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MR0874578 Reviewed Demailly, J.-P. Mesures de Monge-Ampère et mesures pluriharmoniques. (French) [Monge-Ampère measures and pluriharmonic measures] *Séminaire sur les équations aux dérivées partielles, 1985–1986,* Exp. No. XIX, 15 pp., *École Polytech., Palaiseau,* 1986. (Reviewer: M. Klimek) 32F05 (32H15)

MR0813252 Reviewed Demailly, Jean-Pierre Mesures de Monge-Ampère et caractérisation géométrique des variétés algébriques affines. (French) [Monge-Ampère measures and geometric characterization of affine algebraic varieties] *Mém. Soc. Math. France (N.S.)* No. 19 (1985), 124 pp. (Reviewer: G. M. Khenkin) 32H35 (32C10 32F05)

MR0812325 Reviewed Demailly, Jean-Pierre Champs magnétiques et inégalités de Morse pour la d''-cohomologie. (French) [Magnetic fields and Morse inequalities for the d''-cohomology] *Ann. Inst. Fourier (Grenoble)* 35 (1985), no. 4, 189–229. (Reviewer: N. J. Hitchin) 58G25 (32J25 58E05)

MR0803576 Reviewed Demailly, Jean-Pierre Sur le calcul numérique de la constante d'Euler. (French) [The numerical computation of the Euler constant] *Gaz. Math.* No. 27 (1985), 113–126. (Reviewer: H. J. Godwin) 11Y60 (65D20 68Q25)

MR0800173 Reviewed Demailly, Jean-Pierre Propagation des singularités des courants positifs fermés. (French) [Propagation of singularities of closed positive currents] *Ark. Mat.* 23 (1985), no. 1, 35–52. (Reviewer: G. M. Khenkin) 32D20 (32C30)

MR0799607 Reviewed Demailly, Jean-Pierre Champs magnétiques et inégalités de Morse pour la d''-cohomologie. (French) [Magnetic fields and Morse inequalities for d''-cohomology] *C. R. Acad. Sci. Paris Sér. I Math.* 301 (1985), no. 4, 119–122. 32L10 (58G10)

MR0773103 Reviewed Demailly, Jean-Pierre Sur la propagation des singularités des courants positifs fermés. (French) [On the propagation of singularities of closed positive currents] *Complex analysis (Toulouse, 1983),* 53–64, Lecture Notes in Math., 1094, *Springer, Berlin,* 1984. (Reviewer: Aline Bonami) 32D15 (32C30 32F05)

MR0768075 Reviewed Demailly, Jean-Pierre Sur les transformées de Fourier de fonctions continues et le théorème de de Leeuw-Katznelson-Kahane. (French) [On Fourier transforms of continuous functions and a theorem of de Leeuw, Katznelson and Kahane] *C. R. Acad. Sci. Paris Sér. I Math.* 299 (1984), no. 10, 435–438. 43A25

MR0774974 Reviewed Demailly, Jean-Pierre; Gaveau, Bernard Majoration statistique de la courbure d'une variété analytique. (French) [Statistical upper bound for the curvature of an analytic manifold] *P. Lelong-P. Dolbeault-H. Skoda analysis seminar, 1981/1983,* 96–124, Lecture Notes in Math., 1028, *Springer, Berlin,* 1983. (Reviewer: Hung-Hsi Wu) 32F15 (53C20)

MR0774973 Indexed Demailly, Jean-Pierre Constructibilité des faisceaux de solutions des systèmes différentiels holonômes d'après Masaki Kashiwara. (French) [Constructibility of sheaves of solutions of holonomic differential systems after Masaki Kashiwara] *P. Lelong-P. Dolbeault-H. Skoda analysis seminar, 1981/1983,* 83–95, Lecture Notes in Math., 1028, *Springer, Berlin,* 1983. 58G07 (32C38)

MR0748313 Reviewed Demailly, J.-P. Sur la structure des courants positifs fermés. (French) [The structure of closed positive currents] *Conference on complex analysis, Nancy 82 (Nancy, 1982),* 52–62, Inst. Élie Cartan, 8, *Univ. Nancy, Nancy,* 1983. (Reviewer: Jürgen Leiterer) 32C30

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MR3445519 32J25 14Cxx 32Q45

Demailly, Jean-Pierre (F-GREN-F)

Towards the Green-Griffiths-Lang conjecture. (English summary)

Analysis and geometry, 141–159, *Springer Proc. Math. Stat.*, 127, *Springer*, *Cham*, 2015.

{A review for this item is in process.}
{For the collection containing this paper see MR3379815}

MR3446751 (Review) 32Q15 53C55

Demailly, Jean-Pierre (F-GREN-F)

Structure theorems for compact Kähler manifolds with nef anticanonical bundles. (English summary)

Complex analysis and geometry, 119–133, *Springer Proc. Math. Stat.*, 144, *Springer*, *Tokyo*, 2015.

Summary: "This survey presents various results concerning the geometry of compact Kähler manifolds with numerically effective first Chern class: structure of the Albanese morphism of such manifolds, relations tying semipositivity of the Ricci curvature with rational connectedness, positivity properties of the Harder-Narasimhan filtration of the tangent bundle."

{For the collection containing this paper see MR3446743}

MR3380444 (Review) 53C55 14M22
Campana, F. [**Campana, Frédéric**] (F-LOR-IEC);
Demailly, J.-P. [**Demailly, Jean-Pierre**] (F-GREN-F);
Peternell, Th. [**Peternell, Thomas**] (D-BAYR-IM)

Rationally connected manifolds and semipositivity of the Ricci curvature. (English summary)

Recent advances in algebraic geometry, 71–91, *London Math. Soc. Lecture Note Ser.*, 417, *Cambridge Univ. Press*, *Cambridge*, 2015.

The celebrated Beauville-Bogomolov decomposition theorem [A. Beauville, J. Differential Geom. **18** (1983), no. 4, 755–782 (1984); MR0730926] states (in one of its forms) that given a compact Kähler manifold X with zero real first Chern class, the universal cover $\tilde{X}$ of X splits holomorphically (and isometrically, once a Ricci-flat Kähler metric is chosen) into the product of a flat factor $\mathbb{C}^q$ and simply connected compact Kähler manifolds with special unitary or compact symplectic holonomy (i.e., Calabi-Yau or hyper-Kähler manifolds).

The main result of the paper under review is a decomposition theorem in the same spirit as the one above for compact Kähler manifolds with semi-positive first Chern class, or equivalently, with a Kähler metric whose Ricci curvature is non-negative. In this more general setting, a new type of factor may appear in the decomposition of the universal cover, namely, simply connected compact Kähler manifolds with merely unitary holonomy. In order to have a nice characterization of these factors, which are indeed shown to be rationally connected, the authors prove a new characterization of rationally connected compact Kähler manifolds, in the spirit of Mumford's conjecture.

The characterization is as follows. Let X be a compact Kähler manifold. Then, X is rationally connected if and only if for every invertible subsheaf $\mathcal{L} \subseteq \Omega^p_X$, $1 \leq p \leq n$, (resp. $\subseteq \mathcal{O}_X((T^*_X)^{\otimes m})$, $m \geq 1$), $\mathcal{L}$ is not pseudoeffective; moreover, in this case X is projective and the above is equivalent to the fact that for any ample line bundle A on X there exists a constant C such that

$$H^0\big(X, (T^*_X)^{\otimes m} \otimes A^{\otimes k}\big) = \{0\}, \quad \text{for all } m \geq Ck$$

(recall that Mumford's conjecture is exactly the last statement without the auxiliary ample line bundle).

The proof of the above criterion relies upon a "generalized holonomy principle" which is stated for holomorphic Hermitian vector bundles on compact Hermitian manifolds, with semi-positive mean curvature (i.e. the trace of the Chern curvature with respect to the fixed Hermitian metric on the manifold). This generalized holonomy principle states, among other things, that if the restricted holonomy of the Hermitian vector bundle in question is unitary, then no invertible subsheaf of any tensor power of the dual of the vector bundle can be pseudoeffective.

The paper ends with an appendix by the second-named author, in which he gives a related version of the generalized holonomy principle over flag varieties.

{For the collection containing this paper see MR3380552}

Simone Diverio

MR3329185 01A70

Demailly, Jean-Pierre; **van der Geer, Gerard**;
Hacon, Christopher [**Hacon, Christopher Derek**]; **Kawamata, Yujiro**;
Kobayashi, Toshiyuki [**Kobayashi, Toshiyuki**[1]]; **Miyaoka, Yoichi**;
Schmid, Wilfried (1-HRV-NDM)

Foreword [In commemoration of Professor Kunihiko Kodaira's centennial birthday, March 16, 2015].

J. Math. Sci. Univ. Tokyo **22** (2015), *no. 1, iii–iv.*

MR3444126 (Review) 65L09 70B05 93B30 93C15

Condaminet, Vincent (F-CAEN-LM); **Delvare, Franck** (F-CAEN-LM);
Choï, Daniel (F-CAEN-LM); **Demailly, Hélène** (F-DGA);
Grignon, Christophe (F-DGA); **Heddadj, Settie** (F-NEXTER)

Identification of aerodynamic coefficients of a projectile and reconstruction of its trajectory from partial flight data. (English summary)

Comput. Assist. Methods Eng. Sci. **21** (2014), *no. 3-4,* 177–186.

Summary: "Several optimization techniques are proposed both to identify the aerodynamic coefficients and to reconstruct the trajectory of a fin-stabilized projectile from partial flight data. A reduced ballistic model is used instead of a more general six degree of freedom (6DOF) ballistic model to represent the flight of the projectile. Optimization techniques are proposed in order to identify the set of aerodynamic coefficients. These techniques are compared when identifying the aerodynamic coefficients from both exact and noisy simulated partial flight data."

MR3238108 (Review) 32J17 32J25 32J27
Campana, Frédéric (F-NANC-IE); **Demailly, Jean-Pierre** (F-GREN-F);
Verbitsky, Misha (RS-HSE-ALG)
Compact Kähler 3-manifolds without nontrivial subvarieties. (English summary)
Algebr. Geom. **1** (2014), *no. 2,* 131–139.

In the paper, the authors give an interesting characterization of a compact Kähler threefold which has no nontrivial complex subvarieties, namely that such a compact Kähler threefold should be a torus. This result yields an essential step toward the classification of compact Kähler threefolds. The authors also prove the following result: if X is a normal compact Kähler threefold such that (1) X has only terminal singularities, (2) the canonical bundle of X is nef and (3) X has no effective divisor, then X is a cyclic quotient of a simple non-projective torus. *Atsushi Moriwaki*

MR3222365 01A70 32-03
Skoda, Henri (F-PARIS6-IMJ); **Demailly, Jean-Pierre** (F-GREN-FM);
Siu, Yum-Tong [Siu, Yum Tong] (1-HRV)
In memory of Pierre Lelong.
Henri Skoda, coordinating editor.
Notices Amer. Math. Soc. **61** (2014), *no. 6,* 586–595.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3191972 (Review) 32W20 32Q15 32U05 32U15 32U40 35B65 35J96 53C55
Demailly, Jean-Pierre (F-GREN-F); **Dinew, Sławomir** (1-RTG2-NDM);
Guedj, Vincent (F-TOUL3-IM);
Pham, Hoang Hiep [Phạm, Hoàng Hiệp] (VN-HNUE);
Kołodziej, Sławomir (PL-JAGLMC);
Zeriahi, Ahmed [Zériahi, Ahmed] (F-TOUL3-IM)

Hölder continuous solutions to Monge-Ampère equations. (English summary)

J. Eur. Math. Soc. (JEMS) **16** (2014), *no. 4,* 619–647.

Let (X,ω) be a compact n-dimensional Kähler manifold. The authors study the following complex Monge-Ampère equation on X:

$$\mathrm{MA}(u) := \frac{1}{\int_X \omega^n}(\omega + dd^c u)^n = f\omega^n, \text{ where } f \in L^p,\ p > 1.$$

The authors show that the solution to the above Monge-Ampère equation is Hölder continuous with exponent α arbitrarily close to $2/(1+nq)$, where q denotes the conjugate exponent of p. Furthermore, they obtain an analogue of this result when the cohomology class is semi-positive.

They also obtain a uniform Hölder continuity result in the case of uniformly bounded geometries if the L_p norms of the right-hand sides are uniformly bounded.

Next the authors obtain some properties of the range of Hölder continuous ω-plurisubharmonic functions under the complex Monge-Ampère operator, $\mathrm{MAH}(X,\omega) = \mathrm{MA}(X,\omega) \cup \mathrm{Hölder}(X,\mathbb{R})$.

Theorem 1. If $\mu \in \mathrm{MAH}(X,\omega)$ and $0 < f \in L^p(\mu)$ with $p > 1$ and $\int_X f d\mu = 1$, then $f\mu \in \mathrm{MAH}(X,\omega)$. In particular, the set $\mathrm{MAH}(X,\omega)$ is convex.

They further characterize the probability measures in $\mathrm{MAH}(X,\omega)$ which have finitely many isolated singularities of radial or toric type in terms of an integrability condition.

Theorem 2. Let μ be a probability measure with finitely many isolated singularities of radial or toric type. Then μ belongs to $\mathrm{MAH}(X,\omega)$ if and only if the following strong integrability condition holds:

$$\exp(-\varepsilon\,\mathrm{PSH}(X,\omega)) \subset L^1(\mu) \text{ for some } \varepsilon > 0.$$

Note that the strong integrability condition completely characterizes $\mathrm{MAH}(X,\omega)$ in dimension 1. It is not known whether the strong integrability condition characterizes $\mathrm{MAH}(X,\omega)$ when $n > 1$.

The paper is very well written and gives insight into the subject.

Muhammed Ali Alan

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3179606 (Review) 32W20 32C99 32U05 32U25
Demailly, Jean-Pierre (F-GREN-F); **Phạm, Hoàng Hiệp** (VN-HNUE-M)
A sharp lower bound for the log canonical threshold.
Acta Math. **212** (2014), *no. 1,* 1–9.

The purpose of this short article is to prove a sharp lower bound for the log canonical threshold of a plurisubharmonic function φ with an isolated singularity in an open subset of $\mathbb{C}^n$.

This threshold is defined as the supremum of constants $c>0$ such that $e^{-2c\varphi}$ is integrable in a neighborhood of the singular point, say 0.

The authors relate $c(\varphi)$ to the intermediate multiplicity numbers $e_j(\varphi)$ defined as the Lelong numbers of $(dd^c\varphi)^j$ at 0 (in particular, $e_0(\varphi)=1$ and $e_1(\varphi)$ is the Lelong number of the psh function φ).

The main result of the article is the sharp inequality

$$c(\varphi) \geq \sum_{j=0}^{n-1} \frac{e_j(\varphi)}{e_{j+1}(\varphi)}.$$

This improves an old result of H. Skoda, $c(\varphi) \geq 1/e_1(\varphi)$ [see Bull. Soc. Math. France **100** (1972), 353–408; MR0352517], as well as previous lower estimates [P. Åhag et al., Adv. Math. **222** (2009), no. 6, 2036–2058; MR2562773] and $c(\varphi) \geq n/e_n(\varphi)^{1/n}$ [see T. de Fernex, L. Ein and M. Mustaţă, J. Algebraic Geom. **13** (2004), no. 3, 603–615; MR2047683], which have been important in applications to birational geometry in recent years [see A. V. Pukhlikov, Izv. Ross. Akad. Nauk Ser. Mat. **66** (2002), no. 6, 159–186; MR1970356; I. Cheltsov, Uspekhi Mat. Nauk **60** (2005), no. 5(365), 71–160; MR2195677].

The proof consists of a reduction to the toric case. *Vincent Guedj*

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MR3089070 (Review) 32-02 14C30 32J25 32L20 32Q15 32U05 32U40 32W20
Demailly, Jean-Pierre (F-GREN-F)

Applications of pluripotential theory to algebraic geometry. (English summary)

Pluripotential theory, 143–263, *Lecture Notes in Math.*, 2075, *Springer, Heidelberg*, 2013.

This nice survey article is based on lectures by the author in 2011 and explains how transcendental techniques, particularly those coming from pluripotential theory, can be used as a powerful tool for the study of many problems in algebraic geometry.

The article is divided into four sections. In the first, the holomorphic Morse inequalities are studied, in particular their connections with Monge-Ampère operators and intersection theory.

The second section is focused on regularization of positive closed currents using Bergman kernel techniques, ultimately arising from the Ohsawa-Takegoshi theorem. These regularization results are used to study natural convex cones defined by suitable positivity conditions, in the cohomology space of a compact Kähler manifold, as well as their algebraic counterparts in the Néron-Severi space in the case of a smooth complex projective variety. The author discusses many important topics here: approximate

Zariski decompositions, the mobile intersection product introduced by Boucksom, the dual of the pseudoeffective cone, and more. He uses these techniques in Section 3 to study the asymptotic cohomology functionals introduced by Küronya, and some natural transcendental analogues.

Finally, Section 4 is devoted to the Green-Griffiths-Lang conjecture on the structure of entire curves in a projective manifold X (or, more generally, a directed projective manifold (X, V)) satisfying a suitable "general type" condition.

{For the collection containing this paper see MR3089067}

Mattias Jonsson

MR3089067 (Review) 32-06 32Uxx

Patrizio, Giorgio (I-FRNZ); **Błocki, Zbigniew** (PL-JAGL);
Berteloot, François (F-TOUL3-IM); **Demailly, Jean-Pierre** (F-GREN-FM)

★**Pluripotential theory.**

Lectures from the Centro Internazionale Matematico Estivo (CIME) Session held in Cetraro, 2011.

Edited by Filippo Bracci and John Erik Fornæss.

Lecture Notes in Mathematics, 2075.

Fondazione CIME/CIME Foundation Subseries.

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MR3070567 (Review) 14E30
Demailly, Jean-Pierre (F-GREN-NDM);
Hacon, Christopher D. [Hacon, Christopher Derek] (1-UT-NDM);
Păun, Mihai (F-NANC-NDM)

Extension theorems, non-vanishing and the existence of good minimal models. (English summary)

Acta Math. **210** (2013), *no. 2,* 203–259.

Let (X, Δ) be a log canonical pair in characteristic zero. One of the goals of the Minimal Model Program is to show, when the divisor $K_X + \Delta$ is pseudoeffective, that the pair (X, Δ) has a suitable birational model (called a good minimal model) on which the proper transform of $K_X + \Delta$ becomes semiample. In particular, conjecturally some multiple of $K_X + \Delta$ has sections, and this problem is known as nonvanishing. Even when one knows nonvanishing, it is a very difficult problem to show that a good minimal model exists. An essential ingredient in recent advances in the Minimal Model Program, related to both of the above-mentioned issues, is to find a result which enables one to extend sections from a suitable (prime) divisor on X, and then use induction on the dimension.

The paper under review finds such an extension result under some additional hypotheses. The main result of the paper is too technical to state here; however, the main application is in the case when the adjoint divisor $K_X + \Delta$ is nef, and it has the following simple form.

Corollary 1.8. Let $(X, S + B)$ be a plt pair, where S is a prime divisor and the coefficients of B lie in the interval $(0, 1)$. Assume that $K_X + S + B$ is nef, and that there exists an effective $\mathbb{Q}$-divisor $D \sim_{\mathbb{Q}} K_X + S + B$ with $S \subseteq \mathrm{Supp}(D) \subseteq \mathrm{Supp}(S + B)$. Then the restriction map

$$H^0(X, \mathcal{O}_X(m(K_X + S + B))) \to H^0(S, \mathcal{O}_S(m(K_X + S + B)))$$

is surjective for all sufficiently divisible positive integers m.

The proof is analytic, and it is a skillful use of an extension of the classical Ohsawa-Takegoshi theorem. The assumptions on the supports of D and $S + B$ are natural from the point of view of the Minimal Model Program. Further, previous results of a similar form assumed certain positivity of the divisor B, and this is the first extension result which works for any B, albeit at the expense of assuming that some multiple of $K_X + S + B$ has sections. However, the assumption that the pair is plt is very difficult to achieve in practice.

Section 8 of the paper contains very interesting applications of the main result of [C. D. Hacon, J. McKernan and C. Xu, "ACC for log canonical thresholds", preprint, arXiv:1208.4150, Ann. of Math. (2), to appear], which are of independent interest.

Vladimir Lazić

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Demailly, Jean-Pierre (F-GREN-IF)

Pierre Lelong: une œuvre fondatrice en analyse complexe et en géométrie analytique. (French) [Pierre Lelong: foundational work in complex analysis and analytic geometry]

Gaz. Math. No. 135 (2013), 63–66.

MR3019449 (Review) 14E99 32J18 32L05 53C26
Boucksom, Sébastien (F-PARIS7-IM); **Demailly, Jean-Pierre** (F-GREN-F); **Păun, Mihai** (F-STRAS); **Peternell, Thomas** (D-BAYR-IM)
The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension. (English summary)
J. Algebraic Geom. **22** (2013), *no. 2,* 201–248.

This is a paper whose results were reproduced in a monograph [R. K. Lazarsfeld, *Positivity in algebraic geometry. II*, Ergeb. Math. Grenzgeb. (3), 49, Springer, Berlin, 2004; MR2095472], re-proven in a new language [S. Boucksom, C. Favre and M. Jonsson, J. Algebraic Geom. **18** (2009), no. 2, 279–308; MR2475816] and applied in several other papers [for example F. Campana and T. Peternell, Bull. Soc. Math. France **139** (2011), no. 1, 41–74; MR2815027; C. D. Hacon and J. McKernan, Duke Math. J. **138** (2007), no. 1, 119–136; MR2309156] before it was officially published. Still, its publication is not only justified by setting the historic records right, but also by the fact that the paper is written in a setting different from its successors and contains discussions published nowhere else, on questions that follow naturally from its main result:

Theorem 1. On a non-uniruled projective complex manifold X the canonical bundle K_X is pseudoeffective.

This statement should be seen as a first fundamental step towards the Abundance Conjecture in arbitrary dimensions:

Conjecture 1. A projective complex manifold X has Kodaira dimension $\kappa(X)=-\infty$ if and only if X is uniruled.

The theorem is a consequence of a uniruledness criterion of Y. Miyaoka and S. Mori [Ann. of Math. (2) **124** (1986), no. 1, 65–69; MR0847952] and a general fact on pseudoeffective line bundles:

Theorem 2. A line bundle L on an n-dimensional projective complex manifold X is pseudoeffective if and only if $L\cdot C\geq 0$ for all *strongly moving curves* C that are curves $C=\mu(\widetilde{A}_1\cap\cdots\cap\widetilde{A}_{n-1})$ where $\mu\colon\widetilde{X}\to X$ is a modification of X and the $\widetilde{A}_i$ are sufficiently general very ample divisors on $\widetilde{X}$.

This theorem is one of several duality statements on convex cones in the spaces of curves and divisors on projective complex manifolds, or more generally Kähler manifolds:

- $\overline{NE}(X)$, the closed convex cone generated by irreducible curves, is dual to the convex cone $\overline{K}_{NS}(X)$ generated by nef divisors (a classical result by Kleiman [see R. Hartshorne, *Ample subvarieties of algebraic varieties*, Lecture Notes in Mathematics, Vol. 156, Springer, Berlin, 1970; MR0282977]).
- On an n-dimensional Kähler manifold X, the closed convex cone $\overline{K}(X)$ generated by Kähler forms on X is dual to the convex cone N generated by classes $[Y]\cap\omega^{p-1}$, where $Y\subset X$ ranges over p-dimensional analytic subsets of X, $p=1,2,\dots,n$, and ω ranges over the Kähler forms (a generalization of the Nakai-Moishezon criterion, shown by J.-P. Demailly and M. Păun [Ann. of Math. (2) **159** (2004), no. 3, 1247–1274; MR2113021]).
- Theorem 2 shows that the convex cone $SME(X)$ generated by all strongly movable curves is dual to the closed convex cone $\mathrm{Eff}(X)$ generated by all effective divisors on X. Since $\overline{ME}(X)$, the closed convex cone generated by all irreducible curves moving in a family covering X, is contained in $\mathrm{Eff}(X)^\vee=\overline{SME}(X)$, it is also true that $\overline{ME}(X)=\overline{SME}(X)$.
- The authors conjecture that Theorem 2 also holds in the Kähler setting:

  Conjecture 2. On an n-dimensional Kähler manifold the pseudoeffective cone $\mathcal{E}(X)$ of classes represented by positive $(1,1)$-currents is dual to the closed convex cone $\mathcal{M}(X)$ generated by classes of currents of the form $\mu_*(\widetilde{\omega}_1 \wedge \cdots \wedge \widetilde{\omega}_{n-1})$, where $\mu\colon \widetilde{X} \to X$ is a modification of X and the $\widetilde{\omega}_i$ are Kähler forms on $\widetilde{X}$.

In the paper this is shown for compact hyper-Kähler manifolds but remains wide open in the general case (see also the remarks below).

The main tool in the proof of Theorem 2 is Fujita's approximative Zariski decomposition of a big line bundle L on a projective complex manifold X: For arbitrary $\epsilon > 0$ there exist a modification $\mu_\epsilon\colon \widetilde{X}_\epsilon \to X$ and a decomposition $\mu_\epsilon^* L = E_\epsilon + D_\epsilon$ into an effective divisor E_ϵ and a big and nef divisor D_ϵ such that the volumes of L and D_ϵ differ at most by ϵ (proven by T. Fujita in [Kodai Math. J. **17** (1994), no. 1, 1–3; MR1262949] and by Demailly, L. Ein and Lazarsfeld in [Michigan Math. J. **48** (2000), 137–156; MR1786484]). A Kähler version of an approximative Zariski decomposition follows from a result of Demailly in [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622].

The main idea in the proof of Theorem 2 is that the fixed part E_ϵ and the moving part D_ϵ of such an approximative Zariski decomposition are almost orthogonal, with an estimate

$$D_\epsilon^{n-1} \cdot E_\epsilon \leq C \cdot \epsilon,$$

where C is a constant independent of ϵ. In turn, the proof of this orthogonality estimate on projective complex manifolds X relies on the inequality

$$(*) \qquad \mathrm{Vol}(A - B) \geq A^n - n \cdot A^{n-1} \cdot B$$

for two nef divisors A, B on X. There are simple proofs of $(*)$ on projective complex manifolds, but they also follow from the holomorphic Morse inequalities [see J.-P. Demailly, in *School on Vanishing Theorems and Effective Results in Algebraic Geometry (Trieste, 2000)*, 1–148, ICTP Lect. Notes, 6, Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2001; MR1919457]. On a Kähler manifold, $(*)$, with the optimal coefficient n, would follow from a conjectural transcendental version of these Morse inequalities. In the paper a version of $(*)$ with a coefficient quadratic in n is shown.

The approximative Zariski decomposition is also used to define a "movable intersection product" on the cone of pseudoeffective classes, with values $\langle \alpha_1 \cdots \alpha_k \rangle$ in the cone of non-negative (k,k)-classes. In particular, for $k = 1$ this product yields a "divisorial Zariski decomposition" of a pseudoeffective class α on a Kähler manifold X,

$$\alpha = \langle \alpha \rangle + N(\alpha)$$

where $N(\alpha)$ is a finite sum of effective $\mathbb{R}$-divisors from a countable set of irreducible divisors determined by X.

Furthermore, a *numerical dimension* of a pseudoeffective class α may be defined as

$$\mathrm{nd}(\alpha) = \max\{p \in \mathbb{N} : \langle \alpha^p \rangle \neq 0\},$$

and a generalized Abundance Conjecture can be stated:

Conjecture 3. On a projective complex manifold, or even a Kähler manifold X,

$$\kappa(X) = \mathrm{nd}(X) := \mathrm{nd}(c_1(X)).$$

This development of a movable intersection product closely follows Boucksom's thesis. A completely algebraic approach was taken by Boucksom, Favre and Jonsson [op. cit.], leading to the same results.

Next, the paper deals with natural extensions of pseudoeffectivity of line bundles to holomorphic vector bundles E on projective complex manifolds X. The vector bundle E is called *almost nef* if the restrictions $E|_C$ to curves $C \subset X$ not contained in a countable union of algebraic subsets of X are always nef. E is called *pseudoeffective* if $\mathcal{O}_{\mathbb{P}(E)}(1)$ is a pseudoeffective line bundle on $\mathbb{P}(E)$, and the non-nef locus of $\mathcal{O}_{\mathbb{P}(E)}(1)$ is not projected onto X. The non-nef locus $L_{\mathrm{nonnef}}(\alpha)$ of a pseudo-effective class α is constructed as the union of all logarithmic singularities that currents in perturbed classes $\alpha + \delta\omega$ have in common, where $\delta > 0$ and ω is a Kähler form. Note that $L_{\mathrm{nonnef}}(\alpha)$ contains all irreducible algebraic curves C such that $\alpha \cdot C < 0$, but may be bigger: an example on $\mathbb{P}^2$ blown up in several points is provided.

If E is almost nef then $\mathcal{O}_{\mathbb{P}(E)}(1)$ is pseudoeffective, by Theorem 2. But contrary to a claim of Demailly, T. Peternell and M. H. Schneider in [Internat. J. Math. **12** (2001), no. 6, 689–741; MR1875649] it is not certain whether an almost nef vector bundle is also pseudoeffective. In the paper this implication is at least shown for vector bundles of rank at most 3 with vanishing first Chern class and then applied to the tangent bundle T_X of a K3 surface X: neither is T_X almost nef, nor is $\mathcal{O}_{\mathbb{P}(T_X)}(1)$ pseudoeffective. The last statement was also shown by N. Nakayama [*Zariski-decomposition and abundance*, MSJ Mem., 14, Math. Soc. Japan, Tokyo, 2004; MR2104208].

Finally, the authors prove the Abundance Conjecture for projective 4-folds under the additional assumption that K_X is numerically trivial when restricted to curves in a large family:

Theorem 3. If the canonical bundle K_X on a projective 4-fold X is pseudoeffective and if there is a good covering family (C_t) of curves on X such that $K_X \cdot C_t = 0$ then $\kappa(X) \geq 0$.

A *good covering family* (C_t) of (generically irreducible) curves on X is either *non-connecting* or *strongly connecting*, that is, either two general points on X cannot be connected by a chain of curves C_t, or they can be connected by such a chain, furthermore avoiding any given codim ≥ 2 algebraic subset. Strong connectedness is needed to show that $C_t \cdot L = 0$ implies $\mathrm{nd}(L) = 0$ for a pseudo-effective divisor L. In that case $\kappa(L) \geq 0$ follows from the results of Campana and Peternell [op. cit.]. An example is constructed where $C_t \cdot L = 0$ does not imply $\mathrm{nd}(L) = 0$ if the family (C_t) is only connected; however this can (but need not) occur for $L = K_X$ only on very special X.

If $C_t \cdot K_X > 0$ for all strongly connected families (C_t) of curves, X should be of general type. The authors are at least able to show that $\kappa(X) \leq 0$ or $= \dim X$.

If C_t is a non-connecting family of curves then the proof of Theorem 3 applies the $C_{n,m}$-conjectures on the reduction map associated to the family (C_t) (see the results of Campana [Bull. Soc. Math. France **122** (1994), no. 2, 255–284; MR1273904; Ann. Inst. Fourier (Grenoble) **54** (2004), no. 3, 631–665; MR2097417]) and the log MMP to the basis of this reduction map. Both applications restrict the theorem to dimension 4. These methods resemble and sometimes use results from F. Ambro in [Math. Ann. **330** (2004), no. 2, 309–322; MR2089428]. *Thomas Eckl*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3014194 (Review) 32-03 32A10 32B10 32C35 32H99
Demailly, Jean-Pierre (F-GREN-F)
Henri Cartan et les fonctions holomorphes de plusieurs variables. (French)
[Henri Cartan and multivariate holomorphic functions]
*Henri Cartan & André Weil, mathématiciens du XX*e *siècle*, 99–168, *Ed. Éc. Polytech., Palaiseau*, 2012.

The paper is a review of some fundamental results on holomorphic functions of several variables, and of the role of Henri Cartan in the development of this theory, in particular in the theory of coherent sheaves which he developed and which stands now as one of the most fundamental tools in complex geometry and in algebraic geometry. The exposition is concise but self-contained and the stress is on the essential facts. A few historical remarks are useful for understanding the motivations behind the ideas. All this makes the text much more attractive than many other texts written on the subject. The bibliographical references organized in sections are also very useful.

{For the collection containing this paper see MR3059568}

Athanase Papadopoulos

MR3058660 (Review) 32Q45 14J70 32H30 32L10 32S65
Demailly, Jean-Pierre (F-GREN-F)
Hyperbolic algebraic varieties and holomorphic differential equations.
Acta Math. Vietnam. **37** (2012), *no. 4,* 441–512.

In this survey on Kobayashi hyperbolicity of complex varieties, the author updates his earlier notes [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539].

First, the author reviews the general techniques used in the study of entire curves in complex varieties in the general framework of directed manifolds. In comparison with the previous article, the author not only considers holomorphic subbundles of the tangent bundle but also saturated coherent subsheaves. This allows one to work in particular with singular holomorphic foliations. An interesting generalized Green-Griffiths-Lang conjecture is stated, claiming the algebraic degeneracy of entire curves in projective manifolds tangent to distributions of general type.

Then the author concentrates on one of the main new tools used to attack these problems: the holomorphic Morse inequalities. He gives a detailed account of his recent result [Pure Appl. Math. Q. **7** (2011), no. 4, Special Issue: In memory of Eckart Viehweg, 1165–1207; MR2918158] on the existence of many algebraic differential equations satisfied by entire curves on directed projective varieties of general type.

Finally, the author gives some applications in the setting of projective hypersurfaces of large degree, recovering a result of S. Diverio, J. Merker and E. Rousseau [Invent. Math. **180** (2010), no. 1, 161–223; MR2593279] with a better bound on the minimal degree ensuring that the Green-Griffiths-Lang conjecture is satisfied in this case.

Erwan Rousseau

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2978333 (Review) 32-02 14C30 14F18 32J25 32Q15 32U40
Demailly, Jean-Pierre (F-GREN-FM)
★**Analytic methods in algebraic geometry.**
Surveys of Modern Mathematics, 1.
International Press, Somerville, MA; Higher Education Press, Beijing, 2012. *viii*+231 *pp. ISBN* 978-1-57146-234-3

This book is a compilation and expansion of lecture notes that the author has written in recent years, most notably the Park City 2008 notes [in *Analytic and algebraic geometry*, 295–370, IAS/Park City Math. Ser., 17, Amer. Math. Soc., Providence, RI, 2010; MR2743818], which have a considerable overlap with this book. This is arguably the first written textbook on applications of analysis to problems in algebraic geometry; it is written by one of the leaders and developers of this field, and is aimed at advanced graduate students and other researchers who have already a working knowledge of the basic concepts of complex geometry.

This book is divided into twenty chapters. The first four chapters contain mostly introductory material about complex manifolds, Dolbeault cohomology, closed positive currents and Lelong numbers, Monge-Ampère operators, Hermitian vector bundles, the Bochner technique and vanishing theorems.

Chapter 5, "L^2 estimates and existence theorems", gives a quick introduction to the Hörmander L^2 estimates for $\overline{\partial}$ and to multiplier ideal sheaves, including Nadel's vanishing theorem.

Chapter 6, "Numerically effective and pseudo-effective line bundles", introduces several different notions of positivity for line bundles, their corresponding positive cones in cohomology, and proves the Kawamata-Viehweg vanishing theorem. It also covers analytic Zariski decompositions and Siu's uniform global generation theorem.

Chapter 7, "A simple algebraic approach to Fujita's conjecture", discusses an algebraic approach of Siu towards Fujita's conjecture.

Chapter 8, "Holomorphic Morse inequalities", describes (without proof) these inequalities, and then discusses their algebraic versions, asymptotic cohomology and a conjectural transcendental version of these inequalities.

Chapter 9, "Effective version of Matsusaka's big theorem", contains an exposition of Siu's proof of this theorem, with some simplifications and improvements due to the author.

Chapter 10, "Positivity concepts for vector bundles", discusses the relationship between Griffiths and Nakano positivity for vector bundles.

Chapter 11, "Skoda's L^2 estimates for surjective bundle morphisms", proves Skoda's division theorem, and the Briançon-Skoda theorem as an application.

Chapter 12, "The Ohsawa-Takegoshi L^2 extension theorem", proves this celebrated result, and rederives the Skoda division theorem from it.

Chapter 13, "Approximation of closed positive currents by analytic cycles", contains an exposition of the author's celebrated regularization procedure for closed positive currents. There is also a proof of the fact that a compact complex manifold is bimeromorphic to Kähler iff it supports a Kähler current. The author also introduces the complex singularity exponents of plurisubharmonic functions, relates them to log canonical thresholds, gives a sketch of proof of their semicontinuity, and finally discusses the relation between the Hodge conjecture and the approximation of (p, p) currents by algebraic cycles.

Chapter 14, "Subadditivity of multiplier ideals and Fujita's approximate Zariski decomposition", is an exposition of results due to Demailly-Ein-Lazarsfeld on this topic.

Chapter 15, "Hard Lefschetz theorem with multiplier ideal sheaves", proves a version of the Hard Lefschetz theorem for pseudo-effective line bundles.

Chapter 16, "Invariance of plurigenera of projective varieties", gives Păun's simplified proof of Siu's celebrated result.

Chapter 17, "Numerical characterization of the Kähler cone", is an exposition of this groundbreaking result of Demailly-Păun.

Chapter 18, "Structure of the pseudo-effective cone and mobile intersection theory", discusses results of Boucksom-Demailly-Păun-Peternell characterizing the dual of the pseudoeffective cone, and thus shows that a projective manifold is uniruled iff its canonical bundle is not pseudo-effective.

Chapter 19, "Super-canonical metrics and abundance", deals with recent results of Berman-Demailly on these topics, and ends with a discussion of Tsuji's approach towards the abundance conjecture.

Chapter 20, "Siu's analytic approach and Păun's non vanishing theorem", contains a very short introduction to Siu's analytic proof of the finite generation of the canonical ring, and of Păun's improved non-vanishing theorem.

To summarize, this book is essentially a panoramic view of Demailly's work on analytic geometry over the past 30 years, with an emphasis on very recent results and developments. The writing style is quite condensed, which makes it a demanding but extremely rewarding read for students interested in entering this exciting field of research, and also a very useful reference book for more advanced readers. The great variety of results covered (some of which appear here for the first time in book form) and the numerous open questions and conjectures that are scattered throughout this book make it an especially useful resource. *Valentino Tosatti*

MR2884031 (2012m:32035) 32U05 32Q15 32U40 32W20

Berman, Robert [Berman, Robert J.] (S-CHAL);
Demailly, Jean-Pierre (F-GREN-F)

Regularity of plurisubharmonic upper envelopes in big cohomology classes. (English summary)

Perspectives in analysis, geometry, and topology, 39–66, *Progr. Math.*, 296, *Birkhäuser/Springer*, *New York*, 2012.

The goal of this article is to establish a fundamental regularity result for upper envelopes of quasi-plurisubharmonic (q-psh) functions.

Assume that X is a compact complex manifold in the Fujiki class $\mathcal{C}$ (i.e. bimeromorphic to a Kähler manifold) and let $\{\alpha\} \in H^{1,1}(X, \mathbb{R})$ be a big cohomology class, i.e. a class that can be represented by a positive current which dominates a Hermitian form (it follows from the work of the second author and M. Păun [Ann. of Math. (2) **159** (2004), no. 3, 1247–1274; MR2113021] that such classes exist precisely on these manifolds).

Let α be a smooth closed $(1,1)$-form in $\{\alpha\}$. A function $\psi = X \to \mathbb{R} \cup \{-\infty\}$ is said to be α-plurisubharmonic if it is q-psh (i.e. locally given as the sum of a smooth and a plurisubharmonic function) and such that $\alpha + dd^c\psi \geq 0$ in the sense of currents. Let

PSH(X,α) denote the set of all α-psh functions and set

$$V_\alpha(x) := \sup\{\psi(x) \mid \psi \in \mathrm{PSH}(X,\alpha),\ \psi \le 0\}.$$

The authors show that V_α has locally bounded mixed complex derivatives $\partial^2 V_\alpha/\partial z_i \partial \overline{z_j}$ in the ample locus of α (a Zariski open subset) and derive several consequences.

The main technical tool is a regularization result of the second author [in *Contributions to complex analysis and analytic geometry*, 105–126, Aspects Math., E26, Friedr. Vieweg, Braunschweig, 1994; MR1319346]. A slightly stronger result was obtained by the first author in [Amer. J. Math. **131** (2009), no. 5, 1485–1524; MR2559862] when the cohomology class $\{\alpha\}$ is the first Chern class of a big line bundle. Here, however, the proof is somewhat simpler and will without any doubt find several other applications.

{For the collection containing this paper see MR2867634}

Vincent Guedj

MR2918158 (Review) 32Q45 14C30 32L20
Demailly, Jean-Pierre (F-GREN)

Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture. (English, French summaries)

Pure Appl. Math. Q. **7** (2011), *no. 4, Special Issue: In memory of Eckart Viehweg,* 1165–1207.

Consider an irreducible projective complex n-dimensional variety X of general type. According to the Green-Griffiths-Lang conjecture, the locus of rational, elliptic or more generally entire curves is expected to be contained in a strict Zariski closed subset of X.

The current strategy towards this result, originating in the works of A. Bloch [J. Math. Pures Appl. (9) **5** (1926), 19–66; JFM 52.0373.04], M. L. Green and P. A. Griffiths [in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557], J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539] and Y. T. Siu and S.-K. Yeung [Invent. Math. **124** (1996), no. 1-3, 573–618; MR1369429], is to first find a differential equation fulfilled by all the entire curves in X and then, as a second step, to derive from this differential equation an algebraic equation fulfilled by all such curves. Some special cases of the first issue have been worked out by Green and Griffiths [op. cit.] for surfaces, and by S. Diverio [Math. Ann. **344** (2009), no. 2, 293–315; MR2495771], J. Merker ["Complex projective hypersurfaces of general type: toward a conjecture of Green and Griffiths", preprint, arXiv:1005.0405] and G. Bérczi [in *Contributions to algebraic geometry*, 141–167, EMS Ser. Congr. Rep., Eur. Math. Soc., Zürich, 2012; MR2976941] for high degree hypersurfaces in projective spaces. The whole program was successfully worked out in the case of generic hypersurfaces of high degree in projective spaces by Siu [in *The legacy of Niels Henrik Abel*, 543–566, Springer, Berlin, 2004; MR2077584] and Diverio, Merker and E. Rousseau [Invent. Math. **180** (2010), no. 1, 161–223; MR2593279].

The main achievement of the present work is to settle the first issue in full generality. More precisely, for an irreducible projective complex variety X and a big subbundle V

of the tangent bundle of X, up to taking high enough order, the number of differential equations vanishing on an ample divisor which are fulfilled by all entire curves tangent to V has maximal growth with respect to the degree. The proof here goes through precise curvature estimates for high jet bundles. This provides new tools for working on jet spaces.

The author also offers some potential strategies for the second issue, which, in view of an example of Diverio and Rousseau showing that the exceptional set and the Green-Griffiths locus do not always coincide, may require some additional non-rigidity properties. *Christophe Mourougane*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2858170 (2012j:32021) 32L20 32L10 32W20
Demailly, Jean-Pierre (F-GREN-F)
A converse to the Andreotti-Grauert theorem. (English, French summaries)
Ann. Fac. Sci. Toulouse Math. (6) **20** (2011), *Fascicule Spécial*, 123–135.

Let X be a compact complex manifold, and L a line bundle over X. The Andreotti-Grauert vanishing theorem states that if for some integer q and some $u \in c_1(L)$ the form $u(z)$ has at least $n-q+1$ positive eigenvalues everywhere then $H^j(X, L^{\otimes k}) = 0$ for $j \geq q$ and $k >> 1$. The holomorphic Morse inequalities [in J.-P. Demailly, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325] establish an upper bound for the q-th asymptotic cohomology functional defined as

$$\widehat{h}^q(X, L) := \limsup_{k\to+\infty} \frac{n!}{k^n} h^q(X, L^{\otimes k})$$

and for the q-th asymptotic holomorphic Morse sum of L defined as

$$\widehat{h}^{\leq q}(X, L) := \limsup_{k\to+\infty} \frac{n!}{k^n} \sum_{0\leq j\leq q} (-1)^{q-j} h^j(X, L^{\otimes k}),$$

of the type $\widehat{h}^q(X, L), \widehat{h}^{\leq q}(X, L) \leq \inf_{u\in c_1(L)} \int_D (-1)^q u^n$, where $u \in c_1(L)$ and, in the first case, D is the open set of X made of points where u has signature $(n-q, q)$ while, in the second case, it is the open set of points of X where u has signature $(n-j, j)$ with $0 \leq j \leq q$.

In the paper under review, the author asks for the following converse of the Andreotti-Grauert vanishing theorem. If it is known that the cohomology groups are asymptotically small in a certain degree q, is it true that there exists a Hermitian metric on L with suitable curvature, i.e., with almost no q-index points? This question is clearly related to having equality in the previous holomorphic Morse inequalities. In fact, the author proves that, if X is projective, then $\mathrm{Vol}(X, L) := \widehat{h}^0(X, L) = \inf_{u\in c_1(L)} \int_D (-1)^q u^n$.

In the case of higher cohomology groups, the author proves that if X is a complex projective surface, then the limsups involved in the definition of $\widehat{h}^q(X, L)$ and $\widehat{h}^{\leq q}(X, L)$ are in fact limits and equalities occur in the holomorphic Morse inequalities.

Filippo Bracci

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MR2743818 (2012g:32001) 32-02 14F18 14J40 32C30 32J25 32Q15 32U40
Demailly, Jean-Pierre (F-GREN-F)
Structure theorems for projective and Kähler varieties.
Analytic and algebraic geometry, 295–370, *IAS/Park City Math. Ser.*, 17, *Amer. Math. Soc.*, *Providence*, *RI*, 2010.

This is a thorough survey of analytic approaches to problems in complex algebraic geometry, exemplified by the results of Păun, Boucksom, Tsuji, Campana and the author on positivity.

The paper is organised in eight sections. Section 1, 'Numerically effective and pseudo-effective $(1,1)$ classes', gives the basic definitions of such concepts as nefness and other types of positivity, and multiplier ideal sheaves. It then covers analytic Zariski decomposition; vanishing; Siu's uniform global generation theorem; and Hard Lefschetz in its multiplier ideal sheaf version. Section 2 is entitled 'Holomorphic Morse inequalities': it states these results, asymptotic estimates for $h^q(X, E \otimes \mathcal{O}(kL))$, first in terms of curvature integrals and then algebraically. Section 3 is called 'Approximation of closed positive $(1,1)$-currents by divisors' and goes into rather more analytic detail than we find elsewhere in the paper. One reward is an essentially self-contained proof that a compact complex manifold is of class $\mathcal{C}$ (that is, bimeromorphic to a Kähler manifold) if and only if it admits a Kähler current. Another is a technical result that essentially means that the cone of closed positive currents is a completion of the cone of effective $\mathbb{Q}$-divisors. This, and the fact that the cone of currents is locally compact, is what makes the study of currents useful for the study of asymptotic behaviour of linear systems. This section also contains a sketch of Tian's α invariant and the associated criterion for the existence of Kähler-Einstein metrics.

Section 4, 'Subadditivity of multiplier ideals and Fujita's approximate Zariski decomposition', deals with those topics and gives a geometric interpretation of the volume of a line bundle as the growth of the moving self-intersection of the linear system $|kL|$. Section 5 is called 'Numerical characterization of the Kähler cone': it raises some hard questions about the duality of positive cones in $H^{p,p}$ and $H^{n-p,n-p}$ but considers only the cases $p=1$ and $n-p=1$, the others being at present out of reach. In those cases there are rather satisfactory results, including the theorem that if X is projective then the Kähler cone is the same as the cone of numerically positive real $(1,1)$-classes: this was a completely new result when it was proved by the author and Păun in 2004, even though it is a special case of their more general result for Kähler manifolds. A consequence is the invariance of the Kähler cone under very general deformations.

Section 6, 'Structure of the pseudo-effective cone and mobile intersection theory', deals with the results of Boucksom and compares various positive cones in $H^{n-k,n-k}(X)$ (mostly real, mostly $k=1$), interpreting some of them in terms of mobility of linear systems, Zariski decomposition, etc., and ending with a generalised version of the abundance conjecture (also apparently out of reach at present). This leads into Section 7, 'Super-canonical metrics and abundance', which outlines the recent work of the author and Boucksom on these topics and also outlines Tsuji's ideas about the positivity of relative canonical divisors and the invariance of plurigenera. In addition, it contains a conjectural approach to abundance also suggested by Tsuji. The final Section 8, 'Siu's analytic approach and Păun's non vanishing theorem', covers some recent developments very briefly—although Siu's approach (to the results of Birkar, Cascini, Hacon and McKernan) was announced in 2006, the details are not yet fully worked out.

The paper is thorough and covers a wide range of results. A few misprints have crept in: the mathematical ones are mostly easily corrected, but a more serious error, for a paper likely to be given to research students as background, is that the bibliography is badly incomplete. Many important papers are cited in the text but not listed. The experts will identify them easily, and may find their students asking them to do so.

{For the collection containing this paper see MR2742533}

G. K. Sankaran

MR2667492 01A70

Demailly, Jean-Pierre (F-GREN); **Kobayashi, Shoshichi** (1-CA); **Narasimhan, Raghavan** (1-CHI); **Siu, Yum-Tong** (1-HRV)

Cartan and complex analytic geometry.

Notices Amer. Math. Soc. **57** (2010), *no. 8,* 952–960.

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MR2684780 (2012a:32017) 32L10 32C35 32W20
Demailly, Jean-Pierre (F-GREN-F)

Holomorphic Morse inequalities and asymptotic cohomology groups: a tribute to Bernhard Riemann. (English, French summaries)

Milan J. Math. **78** (2010), *no. 1,* 265–277.

This expository paper presents several interesting results on the asymptotic q-cohomology functions for tensor powers of line bundles in connection with the author's previous work on holomorphic Morse inequalities [Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325]. The asymptotic q-cohomology function is an upper semi-continuous, positively homogeneous function defined on the real Néron-Severi subspace $\mathrm{NS}_{\mathbf{R}}(X)$ of the Bott-Chern cohomology group $H^{1,1}_{\mathrm{BC}}(X, \mathbf{R})$ of a compact complex manifold. It is shown that these functions are locally Lipschitz continuous on the divisorial Néron-Severi subspaces $\mathrm{DNS}_{\mathbf{R}}(X)$, and they are a natural generalization of the notion of volume of a line bundle. This paper also contains several open questions relating these functions to certain Monge-Ampère integrals. *Siqi Fu*

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MR2647006 (2012e:32039) 32Q20 32J27 32W20
Demailly, Jean-Pierre (F-GREN-F); **Pali, Nefton** (F-PARIS11)
Degenerate complex Monge-Ampère equations over compact Kähler manifolds. (English summary)
Internat. J. Math. **21** (2010), *no. 3,* 357–405.

The authors obtain the existence and uniqueness of the solutions of a quite general type of degenerate complex Monge-Ampère equations and investigate their regularity. They use some techniques developed by Yau, Bedford-Taylor (BT) and Tsuji in order to obtain some very general and sharp results on the above problems. Theorem 1.2: Let X be a compact Kähler manifold of complex dimension n and let χ be a $(1,1)$-cohomology class admitting a smooth closed semi-positive $(1,1)$-form ω such that $\int_X \omega^n > 0$. (A) For any $L\ \log^{n+\varepsilon}\ L$-density $v \geq 0$, $\varepsilon > 0$, such that $\int_X v = \int_X\ \chi^n$, there exists a unique closed positive current $T \in \mathrm{BT}_\chi$ such that $T^n = v$. Moreover, this current possesses bounded local potentials over X and continuous local potentials outside a complex analytic set $\Sigma_\chi \subset X$. This set depends only on the class χ and can be taken to be empty if the class χ is Kähler. (B) In the special case of a density $v \geq 0$ possessing complex analytic singularities the current T is also smooth outside the complex analytic subset $\Sigma_\chi \cup Z(v) \subset X$, where $Z(v)$ is the set of zeros and poles of v. Theorem 1.3: Let X be a smooth complex projective variety of general type. If the canonical bundle is nef, then there exists a unique closed positive current $\omega_E \in \mathrm{BT}^{\log}_{2\pi c_1(K_X)}$ solution of the Einstein equation $\mathrm{Ric}(\omega_E) = -\omega_E$. This current possesses bounded local potentials over X and defines a smooth Kähler metric outside a complex analytic subset Σ which is empty if and only if the canonical bundle is ample. Theorem 1.4: Let X be a smooth variety of general type and let $SB \subset \Sigma$ be respectively the stable and augmented stable base locus of the canonical bundle K_X. Then there exists a closed positive current $\omega_E \in 2\pi c_1(K_X)$ over X with locally bounded potentials over $X \smallsetminus SB$, and a solution of the Einstein equation $\mathrm{Ric}(\omega_E) = -\omega_E$ over $X \smallsetminus SB$ which restricts to a smooth (nondegenerate) Kähler-Einstein metric over $X \smallsetminus \Sigma$. The L^∞-estimate used by the authors allows them to solve the conjecture of Tian: Let (X, ω_X) be a polarized compact connected Kähler manifold of complex dimension n, (Y, ω_Y) be a compact irreducible Kähler space of complex dimension $m \leq n$, $\pi\colon X \to Y$ be a surjective holomorphic map and $0 \leq f \in L\ \log^{n+\varepsilon}\ L(X, \omega_X^n)$, for some $\varepsilon > 0$ such that $1 = \int_X f\omega_X^n$. Set $K_t := \{\pi^*\omega_Y + t\omega_X\}^n > 0$ for $t \in (0,1)$. Then the solution of the complex Monge-Ampère equations $(\pi^*\omega_Y + t\omega_X + i\partial\overline{\partial}\psi_t)^n = K_t f \omega_X^n$ satisfy the uniform L^∞-estimate $\mathrm{Osc}(\psi)_t := \sup_X \psi_t - \inf_X \psi_t \leq C < +\infty$ for all $t \in (0,1)$. *V. Oproiu*

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MR2742678 (2011j:32055) 32W20
Demailly, Jean-Pierre (F-GREN-M2)

Estimates on Monge-Ampère operators derived from a local algebra inequality. (English, French summaries)

Complex analysis and digital geometry, 131–143, *Acta Univ. Upsaliensis Skr. Uppsala Univ. C Organ. Hist.*, 86, *Uppsala Universitet, Uppsala*, 2009.

The author proves an integrability theorem for plurisubharmonic (psh) functions. Let K be a compact subset of a domain Ω in $\mathbb{C}^n$, and u a psh function in Ω which off K takes values between $-A$ and 0. Suppose

$$\int_\Omega (dd^c u)^n \le M < n^n \quad \left(d^c = \frac{1}{2\pi}(\overline{\partial} - \partial)\right).$$

Then

$$\int_K e^{-2u}\, dV \le C \quad (dV \text{ the Lebesgue measure})$$

with C depending only on Ω, K, A, M. This is no longer true if $M = n^n$ since for $u(z) = n \log|z|$ the function e^{-2u} is not integrable and the Monge-Ampère mass of u is exactly n^n. The proof is carried out by reduction to an inequality

$$\mathrm{lc}(I) \ge n e(I)^{-1/n},$$

relating log-canonical thresholds $\mathrm{lc}(I)$ and the Hilbert-Samuel multiplicity $e(I)$ of an ideal I of germs of holomorphic functions near zero with zero variety $V(I)$ equal to $\{0\}$. This inequality is due to A. Corti [in *Explicit birational geometry of 3-folds*, 259–312, London Math. Soc. Lecture Note Ser., 281, Cambridge Univ. Press, Cambridge, 2000; MR1798984] for $n = 2$, and to T. de Fernex, L. M. H. Ein and M. Mustaţă [J. Algebraic Geom. **13** (2004), no. 3, 603–615; MR2047683] in the general case. The reduction uses an approximation (due to the author and based on the Ohsawa-Takegoshi theorem) of general u by psh functions of the form $\log\sum|g_j|$, g_j holomorphic, and the semicontinuity theorem for complex singularity exponents of psh functions from [J.-P. Demailly and J. Kollár, Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 4, 525–556; MR1852009]. The author also indicates that by having an independent, “analytic” proof of the integrability statement one could use the arguments of his proof to show the inequality of Corti, de Fernex, Ein and Mustaţă. The analytic proof, relying on pluripotential theory, was later given in [P. Åhag et al., Adv. Math. **222** (2009), no. 6, 2036–2058; MR2562773].

{For further information pertaining to this item see [A. Zériahi, in *Complex analysis and digital geometry*, 144–146, Acta Univ. Upsaliensis Skr. Uppsala Univ. C Organ. Hist., 86, Uppsala Universitet, Uppsala, 2009; MR2742763].}

{For the collection containing this paper see MR2742180}

Sławomir Kołodziej

MR2393263 (2009b:32039) 32Q57 14K99 32L05 32Q15
Demailly, Jean-Pierre (F-GREN-F); **Hwang, Jun-Muk** (KR-AIST-SM);
Peternell, Thomas (D-BAYR-IM)

Compact manifolds covered by a torus. (English summary)

J. Geom. Anal. **18** (2008), *no. 2,* 324–340.

Suppose that A is a complex torus and $f\colon A \to X$ is a surjective map to a complex manifold. What can be said about the structure of X? O. Debarre [C. R. Acad. Sci. Paris Sér. I Math. **309** (1989), no. 2, 119–122; MR1004953] showed that X is $\mathbb{P}^n$ if A is a simple abelian n-fold and f is not an isogeny. Hwang and N. Mok [Math. Z. **238** (2001), no. 1, 89–100; MR1860736] extended this to general abelian varieties, showing that X is an iterated projective space bundle over an étale quotient Y of an abelian variety.

Here A is allowed to be any complex torus (not necessarily algebraic), and it is shown that X is Kähler, and that there is an étale map $X' \to X$ such that X' is a product $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_k} \times B$ of projective spaces and its Albanese torus B. Moreover, X is an étale quotient of X', so X is a $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_k}$-bundle over an étale quotient of B.

The fact that X is Kähler is proved by using the easy fact that f is equidimensional (it factors as $A \to A/S \to X$, where S is a subtorus and $A/S \to X$ is finite). It is then a general fact, due to J. Varouchas, that if Y is a Kähler manifold and $g\colon Y \to Z$ is a proper surjective holomorphic map to a complex manifold Z, then Z is Kähler. Varouchas [Math. Ann. **283** (1989), no. 1, 13–52; MR0973802] proved this for complex spaces, with only weak conditions on X, and the authors here give a quick proof for the case of g finite, which is all that they need. In an appendix they also give a short proof of Varouchas's result under an extra assumption on the singularities of Z.

Back to the main stream of the paper and the bundle structure of X. The map $f\colon A \to X$ has ramification divisor $R = f^*\mathcal{O}_X(-K_X)$, so the anticanonical morphism of X induces a morphism from A whose image is a quotient torus V. The trick is to replace X by an étale cover $\tilde{X}$ of largest possible irregularity. Then $\tilde{X}$ is covered by a complex torus $\tilde{A}$, which is isogenous to the product of $\mathrm{Alb}(\tilde{X})$ and a torus $\tilde{V}$ covering the anticanonical image of $\tilde{X}$. Under these circumstances the fibres of the Albanese map of $\tilde{X}$ are products of projective spaces. These methods, and some of the intermediate results, come from the techniques of Demailly, Peternell and M. H. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325], though the context there is rather different because here the tangent bundle of X is not a priori nef. *G. K. Sankaran*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2344027 01A70 01A60 46-03
Demailly, Jean-Pierre; **Kosarew, Siegmund**; **Malgrange, Bernard** (F-GREN-F)
Adrien Douady et les espaces analytiques banachiques. (French) [Adrien Douady and Banach analytic spaces]
Gaz. Math. No. 113 (2007), 35–38.

MR2334190 (2008k:32057) 32J25 14C30 32L20 32Q15
Demailly, Jean-Pierre (F-GREN-F)
Kähler manifolds and transcendental techniques in algebraic geometry. (English summary)
International Congress of Mathematicians. Vol. I, 153–186, *Eur. Math. Soc.*, *Zürich*, 2007.

Written from the perspective of one of the leading contemporary authorities in the field, this survey traces the development of transcendental algebraic geometry from the early decades of the last century through to the present, with ample indication of its future directions. It begins with the work of Hodge, and the famous conjecture concerning the role of algebraic cycles with rational coefficients as generators of cohomology for projective manifolds. The author points to the impetus it has given the development of Kähler geometry as a more general framework for the study of complex varieties, this in spite of the fact that the conjecture is well-known to be false for Kähler manifolds in general. A brief summary is given of the L^2-existence theory of the Cauchy-Riemann equation for (p,q)-forms with coefficients in a Hermitian-holomorphic vector bundle, associated prominently with the names of Bochner, Kodaira, Kohn, Andreotti-Vesentini, Hörmander and Skoda among many others, as well as some of the theory's most striking consequences, such as the vanishing theorems of Kodaira, Nakano, Kawamata-Viehweg and Nadel, the extension theorem of Ohsawa and Takegoshi, and the embedding theorem of Kodaira. The significance of this last result for the recent development of transcendental algebraic geometry rests largely on the equivalence it establishes between the existence of very ample line bundles and that of Kähler metrics representing classes in integer-valued cohomology. With the introduction of the Kähler and pseudo-effective cones in $H^{1,1}(X,\mathbf{R})$ for a compact Kähler manifold X, there is a shift in emphasis towards the study of closed positive currents which characterizes the contemporary theory. The intersection of the Kähler cone with the Néron-Severi lattice is generated by ample divisors, while the "numerically effective" , or "nef", divisors generate its closure. The term "big divisor" is used to refer to the generators of the interior of the Néron-Severi part of the pseudo-effective cone. Major contributions to the theory of approximation of currents and their Zariski decomposition have been made by the author and S. Boucksom, a notable corollary being the fact that a compact complex manifold is seen to be bimeromorphically equivalent to a Kähler manifold if and only if it carries a Kähler current. Further collaboration between the author and M. Paun has led to a natural generalization of the Nakai-Moishezon criterion which yields a numerical characterization of the Kähler cone. The deformation theory of Kähler structures also begins with a result of Kodaira, to the effect that every Kähler surface is a deformation-limit of algebraic surfaces. By contrast, a series of decisive counterexamples to such a property in higher dimension have been constructed by C. Voisin. Another theorem of Kodaira and Spencer refers to the local deformation-stability of Kähler structures. A global stability theorem for Kähler deformations due to the author and Paun is foreshadowed in dimension two by independent work of Buchdahl and Lamari. These results support the conjecture that all non-Kähler fibres in the deformation space of a Kähler manifold are parametrized by a finite (or perhaps countable) union of analytic substrata of the base space. The last two sections survey work of the author and his collaborators on the duality theory of closed positive currents and their positive cones. In particular, the concepts of "volume" and "numerical dimension" of classes (the latter being conjectured to be equivalent, in the case of the Chern class of the canonical bundle, to the Kodaira dimension) are outlined as tools for the characterization of ampleness. One striking consequence is that any projective manifold which is not uniruled has a pseudo-effective canonical divisor. A synopsis of the current state of the minimal models program in higher dimension and its relationship to Siu's work on deformation-invariance of plurigenera is also covered in conclusion.

{For the collection containing this paper see MR2334180}

Adam Gregory Harris

MR2142242 (2006d:32033) 32Q15 32G20 32J27
Demailly, Jean-Pierre (F-GREN-F); **Eckl, Thomas** (D-KOLN);
Peternell, Thomas (D-BAYR-IM)

Line bundles on complex tori and a conjecture of Kodaira.

Comment. Math. Helv. **80** (2005), *no. 2,* 229–242.

A compact Kähler manifold is called almost algebraic if it can be approximated by smooth projective varieties. K. Kodaira proved in [Ann. of Math. (2) **78** (1963), 1–40; MR0184257] that every Kähler surface is almost algebraic. The statement that this should be true also in higher dimensions is known as the Kodaira conjecture. Recently, C. Voisin ["On the homotopy types of Kähler manifolds and the birational Kodaira problem", preprint, arxiv.org/abs/math/0410040] and K. Oguiso ["Automorphisms of hyperkähler manifolds in the view of topological entropy", preprint, arxiv.org/abs/math/0407476] constructed counterexamples by constructing rigid non-algebraic Kähler threefolds. The present paper, which was completed before the counterexamples appeared, gives some observations concerning the Kodaira conjecture. A certain blow-up of a $\mathbb{P}^3_1$-bundle over a 3-dimensional complex torus with Picard number ≥ 3 is shown to be rigid. It turns out, however, that these complex tori are algebraic. Some interesting generalizations are also considered.

H. Lange

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2112584 (2006b:14011) 14C25 32J27
Demailly, Jean-Pierre (F-GREN-IF)

On the geometry of positive cones of projective and Kähler varieties. (English summary)

The Fano Conference, 395–422, *Univ. Torino*, *Turin*, 2004.

Some of the more fundamental problems and results in complex geometry revolve around such questions as when a complex manifold would be projective or Kähler, or how much of its geometry could be determined by divisors and curves. All projective manifolds are Kähler, and a famous theorem of Kodaira proves that a Kähler manifold (X, ω) is projective precisely when the class $[\omega] \in H^{1,1}(X) \subset H^2(X, \mathbb{R})$ moreover represents a class in $H^2(X, \mathbb{Z})$. Kodaira had also conjectured that a compact complex surface admits a Kähler metric if and only if the first Betti number is even. A closely related question concerns the ampleness of holomorphic line bundles L on X. When X is projective, the Nakai-Moishezon criterion establishes that ampleness of L is equivalent to having a strictly positive integral for the p-th exterior power of the Chern class of L over any algebraic subset of dimension p for $1 \leq p \leq n = \dim(X)$. Mori's theory of complex three-manifolds brought new techniques to bear on the projective context via the geometry of cones of divisors and curves lying within their respective cohomology groups. For example, a conjecture of Fano asserts that a projective X is "uniruled" by rational curves precisely when the Chern class of the canonical line bundle lies outside the closure of the cone of effective divisors. The article under review is a survey of relatively recent achievements of the author and his collaborators, S. Boucksom, M. Paun and T. Peternell, in further unifying and extending the theory surrounding these questions. Central to their programme are the powerful techniques associated with positive currents of type $(1,1)$ on compact Kähler manifolds, and the interplay between the open convex cone of Kähler forms and the enveloping closed convex cone of positive $(1,1)$-currents (the "pseudo-effective" cone). While some basic familiarity with Kähler geometry and the theory of currents is assumed, the author's exposition is designed to be informative to the non-specialist. Among the results surveyed, some highlights are a generalization of the Nakai-Moishezon criterion and its application to the characterisation of Kähler currents on compact complex manifolds, as well as a theory of Poincaré duality between cones of positive currents of type $(1,1)$ and $(n-1, n-1)$, which leads in particular to a proof of Fano's conjecture.

{For the collection containing this paper see MR2112562}

Adam Gregory Harris

MR2113021 (2005i:32020) 32J27 32Q15
Demailly, Jean-Pierre (F-GREN); **Paun, Mihai** (F-STRAS)
Numerical characterization of the Kähler cone of a compact Kähler manifold. (English summary)
Ann. of Math. (2) **159** (2004), *no. 3,* 1247–1274.

This article gives a beautiful solution of a long-standing basic problem in Kähler geometry and, as such, can be viewed as a classic. It is likely to have a lasting impact on the field.

The problem was to generalize the classical Nakai-Moishezon criterion of ampleness to a numerical characterization of the Kähler cone of a compact Kähler manifold.

Let us first recall the statement of the Nakai-Moishezon theorem. Let k be a field and X be a projective scheme over k. Let L be a Cartier divisor on X. Then L is ample iff for every positive dimensional reduced closed subscheme $Z \subset X$, $L^{\dim Z} \cdot Z > 0$.

If $k = \mathbf{C}$ and X is smooth, we can reformulate this using Kodaira's theorem that ample divisors L on X are characterized by the existence of a smooth Hermitian metric of positive curvature or, in an equivalent fashion, by the fact that the first Chern class $c_1(L)$, as an element of the vector space $H^{1,1}(X)$ of degree 2 de Rham real cohomology classes represented by closed $(1,1)$-forms, has a Kähler representative. The open convex cone in $H^{1,1}(X)$ consisting of classes with a Kähler representative is called the Kähler cone and will be denoted by $\mathcal{K}(X)$.

Thus, we get the following statement: Let X be a complex projective manifold. Let $\mathrm{NS}(X) \subset H^{1,1}(X)$ be the subset of $H^{1,1}(X)$ consisting of classes with integral periods. A class $\omega \in \mathrm{NS}(X)$ lies in $\mathcal{K}(X)$ iff $\int_Z \omega^{\dim Z} > 0$ for every positive-dimensional closed analytic subset Z of X.

We will denote by $\mathcal{P}(X)$ the set of classes ω cut out by the conditions that $\int_Z \omega^{\dim Z} > 0$ for every positive-dimensional closed analytic subset Z of X.

It was widely believed that a similar result holds for general real $(1,1)$ classes, namely that $\mathcal{K}(X) = \mathcal{P}(X)$.

The article under review confirms this conjecture in the more general case of compact Kähler manifolds. Here the statement should be modified to the effect that $\mathcal{K}(X)$ is a connected component of $\mathcal{P}(X)$.

The proof consists in a reduction to the nef case and a subtle application of Yau's fundamental work on the solution of the inhomogeneous complex Monge-Ampère equation in which the volume form acquires a singularity. *Philippe P. Eyssidieux*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2039988 (2005f:32033) 32L20 32J27
Demailly, Jean-Pierre (F-GREN-F); **Peternell, Thomas** (D-BAYR-IM)

A Kawamata-Viehweg vanishing theorem on compact Kähler manifolds. (English summary)

Surveys in differential geometry, Vol. VIII (*Boston, MA*, 2002), 139–169, *Surv. Differ. Geom.*, *VIII*, *Int. Press*, *Somerville*, *MA*, 2003.

In this article the authors prove a partial generalization of the Kawamata-Viehweg vanishing theorem for a normal compact Kähler space X of dimension n: if L is a nef divisor with $L^2 \neq 0$, then $H^q(X, \mathcal{O}_X(K_X + L)) = 0$ for $q \geq n-1$.

As an application of this vanishing result, the authors derive the following corollary: Let X be a **Q**-Gorenstein minimal Kähler threefold. Then $\kappa(X) \geq 0$. Recall that **Q**-Gorenstein means that mK_X is Cartier for some $m > 0$ and a minimal Kähler threefold is a Kähler threefold with only terminal singularities such that K_X is nef.

The main importance of this result is that it applies to simple Kähler threefolds that are not Kummer. The interesting twist involved in this case is that it is expected that a simple minimal Kähler threefold is always Kummer. Now it may appear that because of this the above case is not that interesting, but the situation is the exact opposite. The above result reduces the problem of non-existence of non-Kummer simple minimal Kähler threefolds to the case of Kodaira dimension zero.

Furthermore, besides the main application mentioned above, vanishing theorems have been proven extremely useful in general, so one expects that this theorem will be helpful in many different ways.

{For the collection containing this paper see MR2039983}

Sándor J. Kovács

MR2015548 (2004m:32040) 32L20 32J27
Demailly, Jean-Pierre (F-GREN-F); **Peternell, Thomas** (D-BAYR-IM)
A Kawamata-Viehweg vanishing theorem on compact Kähler manifolds. (English summary)
J. Differential Geom. **63** (2003), *no. 2,* 231–277.

The purpose of this work is to extend to the compact Kähler case some important results in the Mori theory of projective complex varieties.

The authors first obtain a Kawamata-Viehweg type vanishing theorem for the cohomology group $H^{n,n-1}(X,L)$ of a nef line bundle L with $L^2 \neq 0$ on a normal compact Kähler space X of dimension n. This is done by first showing the vanishing of the map $H^{n-1}(X,K_X\otimes L\otimes \mathcal{J}) \to H^{n-1}(X,K_X\otimes L)$ for a well-chosen Nadel ideal sheaf and then showing that $\mathcal{O}(-L+D)$ has no nonzero holomorphic section on the divisorial part D of the variety of $\mathcal{J}$.

Then the authors obtain an abundance result on the existence of a pluricanonical section on a minimal Kähler 3-fold with terminal singularities. In order to apply the Riemann-Roch formula, they first prove a stability type inequality between Chern numbers and then a bound on $h^2(X,mK_X)$ thanks to the first part.

Christophe Mourougane

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MR1924513 (2003g:14009) 14C30 14D07 14J32 32G20 32J25

Bertin, José (F-GREN); **Demailly, Jean-Pierre** (F-GREN);
Illusie, Luc (F-PARIS11); **Peters, Chris [Peters, Chris A. M.]** (F-GREN)

★**Introduction to Hodge theory. (English summary)**

Translated from the 1996 French original by James Lewis and Peters.
SMF/AMS Texts and Monographs, 8.
American Mathematical Society, Providence, RI; Société Mathématique de France, Paris, 2002. x+232 *pp.* \$65.00. *ISBN* 0-8218-2040-0

The French original has been reviewed [*Introduction à la théorie de Hodge*, Soc. Math. France, Paris, 1996; MR1409818].

MR1922099 (2003f:32029) 32Q15 32J27

Demailly, Jean-Pierre (F-GREN-F)

On the Frobenius integrability of certain holomorphic p-forms. (English summary)

Complex geometry (*Göttingen*, 2000), 93–98, *Springer*, *Berlin*, 2002.

Let X be a compact Kähler manifold, L a line bundle on X and $\theta \in H^0(X, \Omega^p_X \otimes L^{-1})$ a holomorphic p-form with values in L^{-1}. Finally, let S_θ be the subsheaf of germs of vector fields ξ in the tangent sheaf T_X such that the contraction $i_\xi\theta$ vanishes. In this paper, the author asks if the sheaf S_θ is integrable, i.e. if S_θ is closed under the Lie bracket $[S_\theta, S_\theta] \subset S_\theta$. He shows that S_θ is integrable if L is pseudo-effective.

For an application of this result, assume that X is a contact manifold, i.e. a manifold of odd complex dimension $\dim X = 2n+1$ that carries a form $\theta \in H^0(X, \Omega^1_X \otimes L^{-1})$ such that $\theta \wedge (d\theta)^n \in H^0(X, K_X \otimes L^{-(n+1)})$ does not have any zeros—it is an elementary computation to see that the expression $\theta \wedge (d\theta)^n$ is well-defined even if θ is an L^{-1}-valued form. It was long conjectured that contact manifolds with $b_2(X) = 1$ must be Fano. Demailly's result immediately gives a positive answer to this conjecture. Together with the results of [Invent. Math. **142** (2000), no. 1, 1–15; MR1784795], this implies that a projective contact manifold with $b_2(X) > 2$ is always isomorphic to the projectivized hyperplane bundle $X = \mathbf{P}(T^*_Y)$ associated with a projective manifold Y.

This paper is amazingly short and very well written. After the statement of the result, a brief review of contact manifolds and a list of known results, the actual proof takes a little less than two pages.

{For the collection containing this paper see MR1922091}

Stefan Kebekus

MR1919457 (2003f:32020) 32J25 32L10 32Q15
Demailly, Jean-Pierre (F-GREN-F)
Multiplier ideal sheaves and analytic methods in algebraic geometry.
School on Vanishing Theorems and Effective Results in Algebraic Geometry (Trieste, 2000), 1–148, *ICTP Lect. Notes*, 6, *Abdus Salam Int. Cent. Theoret. Phys.*, *Trieste*, 2001.

This is an extended version of the CIME lectures by the same author [in *Transcendental methods in algebraic geometry (Cetraro, 1994)*, 1–97, Lecture Notes in Math., 1646, Springer, Berlin, 1996; MR1603616].

The main changes have been made to present Siu's result on deformation invariance of plurigenera of varieties of general type. Along the way, a complete picture of the theory of multiplier ideal sheaves is drawn from the point of view of analytic geometry: Nadel vanishing theorem, subadditivity properties, global generation properties, Hard Lefschetz type theorem, semicontinuity properties. This theory hence provides new formulations of classical theorems such as the Skoda division theorem, the Ohsawa-Takegoshi-Manivel extension theorem, and the approximation of plurisubharmonic functions.

These notes written for non-specialists collect material published elsewhere. They serve as a nice display of the modern analytic toolbox in algebraic geometry.

{For the collection containing this paper see MR1919456}

Christophe Mourougane

MR1875649 (2003a:32032) 32J27 32Q15 32Q57
Demailly, Jean-Pierre (F-GREN); **Peternell, Thomas** (D-BAYR);
Schneider, Michael [Schneider, Michael Hellmut] (D-BAYR)
Pseudo-effective line bundles on compact Kähler manifolds. (English summary)
Internat. J. Math. **12** (2001), *no. 6,* 689–741.

This work is intended first to study the classification theory of compact Kähler manifolds with canonical bundle having weak positivity or negativity properties. The usual features of Kodaira dimension, Albanese map and fundamental group are studied in detail. (The case of projective 3-folds is carried further thanks to Mori theory.)

Those results are obtained from a nice generalization of the Hard Lefschetz theorem for the cohomology with values in a pseudo-effective line bundle. Although very technical, the heart of the work is a regularization process for quasi-pluri-subharmonic functions, for it enables one to deal with singular metrics via the Bochner technique.

Pseudoeffective line bundles and vector bundles are also studied on their own and especially with regard to their nefness properties. *Christophe Mourougane*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1861061 (2002k:32033) 32J25 32C35
Campana, Frédéric (F-NANCS); **Demailly, Jean-Pierre** (F-GREN-F)
Cohomologie L^2 sur les revêtements d'une variété complexe compacte. (French)
[L^2-cohomology on the coverings of a compact complex manifold]
Ark. Mat. **39** (2001), *no. 2,* 263–282.

Let X be a complex analytic space and $\tilde{X} \to X$ an unramified covering space (possibly with infinite fibers). Any coherent analytic sheaf F on X lifts to a coherent analytic sheaf on $\tilde{X}$. The article under review defines L^2 cohomology groups on $\tilde{X}$ with values in $\tilde{F}$ and gives a proof of its expected properties: cohomology exact sequences, Leray spectral sequences, Serre duality, vanishing theorems, finiteness of Γ-dimension and a variant of Atiyah's L^2 index theorem for Galois coverings [M. F. Atiyah, in *Colloque "Analyse et Topologie" en l'Honneur de Henri Cartan (Orsay, 1974)*, 43–72. Astérisque, 32-33, Soc. Math. France, Paris, 1976; MR0420729]. When X is a compact manifold and F is free these L^2 cohomology groups can be computed by the L^2 Dolbeault complex. Similar results with a slightly different perspective have been independently developed by the reviewer [Math. Ann. **317** (2000), no. 3, 527–566 MR1776117 (2003c:32020)].

Philippe P. Eyssidieux

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MR1852009 (2002e:32032) 32Q20 32U05
Demailly, Jean-Pierre (F-GREN-F); **Kollár, János** (1-PRIN)
Semi-continuity of complex singularity exponents and Kähler-Einstein metrics on Fano orbifolds. (English, French summaries)
Ann. Sci. École Norm. Sup. (4) **34** (2001), *no. 4,* 525–556.

Summary: "We introduce complex singularity exponents of plurisubharmonic functions and prove a general semicontinuity result for them. This concept contains as a special case several similar concepts which have been considered, e.g., by Arnol′d and Varchenko, mostly for the study of hypersurface singularities. The plurisubharmonic version is somehow based on a reduction to the algebraic case, but it also takes into account more quantitative information of great interest for complex analysis and complex Einstein metrics on certain Fano orbifolds, following Nadel's original ideals (but with a drastic simplification in the technique, once the semicontinuity result is taken for granted). In this way, three new examples of rigid Kähler-Einstein del Pezzo surfaces with quotient singularities are obtained." *Lin Weng*

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MR1786484 (2002a:14016) 14E99 14J17
Demailly, Jean-Pierre (F-GREN-FM); **Ein, Lawrence** (1-ILCC);
Lazarsfeld, Robert (1-MI)
A subadditivity property of multiplier ideals.
Dedicated to William Fulton on the occasion of his 60th birthday.
Michigan Math. J. **48** (2000), 137–156.

Given an effective fractional coefficient divisor D on a smooth projective complex algebraic variety X, the multiplier ideal is an ideal sheaf on the variety which reflects subtle features of the singularities of the divisor. For example, the closed subset defined by (the radical of) this ideal is precisely the locus of points where the pair (X, D) fails to be log-terminal. Thus the multiplier ideal endows the non-log terminal locus with a scheme structure that somehow quantifies the singularities of the pair. Multiplier ideals are playing an increasingly important role in higher-dimensional birational geometry, in part because of their strong vanishing properties.

This paper establishes an interesting relationship for the multiplier ideals associated to two different fractional coefficient divisors, D and E. Specifically, it is shown that the multiplier ideal of the sum $D+E$ is contained in the product of the multiplier ideal of D and the multiplier ideal of E. Proofs are presented both in the analytic framework, yielding the corresponding "subadditivity property" for multiplier ideals associated to plurisubharmonic functions, and in the algebraic framework, yielding the same subadditivity property also for multiplier ideals of ideals.

As an application, the authors re-prove a result of Fujita which gives a sort of numerical form of an approximate Zariski decomposition for big divisors on a smooth projective variety. Further applications of the "subadditivity" property appeared shortly thereafter in [L. M. H. Ein, R. K. Lazarsfeld and K. E. Smith, Invent. Math. **144** (2001), no. 2, 241–252 MR1826369 (2002b:13001)]. The subadditivity property seems destined to become a fundamental result in the subject.

{For the collection containing this paper see MR1778979}

Karen E. Smith

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MR1782659 (2001m:32041) 32L10 32D15 32J25 32U05
Demailly, Jean-Pierre (F-GREN-F)
On the Ohsawa-Takegoshi-Manivel L^2 extension theorem. (English, French summaries)
Complex analysis and geometry (Paris, 1997), 47–82, *Progr. Math.*, 188, *Birkhäuser, Basel*, 2000.

This paper provides a rather deep insight into the current status of L^2 extension techniques for sections [resp. $(0,q)$-forms] of vector bundles over complex analytic submanifolds. The fundamental extension theorem of T. Ohsawa and K. Takegoshi [Math. Z. **195** (1987), no. 2, 197–204; MR0892051], refined in many ways by Ohsawa, and in a more geometric setting by L. Manivel [Math. Z. **212** (1993), no. 1, 107–122; MR1200166], is proven here in its most general form. Unfortunately, a gap in the proof of Manivel is pointed out, regarding the regularity of the extension in the case of $(0,q)$-forms when $q>0$. It thus reappears here as a conjecture, which is discussed in detail, but without being settled.

This theorem can yield powerful constructions that have been used in transcendental algebraic geometry. First, any psh function on a pseudoconvex open set in $\mathbb{C}^n$ can be approximated accurately with functions of the form $c\log|f|$ where f is a holomorphic function; this can be applied for instance to approximate the curvature current of a singular metric by divisors with multiplicities controlled by the Lelong numbers of the current. Other implications are detailed, among them a Briançon-Skoda theorem for multiplier ideal sheaves, and an analytical proof of Fujita's approximate Zariski decomposition for big line bundles.

{For the collection containing this paper see MR1782699}

Thierry Bouche

MR1772670 (2001m:32042) 32L10 14C20 32J25
Demailly, Jean-Pierre (F-GREN-F)
Méthodes L^2 et résultats effectifs en géométrie algébrique. (French. French summary) [L^2-methods and effective results in algebraic geometry]
Séminaire Bourbaki, Vol. 1998/99.
Astérisque No. 266 (2000), *Exp. No.* 852, 3, 59–90.

This paper surveys the recent work that has been done by Demailly, Siu and Nadel among others about effective results in algebraic geometry obtained through Hörmander's L^2-methods for the $\overline{\partial}$ equation with singular metrics. A first section provides good insight into the basic tools, which are defined, and the results, whose proofs are outlined: singular metrics of holomorphic line bundles over complex analytic manifolds, the Bochner-Kodaira-Nakano identity for the antiholomorphic Laplace-Beltrami operator (in the case where the metric is smooth), L^2 estimates with singular metrics, Nadel's multiplier ideal sheaves and the corresponding vanishing theorem. Two important applications of these techniques are then described in detail: the Fujita conjecture [see Y. T. Siu, in *Modern methods in complex analysis (Princeton, NJ, 1992)*, 291–318, Ann. of Math. Stud., 137, Princeton Univ. Press, Princeton, NJ, 1995; MR1369144] and Siu's theorem about the invariance of plurigenera under deformation [see Y. T. Siu, Invent. Math. **134** (1998), no. 3, 661–673; MR1660941].

{For the collection containing this paper see MR1772667}

Thierry Bouche

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MR1759887 (2001f:32045) 32Q45 14J29 14J70
Demailly, Jean-Pierre (F-GREN-F); **El Goul, Jawher** (F-TOUL3)
Hyperbolicity of generic surfaces of high degree in projective 3-space. (English summary)
Amer. J. Math. **122** (2000), *no. 3,* 515–546.

S. Kobayashi [*Hyperbolic manifolds and holomorphic mappings*, Dekker, New York, 1970; MR0277770] conjectured that a generic hypersurface of dimension n in the projective space $\mathbf{P}^{n+1}$ is hyperbolic, i.e., every holomorphic map from the affine complex line $\mathbf{C}$ into such a hypersurface is constant. In the paper under review the authors verify the above conjecture for a very generic surface in $\mathbf{P}^3$ of degree $d \geq 21$ (i.e., away from a possible countable union of subvarieties in the moduli space of surfaces of degree d). More precisely, the surfaces for which the claim holds are of general type, have Picard number 1 and their Chern classes satisfy certain inequalities. The methods and techniques developed and used in the paper might be of independent interest but they are far too elaborate to be discussed here.. For the purpose of this review we outline briefly the key ideas of the proof which goes as follows. Using the Riemann-Roch theorem one produces a branched covering Z of X living in the projectivized tangent bundle of X. If $f\colon \mathbf{C} \longrightarrow X$ is a non-constant holomorphic map, then its first differential extends to a holomorphic map whose image is contained in a leaf of an algebraic foliation on Z. By a recent argument of M. McQuillan [Inst. Hautes Études Sci. Publ. Math. No. 87 (1998), 121–174; MR1659270], the resulting curve must be algebraically degenerate, i.e., contained in a proper algebraic subvariety of Z. In order to apply this result one is in fact forced to consider 2-jets. Then the closure of the image of f is either a rational or an elliptic curve. On the other hand, by a result of H. Clemens [Ann. Sci. École Norm. Sup. (4) **19** (1986), no. 4, 629–636; MR0875091], a generic surface of degree at least 7 in the projective space contains no rational or elliptic curves which implies that f is in fact constant.

Similar results in a more general context (implying, in particular, the Kobayashi conjecture for generic surfaces of degree at least 36 in $\mathbf{P}^3$) were obtained recently in [M. McQuillan, Geom. Funct. Anal. **9** (1999), no. 2, 370–392; MR1692470].

Tomasz Szemberg

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MR1748605 (2002e:32046) 32U40 32C30 32C37 32F10 53C60

Demailly, Jean-Pierre (F-GREN-F)

Pseudoconvex-concave duality and regularization of currents. (English summary)

Several complex variables (Berkeley, CA, 1995–1996), 233–271, *Math. Sci. Res. Inst. Publ.*, 37, *Cambridge Univ. Press*, *Cambridge*, 1999.

The goal of this paper is to investigate some duality properties connecting pseudoconvexity and pseudoconcavity in a certain perspective to obtain a geometric version of the Serre duality theorem. These duality properties are related to several geometric problems, such as the conjecture of Hartshorne asserting that the complement of a q-codimensional algebraic subvariety Y with ample normal bundle N_Y in a projective algebraic variety X is q-convex in the sense of Andreotti-Grauert. M. Schneider proved the conjecture in the case that the normal bundle is positive in the sense of Griffiths.

Using Sommese's result, the author proves the conjecture in the case that N_Y^* has a strictly convex plurisubharmonic Finsler metric.

Let X be a complex manifold of dimension n and E a holomorphic vector bundle of rank r. Demailly treats the problem of approximation of closed positive $(1,1)$-currents and the attenuation of their singularities. In general a closed positive current T cannot be approximated in the weak topology by smooth closed positive currents. J.-P. Demailly [Ann. Sci. École Norm. Sup. (4) **15** (1982), no. 3, 457–511; MR0690650; J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622; in *Contributions to complex analysis and analytic geometry*, 105–126, Vieweg, Braunschweig, 1994; MR1319346] proved that this approximation is possible if we allow the regularization T_ε to have a small negative part. The main point is to control the negative part in terms of the global geometry of the ambient geometry X. It turns out that more or less optimal bounds can be described in terms of the convexity of a Finsler metric on the tangent bundle T_X. The author gives an easy proof based on the use of symmetric products of Finsler metrics.

{For the collection containing this paper see MR1748597}

Mongi Blel

MR1622747 (99e:32047) 32J17

Campana, Frédéric (F-NANC); **Demailly, Jean-Pierre** (F-GREN-F);
Peternell, Thomas (D-BAYR-IM)

The algebraic dimension of compact complex threefolds with vanishing second Betti number. (English summary)

Compositio Math. **112** (1998), *no. 1,* 77–91.

A compact complex threefold with vanishing second Betti number cannot be algebraic or Kähler. Then the natural question is: What possibilities are there for the algebraic dimension of such manifolds? (Algebraic dimension is the transcendence degree of the field of meromorphic functions over **C**.)

The main result of this article is that if the algebraic dimension is positive, then the topological Euler characteristic is 0 and then either $b_1 = 0$ and $b_3 = 2$ or $b_1 = 1$ and $b_3 = 0$. An interesting corollary is that S^6 does not admit a complex structure with a non-constant meromorphic function. The authors deduce the main result as a straightforward consequence of a vanishing theorem for vector bundles twisted by generic elements of Pic^0. Examples of threefolds with positive algebraic dimension and vanishing second Betti number and topological Euler characteristics are also given showing that the result is optimal.

The authors also investigate more deeply threefolds with vanishing second Betti number and whose algebraic dimension is 1.

This is another interesting article from these distinguished authors presenting ideas of great interest to algebraic and complex analytic geometers alike. *Sándor J. Kovács*

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spaces. Wiley, 1976. MR0463470
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MR1492596 (99a:32033) 32H20 32J10 32L05
Demailly, Jean-Pierre (F-GREN-F)
Variétés projectives hyperboliques et équations différentielles algébriques.
(French) [Hyperbolic projective varieties and algebraic differential equations]
Journée en l'Honneur de Henri Cartan, 3–17, *SMF Journ. Annu.*, 1997, *Soc. Math. France*, *Paris*, 1997.

This is a very well-written survey of some of the more recent developments in the theory of holomorphic curves in algebraic varieties, a holomorphic curve in an algebraic variety being a holomorphic mapping from the complex plane to the variety. The author's survey concentrates in particular on the recent work of Y. T. Siu and S.-K. Yeung [Amer. J. Math. **119** (1997), no. 5, 1139–1172; MR1473072]. An extensive bibliography is also provided. Although probably best suited for those readers already familiar with the language of complex differential geometry, and in particular the language used when working with Hermitian vector bundles and meromorphic connections, the survey is for the most part a very accessible introduction to some of the latest developments in the field and assumes little prior knowledge of Nevanlinna theory, algebraic geometry, or the other techniques commonplace in the study of holomorphic curves.

The reviewer's translation of the author's first paragraph reads as follows: "The goal of this text is to offer an introduction, which is as elementary as possible, to an important result concerning the geometry of the images of holomorphic curves in complex algebraic varieties. This result finds its origin in the fundamental work of A. Bloch [J. Math. Pures Appl. (9) **5** (1926), 19–66; JFM 52.0373.05] and in the thesis of H. Cartan [Ann. Sci. École Norm. Sup. **45** (1928), 255–346; JFM 54.0357.06]. The proof that we give here is a very recent contribution by Siu and Yeung [op. cit.]. It proceeds in a relatively simple manner with help from classical estimates in Nevanlinna theory, like the lemma on the logarithmic derivative, and by making use of differential operators such as Wronskians, all ideas whose germs were already sown in Henri Cartan's thesis [op. cit.]."

More specifically, the author explains techniques for showing that a holomorphic curve in an algebraic variety is algebraically degenerate, meaning that its image is contained in a proper algebraic subvariety. A fundamental conjecture along these lines is the conjecture of Green and Griffiths stating that a holomorphic curve in a variety of general type must be algebraically degenerate. The survey is centered around the following fundamental vanishing theorem. If $f\colon \mathbf{C} \to X$ is a holomorphic curve in a projective variety X, if L is a positive line bundle on X, and if P is an algebraic differential operator on X with values in L^{-1}, then P applied to f is zero. For hypersurfaces in projective space, this theorem can be applied to Wronskian-like differential operators coming from explicitly constructed meromorphic connections, as in the work of A. M. Nadel [Duke Math. J. **58** (1989), no. 3, 749–771; MR1016444]. This results in specific examples of general type projective varieties in which every holomorphic curve is algebraically degenerate. This method also proves that in some of these varieties, the image of every holomorphic curve must be constant; such varieties are called hyperbolic.

{For the collection containing this paper see MR1492594}

William A. Cherry

MR1492594 (98h:00041) 00B30

Hirzebruch, Friedrich [Hirzebruch, Friedrich Ernst Peter];
Demailly, Jean-Pierre (F-GREN-F); **Lannes, Jean**

★**Journée en l'Honneur de Henri Cartan. (French) [Conference in Honor of Henri Cartan]**

SMF Journée Annuelle [SMF Annual Conference], 1997.
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Contents:

{Most of the papers are being reviewed individually.}

MR1492539 (99b:32037) 32H20 14J40 32L10
Demailly, Jean-Pierre (F-GREN-F)
Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. **(English summary)**
Algebraic geometry—Santa Cruz 1995, 285–360, *Proc. Sympos. Pure Math.*, 62, *Part* 2, *Amer. Math. Soc.*, *Providence*, *RI*, 1997.

The article under review is an expanded version of five lectures delivered at the Santa Cruz AMS Summer School on Algebraic Geometry. It proposes an important framework for solving several geometry questions related to hyperbolicity in the sense of Kobayashi. This framework was initiated by M. Green and P. Griffiths [in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557]. Aiming, among other things, to fix a gap in Green-Griffiths' proof of the pointwise version of the Ahlfors-Schwarz lemma for jet differentials, Demailly introduces the concept of "directed manifold" and an associated tower of projective bundles over X (called Semple jet bundles). The Ahlfors-Schwarz lemma is then established in this setting, and the proof of Bloch's theorem is recovered following the approach of Green and Griffiths. Several important new results are also obtained in this paper. Moreover, the author believes that the Semple bundle construction should be an efficient tool to calculate the case locus; therefore several important open problems in the theory of complex hyperbolicity hopefully could be settled under this framework. It should be noted that, since the appearance of this article, Demailly has proved jointly with J. El Goul that every generic surface in $\mathbf{P}^3$ of degree greater than or equal to 42 is Kobayashi hyperbolic. It has been conjectured by Kobayashi that every generic surface in $\mathbf{P}^3$ of degree greater than or equal to 5 is Kobayashi hyperbolic.

This paper is a quite important contribution to the theory of complex hyperbolicity. The paper is self-contained and the exposition is excellent. It is highly recommended to the experts in this field, as well as to anyone who desires a general overview of this subject.

We will now try to outline this article. A complex directed manifold is a pair (X,V) where X is a complex manifold and V is a holomorphic subbundle of T_X; here T_X is the tangent bundle of X. To study the complex hyperbolicity of (X,V), a well-known major technique is the so-called "negative curvature method". The method is based on the following observation: by the Ahlfors-Schwarz lemma, the existence of a Hermitian metric on the line bundle $\mathcal{O}_{\mathbf{P}(V)}(-1)$ over $\mathbf{P}(V)$ (i.e. a Finsler metric on V) with negative curvature implies that (X,V) is hyperbolic. Let us recall here how to construct such a metric. Assume that V^* is "very big" in the following sense: there exist an ample line bundle L and a sufficiently large integer m such that the global sections $H^0(X, S^mV^*\otimes L^{-1})$ generate all fibers over $X\smallsetminus Y$, for some analytic subset $Y\subset X$. Let $\sigma_1,\cdots,\sigma_N$ be such global sections, and define

$$N(\xi)=\Big(\sum_{1\le j\le N}|\sigma_j(x)\cdot\xi^m|^2\Big)^{1/2m},\quad \xi\in V_x^*;$$

N then gives rise to such a metric. Therefore we have the following result: Let (X,V) be a directed complex manifold. Assume that V^* is "very big". Then every entire curve $f\colon \mathbf{C}\to X$ tangent to V satisfies $f(\mathbf{C})\subset Y$, where Y is the subset of X defined above. In particular, if V^* is ample, then (X,V) is hyperbolic.

The heart of the article consists of Chapters 4 to 7. They are devoted to extending the above result to k-jet differentials. The idea is based on the important fact, first observed by Green and Griffiths, that the Ahlfors-Schwarz lemma still works for k-jet differentials, and thus k-jet negativity also implies hyperbolicity. Unfortunately, there is a slight technical gap in Green and Griffiths' approach in the step proving the pointwise Ahlfors-Schwarz lemma for jet differentials. In his paper, Demailly fills the gap in the case of invariant jet differentials, and also extends the result to the more general situation of directed manifolds. (Note: Another solution has been provided later by Y. T. Siu and S.-K. Yeung by means of Nevanlinna's second main theorem [see Amer. J. Math. **119** (1997), no. 5, 1139–1172; MR1473072].)

To do this, Demailly introduces a canonical tower of projective bundles (also called Semple jet bundles). Given a complex directed manifold (X, V), a new complex directed manifold $(\widetilde{X}, \widetilde{V})$ is produced as follows. Let $\widetilde{X} = \mathbf{P}(V)$ be the projectivized bundle of lines of V, and let $\widetilde{V} \subset T_{\widetilde{X}}$ be the subbundle of $T_{\widetilde{X}}$ defined as follows: for every point $(x, [v]) \in \widetilde{X}$ associated with a vector $v \in V_x \smallsetminus \{0\}$,

$$\widetilde{V}_{(x,[v])} = \left\{\xi \in T_{\widetilde{X},(x,[v])} \colon \pi_*\xi \in \mathbf{C}\right\}, \quad V_x \subset T_{X,x},$$

where $\pi\colon \widetilde{X} = \mathbf{P}(V) \to X$ is the natural projection. The projectivized k-jet bundle $\mathbf{P}_k V = X_k$ (or Semple k-jet bundle) and the associated subbundle $V_k \subset T_{X_k}$ are defined inductively by $(X_0, V_0) = (X, V)$, $(X_k, V_k) = (\widetilde{X}_{k-1}, \widetilde{V}_{k-1})$. Every non-constant tangent trajectory $f\colon \Delta_R \to X$ of (X, V) lifts to a well-defined and unique tangent trajectory $f_{[k]}\colon \Delta_R \to X_k$ of (X_k, V_k).

The author shows that the Ahlfors-Schwarz lemma works at each level of the tower of projective bundles. That is: If (X, V) has a k-jet metric h_k on the line bundle $\mathcal{O}_{\mathbf{P}_k V}(-1)$ (i.e. a Finsler metric on the vector bundle V_{k-1} over $\mathbf{P}_{k-1}V$), with negative jet curvature, then every entire curve $f\colon \mathbf{C} \to X$ tangent to V satisfies $f_{[k]}(\mathbf{C}) \subset \Sigma_{h_k}$, where Σ_{h_k} is the singularity set of the metric h_k.

To produce such metrics h_k, one uses global sections of $H^0(\mathbf{P}_k V, \mathcal{O}_{\mathbf{P}_k V}(m) \otimes \pi_{0,k}^* L^{-1})$, where L is an ample line bundle on X. The author also shows that the direct images $(\pi_{0,k})_* \mathcal{O}_{\mathbf{P}_k V}(m)$ can be viewed as bundles of algebraic differential operators of order k and degree m, acting on germs of curves and invariant under reparametrization. This bundle is denoted by $E_{k,m}(V^*)$. Therefore $H^0(\mathbf{P}_k V, \mathcal{O}_{\mathbf{P}_k V}(m) \otimes \pi_{0,k} L^{-1}) \simeq H^0(X, E_{k,m}(V^*) \otimes L^{-1})$.

The above discussion leads to the following result: Assume that there exist integers $k, m > 0$ and an ample line bundle L on X such that $H^0(X, E_{k,m}(V^*) \otimes L^{-1})$ has nonzero sections $\sigma_0, \cdots, \sigma_N$. Let $Z \subset \mathbf{P}_k V$ be the base locus of these sections. Then every entire curve $f\colon \mathbf{C} \to X$ tangent to V satisfies $f_{[k]}(\mathbf{C}) \subset Z$. In other words, for every global parametrization invariant polynomial differential operator P with values in L^{-1}, every entire curve f as above must satisfy the algebraic differential equation $P(f) = 0$.

The dimension

$$h^0(X, E_{k,m}(V^*) \otimes L^{-1}) = \dim H^0(X, E_{k,m}(V^*) \otimes L^{-1})$$

can be computed by using the Riemann-Roch theorem and a vanishing theorem due to Bogomolov. In particular, in the surface case, the Riemann-Roch theorem yields the following (see Chapter 13, Corollary 13.9): If X is an algebraic surface of general type and L an ample line bundle over X, then

$$h^0(X, E_{2,m} T^* X \otimes \mathcal{O}(-L)) \geq \frac{m^4}{648}(13c_1^2 - 9c_2) - O(m^3).$$

In particular, every smooth surface $X \subset \mathbf{P}^3$ of degree $d \geq 15$ admits a nontrivial section, and every entire function $f\colon \mathbf{C} \to X$ must satisfy the corresponding algebraic differential equations.

However, it seems very difficult to conclude that f satisfies an algebraic equation. The author suggests in Chapter 13 that the Riemann-Roch calculations might be helpful to locate the base locus, thus to conclude the algebraic degeneracy.

Another important part of this article is Chapter 2 and Chapter 9, where Demailly shows that Kobayashi hyperbolicity is related to other properties of a more algebraic nature. A projective directed manifold (X,V) is called algebraically hyperbolic if there exists $\varepsilon > 0$ such that every algebraic curve $C \subset X$ tangent to V satisfies $2\,g(\overline{C}) - 2 \geq \varepsilon \deg_\omega(C)$ ($\overline{C}$ is the normalization of C). The main result of Chapter 2 is that if (X,V) is hyperbolic, then (X,V) is algebraically hyperbolic. Chapter 9 extends this result to k-jet metrics and shows that the negativity of k-jet curvature implies strong restrictions of an algebraic nature on curve genera and their singularity indices.

Chapter 11 recalls the "meromorphic connection" method introduced by Siu [Y. T. Siu, Duke Math. J. **55** (1987), no. 1, 213–251; MR0883671; A. M. Nadel, Duke Math. J. **58** (1989), no. 3, 749–771; MR1016444]. Using this method, the author reports on a joint work with J. El Goul, where examples of hyperbolic surfaces in $\mathbf{P}^3$ are produced for any degree ≥ 11.

{For the collection containing this paper see MR1492532}

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MR1462789 (98j:32027) 32H20 14J99 32H30

Demailly, Jean-Pierre (F-GREN)

Variétés hyperboliques et équations différentielles algébriques. **(French)**
[Hyperbolic manifolds and algebraic differential equations]

Gaz. Math. No. 73 (1997), 3–23.

Let X be a projective algebraic variety and $f\colon \mathbf{C} \to X$ be a nonconstant entire curve. Then for every algebraic differential operator P with values in the dual L^* of a holomorphic line bundle L over X with positive curvature, one has $P(f', \cdots, f^{(k)}) \equiv 0$.

This theorem, which was stated by M. Green and P. Griffiths [in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557], plays a key role if one wants to prove hyperbolicity of X. After Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997 MR1492539 (99b:32037)] had proved a slightly weaker version, Y. T. Siu and S.-K. Yeung [Amer. J. Math. **119** (1997), no. 5, 1139–1172; MR1473072] gave a convincing proof.

The paper under review is an extended version of the author's talk given in honor of Henri Cartan. The first part deals with another proof of the theorem, the idea of which was also apparently given by Siu. The main tool used in this proof is Nevanlinna theory. The presentation is self contained and very elegant. The second part of the paper follows Demailly and J. El Goul [C. R. Acad. Sci. Paris Sér. I Math. **324** (1997), no. 12, 1385–1390; MR1457092; see the preceding review]. Here the author shows how to use the theorem in combination with a Wronskian differential operator and the author's concept of partial projective connections to obtain families of smooth hyperbolic surfaces $Y \subset$

$\mathbf{P}^3$ of any degree $d \geq 11$. The hyperbolicity of the same families was also obtained by Siu and Yeung [op. cit.] with different methods. *Gerd Dethloff*

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Demailly, Jean-Pierre (F-GREN-FM); **El Goul, Jawher** (F-GREN-FM)
Connexions méromorphes projectives partielles et variétés algébriques hyperboliques. (French. English, French summaries) [Partial projective meromorphic connections and hyperbolic projective varieties]
C. R. Acad. Sci. Paris Sér. I Math. **324** (1997), *no. 12,* 1385–1390.

This paper presents one more exposition on the hyperbolicity of special hypersurfaces in $\mathrm{P}^3(\mathbf{C})$ and related topics [see A. M. Nadel, Duke Math. J. **58** (1989), no. 3, 749–771; MR1016444; J. El Goul, Manuscripta Math. **90** (1996), no. 4, 521–532; MR1403721].

The main result of Nadel and the scheme of its proof are not modified in the paper under review. One can single out two new aspects of the exposition. The authors indicate a new possibility for proving the main auxiliary result, which following Nadel is usually named "the Siu degeneration theorem", and introduce a notion of partial projective connections.

As for possible development of the subject, it seems that a generalization of the Siu degeneration theorem for the case of singular projective varieties would give substantial progress in this area. Till now the classical second main theorem of H. Cartan [Mathematica (Cluj) **7** (1933), 5–31; Zbl 007.41503 (p. 12)] enables one to obtain more general and sharper results than that of Nadel-El Goul-Demailly. For the details see the review of the above-cited paper of El Goul. *Evgenii Nochka*