot C \leq n + 1$? Again, if X is projective, the answer is known to be positive.

PROBLEM 3: *Deformations of varieties in the class $\mathcal{C}$.*
Recall that a compact complex space is said to be in the class $\mathcal{C}$ if it is bimeromorphic to some compact complex Kähler manifold. It follows from the theory of Kodaira and Spencer (see Ann. of Math. 71, 1960) that small deformations of a compact Kähler manifold are again Kähler. However an example of F. Campana (see F. Campana, Math. Ann. 290, (1991)) shows that there exists a Moishezon (hence in the class $\mathcal{C}$), non Kähler threefold, wich has arbitrarily small deformations wich are not in the class $\mathcal{C}$.

Let $\mathcal{X} \to \Delta$ a deformation of compact complex manifolds over the unit disc in $\mathbb{C}$. Assume that X_s, the fiber over s, is Kähler for every $s \in \Delta \setminus \{0\}$. Does this imply that X_0 belongs to the class $\mathcal{C}$?

A positive answer is known under some restrictive assumptions on the family of Kähler metrics ω_s on the fibers X_s, $s \in \Delta \setminus 0$. Namely, the metrics should remain uniformly bounded, while their volume is bounded away from 0 when approaching the central fiber. Can one remove these restrictive assumptions ?