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Demailly, Jean-Pierre; Gaussier, Hervé
Algebraic embeddings of smooth almost complex structures. (English) Zbl 06802929
J. Eur. Math. Soc. (JEMS) 19, No. 11, 3391-3419 (2017).

Summary: The goal of this work is to prove an embedding theorem for compact almost complex manifolds into complex algebraic varieties. It is shown that every almost complex structure can be realized by the transverse structure to an algebraic distribution on an affine algebraic variety, namely an algebraic subbundle of the tangent bundle. In fact, there even exist universal embedding spaces for this problem, and their dimensions grow quadratically with respect to the dimension of the almost complex manifold to embed. We give precise variation formulas for the induced almost complex structures and study the related versality conditions. At the end, we discuss the original question raised by F. Bogomolov: can one embed every compact complex manifold as a $\mathcal{C}^\infty$ smooth subvariety that is transverse to an algebraic foliation on a complex projective algebraic variety?

MSC:

32Q60 Almost complex manifolds
32Q40 Embedding theorems
32G05 Deformations of complex structures
53C12 Foliations (differential geometry)

Keywords:

deformation of complex structures; almost complex manifolds; complex projective variety; Nijenhuis tensor; transverse embedding; Nash algebraic map

Full Text: DOI

Demailly, Jean-Pierre
Variational approach for complex Monge-Ampère equations and geometric applications. (English) Zbl 06784933
Séminaire Bourbaki. Volume 2015/2016. Exposés 1104–1119. Avec table par noms d'auteurs de 1948/49 à 2015/16. Paris: Société Mathématique de France (SMF) (ISBN 978-2-85629-855-8/pbk). Astérisque 390, 245-275, Exp. No. 1112 (2017).

For the entire collection see [Zbl 1370.00002].

MSC:

32W20 Complex Monge-Ampère operators
32Q25 Calabi-Yau theory
53C55 Hermitian and Kählerian manifolds (global differential geometry)

Cao, JunYan; Demailly, Jean-Pierre; Matsumura, Shin-ichi
A general extension theorem for cohomology classes on non reduced analytic subspaces. (English) Zbl 1379.32017
Sci. China, Math. 60, No. 6, 949-962 (2017).

The authors generalize the Ohsawa-Takegoshi extension theorem with the goal to prove it with the weakest possible hypothesis. The main theorem goes as follows:

Let E be a holomorphic line bundle over a holomorphically convex Kähler manifold X. Let h be a (possibly) singular Hermitian metric on E, ψ a quasi-plurisubharmonic function with neat analytic singularities on X. If there exists a continuous function $\delta > 0$ on X such that

$$\Theta_{E,h} + (1+\alpha\delta) i\partial\bar{\partial}\psi \geq 0$$

in the sense of currents for all $\alpha \in [0,1]$, then the morphism induced by the natural inclusion $\mathcal{I}(he^{-\psi}) \to \mathcal{I}(h)$, namely

$$H^q(X, K_X \otimes E \otimes \mathcal{I}(he^{-\psi})) \to H^q(X, K_X \otimes E \otimes \mathcal{I}(h)),$$

is injective for every $q \geq 0$.

It is noted that most of the argument carries on to the case when X is only weakly pseudoconvex, yet at one place there is a problem and hence this case remains open.

An alternative proof using an idea of the third author is presented.

Reviewer: Zywomir Dinew (Kraków)

MSC:

32L10 Sections of holomorphic vector bundles
32Q15 Kähler manifolds
32E05 Holomorphically convex complex spaces, reduction theory

Keywords:

holomorphically convex Kähler manifold; Ohsawa-Takegoshi extension theorem; singular Hermitian metric; multiplier ideal sheaf

Full Text: DOI

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[5] Demailly, J-P, Extension of holomorphic functions defined on non reduced analytic subvarieties, (2015) · Zbl 0581.32036 · doi:10.2977/prims/1195181609

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[15] Matsumura S. An injectivity theorem with multiplier ideal sheaves of singular metrics with transcendental singularities. J Algebraic Geom, in press, arXiv:1308.2033v4, 2013 · Zbl 06837517 · doi:10.1007/978-3-319-20337-9_4

[16] Matsumura S. An injectivity theorem with multiplier ideal sheaves for higher direct images under K\"ahler morphisms. ArXiv:1607.05554v1, 2016 · Zbl 1271.14001 · doi:10.4007/annals.2015.182.2.5

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[18] Ohsawa, T; Takegoshi, K, On the extension of L\^{}\{2\} holomorphic functions, Math Z, 195, 197-204, (1987) · Zbl 0625.32011 · doi:10.1007/BF01166457

Demailly, Jean-Pierre

Extension of holomorphic functions defined on non reduced analytic subvarieties. (English) Zbl 1360.14025

Ji, Lizhen (ed.) et al., The legacy of Bernhard Riemann after one hundred and fifty years. Volume I. Somerville, MA: International Press; Beijing: Higher Education Press (ISBN 978-1-57146-318-0/pbk; 978-1-57146-316-6/set). Advanced Lectures in Mathematics (ALM) 35, 1, 191-222 (2016).

The article generalizes results and methods of *T. Ohsawa* and *K. Takegoshi* theorem from [Math. Z. 195, 197–204 (1987; Zbl 0625.32011)] to the case when the subvariety Y is not necessarily reduced, by using a multiplier ideal sheaf and jumping numbers. For a holomorphic vector bundle E on a complex manifold X one can discuss the existence of global holomorphic extensions $F \in H^0(X, E)$ of a section $f \in H^0(Y, E|_Y)$ together with L^2 approximations. The article considers this problem when X is a weakly pseudoconvex Kähler manifold with Kähler metric ω and when the holomorphic vector bundle E is equipped with a (possibly singular) hermitian metric $h = e^{-\varphi}$. Let ψ denote a quasi-psh function on X with neat analytic singularities and with log canonical singularities along a analytic subvariety $Y = V(\mathcal{I}(\psi))$ (so that Y is reduced). If the Chern curvature tensor $\Theta_{E,h}$ has the property that $i\Theta_{E,h} + \alpha i\partial\overline{\partial} \otimes Id_E$ is Nakano semipositive for all $\alpha \in [1, 1+\delta]$ and some $\delta > 0$, then for every section $f \in H^0(Y^0, (K_X \otimes E)|_{Y^0})$ on $Y^0 = Y_{\text{reg}}$ such that
$$\int_{Y_0} |f|^2_{\omega,h} dV_{Y^0,\omega}[\psi] < +\infty$$
there exists an extension $F \in H^0(X, K_X \otimes E)$ whose restriction to Y^0 is equal to f, such that
$$\int_X \gamma(\delta\psi)|F|^2_{\omega,h} e^{-\psi} dV_{X,\omega} < \frac{34}{\delta} \int_{Y_0} |f|^2_{\omega,h} dV_{Y^0,\omega}[\psi]$$
The remark states that if F is a $(n, 0)$-form then the product $|F|^2_{\omega,h} dV_{X,\omega}$ does not depend on ω. The author claims that the constant $\frac{34}{\delta}$ in the inequality is not optimal.

The concept of the multiplier ideal sheaf used in the proof is parallel yet more general than the one presented by *D. Popovici* [Nagoya Math. J. 180, 1–34 (2005; Zbl 1116.32017)].

For the entire collection see [Zbl 1343.01006].

Reviewer: Malgorzata Marciniak (Flushing)

MSC:

14C30 Transcendental methods, Hodge theory, Hodge conjecture
14F05 Sheaves, derived categories of sheaves, etc.
32C35 Analytic sheaves and cohomology groups

Cited in **1** Document

Keywords:

holomorphic function; plurisubharmonic function; multiplier ideal sheaf; L^2 extension theorem; Ohsawa-Takegoshi theorem; log canonical singularities; non reduced subvariety Kähler metric; multiplier ideal sheaf; jumping numbers

Demailly, Jean-Pierre

Numerical analysis and differential equations. 4th edition. (Analyse numérique et équations différentielles.) (French) Zbl 1362.65001

Grenoble Sciences. Les Ulis: EDP Sciences (ISBN 978-2-7598-1926-3/pbk; 978-2-7598-2004-7/ebook). vii, 368 p. (2016).

Publisher's description: Cet ouvrage est la quatrième édition d'un livre devenu aujourd'hui un classique sur la théorie des équations différentielles ordinaires. Le cours théorique de base est accompagné d'un exposé détaillé des méthodes numériques qui permettent de résoudre ces équations en pratique.

De multiples techniques de l'analyse numérique sont présentées : interpolation polynomiale, intégration numérique, méthodes itératives pour la résolution d'équations. Suit un exposé rigoureux des résultats sur l'existence, l'unicité et la régularité des solutions des équations différentielles, avec étude détaillée des équations du premier et du second ordre, des équations et systèmes linéaires à coefficients constants. Enfin, sont décrites les méthodes numériques à un pas ou multi-pas, avec étude comparative de la stabilité et du coût en temps de calcul. De nombreux exemples concrets, des exercices et problèmes d'application en fin de chapitre facilitent l'apprentissage.

Plusieurs améliorations ont été apportées dans cette dernière version. De nouveaux problèmes ou exercices ont été introduits dans presque tous les chapitres. La principale nouveauté est que l'ouvrage est maintenant un pap-ebook : le site compagnon en accès libre propose au lecteur des compléments théoriques et pratiques, ainsi que la correction d'un grand nombre d'exercices.

Cet ouvrage accessible aux L3, M1 et M2 de mathématiques est très utilisé pour la préparation aux concours de l'enseignement. Il constitue un outil de référence pour les enseignants, chercheurs et scientifiques d'autres disciplines.

For the previous edition see [Zbl 0869.65041]. See the review of the German edition in [Zbl 0869.65042].

MSC:

65-01 Textbooks (numerical analysis)
34-01 Textbooks (ordinary differential equations)
65L05 Initial value problems for ODE (numerical methods)
65L06 Multistep, Runge-Kutta, and extrapolation methods
65D32 Quadrature and cubature formulas (numerical methods)
65H10 Systems of nonlinear equations (numerical methods)

Boman, Jan (ed.); **Sigurdsson, Ragnar (ed.)**; **Lerner, Nicolas**; **Demailly, Jean-Pierre**; **Atiyah, Michael**; **Treves, François**; **Helgason, Sigurdur**; **Grubb, Gerd**; **Bony, Jean-Michel**; **Kiselman, Christer O.**; **Broström, Sofia**

To the memory of Lars Hörmander (1931–2012). (English) Zbl 1338.35005

Notices Am. Math. Soc. 62, No. 8, 890-907 (2015).

MSC:

35-03 Historical (partial differential equations)
46-03 Historical (functional analysis)
47-03 Historical (operator theory)
01A70 Biographies, obituaries, personalia, bibliographies

Full Text: DOI

Demailly, Jean-Pierre

On the cohomology of pseudoeffective line bundles. (English) Zbl 1337.32030

Fornæss, John Erik (ed.) et al., Complex geometry and dynamics. The Abel symposium 2013, Trondheim, Norway, July 2–5, 2013. Cham: Springer (ISBN 978-3-319-20336-2/hbk; 978-3-319-20337-9/ebook). Abel Symposia 10, 51-99 (2015).

Summary: The goal of this survey is to present various results concerning the cohomology of pseudoeffective line bundles on compact Kähler manifolds, and related properties of their multiplier ideal sheaves. In case the curvature is strictly positive, the prototype is the well known Nadel vanishing theorem, which is itself a generalized analytic version of the fundamental Kawamata-Viehweg vanishing theorem of algebraic geometry. We are interested here in the case where the curvature is merely semipositive in the sense of currents, and the base manifold is not necessarily projective. In this situation, one can still obtain interesting information on cohomology, e.g. a Hard Lefschetz theorem with pseudoeffective coefficients,

in the form of a surjectivity statement for the Lefschetz map. More recently, Junyan Cao, in his PhD thesis defended in Grenoble, obtained a general Kähler vanishing theorem that depends on the concept of numerical dimension of a given pseudoeffective line bundle. The proof of these results depends in a crucial way on a general approximation result for closed $(1,1)$-currents, based on the use of Bergman kernels, and the related intersection theory of currents. Another important ingredient is the recent proof by Guan and Zhou of the strong openness conjecture. As an application, we discuss a structure theorem for compact Kähler threefolds without nontrivial subvarieties, following a joint work with F. Campana and M. Verbitsky. We hope that these notes will serve as a useful guide to the more detailed and more technical papers in the literature; in some cases, we provide here substantially simplified proofs and unifying viewpoints.

For the entire collection see [Zbl 1336.32001].

MSC:

32J27 Compact Kähler manifolds: generalizations, classification

Keywords:

pseudoeffective line bundles on Kähler manifolds; multiplier ideal sheaves

Full Text: DOI

References:

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.

Demailly, Jean-Pierre (ed.); van der Geer, Gerard (ed.); Hacon, Christopher (ed.); Kawamata, Yujiro (ed.); Kobayashi, Toshiyuki (ed.); Miyaoka, Yoichi (ed.); Schmid, Wilfried (ed.)

Foreword. (English) Zbl 1332.00108

J. Math. Sci., Tokyo 22, No. 1, iii-iv (2015).

From the text: Professor Kunihiko Kodaira is one of the greatest mathematicians of the twentieth century, and this issue is dedicated to him to commemorate his 100th birthday. The authors of the articles included in this issue belong to various generations.

MSC:

00B30 Festschriften
14-06 Proceedings of conferences (algebraic geometry)
32-06 Proceedings of conferences (several complex variables)

Biographic references:

Kodaira, Kunihiko

Full Text: Link

Demailly, Jean-Pierre

Towards the Green-Griffiths-Lang conjecture. (English) Zbl 1327.14048

Baklouti, Ali (ed.) et al., Analysis and geometry. MIMS-GGTM, Tunis, Tunisia, March 24–27, 2014. Proceedings of the international conference. In honour of Mohammed Salah Baouendi. Cham: Springer (ISBN 978-3-319-17442-6/hbk; 978-3-319-17443-3/ebook). Springer Proceedings in Mathematics & Statistics 127, 141-159 (2015).

Summary: The Green-Griffiths-Lang conjecture stipulates that for every projective variety X of general

type over $\mathbb{C}$, there exists a proper algebraic subvariety of X containing all non constant entire curves $f : \mathbb{C} \to X$. Using the formalism of directed varieties, we prove here that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet semistability property of the tangent bundle T_X. We then give a sufficient criterion for the Kobayashi hyperbolicity of an arbitrary directed variety (X, V).

For the entire collection see [Zbl 1320.00044].

MSC:

14C30 Transcendental methods, Hodge theory, Hodge conjecture
32J25 Transcendental methods of algebraic geometry
14C20 Divisors, linear systems, invertible sheaves

Keywords:

projective algebraic variety; variety of general type; entire curve; jet bundle; semple tower; Green-Griffiths-Lang conjecture; holomorphic morse inequality; semistable vector bundle; Kobayashi hyperbolic

Full Text: DOI

References:

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Demailly, Jean-Pierre

Structure theorems for compact Kähler manifolds with nef anticanonical bundles. (English) Zbl 1326.32007

Bracci, Filippo (ed.) et al., Complex analysis and geometry. KSCV 10. Proceddings of the 10th symposium, Gyeongju, Korea, August 7–11, 2014. Tokyo: Springer (ISBN 978-4-431-55743-2/hbk; 978-4-431-55744-9/ebook). Springer Proceedings in Mathematics & Statistics 144, 119-133 (2015).

Summary: This survey presents various results concerning the geometry of compact Kähler manifolds with numerically effective first Chern class: structure of the Albanese morphism of such manifolds, relations tying semipositivity of the Ricci curvature with rational connectedness, positivity properties of the Harder-Narasimhan filtration of the tangent bundle.

For the entire collection see [Zbl 1328.32001].

MSC:

32-02 Research monographs (several complex variables)
32J27 Compact Kähler manifolds: generalizations, classification
32L05 Holomorphic fiber bundles and generalizations

Keywords:

compact Kähler manifold; anticanonical bundle; semipositive Ricci curvature; Ricci flat manifold; rationally connected variety; holonomy principle

Full Text: DOI

References:

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Skoda, Henri (ed.); Demailly, Jean-Pierre; Siu, Yum-Tong
In memory of Pierre Lelong. (English) Zbl 1338.01059
Notices Am. Math. Soc. 61, No. 6, 586-595 (2014).

MSC:

01A70 Biographies, obituaries, personalia, bibliographies
32-03 Historical (several complex variables and analytic spaces)

Biographic references:

Lelong, Pierre

Full Text: DOI

Huckleberry, Alan (ed.); **Peternell, Thomas (ed.)**; **Siu, Yum-Tong**; **Ohsawa, Takeo**; **Demailly, Jean-Pierre**; **Barlet, Daniel**; **Trautmann, Günther**; **Lieb, Ingo**
A tribute to Hans Grauert. (English) Zbl 1338.01041
Notices Am. Math. Soc. 61, No. 5, 472-483 (2014).

MSC:

01A70 Biographies, obituaries, personalia, bibliographies

Biographic references:

Grauert, Hans

Full Text: DOI

Campana, F.; **Demailly, J.-P.**; **Peternell, T.**
Rationally connected manifolds and semipositivity of the Ricci curvature. (English) Zbl 1369.53052
Hacon, Christopher D. (ed.) et al., Recent advances in algebraic geometry. A volume in honor of Rob Lazarsfeld's 60th birthday. Based on the conference, Ann Arbor, MI, USA, May 16–19, 2013. Cambridge: Cambridge University Press (ISBN 978-1-107-64755-8/pbk; 978-1-107-41600-0/ebook). London Mathematical Society Lecture Note Series 417, 71-91 (2014).

Summary: This paper establishes a structure theorem for compact Kähler manifolds with semipositive anticanonical bundle. Up to finite étale cover, it is proved that such manifolds split holomorphically and isometrically as a product of Ricci flat varieties and of rationally connected manifolds.

The proof is based on a characterization of rationally connected manifolds through the nonexistence of certain twisted contravariant tensor products of the tangent bundle, along with a generalized holonomy principle for pseudoeffective line bundles. A crucial ingredient for this is the characterization of uniruledness by the property that the anticanonical bundle is not pseudoeffective.

For the entire collection see [Zbl 1318.14002].

MSC:

53C55 Hermitian and Kählerian manifolds (global differential geometry)
14M22 Rationally connected varieties

Demailly, Jean-Pierre; **Phạm, Hoàng Hiệp**
A sharp lower bound for the log canonical threshold. (English) Zbl 1298.14006
Acta Math. 212, No. 1, 1-9 (2014).

The log canonical threshold of a plurisubharmonic function φ with an isolated singularity at 0 in an open subset of $\mathbb{C}^n$ is the the supremum of $c > 0$ such that $\exp(-2c\varphi)$ is integrable on a neighborhood of the origin. The main result is the sharp lower bound $c(\phi) \geq \sum_{j=0}^{n-1} e_j(\varphi)/e_{j+1}(\varphi)$, where the intersection numbers $e_j(\varphi)$ are the Lelong numbers of $(dd^c\varphi)^j$ at 0.

Reviewer: Jan Stevens (Göteborg)

MSC:

14B05 Singularities (algebraic geometry)
32U05 Plurisubharmonic functions and generalizations
32U25 Lelong numbers
14C20 Divisors, linear systems, invertible sheaves

Keywords:

log canonical threshold; plurisubharmonic functions; Lelong number

Full Text: DOI

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Demailly, Jean-Pierre; Dinew, Sławomir; Guedj, Vincent; Hiep, Pham Hoang; Kołodziej, Sławomir; Zeriahi, Ahmed

Hölder continuous solutions to Monge-Ampère equations. (English) Zbl 1296.32012

J. Eur. Math. Soc. (JEMS) 16, No. 4, 619-647 (2014).

Let (X,ω) be a compact Kähler manifold of dimension n. The authors study the following complex Monge-Ampère operator

$$\mathrm{MA}(u) := \frac{1}{V_\omega}(\omega + dd^c u)^n, \quad \text{where} \quad V_\omega = \int_X \omega^n,$$

acting on ω-plurisubharmonic functions, $u \in \mathrm{PSH}(X,\omega)$, which are Hölder continuous, $u \in \text{Hölder}(X,\mathbb{R})$.

The following theorem gives a better information about the Hölder exponent of the solution to the complex Monge-Ampère equation than theorems proved recently by *P. Eyssidieux* et al. [J. Am. Math. Soc. 22, No. 3, 607–639 (2009; Zbl 1215.32017)] and *S. Dinew* [J. Inst. Math. Jussieu 9, No. 4, 705–718 (2010; Zbl 1207.32034)].

Theorem A. Let $\mu = f\omega^n = \mathrm{MA}(u)$ be a probability measure absolutely continuous with respect to the Lebesgue measure with density $f \in L^p$, $p > 1$. Then u is Hölder continuous with exponent arbitrary close to $\frac{2}{1+nq}$, where $\frac{1}{p} + \frac{1}{q} = 1$.

The optimal value of the Hölder exponent in Theorem A is still unknown, but it cannot be better than $\frac{2}{nq}$, see [*S. Pliś*, Ann. Pol. Math. 86, No. 2, 171–175 (2005; Zbl 1136.32306)] or [*V. Guedj* et al., Bull. Lond. Math. Soc. 40, No. 6, 1070–1080 (2008; Zbl 1157.32033)].

Moreover, Theorem A is generalized from the Kähler case to the case of big cohomology classes.

The rest of the paper is devoted to the study of the range

$$\mathrm{MAH}(X,\omega) = \mathrm{MA}\big(\mathrm{PSH}(X,\omega) \cap \text{Hölder}(X,\mathbb{R})\big).$$

A complete characterization of the set $\mathrm{MAH}(X,\omega)$ is unknown, but some of its properties are proved in the following theorem.

Theorem B. The set $\mathrm{MAH}(X,\omega)$ has the L^p property, i.e. if $\mu \in \mathrm{MAH}(X,\omega)$, $f \geq 0$, $f \in L^p(\mu)$ with $||f||_p = 1$, then $f\mu \in \mathrm{MAH}(X,\omega)$. In particular $\mathrm{MAH}(X,\omega)$ is a convex set.

T.-C. Dinh et al. [J. Differ. Geom. 84, No. 3, 465–488 (2010; Zbl 1211.32021)] observed that the measures $\mu \in \mathrm{MAH}(X,\omega)$ have the following property

$$\exp\big(-\epsilon \mathrm{PSH}(X,\omega)\big) \subset L^1(\mu), \quad \text{for some} \quad \epsilon > 0.$$

Condition (DNS) gives a full description of the range $\mathrm{MAH}(X,\omega)$ for $n = 1$, see [*T.-C. Dinh* and *N. Sibony*, Comment. Math. Helv. 81, No. 1, 221–258 (2006; Zbl 1094.32005)].

In the article under review it is proved that, for some special class of measures, this characterization is true in higher dimensions.

Theorem C. Let μ be a probability measure with finitely many isolated singularities which is radial or toric. Then $\mu \in \mathrm{MAH}(X,\omega)$ if and only if condition (DNS) is satisfied.

Reviewer: Rafał Czyz (Krakow)

MSC:

32U05 Plurisubharmonic functions and generalizations

32U40 Currents

53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords:
Monge-Ampère operator; Kähler manifold; pluripotential theory; Hölder continuity

Full Text: DOI arXiv

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Campana, Frédéric; **Demailly, Jean-Pierre**; **Verbitsky, Misha**
Compact Kähler 3-manifolds without nontrivial subvarieties. (English) Zbl 1293.32028
Algebr. Geom. 1, No. 2, 131-139 (2014).

This is a very interesting paper. The authors prove that compact Kähler threefolds without nontrivial subvariety are tori.

We describe below the proof in four steps:

Step 1 (Corollary 4.3): If X is a compact simple Kähler threefold, then K_X is pseudo-effective. That is the Chern class of the canonical line bundle has a positive $(1, 1)$-current.

X is simple if there is an intersection A of a set of accountable dense Zariski open sets such that each point in A is not contained in any nontrivial subvariety. The description on page 131 of the "simple" is a little bit confusing. The authors use the celebrated result of Brunella Theorem 4.1. They notice that $h^{2,0} > 0$, otherwise X is projective.

Step 2 (Corollary 2.6): If X is a compact Kähler manifold without nontrivial subvariety, then K_X is nef but not big. This comes from Theorem 2.5.

Step 3 (Corollary 3.3): $\mathcal{X}(\mathcal{O}) = 0$. This comes from a manipulation of the Riemann-Roch formula with a version of the Hard Lefschetz Theorem 3.1.

Step 4 (Lemma 1.4.): Another application of the Riemann-Roch formula noticing that $h^{3,0} \leq 1$.

See also the authors' summary.

Reviewer: Daniel Guan (Riverside)

MSC:

32J17 Compact 3-folds (analytic spaces)
32J27 Compact Kähler manifolds: generalizations, classification
32J25 Transcendental methods of algebraic geometry

Keywords:

compact Kähler threefolds; simple manifolds; holomorphic foliations; complex torus; hyperkähler manifolds

Full Text: DOI

Demailly, Jean-Pierre

Pierre Lelong: a fundamental work in complex analysis and analytical geometry. (Pierre Lelong: une œuvre fondatrice en analyse complexe et en géométrie analytique.) (French) Zbl 1296.01027

Gaz. Math., Soc. Math. Fr. 135, 63-66 (2013).

MSC:

01A70 Biographies, obituaries, personalia, bibliographies
32-03 Historical (several complex variables and analytic spaces)

Cited in **1** Document

Biographic references:

Lelong, Pierre

Demailly, Jean-Pierre

Episciences: a publishing platform for open archive overlay journals. (English) Zbl 1290.01031

Eur. Math. Soc. Newsl. 87, 31-32 (2013).

MSC:

01A80 Sociology (and profession) of mathematics

Demailly, Jean-Pierre; **Hacon, Christopher D.**; **Păun, Mihai**

Extension theorems, non-vanishing and the existence of good minimal models. (English) Zbl 1278.14022

Acta Math. 210, No. 2, 203-259 (2013).

Let X be a complex projective manifold (or normal complex projective variety with mild singularities). The aim of the minimal model program is to construct a birational model $X \dashrightarrow X'$ such that either X' admits a fibration with general fibre a Fano variety or X' is a good minimal model, that is some positive multiple of the canonical divisor $K_{X'}$ defines a morphism. If X is covered by rational curves or X is of general type (that is some positive multiple of K_X defines a birational map) the minimal model program is completed in the landmark paper by *C. Birkar* et al., [J. Am. Math. Soc. 23, No. 2, 405–468 (2010; Zbl 1210.14019)]. Thus the main challenge is now to study projective manifolds X that are not covered by rational curves and not of general type. By a fundamental result of *S. Boucksom* et al. [J. Algebr. Geom.

22, No. 2, 201–248 (2013; Zbl 1267.32017)], the canonical divisor K_X is then pseudoeffective, that is K_X is a limit of effective divisor. However the nonvanishing conjecture claims that some positive multiple of the canonical divisor is actually effective. Once we know that there exists at least one effective pluricanonical divisor D one can hope to establish the existence of good minimal models inductively by proving that the restriction morphism
$$H^0(X, \mathcal{O}_X(mK_X)) \to H^0(D, \mathcal{O}_D(mK_X))$$
is surjective for $m \gg 0$. A similar extension result played a crucial role in the proof of the existence of flips by *C. D. Hacon* and *J. McKernan* [J. Am. Math. Soc. 23, No. 2, 469–490 (2010; Zbl 1210.14021)]. In the paper under review the authors realise an important step of this strategy by proving the following "plt" extension theorem:
Let X be a projective manifold and $S+B$ a $\mathbb{Q}$-divisor with simple normal crossings such that
1) $(X, S+B)$ is plt (i.e. S is a prime divisor with $\mathrm{mult}_S(S+B)=1$ and $\lfloor B \rfloor = 0$), and
2) there exists an effective $\mathbb{Q}$-divisor $D \sim_{\mathbb{Q}} K_X + S + B$ such that
$$S \subset \mathrm{Supp}(D) \subset \mathrm{Supp}(S+B),$$
and
3) for any ample divisor A and any rational number $\epsilon > 0$, there is an effective $\mathbb{Q}$-divisor $D \sim_{\mathbb{Q}} K_X + S + B + \epsilon A$ whose support does not contain S).
Consider $\pi : \tilde{X} \to X$ a log-resolution of $(X, S+B)$, so that we have
$$K_{\tilde{X}} + \tilde{S} + \tilde{B} = \pi^*(K_X + S + B) + \tilde{E}$$
where $\tilde{S}$ is the strict transform of S. Let m be an integer, such that $m(K_X+S+B)$ is Cartier, and let u be a section of $m(K_X+S+B)|_S$, such that
$$Z_{\pi^*(u)} + m\tilde{E}|_{\tilde{S}} \geq m\Xi,$$
where $Z_{\pi^*(u)}$ is the zero divisor of the section $\pi^*(u)$ and Ξ the extension obstruction divisor (cf. [Zbl 1210.14021]). Then u extends to X.
The main achievement of this theorem compared to earlier extension results is that one does not assume B to be strictly positive (i.e. ample or big). The authors conjecture that their statement also holds under the weaker assumption that the pair $(X, S+B)$ is dlt. This stronger extension result would then reduce the minimal model conjecture to the nonvanishing problem. More precisely the authors prove the following theorem: Suppose that the "dlt" extension theorem holds in dimension n. Suppose also that the non-vanishing conjecture holds for semi-log-canonical pairs of dimension n. Then every n-dimensional projective manifold that is not covered by rational curves has a good minimal model.

Reviewer: Andreas Höring (Nice)

MSC:

14E30 Minimal model program (Mori theory, extremal rays)
14J40 Algebraic n-folds ($n > 4$)
32J25 Transcendental methods of algebraic geometry

Keywords:

extension theorem; minimal model; MMP; abundance conjecture; nonvanishing conjecture

Full Text: DOI

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Demailly, Jean-Pierre

Applications of pluripotential theory to algebraic geometry. (English) Zbl 1271.32037

Bracci, Filippo (ed.) et al., Pluripotential theory. Lectures of the CIME course, Cetraro, Italy, 2011. Berlin: Springer; Florence: Fondazione CIME (ISBN 978-3-642-36420-4/pbk; 978-3-642-36421-1/ebook). Lecture Notes in Mathematics 2075. CIME Foundation Subseries, 143-263 (2013).

Summary: These lectures are devoted to the study of various contemporary problems of algebraic geometry, using fundamental tools from complex potential theory, namely plurisubharmonic functions, positive currents and Monge-Ampère operators. Since their inception by Oka and Lelong in the mid 1940s, plurisubharmonic functions have been used extensively in many areas of algebraic and analytic geometry, as they are the function theoretic counterpart of pseudoconvexity, the complexified version of convexity. One such application is the theory of L^2 estimates via the Bochner-Kodaira-Hörmander technique, which provides very strong existence theorems for sections of holomorphic vector bundles with positive curvature. One can mention here the foundational work achieved by Bochner, Kodaira, Nakano, Morrey, Kohn, Andreotti-Vesentini, Grauert, Hörmander, Bombieri, Skoda and Ohsawa-Takegoshi in the course of more than four decades. Another development is the theory of holomorphic Morse inequalities (1985), which relate certain curvature integrals with the asymptotic cohomology of large tensor powers

of line or vector bundles, and bring a useful complement to the Riemann-Roch formula.We describe here the main techniques involved in the proof of holomorphic Morse inequalities (Sect. 1) and their link with Monge-Ampère operators and intersection theory. Section 2, especially, gives a fundamental approximation theorem for closed (1, 1)-currents, using a Bergman kernel technique in combination with the Ohsawa-Takegoshi theorem. As an application, we study the geometric properties of positives cones of an algebraic variety (nef and pseudo-effective cone), and derive from there some results about asymptotic cohomology functionals in Sect. 3. The last Sect. 4 provides an application to the study of the Green-Griffiths-Lang conjecture. The latter conjecture asserts that every entire curve drawn on a projective variety of general type should satisfy a global algebraic equation; via a probabilistic curvature estimate, holomorphic Morse inequalities imply that entire curves must at least satisfy a global algebraic differential equation.

For the entire collection see [Zbl 1266.31001].

MSC:

32U05 Plurisubharmonic functions and generalizations
32W20 Complex Monge-Ampère operators
32U40 Currents

Keywords:

plurisubharmonic functions; positive currents; Monge-Ampère operator; holomorphic Morse inequalities; Ohsawa-Takegoshi theorem

Full Text: DOI

Boucksom, Sébastien; **Demailly, Jean-Pierre**; **Păun, Mihai**; **Peternell, Thomas**

The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension. (English) Zbl 1267.32017

J. Algebr. Geom. 22, No. 2, 201-248 (2013).

Summary: We prove that a holomorphic line bundle on a projective manifold is pseudo-effective if and only if its degree on any member of a covering family of curves is non-negative. This is a consequence of a duality statement between the cone of pseudo-effective divisors and the cone of "movable curves", which is obtained from a general theory of movable intersections and approximate Zariski decomposition for closed positive $(1,1)$-currents. As a corollary, a projective manifold has a pseudo-effective canonical bundle if and only if it is not uniruled. We also prove that a 4-fold with a canonical bundle which is pseudo-effective and of numerical class zero in restriction to curves of a good covering family, has non-negative Kodaira dimension.

MSC:

32Q15 Kähler manifolds
32J27 Compact Kähler manifolds: generalizations, classification

Keywords:

compact Kähler manifold; projective Kähler manifold; Kodaira dimension; uniruled manifold

Full Text: DOI

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Demailly, Jean-Pierre (ed.); **Hulek, Klaus (ed.)**; **Peternell, Thomas (ed.)**

Complex analysis. Abstracts from the workshop held September 2–8, 2012. (Komplexe Analysis.) (English) Zbl 1349.00135

Oberwolfach Rep. 9, No. 3, 2597-2656 (2012).

Summary: The aim of this workshop was to discuss recent developments in several complex variables and complex geometry. Special emphasis was put on the interaction of analytic and algebraic methods. Topics included Kähler geometry, Ricci-flat manifolds, moduli theory and themes related to the minimal model program.

MSC:

00B05 Collections of abstracts of lectures
00B25 Proceedings of conferences of miscellaneous specific interest
32-06 Proceedings of conferences (several complex variables)
14-06 Proceedings of conferences (algebraic geometry)

Full Text: DOI

References:

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.

Demailly, Jean-Pierre
Henri Cartan and multivariate holomorphic functions. (Henri Cartan et les fonctions holomorphes de plusieurs variables.) (French) Zbl 1294.32002
Harinck, Pascale (ed.) et al., Henri Cartan et André Weil. Mathématiciens du XX^{e} siècle. Journées mathématiques X-UPS, Palaiseau, France, May 3–4, 2012. Palaiseau: Les Éditions de l'École Polytechnique (ISBN 978-2-7302-1610-4/pbk). 99-168 (2012).

The paper is a review of some fundamental results on holomorphic functions of several variables, and of the role of Henri Cartan in the development of this theory, in particular in the theory of coherent sheaves which he developed and which stands now as one of the most fundamental tools in complex geometry and in algebraic geometry. The exposition is concise but self-contained and the stress is on the essential facts. A few historical remarks are useful for understanding the motivations behind the ideas. All this makes the text much more attractive than many other texts written on the subject. The bibliographical references organized in sections are also very useful.

For the entire collection see [Zbl 1270.01009].

Reviewer: Athanase Papadopoulos (MR3014194)

MSC:

32-03 Historical (several complex variables and analytic spaces)
32A10 Holomorphic functions (several variables)
32B10 Germs of analytic sets, local parametrization
32C35 Analytic sheaves and cohomology groups
32H99 Holomorphic mappings on analytic spaces

Keywords:

holomorphic functions of several variables; coherent sheaves; complex geometry; algebraic geometry

Full Text: Link

Demailly, Jean-Pierre
Hyperbolic algebraic varieties and holomorphic differential equations. (English) Zbl 1264.32022
Acta Math. Vietnam. 37, No. 4, 441-512 (2012).

The long and exhaustive article reviews and explains recent methods and results in the theory of hyperbolic algebraic varieties, including several new results of the author which are focussed on some of the most challenging conjectures about hyperbolicity of algebraic manifolds of general type, the conjectures of Green-Griffiths and Lang. The conjectures of Green-Griffiths state that a projective algebraic variety X is of general type if and only if X is measure hyperbolic, and that X then contains a proper subvariety Y such that $f(\mathbb{C}) \subset Y$ for every non-constant holomorphic map $f : \mathbb{C} \to X$. An affirmative solution for these conjectures would imply that every very generic hypersurface in $\mathbb{P}^{n+1}$, $n \geq 3$, of degree $d \geq 2n+1$ is hyperbolic, see [*L. Ein*, Invent. Math. 94, No. 1, 163–169 (1988; Zbl 0701.14002); Math. Ann. 289, No. 3, 465–471 (1991; Zbl 0746.14019)] and [*C. Voisin*, J. Differ. Geom. 44, No. 1, 200–213 (1996; Zbl 0883.14022)].

The author studies hyperbolicity problems in a more general setting. He works in the class of directed manifolds and transfers classical hyperbolicity concepts for complex varieties to this category. Special emphasis lies on the application of the Ahlfors-Schwarz lemma, jets of curves, Semple jet bundles, k-jet bundles, holomorphic Morse inequalities, jet differentials and k-jet metrics with negative curvature. A (compact resp. projective) directed manifold is understood as a pair (X, V) of a (compact resp. projective) connected complex manifold X and an irreducible closed analytic subspace V of the holomorphic tangent bundle T_X such that $V \cap T_{X,x}$ is a linear subspace of $T_{X,x}$ for every $x \in X$. Based on the Brody criterion it turns out that a compact directed manifold (X, V) is hyperbolic iff every holomorphic map $f : \mathbb{C} \to X$ with $f'(\mathbb{C}) \subset V$ is constant. A projective directed manifold (X, V) is by definition algebraic hyperbolic if X admits an Hermitian metric with fundamental form ω such that for some $\epsilon > 0$ the inequality $-\chi(\overline{C}) \geq \epsilon \int_C \omega$ is satisfied for every irreducible closed algebraic curve $C \subset X$ tangent to V and normalized by $\overline{C}$. Hyperbolic projective directed manifolds are algebraic hyperbolic, but the converse is not known.

The main results of the paper are partial answers to a generalized version of the conjectures mentioned above. Assume that (X, V) is a projective directed manifold and that the canonical bundle of V is big. The generalized Green-Griffiths-Lang conjecture claims the existence of a proper subvariety Y of X with $f(\mathbb{C}) \subset Y$ for every non-constant holomorphic $f : \mathbb{C} \to X$ with $f'(\mathbb{C}) \subset V$. Under the assumption that X is of general type the author proves the existence of global algebraic differential operators P on X with $P(f, f', \dots, f^{(k)}) = 0$ for every such f.

Another interesting result of the author is related to the hyperbolicity of generic algebraic hypersurfaces in $\mathbb{P}^{n+1}$ of sufficiently high degree. *Y.-T. Siu* ["Hyperbolicity of generic high-degree hypersurfaces in complex projective spaces", Preprint, `arXiv:1209.2723`] has shown that there exists a sequence (d_n) in $\mathbb{N}$ with the property that a generic algebraic hypersurface in $\mathbb{P}^{n+1}$ of degree $d \geq d_n$, $n \geq 2$, is hyperbolic. *S. Diverio, J. Merker* and *E. Rousseau* [Invent. Math. 180, No. 1, 161–223 (2010; Zbl 1192.32014)] proved that $d_n := 2^{n^5}$ does the job. The paper under review yields a considerable improvement of these estimates. The theorem of Siu is fulfilled for $d_2 := 286$, $d_3 := 7316$ and $d_n := \lfloor \frac{n^4}{3}(n \log(n \log(24n)))^n \rfloor$ for $n \geq 4$.

Reviewer: Eberhard Oeljeklaus (Bremen)

MSC:

32Q45	Hyperbolic and Kobayashi hyperbolic manifolds
32L10	Sections of holomorphic vector bundles
53C55	Hermitian and Kählerian manifolds (global differential geometry)
14J40	Algebraic n-folds ($n > 4$)

Keywords:

Kobayashi hyperbolic variety; directed manifold; jet bundle; Chern connection; variety of general type; Green-Griffiths conjecture; Lang conjecture

Full Text: Link

Berman, Robert; Demailly, Jean-Pierre

Regularity of plurisubharmonic upper envelopes in big cohomology classes. (English)

Zbl 1258.32010

Itenberg, Ilia (ed.) et al., Perspectives in analysis, geometry, and topology. On the occasion of the 60th birthday of Oleg Viro. Based on the Marcus Wallenberg symposium on perspectives in analysis, geometry, and topology, Stockholm, Sweden, May 19–25, 2008. Basel: Birkhäuser (ISBN 978-0-8176-8276-7/hbk; 978-0-8176-8277-4/ebook). Progress in Mathematics 296, 39-66 (2012).

The authors deal with the regularity of certain quasiplurisubharmonic upper envelopes. The main theorem says that if X is a compact complex manifold of the Fujiki class $\mathcal{C}$ (these are the smooth varieties that are bimeromorphic to compact Kähler manifolds, or equivalently, they carry a cohomology class $\{\alpha\} \in H^{1,1}(X,\mathbb{R})$ which is big), then let $\{\alpha\}$ be the aforementioned big class and let T_0 be the current obtained from α by adding the dd^c of a quasiplurisubharmonic function ψ_0 with only analytic singularities, such that it dominates some positive multiple of the fixed Hermitian metric on X. Then the following function (called the upper envelope)

$$\varphi = \sup\left\{\psi \leq 0 : \psi \text{ is } \alpha\text{-plurisubharmonic}\right\}$$

is itself quasiplurisubharmonic and has locally bounded second order derivatives outside the analytic set given by the "$-\infty$"-set of ψ_0. The order of blow up near this set is also estimated. The main theorem is then applied to obtain a priori inequalities for the solution of the Dirichlet problem for a degenerate Monge-Ampère operator, to the study of geodesics in the space of Kähler potentials and finally to obtain a logarithmic modulus of continuity for Tsuji's supercanonical metrics.

For the entire collection see [Zbl 1230.00045].

Reviewer: Zywomir Dinew (Kraków)

MSC:

32U05 Plurisubharmonic functions and generalizations

Keywords:

plurisubharmonic function; quasiplurisubharmonic function; upper envelope; Monge-Ampère operator

Full Text: DOI arXiv

Demailly, Jean-Pierre

Analytic methods in algebraic geometry. (English) Zbl 1271.14001

Surveys of Modern Mathematics 1. Somerville, MA: International Press; Beijing: Higher Education Press (ISBN 978-1-57146-234-3/pbk). 231 p. (2012).

The book under review is based upon a series of lectures given by Jean-Pierre Demailly at the Park City Mathematics Institute in 2008, it was partly published in [IAS/Park City Mathematics Series 17, 295–370 (2010; Zbl 1222.32043)]. The main aim is to give a presentation of analytic techniques especially as it relates to positivity of vector bundles.

The book begins by briefly reviewing the concepts of sheaf cohomology, plurisubharmonic functions, currents and other topics. In Chapter 4, it moves on to the Bochner technique and applications to the Akizuki-Nakano-Kodaira vanishing theorem. Chapter 5 covers L^2 estimates and multiplier ideal sheaves. Chapter 6 covers pseudo effective and nef line bundles and Kawamata-Viehweg vanishing and applications. The next chapter covers applications of these ideas, results towards Fujita's conjecture, such as Reider's Theorem and work of Siu. Chapter 8 covers Holomorphic Morse inequalities as introduced by Demailly. Chapter 9 covers effective versions of Matsusaka's big theorem. Chapter 11 covers the question of surjectivity of global sections for maps of vector bundles and an application to the Briancon-Skoda Theorem. Chapter 12 covers the Ohsawa-Takegoshi L^2 Extension Theorem and Skoda's division theorem. Chapter 13 focuses on approximation of positive currents and plurisubharmoic functions and relations to the Hodge Conjecture. The remainder of the book (chapters 15 through 20), cover various topics in higher dimensional algebraic geometry such as Subadditivity of multiplier ideals, invariance of plurigenera, the Kähler cone and Pseudo-efective cone, abundance, and many other topics.

Reviewer: Karl Schwede (University Park)

MSC:

14-02	Research monographs (algebraic geometry)	Cited in **1** Review
14F18	Multiplier ideals	Cited in **16** Documents
14C20	Divisors, linear systems, invertible sheaves	
14J40	Algebraic n-folds ($n > 4$)	
32C30	Integration on analytic sets and spaces, currents	
32U40	Currents	

Keywords:

positivity; vanishing theorem; multiplier ideal; current; extension theorem; line bundles

Demailly, Jean-Pierre

Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture. (English) Zbl 1316.32014

Pure Appl. Math. Q. 7, No. 4, 1165-1207 (2011).

Summary: The goal of this work is to study the existence and properties of non constant entire curves f drawn in a complex irreducible n-dimensional variety X, and more specifically to show that they must satisfy certain global algebraic or differential equations as soon as X is projective of general type. By means of holomorphic Morse inequalities and a probabilistic analysis of the cohomology of jet spaces, we are able to reach a significant step towards a generalized version of the Green-Griffiths-Lang conjecture.

MSC:

32L20	Vanishing theorems (analytic spaces)	Cited in **2** Reviews
14C30	Transcendental methods, Hodge theory, Hodge conjecture	Cited in **12** Documents
32Q45	Hyperbolic and Kobayashi hyperbolic manifolds	

Full Text: DOI arXiv

Demailly, Jean-Pierre

A converse to the Andreotti-Grauert theorem. (English. French summary) Zbl 1228.32020

Ann. Fac. Sci. Toulouse, Math. (6) 20, Spec. Issue, 123-135 (2011).

Summary: The goal of this paper is to show that there are strong relations between certain Monge-Ampère integrals appearing in holomorphic Morse inequalities, and asymptotic cohomology estimates for tensor powers of holomorphic line bundles. Especially, we prove that these relations hold without restriction for projective surfaces, and in the special case of the volume, i.e. of asymptotic 0-cohomology, for all projective manifolds. These results can be seen as a partial converse to the Andreotti-Grauert vanishing theorem.

Reviewer: Viorel Vâjâitu (Bucureşti)

MSC:

32J25	Transcendental methods of algebraic geometry
32L10	Sections of holomorphic vector bundles
14C20	Divisors, linear systems, invertible sheaves
14F99	Homology and cohomology theory (algebraic geometry)

Keywords:

asymptotic cohomology functions; holomorphic Morse inequalities; volume of a line bundle

Full Text: DOI EuDML

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France 90, p. 193-259 (1962). Zbl0106.05501 MR150342 · Zbl 0106.05501 · numdam:BSMF_1962___90___193_0 · eudml:87019

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[5] Demailly (J.-P.).- Estimations L 2 pour l'op\'erateur \partial \? d'un fibr\'e vectoriel holomorphe semi-positif au dessus d'une vari\'et\'e k\"ahl\'erienne compl\'ete, Ann. Sci. \'Ecole Norm. Sup. 15, p. 457-511 (1982). Zbl0507.32021 MR690650 · Zbl 0507.32021 · numdam:ASENS_1982_4_15_3_457_0 · eudml:82103

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[9] Demailly (J.-P.).- Holomorphic Morse inequalities and asymptotic cohomology groups: a tribute to Bernhard Riemann, Milan Journal of Mathematics 78, p. 265-277 (2010). Zbl1205.32017 MR2684780 · Zbl 1205.32017 · doi:10.1007/s00032-010-0118-3

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[11] Demailly (J.-P.), P\u{a}un (M.).- Numerical characterization of the K\"ahler cone of a compact K\"ahler manifold, arXiv: math.AG/0105176 ; Annals of Math. 159, p. 1247-1274 (2004). Zbl1064.32019 MR2113021 · Zbl 1064.32019 · doi:10.4007/annals.2004.159.1247

[12] de Fernex (T.), K\"uronya (A.), Lazarsfeld (R.).- Higher cohomology of divisors on a projective variety, Math. Ann. 337, p. 443-455 (2007). Zbl1127.14012 MR2262793 · Zbl 1127.14012 · doi:10.1007/s00208-006-0044-4

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[15] K\"uronya (A.).- Asymptotic cohomological functions on projective varieties, Amer. J. Math. 128, p. 1475-1519 (2006). Zbl1114.14005 MR2275909 · Zbl 1114.14005 · doi:10.1353/ajm.2006.0044 · muse.jhu.edu

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Demailly, Jean-Pierre

Structure theorems for projective and Kähler varieties. (English) Zbl 1222.32043

McNeal, Jeffery (ed.) et al., Analytic and algebraic geometry. Common problems, different methods. Lecture notes from the Park City Mathematics Institute (PCMI) graduate summer school on analytic and algebraic geometry, Park City, UT, USA, Summer 2008. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-4908-8/hbk). IAS/Park City Mathematics Series 17, 295-370 (2010).

The main purpose of these notes is to describe some basic structure theorems for projective or compact Kähler varieties and their cohomology, using recent techniques of complex analysis and potential theory. One central unifying concept is that one of positivity, which can be viewed either in algebraic terms (positivity of divisors and algebraic cycles) or in more analytic terms (plurisubharmonicity, positive currents, Hermitian connections with positive curvature). In the 20th century, powerful L^2 techniques have emerged, giving rise to an incredible amount of geometric consequences. Here the author refers to these results and points out several topics: algebro-analytic characterizations of the Kaehler cone, the pseudo-effective cone of divisors, concepts of volume and mobile intersections, super-canonical metrics, non vanishing theorem, finiteness of the canonical ring.

For the entire collection see [Zbl 1202.00101].

Reviewer: Gabriela Paola Ovando (Rosario)

MSC:

32Q15 Kähler manifolds

Cited in **1** Review
Cited in **2** Documents

Keywords:

projective varieties; Kähler varieties; positivity

Demailly, Jean-Pierre (ed.); Hulek, Klaus (ed.); Peternell, Thomas (ed.)

Complex analysis. Abstracts from the workshop held August 29th – September 4th, 2010. (Komplexe analysis.) (English) Zbl 1209.00048

Oberwolfach Rep. 7, No. 3, 2283-2333 (2010).

Summary: The aim of this workshop was to discuss recent developments in several complex variables and complex geometry. Special emphasis was put on the interaction between model theory and the classification theory of complex manifolds. Other topics included Kähler geometry, foliations, complex symplectic manifolds and moduli theory.

MSC:

00B05 Collections of abstracts of lectures
32-06 Proceedings of conferences (several complex variables)
14-06 Proceedings of conferences (algebraic geometry)
32Qxx Complex manifolds
14Jxx Surfaces and higher-dimensional varieties
14Dxx Families, fibrations

Full Text: DOI Link

Demailly, Jean-Pierre

Holomorphic Morse inequalities and asymptotic cohomology groups: a tribute to Bernhard Riemann. (English. French summary) Zbl 1205.32017

Milan J. Math. 78, 265-277 (2010).

Summary: The goal of this note is to present the potential relationships between certain Monge-Ampère integrals appearing in holomorphic Morse inequalities, and asymptotic cohomology estimates for tensor powers of line bundles, as recently introduced by algebraic geometers. The expected most general statements, which are still conjectural, certainly owe a debt to Riemann's pioneering work, which led to the concept of Hilbert polynomials and to the Hirzebruch-Riemann-Roch formula during the XX-th century.

MSC:

32L10 Sections of holomorphic vector bundles
14B05 Singularities (algebraic geometry)
14C17 Intersection theory, etc.

Cited in **2** Documents

Keywords:

holomorphic Morse inequalities; Monge-Ampère integrals; Dolbeault cohomology; asymptotic cohomology groups; Riemann-Roch formula; Hermitian metrics; Chern curvature tensor; plurisuharmonic approximation

Full Text: DOI

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Demailly, Jean-Pierre; Kobayashi, Shoshichi; Narasimhan, Raghavan; Siu, Yuan-Tong
Cartan and complex analytic geometry. (English) Zbl 1195.01062
Notices Am. Math. Soc. 57, No. 8, 952-960 (2010).

MSC:

01A70 Biographies, obituaries, personalia, bibliographies
32-03 Historical (several complex variables and analytic spaces)

Cited in **1** Document

Biographic references:

Cartan, Henri

Full Text: Link

Demailly, Jean-Pierre; **Pali, Nefton**
Degenerate complex Monge-Ampére equations over compact Kähler manifolds. (English) Zbl 1191.53029
Int. J. Math. 21, No. 3, 357-405 (2010).

The Calabi conjecture was solved by *S.-T. Yau* [Commun. Pure Appl. Math. 31, 339–411 (1978; Zbl 0362.53049)]. *A. Bedford* and *B. A. Taylor* [Invent. Math. 37, 1–44 (1976; Zbl 0315.31007)] initiated a new method for the study of degenerate complex Monge-Ampére equations. In this paper, the authors prove existence and uniqueness of the solution of some very general type of degenerate complex Monge-Ampére equations, and investigate their regularity. Also the existence and fine regularity properties of the solutions of complex Monge-Ampére equations with respect to a given degenerate metric are proved.

Reviewer: Constantin Călin (Iaşi)

MSC:

53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)
53C55 Hermitian and Kählerian manifolds (global differential geometry)
32J15 Compact surfaces (analytic spaces)

Cited in **20** Documents

Keywords:

complex Monge-Ampère equations; Kähler-Einstein metrics; closed positive currents; plurisubharmonic functions; capacities; Orlicz spaces

Full Text: DOI arXiv

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[5] DOI: 10.1090/S0002-9939-07-08858-2 · Zbl 1116.32024 · doi:10.1090/S0002-9939-07-08858-2
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[35] DOI: 10.1215/S0012-7094-85-05210-X · Zbl 0578.32023 · doi:10.1215/S0012-7094-85-05210-X

Demailly, Jean-Pierre
Estimates on Monge-Ampère operators derived from a local algebra inequality. (English) Zbl 1209.32024
Passare, Mikael (ed.), Complex analysis and digital geometry. Proceedings from the Kiselmanfest, Uppsala, Sweden, May 2006 on the occasion of Christer Kiselman's retirement. Uppsala: Univ. Uppsala (ISBN 978-91-554-7672-4/pbk). 131-143 (2009).

Author's abstract: The goal of this short note is to relate the integrability property of the exponential $e^{-2\varphi}$ of a plurisubharmonic function φ with isolated or compactly supported singularities to a priori bounds for the Monge-Ampère mass of $(dd^c\varphi)^n$. The inequality is valid locally or globally on an arbitrary open subset Ω in $\mathbb{C}^n$. We show that $\int_\Omega (dd\varphi)^n < n^n$ implies $\int_K e^{-2\varphi} < +\infty$ for every compact subset K in Ω, while functions of the form $\varphi(z) = n \log |z - z_0|$, $z_0 \in \Omega$, appear as limit cases. The result is derived from an inequality of pure local algebra, which turns out a posteriori to be equivalent to it, proved by A. Corti in dimension $n = 2$, and later extended by L. Ein, T. De Fernex and M. Mustaţă to arbitrary dimensions.

For the entire collection see [Zbl 1192.00078].

Reviewer: Daniel Barlet (Nancy)

MSC:

32W20	Complex Monge-Ampère operators
32U10	Plurisubharmonic exhaustion functions
32S05	Local singularities (analytic spaces)
14B05	Singularities (algebraic geometry)
14C17	Intersection theory, etc.

Keywords:

Monge-Ampère operator; local algebra; monomial ideal; Hilbert-Samuel multiplicity; log-canonical threshold; plurisubharmonic function; Ohsawa-Takegoshi L^2 extension theorem; approximation of singularities; birational rigidity

Demailly, Jean-Pierre (ed.); **Hulek, Klaus (ed.)**; **Mok, Ngaiming (ed.)**; **Peternell, Thomas (ed.)**
Complex analysis. Abstracts from the workshop held August 24–30, 2008. (Komplexe Analysis.) (English) Zbl 1177.14011
Oberwolfach Rep. 5, No. 3, 2165-2218 (2008).

Introduction: The workshop *Komplexe Analysis*, organised by Jean-Pierre Demailly (Grenoble), Klaus Hulek (Hannover), Ngaiming Mok (Hong Kong) and Thomas Peternell (Bayreuth) was held August 24th–August 30, 2008. This meeting was well attended with 46 participants from Europe, US, and the Far East. The participants included several leaders in the field as well as many young (non-tenured) researchers.

The aim of the meeting was to present recent important results in several complex variables and complex geometry with particular emphasis on topics linking different areas of the field, as well as to discuss new directions and open problems. Altogether there were nineteen talks of 60 minutes each, a programme which left sufficient time for informal discussions and joint work on research projects. One of the topics at the center of the conference was the classification theory of higher dimensional varieties. Y. Kawamata lectured on the connections between the minimal model programme and derived categories; A. Corti discussed an approach to the finite generation of the canonical ring without minimal models, but still in connection with the seminal work which was presented by J. McKernan in the last Complex Analysis meeting in Oberwolfach 2006, where the finite generation of the canonical ring of varieties of general type was announced. Extension theorems, non vanishing and positivity result for certain direct image sheaves play a role in the global classification of complex manifolds. This was largely discussed by M. Paun and B. Berndtsson. In their work analytic methods are central, whereas the talks by Kawamata and Corti were more of an algebraic nature. Also very much on the analytic side and connected to Berndtsson's talk, H. Tsuji lectured on generalised Kähler-Einstein metrics. Families of projective manifolds over higher-dimensional base spaces were considered in the talk by S. Kebekus. Direct images of coherent sheaves also play a central role in this context. About five years ago, Campana introduced new variations on the concept of "orbifolds"; they were already the suject of talks in past sessions and have turned out to be of increasing interest – in the present session, new results on the hyperbolicity of orbifolds were presented in the talk by E. Rousseau. As to varieties with special geometry, K. Oguiso spoke on non-algebraic hyperkähler manifolds and, with a rather different flavour, F. Catanese on complex and real threefolds fibered by rational curves, with a special emphasis on real algebraic geometry. J. Chen discussed the influence of terminal singularities in three-dimensional geometry, a more algebraic topic. On the analytic side, A. Teleman reported on recent progress in the classification of non-Kähler surfaces in the so called Kodaira class VII, using gauge-theoretical methods, and S. K. Yeung lectured on new results on fake projective planes. Group actions and envelopes of holomorphy were the topics of the talk by X. Zhou. S. Boucksom discussed equidistribution of Fekete points on complex manifolds, in relation with energy functionals for Monge-Ampère operators. R. Lazarsfeld presented a very interesting new approach to study properties of linear systems and line bundles via convex geometry. Overall, moduli spaces appeared to be a central theme in the workshop, and were discussed extensively in at least four talks: V. Gritsenko considered moduli spaces of K3-surfaces; S. Grushevsky spoke on intersection numbers of divisor on the moduli space of curves, and K. Ludwig and G. Farkas lectured on the moduli spaces of spin and Prym curves, their singularities, Kodaira dimension and enumerative geometry.

MSC:

14-06 Proceedings of conferences (algebraic geometry)
14Jxx Surfaces and higher-dimensional varieties
32-06 Proceedings of conferences (several complex variables)
32Qxx Complex manifolds
00B05 Collections of abstracts of lectures

Full Text: DOI Link

Demailly, Jean-Pierre; **Hwang, Jun-Muk**; **Peternell, Thomas**
Compact manifolds covered by a torus. (English) Zbl 1144.14035
J. Geom. Anal. 18, No. 2, 324-340 (2008).

Let X be a compact complex manifold that is the image of a complex torus by a surjective holomorphic map $A \to X$. The main theorem of this paper states that X is a Kähler manifold and that, up to taking a finite étale cover, X is a product of projective spaces and a torus. This very nice statement generalises similar results for projective manifolds by *O. Debarre* [C. R. Acad. Sci., Paris, Sér. I 309, No. 2, 119–122 (1989; Zbl 0699.14050)] and *J.-M. Hwang* and *N. Mok* [Math. Z. 238, No. 1, 89–100 (2001; Zbl 1076.14021)]. Technically speaking, the proof falls into two independent parts:

In the first part, the authors observe that the morphism $A \to X$ is equidimensional. Since a complex torus is Kähler, a difficult theorem of *J. Varouchas* [Math. Ann. 283, No. 1, 13–52 (1989; Zbl 0632.53059)] then implies that X is also Kähler. Note that the authors give a rather short and self-contained proof of Varouchas' theorem in the appendix.

For the second part, we observe that the main theorem implies a-posteriori that the tangent bundle of X is nef. The strategy of the proof is now to show that many of the tools used in the study of manifolds with nef tangent bundle by *J.-P. Demailly, T. Peternell* and *M. Schneider* [J. Algebr. Geom. 3, No. 2, 295–345 (1994; Zbl 0827.14027)] are still available under the hypothesis that X is covered by a torus. For example it is shown that the Albanese map of X is a surjective submersion with connected fibres, and the fundamental group is almost abelian. Furthermore the anticanonical bundle is semi-ample and induces an equidimensional fibration. The main theorem is then established by comparing the Albanese morphism and the anticanonical fibration.

Reviewer: Andreas Höring (Paris)

MSC:

14J40	Algebraic n-folds ($n > 4$)	Cited in **1** Document
14C30	Transcendental methods, Hodge theory, Hodge conjecture	
32J25	Transcendental methods of algebraic geometry	

Keywords:

complex torus; abelian variety; projective space; Kähler manifold; Albanese morphism; fundamental group; étale cover; nef divisor; nef tangent bundle; anticanonical bundle; numerically flat vector bundle

Full Text: DOI arXiv

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[10] Demailly, J.-P., P\v{a}un, M.: Numerical characterization of the K\"ahler cone of a compact K\"ahler manifold. Ann. Math. 159(3), 1247–1274 (2004) · Zbl 1064.32019 · doi:10.4007/annals.2004.159.1247

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Demailly, Jean-Pierre; **Kosarew, Siegmund**; **Malgranqe, Bernard**
Adrien Douady and Banach analytic spaces. (Adrien Douady et les espaces analytiques banachiques.) (French) Zbl 1168.01321
Gaz. Math., Soc. Math. Fr. 113, 35-38 (2007).

MSC:

01A70 Biographies, obituaries, personalia, bibliographies
01A60 Mathematics in the 20th century
46-03 Historical (functional analysis)

Biographic references:

Douady, Adrien

Demailly, Jean-Pierre
Kähler manifolds and transcendental techniques in algebraic geometry. (English) Zbl 1141.14007
Sanz-Solé, Marta (ed.) et al., Proceedings of the international congress of mathematicians (ICM), Madrid, Spain, August 22–30, 2006. Volume I: Plenary lectures and ceremonies. Zürich: European Mathematical Society (EMS) (ISBN 978-3-03719-022-7/hbk). 153-186 (2007).

This paper surveys some of the main recent advances in the study of the geometry of projective or compact Kähler manifolds obtained by using local and global complex analytic methods. After a short introduction, section 2 presents well-known definitions and results, necessary for the paper. Section 3 describes the main results obtained by *J.-P. Demailly* and *M. Păun* [Ann. Math. 159, 1247–1274 (2004; Zbl 1064.32019)], namely:

Theorem 3.1. Let X be a compact Kähler manifold. Let $\mathcal{P}$ be the set of real $(1,1)$ cohomology classes $\{\alpha\}$ which are numerically positive on analytic cycles, i.e. such that $\int_Y \alpha^p > 0$, for every irreducible analytic set Y in X, $p = \dim Y$. Then the Kähler cone $\mathcal{K}$ of X is one of the connected components of $\mathcal{P}$.

Corollary 3.2. If X is projective algebraic, then $\mathcal{K} = \mathcal{P}$.

These results (which are new even in the projective case) can be seen as a generalization of the well-known Nakai–Moishezon criterion. Sketches of the proofs are given. The section 3 ends with a consequence about the dual of the cone $\mathcal{K}$ in $H_{\mathbb{R}}^{n-1,n-1}(X)$.

Section 4 is devoted to deformations of compact Kähler manifolds. Kodaira showed in the 60s that every Kähler surface X is a limit by deformations of algebraic surfaces. The long-standing question whether a similar property holds in higher dimensions was shown in negative by C. Voisin:

Theorem 4.1. (C. Voisin) (i) In any dimension ≥ 4, there exist compact Kähler manifolds which do not have the homotopy type (or even the homology ring) of a complex projective manifold.

(ii) In any dimension ≥ 8, there exist compact Kähler manifolds X such that no compact bimeromorphic model X' of X has the homotopy type of a complex projective manifold.

Then, the behaviour of the Kähler cone of X_t as t approaches the "bad strata" (X_t in a deformation of Kähler manifolds) is given (this is a result of the above paper by Demailly - Păun).

Section 5 presents results on positive cones in $H^{n-1,n-1}(X)$ and Serre duality. We shall give only two results:

Theorem 5.3. (Demailly - Păun) If X is Kähler, then the cones $\bar{\mathcal{K}} \subset H^{1,1}(X,\mathbb{R})$ and $\mathcal{N} \subset H_{\mathbb{R}}^{n-1,n-1}(X)$ are dual by Poincaré duality, and $\mathcal{N}$ is the closed convex cone generated by classes $[Y] \wedge \omega^{p-1}$, where $Y \subset X$ ranges over p-dimensional analytic subsets, $p = 1, 2, \dots, n$, and ω ranges over Kähler forms.

The next result is from the paper of *S. Boucksom, J.-P. Demailly, M. Păun* and *Th. Peternell* [The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension, `arXiv:math/0405285`]:

Theorem 5.14. If X is projective, then a class $\alpha \in NS_{\mathbb{R}}(X)$ is pseudo-effective if (and only if) it is in the dual cone of the cone $SME(X)$ of strongly movable curves.

The sections ends with some applications and conjectures. The final section 6 presents new results around the invariance of plurigenera. We give only one result which is a special case of a result of Păun:

Corollary 6.3. (Siu) For any projective family $t \mapsto X_t$ of algebraic varieties, the plurigenera $p_m(X_t) = h^\circ(X_t, mK_{X_t})$ do not depend on t.

For the entire collection see [Zbl 1111.00009].

Reviewer: Vasile Brînzănescu (Bucureşti)

MSC:

14C30	Transcendental methods, Hodge theory, Hodge conjecture
32C30	Integration on analytic sets and spaces, currents
32L20	Vanishing theorems (analytic spaces)

Cited in **4** Documents

Keywords:

projective variety; Kähler manifold; Hodge theory; positive current

Demailly, Jean-Pierre

Towards a revaluation of mathematics and science teaching: GRIP initiatives and SLECC network classes. (Vers une réévaluation de l'enseignement des mathématiques et des sciences: initiatives du GRIP et réseau de classes SLECC.) (French) Zbl 1343.00016

Gaz. Math., Soc. Math. Fr. 110, 61-64 (2006).

MSC:

00A35	Methodology of mathematics, didactics
97B40	Higher education
97B70	Syllabuses, educational standards

Full Text: Link

Demailly, Jean-Pierre (ed.); **Hulek, Klaus (ed.)**; **Peternell, Thomas (ed.)**

Report 40/2006: Komplexe Analysis (August 27th – September 2nd, 2006). (English) Zbl 1109.14301

Oberwolfach Rep. 3, No. 3, 2399-2446 (2006).

Abstract: The main aim of this workshop was to discuss recent developments in several complex variables and complex geometry. The topics included: classification of higher dimensional varieties, mirror symmetry, hyperbolicity, Kähler geometry and classical geometric questions.

Contributions:

–Michel Brion, Log homogeneous varieties (p. 2403)

–Bernd Siebert (joint with Mark Gross), Tropical games and Mirror Symmetry (p. 2405)

–Fedor Bogomolov (joint with Bruno de Oliveira), Symmetric tensors and the geometry of secant varieties in a projective space (p. 2408)

–Philippe Eyssidieux (joint with Vincent Guedj, Ahmed Zeriahi), Singular Kähler-Einstein metrics (p. 2409)

–Pelham M.H. Wilson, The geometry of Kähler moduli (p. 2411)

–James McKernan (joint with Caucher Birkar, Paolo Cascini, Christopher Hacon), Existence of minimal models for varieties of log general type (p. 2414)

–Jun-Muk Hwang, Rigid targets of surjective holomorphic maps (p. 2419)

–Andreas Gathmann (joint with Hannah Markwig), Tropical enumerative geometry (p. 2421)

–Aleksandr V. Pukhlikov, Birationally rigid Fano fiber spaces (p. 2422)

–Fabrizio Catanese (joint with Sönke Rollenske), The slope of Kodaira fibrations (p. 2424)

–Sándor Kovács (joint with Stefan Kebekus), Viehweg's conjecture for two-dimensional bases (p. 2427)

–Erwan Rousseau, Recent developments on hyperbolicity of complex algebraic varieties (p. 2430)

–Priska Jahnke (joint with C. Casagrande, I. Radloff), On the Picard number of almost Fano threefolds (p. 2433)

–Andrew J. Sommese (joint with Charles W. Wampler), Using fiber products to compute exceptional sets (p. 2435)

–Takeo Ohsawa, Levi flat hypersurfaces in complex manifolds (p. 2436)

–Mihai Păun, Siu's invariance of plurigenera: a one-tower proof (p. 2437)

–Duco van Straten (joint with K. Jung), Arctic computation of monodromy (p. 2439)

–Michael McQuillan, Residues hyperbolicity and abundance (p. 2442)

MSC:

14-XX	Algebraic geometry
32-XX	Several complex variables and analytic spaces
00B05	Collections of abstracts of lectures

Full Text: DOI Link

Demailly, Jean-Pierre; **Eckl, Thomas**; **Peternell, Thomas**
Line bundles on complex tori and a conjecture of Kodaira. (English) Zbl 1078.32014
Comment. Math. Helv. 80, No. 2, 229-242 (2005).

In 1963 Kodaira proved that every smooth compact Kähler surface is almost algebraic in the sense that it can be realized as a deformation of a projective surface. Until recently it was an open problem whether it is true in general that a compact Kähler manifold is almost algebraic. An affirmative answer would have implied that every compact Kähler manifold has the homotopy type of a projective algebraic manifold and is projective if it is rigid. But in 2004 *C. Voisin* [Invent. Math. 157, No. 2, 329–343 (2004; Zbl 1065.32010)] constructed for every dimension greater than three compact Kähler manifolds with homotopy type different from the homotopy type of a projective manifold. Her examples are built from compact complex tori by blowing up processes.

In the paper under review the authors are equally interested in giving an affirmative answer to the above question for a subclass of compact Kähler manifolds and in finding new counterexamples. They discuss different strategies for $\mathbb{P}(V)$-bundles on complex tori. These considerations are based on the fact that the structure of $\mathbb{P}(V)$-bundles or $\mathbb{P}_r$-bundles over a compact complex manifold survives under deformation (Theorem 8). The authors consider the following situation: Let A be a three-dimensional compact complex torus and L_1, L_2, L_3 holomorphic line bundles on A representing three linear independent elements in the Néron-Severi group $\mathrm{NS}(A)$. The manifold $Y = \mathbb{P}(\mathcal{O}_A \oplus L_1) \times_A \mathbb{P}(\mathcal{O}_A \oplus L_2) \times_A \mathbb{P}(\mathcal{O}_A \oplus L_3)$ is a holomorphic $\mathbb{P}_1^3$-bundle over A with a natural holomorphic section Z given by the direct summand $\mathcal{O}_A$ in every factor.

The main result of the paper (Theorem 4) asserts that Y is algebraically approximable by projective Albanese bundles $Y_n \to A_n$ with $Y_n = \mathbb{P}(\mathcal{O}_{A_n} \oplus L_1) \times_{A_n} \mathbb{P}(\mathcal{O}_{A_n} \oplus L_2) \times_{A_n} \mathbb{P}(\mathcal{O}_{A_n} \oplus L_3)$ and $\lim_{n\to\infty} A_n = A$ in the sense of deformation theory. The proof uses an explicit description of $\mathrm{NS}(A)$ in terms of skew-symmetric integer 6×6 matrices and calculations with support of the computer algebra program Macauley 2 [see *D. Eisenbud* et al., Algorithms and Computation in Mathematics. 8. (Berlin: Springer) (2002; Zbl 0973.00017)]. Blowing up in the bundle Y every fiber F in the point $F \cap Z$ gives a compact Kähler manifold X, a holomorphic fiber bundle over A with projective rational fiber. Under the additional assumption that not all of the line bundles L_1, L_2, L_3 remain holomorphic under small deformations of A, the manifold X is rigid (Proposition 3) and could a priori be a counterexample. But the assumption on the L_i forces $A_n = A$ in Theorem 4, hence X is already projective. The authors explain their ideas how modifications of their construction and more general settings could eventually lead to new counter-examples.

Reviewer: Eberhard Oeljeklaus (Bremen)

MSC:

32J27 Compact Kähler manifolds: generalizations, classification

32G05 Deformations of complex structures

32Q15 Kähler manifolds

Cited in **3** Documents

Keywords:

compact Kähler manifold; almost algebraic; algebraically approximable; Kodaira conjecture

Software:

Macaulay2

Full Text: DOI Link arXiv

Demailly, Jean-Pierre (ed.); **Hulek, Klaus (ed.)**; **Peternell, Thomas (ed.)**
Workshop: Complex analysis. (Komplexe Analysis.) (English) Zbl 1078.30501
Oberwolfach Rep. 1, No. 3, Report 42, 2171-2215 (2004).

Contributions:

– Daniel Barlet, Application of complex analysis to oscillating integrals p.2171

– Ingrid C.Bauer (joint with F.Catanese and F.Grunewald), Beauville surfaces without real structures and group theory p.2171

– Michel Brion, Extension of equivariant vector bundles p.2174

– Ciprian S.Borcea, Polygon spaces, tangents to quadrics and special Lagrangians p.2177

– Bertrand Deroin

– Immersed Levi-flat hypersurfaces into non negatively curved complex surfaces p.2179

– Wolfgang Ebeling (joint with Sabir M.Gusein-Zade and José Seade), Indices of 1-forms on singular varieties p.2182

– Akira Fujiki (joint with Massimiliano Pontecorvo), Anti-self-dual hermitian metrics on Inoue surfaces p.2184

– Samuel Grushevsky, Addition formulas for theta functions, and linear systems on abelian varieties p.2186

– Peter Heinzner (joint with Gerald Schwarz), Cartan decomposition of the moment map p.2188

– Jun-Muk Hwang, Bound on the number of curves of a given degree through a general point of a projective variety p.2191

– Priska Jahnke (joint with Thomas Peternell and Ivo Radloff), Threefolds with big and nef anticanonical bundles p.2193

– Shigeyuki Kondo, A complex ball uniformization for the moduli spaces of del Pezzo surfaces via periods of K3 surfaces p.2195

– Michael Lönne, Braid monodromy of hypersurface singularities p.2197

– Laurent Meersseman (joint with Alberto Verjovsky), A foliation of S5 by complex surfaces and its moduli space p.2199

– Keiji Oguiso, Automorphisms of hyperkähler manifolds p.2201

– Edoardo Sernesi (joint with A.Bruno), The non-Petri locus for pencils p.2203

– Andrew J.Sommese (joint with Jan Verschelde and Charles W.Wampler), Numerically decomposing the intersection of algebraic varieties p.2203

– Andrei Teleman (joint with Matei Toma), Complex geometric applications of Gauge Theory p.2204

MSC:

30-06 Proceedings of conferences (functions of one complex variable)
32-06 Proceedings of conferences (several complex variables)
00B05 Collections of abstracts of lectures

Full Text: Link

Demailly, Jean-Pierre; **Paun, Mihai**
Numerical characterization of the Kähler cone of a compact Kähler manifold. (English)
Zbl 1064.32019
Ann. Math. (2) 159, No. 3, 1247-1274 (2004).

The Kähler cone of a compact Kähler manifold is the set of cohomology classes of smooth positive definite clossed $(1,1)$-forms. The authors show that this cone depends only on the intersection product of the cohomology ring, the Hodge structure and the homology classes of analytic cycles: if X is a compact Kähler manifold, the Kähler cone $\mathcal{K}$ of X is one of the connected components of the set $\mathcal{P}$ of real $(1,1)$-cohomology classes $\{\alpha\}$ which are numerically positive on the analytic cycles, i.e. such that $\int_Y \alpha^p > 0$ for every irreducible analytic set in X, $p = \dim Y$. This result can be considered as a generalization of the Nakai-Moishezon criterion, which provide a necessary and sufficient criterion for a line bundle to be ample. If X is projective then $\mathcal{K} = \mathcal{P}$. If X is a compact Kähler manifold, the $(1,1)$-cohomology class α is nef (numerically effective free) if and only if there exists a Kähler metric ω on X such that $\int_Y \alpha^k \wedge \omega^{p-k} \geq 0$ for all irreducible analytic sets Y and all $k = 1, 2, \dots, p = \dim Y$. A $(1,1)$-cohomology class $\{\alpha\}$ on X is nef if and only if for every irreducible analytic set Y in X, $p = \dim Y$, and for every Kähler metric ω on X, one has $\int_Y \alpha \wedge \omega^{p-1} \geq 0$.

First, the authors obtain a sufficient condition for a nef class to contain a Kähler current. Then the main result is obtained by an induction on the dimension.

The obtained result has an important application to the deformation theory of compact Kähler manifolds: consider $\mathcal{X} \to S$ a deformation of compact Kähler manifolds over an irreducible base S. There exists a countable union $S' = \bigcup S_\nu$ of analytic subsets $S_\nu \subset S$, such that the Kähler cones $\mathcal{K}_t \subset H^{1,1}(X_t, \mathbb{C})$ are invariant over $S \setminus S'$ under parallel transport with respect to the $(1,1)$-projection $\nabla^{1,1}$ of the Gauss-Manin connection.

Reviewer: Vasile Oproiu (Iaşi)

MSC:

32Q15 Kähler manifolds
32Q25 Calabi-Yau theory
53C55 Hermitian and Kählerian manifolds (global differential geometry)
32J27 Compact Kähler manifolds: generalizations, classification

Keywords:

Kähler manifolds; Kähler cone; nef cohomology classes; Kähler currents

Full Text: DOI

Demailly, Jean-Pierre
On the geometry of positive cones of projective and Kähler varieties. (English) Zbl 1071.14013
Collino, Alberto (ed.) et al., The Fano conference. Papers of the conference organized to commemorate the 50th anniversary of the death of Gino Fano (1871–1952), Torino, Italy, September 29–October 5, 2002. Torino: Università di Torino, Dipartimento di Matematica. 395-422 (2004).

Summary: The goal of these notes is to give a short introduction to several works by Sébastien Boucksom, Mihai Paun, Thomas Peternell and myself on the geometry of positive cones of projective or Kähler manifolds. Mori theory has shown that the structure of projective algebraic manifolds is – up to a large extent – governed by the geometry of its cones of divisors or curves. In the case of divisors, two cones are of primary importance: the cone of ample divisors and the cone of effective divisors (and the closure of these cones as well). We introduce here the analogous transcendental cones for arbitrary compact Kähler manifolds, and show that these cones depend only on analytic cycles and on the Hodge structure of the base manifold. Also, we obtain new very precise duality statements connecting the cones of curves and divisors via Serre duality. As a consequence, we are able to prove one of the basic conjectures in the classification of projective algebraic varieties – a subject which Gino Fano contributed to in many ways: a projective algebraic manifold X is uniruled (i.e. covered by rational curves) if and only if its canonical class $c_1(K_X)$ does not lie in the closure of the cone spanned by effective divisors.

For the entire collection see [Zbl 1051.00013].

MSC:

14C30	Transcendental methods, Hodge theory, Hodge conjecture	Cited in **1** Document
14C20	Divisors, linear systems, invertible sheaves	
32J27	Compact Kähler manifolds: generalizations, classification	

Keywords:

transcendental cones; analytic cycles; Hodge structure; duality

Demailly, Jean-Pierre; Peternell, Thomas
A Kawamata-Viehweg vanishing theorem on compact Kähler manifolds. (English) Zbl 1077.32504
J. Differ. Geom. 63, No. 2, 231-277 (2003).

This paper appeared earlier under the same title in the proceedings of a conference. See *J.-P. Demailly* and *T. Peternell*, Surv. Differ. Geom. 8, 139–169 (2003; Zbl 1053.32011).

Reviewer: Imre Patyi (Atlanta)

MSC:

32L20	Vanishing theorems (analytic spaces)	Cited in **1** Review
32J27	Compact Kähler manifolds: generalizations, classification	Cited in **8** Documents

Keywords:

Kawamata-Viehweg vanishing theorem; compact Kähler spaces; abundance for threefolds

Full Text: DOI

Demailly, Jean-Pierre; Peternell, Thomas
A Kawamata-Viehweg vanishing theorem on compact Kähler manifolds. (English) Zbl 1053.32011
Yau, S.-T. (ed.), Surveys in differential geometry. Lectures on geometry and topology held in honor of Calabi, Lawson, Siu, and Uhlenbeck at Harvard University, Cambridge, MA, USA, May 3–5, 2002. Somerville, MA: International Press (ISBN 1-57146-114-0/hbk). Surv. Differ. Geom. 8, 139-169 (2003).

This article by well-known experts in complex geometry adds results to the ongoing effort to extend some important parts of Mori's theory of complex projective varieties to the case of compact Kähler manifolds and spaces. It appeared in the proceedings of a prestigious conference, and as a journal paper in [J. Differ. Geom. 63, No. 2, 231–277 (2003; Zbl 1077.32504)] under the same title.

The main results of this long and involved paper are as follows. In claim 0.1 the authors obtain a Kawamata-Viehweg vanishing theorem for the cohomology group $H^q(X, K_X + L) = 0$, $q \geq n-1$, where X is a normal compact Kähler space of dimension n, and $L \to X$ is a nef line bundle with $L^2 \neq 0$.

The proof of claim 0.1 is via demonstrating that the natural coefficient map induces zero in cohomology $H^{n-1}(X, K_X \otimes L \otimes J) \to H^{n-1}(X, K_X \otimes L)$, where J is a suitable multiplier ideal sheaf corresponding to a singular metric h on L. The latter vanishing is reduced to the study of a divisor D associated to h by Siu decomposition, and consists in showing that $H^0(D, (-L+D)|D) = 0$, done by working with Hodge index inequalities.

Then claim 0.1 is applied to abundance for threefolds given in claim 0.3:

If X is a $\mathbb{Q}$-Gorenstein Kähler threefold with only terminal singularities and K_X nef, then $\kappa(X) \geq 0$ for the Kodaira dimension.

The paper is informative and pleasant to read.

For the entire collection see [Zbl 1034.53003].

Reviewer: Imre Patyi (Atlanta)

MSC:

32L20 Vanishing theorems (analytic spaces)
32J27 Compact Kähler manifolds: generalizations, classification

Keywords:

Kawamata-Viehweg vanishing theorem; compact Kähler spaces; abundance for threefolds

Demailly, Jean-Pierre

On the Frobenius integrability of certain holomorphic p-forms. (English) Zbl 1011.32019

Bauer, Ingrid (ed.) et al., Complex geometry. Collection of papers dedicated to Hans Grauert on the occasion of his 70th birthday. Berlin: Springer. 93-98 (2002).

Summary: The goal of this note is to exhibit the integrability properties (in the sense of the Frobenius theorem) of holomorphic p-forms with values in certain line bundles with semi-negative curvature on a compact Kähler manifold. There are in fact very strong restrictions, both on the holomorphic form and on the curvature of the semi-negative line bundle. In particular, these observations provide interesting information on the structure of projective manifolds which admit a contact structure: either they are Fano manifolds or, thanks to results of Kebekus-Peternell-Sommese-Wisniewski, they are biholomorphic to the projectivization of the cotangent bundle of another suitable projective manifold.

For the entire collection see [Zbl 0989.00069].

MSC:

32Q15 Kähler manifolds
32J25 Transcendental methods of algebraic geometry

Keywords:

Frobenius integrability; holomorphic p-forms

Bertin, José; **Demailly, Jean-Pierre**; **Illusie, Luc**; **Peters, Chris**

Introduction to Hodge theory. Transl. from the French by James Lewis and Chris Peters. (English) Zbl 0996.14003

SMF/AMS Texts and Monographs. 8. Providence, RI: American Mathematical Society (AMS). ix, 232 p. (2002).

This book is the English translation of the French original published in 1996 under the title "Introduction à la théorie de Hodge" (1996; Zbl 0849.14002). Back then it appeared as volume 3 in the new series "Panoramas et Synthèses" edited by Société Mathématique de France. Grown out of a set of lectures which the authors had delivered at a conference on the present state of Hodge theory (Grenoble 1994), the aim of the text was to develop a number of fundamental concepts and results of both classical and modern Hodge theory, primarily addressed to graduate students and non-expert researchers in the field. In the English translation of this profound introduction to classical and modern Hodge theory, which discusses the subject in great depth and leads the reader to the forefront of contemporary research in many areas related to Hodge theory, the text has been left entirely unchanged.

Now as five years before, this book provides a masterly guide through Hodge theory and its various applications. It still maintains its unique up-to-date character, within the textbook and survey literature on the subject, as well as its significant role as an indispensible source for active researchers and teachers in the field, together with the additional advantage that its translation into English makes it now accessible to the entire mathematical and physical community worldwide. Without any doubt, this is exactly what both those communities and this excellent book on Hodge theory needed and deserved.

Reviewer: Werner Kleinert (Berlin)

MSC:

14C30 Transcendental methods, Hodge theory, Hodge conjecture

14-02 Research monographs (algebraic geometry)

14F17 Vanishing theorems

14D07 Variation of Hodge structures

81T30 String and superstring theories

58A14 Hodge theory (global analysis)

14D05 Structure of families

14J32 Calabi-Yau manifolds

Cited in **5** Documents

Keywords:

characteristic p; De Rham cohomology algebra; Hodge degeneration; variation of Hodge structures; Gauss-Manin connexion; period domains; Picard-Lefschetz theory; Calabi-Yau manifolds; Higgs bundles; Hodge theory; vanishing theorems; mirror symmetry

Campana, Frédéric; **Demailly, Jean-Pierre**

L^2 -cohomology on the coverings of a compact complex manifold. (French) Zbl 1066.32012

Ark. Mat. 39, No. 2, 263-282 (2001).

The aim of this paper is to define a natural L^2-cohomology on any unramified covering of a complex analytic space X, with values in the lifting of any coherent analytic sheaf on X. This L^2 cohomology has been constructed independently by *P. Eyssidieux* [Math. Ann. 317, 527–566 (2000; Zbl 0964.32008)].

It is seen that the usual properties of sheaf cohomology such as cohomology exact sequences or spectral sequences hold in this L^2-cohomology on X. If X is projective and non-singular there are L^2 vanishing theorems analogous to those of Kodaira-Serre and Kawamata-Viehweg.

When X is compact it is possible to define the Γ-dimension for Galois coverings. This Γ-dimension turns out to be finite in this case. An extension of Atiyah's index theorem is given in this context.

Reviewer: A. Diaz-Cano (Madrid)

MSC:

32C35 Analytic sheaves and cohomology groups

32C99 General theory of analytic spaces

32T99 Pseudoconvex domains

58J20 Index theory and related fixed-point theorems (PDE on manifolds)

Cited in **2** Documents

Full Text: DOI

References:

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AV Andreotti, A. etVesentini, E., Carleman estimates for the Laplace-Beltrami equation in complex manifolds,Inst. Hautes \'Etudes Sci. Publ. Math 25 (1965), 81–130. · Zbl 0138.06604 · doi:10.1007/BF02684398

A Atiyah, M., Elliptic operators, discrete groups and von Neumann algebras, dansElliptic Operators, Discrete Groups and von Neumann Algebras.Colloque "Analyse et Topologie" en l'honneur de Henri Cartan (Orsay, 1974), Ast\'erisque32–33, p. 43–72, Soc. Math. France, Paris, 1976.

C Campana, F., Fundamental group and positivity of cotangent bundles of compact K\"ahler manifolds,J. Algebraic Geom. 4 (1995), 487–502. · Zbl 0845.32027

D Demailly, J.-P.., EstimationsL 2 pour l'op\'erateur $\bar \partial $ d'un fibr\'e vectoriel hermitien semi-positif,Ann. Sci. \'Ecole Norm. Sup. 15 (1982), 457–511.

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E3 Eyssidieux, P., Invariants de von Neumann des faisceaux analytiques coh\'erents,Math. Ann. 317 (2000), 527–566. · Zbl 0964.32008 · doi:10.1007/PL00004413

G Gromov, M., K\"ahler hyperbolicity andL 2-Hodge theory,J. Differential Geom 33 (1991), 263–291. · Zbl 0719.53042

H H\"ormander, L.,An Introduction to Complex Analysis in Several Variables. 3rd ed., North-Holland Math. Library7, Elsevier, Amsterdam, 1990. · Zbl 0685.32001

JZ Jost, J. etZuo, K., Vanishing theorems forL 2-cohomology on infinite coverings of compact K\"ahler manifolds and applications in algebraic geometry,Comm. Anal. Geom. 8 (2000), 1–30.

K Koll\'ar, J.,Shafarevitch Maps and Automorphic Forms, Princeton Univ. Press, Princeton, N. J., 1995.

NR Napier, T. etRamachandran, M.., The $L^2 \bar \partial $ -method, weak Lefschetz theorems, and the topology of K\"ahler manifolds,J. Amer. Math. Soc. 11 (1998), 375–396. · Zbl 0897.32007 · doi:10.1090/S0894-0347-98-00257-4

O Ohsawa, T., A reduction theorem for cohomology groups of very stronglyq-convex K\"ahler manifolds,Invent. Math. 63 (1981), 335–354; Invent. Math.66 (1982) 391–393. · Zbl 0457.32007 · doi:10.1007/BF01393882

P Pansu, P., Introduction toL 2 Betti numbers, dansRiemannian Geometry (Waterloo, Ont., 1993) (Lovri\'c, M., Maung, M.-O. et Wang, M. Y.-K., \'eds) Fields Inst. Monogr.4, p. 53–86, Amer. Math. Soc., Providence, R. I., 1996.

W Weil, A.,Introduction \`a l'\'etude des vari\'et\'es k\"ahl\'eriennes, Hermann, Paris, 1958.

Demailly, Jean Pierre

Refoundation of mathematics in France. (Italian) Zbl 1106.01306

Lett. Mat. Pristem 42, 10-14 (2001).

The young and (even more) promising author of the present study, Jean-Piere Demailly, approaches the problem of mathematics refoundation from very (highly) French positions, by proudly asserting that the "nouvelles maths" have been initiated in France and nowhere else.

The role played in France by the "commission for the long-sum reflexion on the teaching of mathematics" is emphasized, and the name of its president – Jean-Piere Kahane – is mentioned.

Actually, the study is Demailly's contribution to the round table organized in Paris on "Mathematics and the Teaching of Sciences". It is quite a revolutionary text.

Extremely "refreshing" for the reader is the idea of having illustrated the text with coloured, challenging reproductions of Picasso.

Reviewer: Cristina Irimia (Iaşi)

MSC:

01A60 Mathematics in the 20th century

Keywords:

mathematics; France; J. P. Kahane; teaching of science

Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael
Pseudo-effective line bundles on compact Kähler manifolds. (English) Zbl 1111.32302
Int. J. Math. 12, No. 6, 689-741 (2001).

Summary: The goal of this work is to pursue the study of pseudo-effective line bundles and vector bundles. Our first result is a generalization of the Hard-Lefschetz theorem for cohomology with values in a pseudo-effective line bundle. The Lefschetz map is shown to be surjective when (and, in general, only when) the pseudo-effective line bundle is twisted by its multiplier ideal sheaf. This result has several geometric applications, e.g., to the study of compact Kähler manifolds with pseudo-effective canonical or anti-canonical line bundles. Another concern is to understand pseudo-effectivity in more algebraic terms. In this direction, we introduce the concept of an "almost" nef line bundle, and mean by this that the degree of the bundle is nonnegative on sufficiently generic curves. It can be shown that pseudo-effective line bundles are almost nef, and our hope is that the converse also holds. This can be checked in some cases, e.g., for the canonical bundle of a projective 3-fold. From this, we derive some geometric properties of the Albanese map of compact Kähler 3-folds.

MSC:

32J27 Compact Kähler manifolds: generalizations, classification
32Q15 Kähler manifolds
32Q57 Classification theorems

Full Text: DOI

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Demailly, Jean-Pierre
Multiplier ideal sheaves and analytic methods in algebraic geometry. (English) Zbl 1102.14300
Demailly, J.P. (ed.) et al., School on vanishing theorems and effective results in algebraic geometry. Lecture notes of the school held in Trieste, Italy, April 25–May 12, 2000. Trieste: The Abdus Salam International Centre for Theoretical Physics (ISBN 92-95003-09-8/pbk). ICTP Lect. Notes 6, 1-148 (2001).

The lecture notes under review, based on the author's course at the 2000 Trieste school on vanishing theorems and effective results in algebraic geometry, are an extended version of the author's CIME lectures [in: Transcendental methods in algebraic geometry. Lect. 3rd sess. CIME , Cetraro, Italy, 1994. Lect. Notes Math. 1646, 1–97 (1996; Zbl 0883.14005)]. They give a comprehensive survey on the application of analytic methods to algebraic geometry, especially to vanishing theorems. Aimed at non-specialist (in the author's words, they are "written with the idea of serving as an analytic toolbox for algebraic geometers"), they provide a lot of historical and introductory material on the subject, as well as very advanced topics and recent developments.

For a detailed account see the review of [loc. cit.]. The main changes are due to the incorporation of *Y. T. Siu*'s new result on the deformation invariance of plurigenera of varieties of general type [Invent. Math. 134, No.3, 661-673 (1998; Zbl 0955.32017)].

For the entire collection see [Zbl 0986.00053].

Reviewer: Olaf Teschke (Berlin)

MSC:

14F17	Vanishing theorems
32J25	Transcendental methods of algebraic geometry
32L10	Sections of holomorphic vector bundles
14C20	Divisors, linear systems, invertible sheaves
32Q15	Kähler manifolds
32L20	Vanishing theorems (analytic spaces)

Cited in **3** Reviews
Cited in **33** Documents

Demailly, J.P. (ed.); **Göttsche, L. (ed.)**; **Lazarsfeld, R. (ed.)**
School on vanishing theorems and effective results in algebraic geometry. Lecture notes of the school held in Trieste, Italy, April 25–May 12, 2000. (English) Zbl 0986.00053
ICTP Lecture Notes. 6. Trieste: The Abdus Salam International Centre for Theoretical Physics. vii, 393 p. (2001).

The articles of this volume will be reviewed individually.

Indexed articles:

Demailly, Jean-Pierre, Multiplier ideal sheaves and analytic methods in algebraic geometry., 1-148 [Zbl

1102.14300]

Smith, Karen E., Tight closure and vanishing theorems., 149-213 [Zbl 1079.13500]

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Peternell, Thomas, Subsheaves in the tangent bundle: Integrability, stability and positivity, 285-334 [Zbl 1027.14009]

Hwang, Jun-Muk, Geometry of minimal rational curves on Fano manifolds., 335-393 [Zbl 1086.14506]

MSC:

00B25 Proceedings of conferences of miscellaneous specific interest
14-06 Proceedings of conferences (algebraic geometry)

Keywords:

School; Vanishing theorems; Proceedings; Algebraic geometry; Trieste (Italy)

Full Text: Link Link

Demailly, Jean-Pierre; **Kollár, János**
Semi-continuity of complex singularity exponents and Kähler-Einstein metrics on Fano orbifolds. (English) Zbl 0994.32021
Ann. Sci. Éc. Norm. Supér. (4) 34, No. 4, 525-556 (2001).

Let φ be a plurisubharmonic function on a complex manifold X. The complex singularity exponent $c_K(\varphi)$ of φ on a compact set $K \subset X$ is the supremum over $c \geq 0$ such that $\exp(-2c\varphi)$ is integrable on a neighborhood of K. The notion plays an important role in complex analysis and algebraic geometry, and several other characteristics of singularities for analytic objects (holomorphic functions, coherent ideal sheaves, divisors, currents) are its particular cases.

The main results of the paper is lower semicontinuity of the map $\varphi \mapsto c_K(\varphi)$, which means that if $\varphi_j \to \varphi$ in $L^1_{\text{loc}}(X)$ then $\exp(-2 \subset \varphi_j) \to \exp(-2 \subset \varphi)$ in L^1-norm over a neighborhood of K for all positive $c < c_K(\varphi)$.

As a consequence, a comparatively simple proof is given for the existence of Kähler-Einstein metrics on certain Fano orbifolds. In this way, the authors produce three new examples of rigid del Pezzo surfaces with quotient singularities which admit a Kähler-Einstein metric.

Reviewer: Alexandr Yu.Rashkovsky (Khar'kov)

MSC:

32S05 Local singularities (analytic spaces)
14B05 Singularities (algebraic geometry)
14J45 Fano varieties
32U05 Plurisubharmonic functions and generalizations
32U25 Lelong numbers

Keywords:

Arnold multiplicity; multiplier ideal sheaf; Lelong number; complex singularity exponent; Kähler-Einstein metrics; Fano orbifolds; del Pezzo surfaces

Full Text: DOI Numdam EuDML

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Demailly, Jean-Pierre; Ein, Lawrence; Lazarsfeld, Robert

A subadditivity property of multiplier ideals. (English) Zbl 1077.14516

Mich. Math. J. 48, Spec. Vol., 137-156 (2000).

Summary: Given an effective -divisor D on a smooth complex variety, one can associate to D its multiplier ideal sheaf $J(D)$, which measures in a somewhat subtle way the singularities of D. Because of their strong vanishing properties, these ideals have come to play an increasingly important role in higher dimensional geometry. We prove that for two effective -divisors D and E, one has the "subadditivity" relation: $J(D+E) \subseteq J(D).J(E)$. We also establish several natural variants, including the analogous statement for the analytic multiplier ideals associated to plurisubharmonic functions.

As an application, we give a new proof of a theorem of *T. Fujita* [Kodai Math. J. 17, No. 1, 1–3 (1994; Zbl 0814.14006)] concerning the volume of a big linear series on a projective variety. The first section of the paper contains an overview of the construction and basic properties of multiplier ideals from an algebro-geometric perspective, as well as a discussion of the relation between some asymptotic algebraic constructions and their analytic counterparts.

MSC:

14E99 Birational geometry
14J17 Singularities of surfaces

Full Text: DOI arXiv

Demailly, Jean-Pierre
On the Ohsawa-Takegoshi-Manivel L^2 extension theorem. (English) Zbl 0959.32019
Dolbeault, P. (ed.) et al., Complex analysis and geometry. Proceedings of the international conference in honor of Pierre Lelong on the occasion of his 85th birthday, Paris, France, September 22-26, 1997. Basel: Birkhäuser. Prog. Math. 188, 47-82 (2000).

The Ohsawa-Takegoshi-Manivel L^2 extension theorem addresses the following basic problem: Let Y be a complex analytic submanifold of a complex manifold X; given a holomorphic function f on Y satisfying suitable L^2 conditions on Y, find a holomorphic extension F of f to X, together with a good L^2 estimate for F on X.

The first satisfactory solution of this problem has been obtained by T. Ohsawa and K. Takegoshi. The author follows here a more geometric approach due to L. Manivel, which provides a more general extension theorem in the framework of vector bundles and higher cohomology groups. The first goal of this note is to simplify further Manivel's approach, as well as to point out a technical difficulty in Manivel's proof. The author uses a simplified and slightly extended version of the original Ohsawa-Takegoshi a priori inequality. Then the Ohsawa-Takegoshi-Manivel extension theorem is applied to solve several important problems of complex analysis or geometry. The first of these is an approximation theorem for plurisubharmonic functions. It is shown that the approximation can be made with a uniform convergence of the Lelong numbers of the holomorphic functions towards those of the given plurisubharmonic function. This result contains as a special case Siu's theorem on the analyticity of Lelong number sublevel sets. By combining some of the results provided by the proof of that approximation theorem with Skoda's L^2 estimates for the division of holomorphic functions, a Briançon-Skoda type theorem for Nadel's multiplier ideal sheaves is obtained. Using this result and some ideas of R. Lazarsfeld, it is obtained a new proof of a recent result of T. Fujita: the growth of the number of sections of multiples of a big line bundle is given by the highest power of the first Chern class of the numerically effective part in the line bundle Zariski decomposition.

For the entire collection see [Zbl 0940.00031].

Reviewer: A.V.Chernecky (Odessa)

MSC:

32D15 Continuation of analytic objects (several variables)
32U05 Plurisubharmonic functions and generalizations

Keywords:

L^2 extension theorem; a priori inequality; L^2 existence theorem; approximation theorem; multiplier ideal sheaves; Zariski decomposition of big line bundles; plurisubharmonic functions

Demailly, Jean-Pierre; **El Goul, Jawher**
Hyperbolicity of generic surfaces of high degree in projective 3-space. (English)
Zbl 0966.32014
Am. J. Math. 122, No.3, 515-546 (2000).

The main result of this paper is to prove that a very generic surface X in $\mathbb{P}^3$ of degree $d \geq 21$ is Kobayashi hyperbolic, that is there is no nonconstant holomorphic map from $\mathbb{C} \to X$. As a consequence of the proof, they also prove that the complement of a very generic curve in $\mathbb{P}^2$ is hyperbolic and hyperbolically imbedded for all degrees $d \geq 21$. We note that previously, Siu-Yeung proved the hyperbolicity of the complement of a generic smooth curve of high degree in $\mathbb{P}^2$. The approach roughly is divided into the following steps: First use the Riemann-Roch calculations to prove the existence of suitable jet differentials which vanish on an ample divisor; then use Ahlfors-Schwarz lemma to conclude that the image of f sits in the base locus of the global sections of jet differentials; finally, it is hoped to show, by analysing the base locus carefully, that the base locus actually is a proper subvariety of X.

Reviewer: Min Ru (Houston)

MSC:

32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32H30 Value distribution theory in higher dimensions

Keywords:

hyperbolic; jet differentials; Riemann-Roch; surface of general type; Kobayashi hyperbolic

Full Text: DOI Link

Demailly, Jean-Pierre

L^2 methods and effective results in algebraic geometry. (Méthodes L^2 et résultats effectifs en géométrie algébrique.) (French) Zbl 0962.14014

Séminaire Bourbaki. Volume 1998/99. Exposés 850-864. Paris: Société Mathématique de France, Astérisque. 266, 59-90, Exp. No. 852 (2000).

The paper is a review of analytic methods (L^2 Hodge theory) used in algebraic geometry for studying adjoint linear systems, vanishing theorem for algebraic vector bundles and invariance of plurigenera of general type families. Among the topics discussed in the paper are singular metrics, applications to Fujita's conjecture [*T. Fujita* in: Algebraic Geometry, Proc. Symp., Sendai 1985, Adv. Stud. Pure Math. 10, 167–178 (1987; Zbl 0659.14002)] on global generation of adjoint linear systems, and analytic tools in Siu's proof [*Y.-T. Siu*, Invent. Math. 134, No. 3, 661–673 (1998; Zbl 0955.32017)] of invariance of plurigenera for a family of general type.

For the entire collection see [Zbl 0939.00019].

Reviewer: Taras E.Panov (Moskva)

MSC:

14F43 Other algebro-geometric (co)homologies
14C30 Transcendental methods, Hodge theory, Hodge conjecture
14J60 Vector bundles on surfaces and higher-order varieties, and their moduli
14F05 Sheaves, derived categories of sheaves, etc.
14C20 Divisors, linear systems, invertible sheaves
32J25 Transcendental methods of algebraic geometry
14N30 Adjunction problems

Keywords:

adjoint linear systems; line bundle; Fujita conjecture; variety of general type; invariance of plurigenera; Hodge theory

Full Text: Numdam EuDML

Demailly, Jean-Pierre

Pseudoconvex-concave duality and regularization of currents. (English) Zbl 0960.32011

Schneider, Michael (ed.) et al., Several complex variables. Cambridge: Cambridge University Press. Math. Sci. Res. Inst. Publ. 37, 233-271 (1999).

The paper investigates some basic properties of Finsler metrics on holomorphic vector bundles in the perspective of obtaining geometric versions of the Serre duality theorem. A duality framework under which pseudo-convexity and pseudo-concavity properties get exchanged is established. These duality properties are related to several geometric problems, e.g., the conjecture of Hartshorne and Schneider.

Finally, a new shorter and more geometric proof of a basic regularization theorem for closed $(1,1)$-currents is shown.

For the entire collection see [Zbl 0933.00014].

Reviewer: Viorel Vâjâitu (Bucureşti)

MSC:

32F10 q-convexity, q-concavity
32C30 Integration on analytic sets and spaces, currents
32J25 Transcendental methods of algebraic geometry

Cited in **5** Documents

Keywords:

Hartshorne-Schneider conjecture; plurisubharmonic function; regularization of currents; Finsler metrics; pseudoconvexity; pseudoconcavity

Full Text: Link

Campana, Frédéric; Demailly, Jean-Pierre; Peternell, Thomas
The algebraic dimension of compact complex threefolds with vanishing second Betti number. (English) Zbl 0910.32032
Compos. Math. 112, No.1, 77-91 (1998).

The abstract of the authors describes the content of the paper quite precisely. It reads (with very small changes) as follows: "This note investigates compact complex manifolds X of dimension three with second Betti number $b_2 = 0$. If X admits a nonconstant meromorphic function, then the authors prove that either $b_1(X) = 1$ and $b_3(X) = 0$ or that $b_1(X) = 0$ and $b_3(X) = 2$. The main idea is to show that $c_3(X) = 0$ by means of a vanishing theorem for generic line bundles on X. As a consequence a compact complex threefold homeomorphic to the 6-Sphere S^6 cannot admit a non-constant meromorphic function. Furthermore they investigate the structure of threefolds with $b_2 = 0$ and algebraic dimension one, in the case when the algebraic reduction $X \to_1$ is holomorphic".

Reviewer: E.Oeljeklaus (Bremen)

MSC:

32J17 Compact 3-folds (analytic spaces)
14C20 Divisors, linear systems, invertible sheaves

Cited in **1** Review
Cited in **7** Documents

Keywords:

algebraic reduction; generic vanishing theorem; topological Euler characteristic; algebraic dimension

Full Text: DOI

Demailly, Jean-Pierre
Hyperbolic projective varieties and algebraic differential equations. (Variétés projectives hyperboliques et équations différentielles algébriques.) (French) Zbl 0937.32012
Hirzebruch, Friedrich et al., Journée en l'honneur de Henri Cartan. Paris: Société Mathématique de France, SMF Journ. Annu. 1997, 3-17 (1997).

From the introduction (translated from the French): "The aim of the text is to offer an introduction, as elementary as possible, to an important result concerning the geometry of the images of holomorphic curves in complex algebraic varieties".

For the entire collection see [Zbl 0932.00086].

MSC:

32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32J10 Algebraic dependence theorems (compact analytic spaces)
32L05 Holomorphic fiber bundles and generalizations
32H30 Value distribution theory in higher dimensions
32-02 Research monographs (several complex variables)

Cited in **2** Documents

Keywords:

Nevanlinna theory; geometry of the images of holomorphic curves; complex algebraic varieties

Hirzebruch, Friedrich; **Demailly, Jean-Pierre**; **Lannes, Jean**
Conference in honor of Henri Cartan. (Journée en l'honneur de Henri Cartan.) (French)
Zbl 0932.00086
SMF Journée Annuelle. 1997. Paris: Société Mathématique de France, iv, 27 p. (1997).

The articles of this volume will be reviewed individually.

Indexed articles:

Hirzebruch, F., Learning complex analysis in Münster–Paris, Zürich and Princeton from 1945 to 1953., 1-2 [Zbl 1071.01500]

Demailly, Jean-Pierre, Hyperbolic projective varieties and algebraic differential equations, 3-17 [Zbl 0937.32012]

Lannes, Jean, Diverse aspects of the Steenrod operations, 18-27 [Zbl 0934.55002]

MSC:

00B30 Festschriften
00B15 Collections of articles of miscellaneous specific interest
32-06 Proceedings of conferences (several complex variables)

Keywords:

Journée; Honneur; Dedication; Conference

Biographic references:

Cartan, Henri

Demailly, Jean-Pierre
Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. (English)
Zbl 0919.32014
Kollár, János (ed.) et al., Algebraic geometry. Proceedings of the Summer Research Institute, Santa Cruz, CA, USA, July 9–29, 1995. Providence, RI: American Mathematical Society. Proc. Symp. Pure Math. 62(pt.2), 285-360 (1997).

This are notes of a series of lectures delivered at the Santa Cruz AMS Summer School on Algebraic Geometry. They are mainly devoted to the study of complex varieties through a few geometric questions related to hyperbolicity in the sense of Kobayashi. A convenient framework for this is the category of "directed manifolds", that is, the category of pairs (X, V) where X is a complex manifold and V a holomorphic subbundle of T_X. If X is compact, the pair (X, V) is hyperbolic if and only if there are no nonconstant entire holomorphic curves $f :\to X$ tangent to V (Brody's criterion). The author describes a construction of projectivized k-jet bundles P_kV, which generalizes a construction made by Semple in 1954 and allows to analyze hyperbolicity in terms of negativity properties of the curvature.

An overview information on the lecture notes is given by their contents.

1. Hyperbolicity concepts and directed manifolds
2. Hyperbolicity and bounds for the genus of curves
3. The Ahlfors-Schwarz lemma for metrics of negative curvature
4. Projectivization of a directed manifold
5. Jets of curves and semple jet bundles
6. Jet differentials
7. k-Jet metrics with negative curvature
8. Algebraic criterion for the negativity of jet curvature
9. Proof of the Bloch theorem
10. Logarithmic jet bundles and a conjecture of Lang
11. Projective meromorphic connections and Wronskians
12. Decomposition of jets in irreducible representations
13. Riemann-Roch calculations and study of the base locus
14. Appendix: A vanishing theorem for holomorphic tensor fields.

For the entire collection see [Zbl 0882.00033].

Reviewer: J.Eichhorn (Greifswald)

MSC:

32Q45	Hyperbolic and Kobayashi hyperbolic manifolds	Cited in **8** Reviews
32L10	Sections of holomorphic vector bundles	Cited in **30** Documents
14J40	Algebraic n-folds ($n > 4$)	
53C55	Hermitian and Kählerian manifolds (global differential geometry)	

Keywords:

Kobayashi hyperbolicity

Demailly, Jean-Pierre

Hyperbolic varieties and algebraic differential equations. (Variétés hyperboliques et équations différentielles algébriques.) (French) Zbl 0901.32019

Gaz. Math., Soc. Math. Fr. 73, 3-23 (1997).

In this survey article, the author presents the relationship between the existence of entire curves (i.e. holomorphic curves $f :\to X$) on an algebraic variety X and global algebraic differential operators on the variety X. We mention that the nonexistence of non constant entire curves is equivalent to the Kobayashi's hyperbolicity.

The author gives a complete proof of the following vanishing result of *M. Green* and *Ph. Griffiths*, presented with an incomplete proof in Proc. Int. Chern Symp., Berkely 1979, 41-74 (1980; Zbl 0508.32010)]: "Let X be a projective algebraic variety and let $f :\to X$ be a non constant entire curve. Then $P(f', \dots, f^{(k)}) \equiv 0$ for any algebraic differential operator P with values in the dual L^* of a holomorphic line bundle L on X, with positive curvature". As an application one obtaines explicit examples of hyperbolic algebraic surfaces of small degree by applying the above vanishing result to wronskian operators.

Reviewer: Vasile Brînzănescu (Bucureşti)

MSC:

32Q45	Hyperbolic and Kobayashi hyperbolic manifolds	Cited in **3** Documents
32-02	Research monographs (several complex variables)	
32H30	Value distribution theory in higher dimensions	
32A22	Nevanlinna theory (local); growth estimates; other inequalities (several complex variables)	

Keywords:
hyperbolic varieties; projective varieties of general type; wronskian

Demailly, Jean-Pierre; El Goul, Jawher
Meromorphic partial projective connections and hyperbolic projective varieties. (Connexions méromorphes projectives partielles et variétés algébriques hyperboliques.) (French. Abridged English version) Zbl 0898.32016
C. R. Acad. Sci., Paris, Sér. I 324, No. 12, 1385-1390 (1997).

S. Kobayashi conjectured in [Hyperbolic manifolds and holomorphic mappings, Marcel Dekker, NY (1970; Zbl 0207.37902)] that a generic hypersurface of $\mathbf{CP}^n$ of sufficiently high degree d (where the expected bound is $d \geq 2n-1$) is hyperbolic. The conjecture is true for $\mathbf{CP}^2$, but for $n \geq 3$ a few number of examples are known. For $\mathbf{CP}^3$ (where the expected bound is 5) the first example of a smooth hyperbolic surface in $\mathbf{CP}^3$ of any degree $d \geq 50$ was obtained by *R. Brody* and *M. Green* [Duke Math. J. 44, 873-874 (1977; Zbl 0383.32009)] and *A. M. Nadel* [Duke Math. J. 58, No. 3, 749-771 (1989; Zbl 0686.32015)] obtained examples of degree $d \geq 21$ and the second author [Manuscr. Math. 90, No. 4, 521-532 (1996)] gave examples of degree $d \geq 14$. In this paper, following some ideas of *Y. T. Siu* [Duke Math. J. 55, 213-251 (1987; Zbl 0623.32018)] and A. Nadel, the authors introduce the concept of meromorphic connection and construct Wronskian operators acting on jets of holomorphic curves. Then using some results, the authors give examples of hyperbolic algebraic surfaces in $\mathbf{CP}^3$ with arbitrary degree $d \geq 11$.

Reviewer: Raul Ibañez (Bilbão)

MSC:

32Q45	Hyperbolic and Kobayashi hyperbolic manifolds	Cited in **2** Documents
14H10	Families, algebraic moduli (curves)	
32A20	Meromorphic functions (several variables)	
32C25	Analytic subsets and submanifolds	
53A20	Projective differential geometry	
53C55	Hermitian and Kählerian manifolds (global differential geometry)	

Keywords:
meromorphic connection; Wronskian operator; hyperbolic surfaces; complex projective space

Full Text: DOI

Demailly, Jean-Pierre
Numerical analysis and differential equations. Nouvelle éd. (Analyse numérique et équations différentielles.) (French) Zbl 0869.65041
Grenoble: Presses Univ. de Grenoble. 309 p. (1996).

See the review of the German translation (1994; Zbl 0869.65042).

MSC:

65L05	Initial value problems for ODE (numerical methods)	Cited in **2** Reviews Cited in **1** Document
65L06	Multistep, Runge-Kutta, and extrapolation methods	
65D32	Quadrature and cubature formulas (numerical methods)	
65H10	Systems of nonlinear equations (numerical methods)	
65-01	Textbooks (numerical analysis)	
34-01	Textbooks (ordinary differential equations)	

Keywords:
ordinary differential equation; initial value; problem; roundoff problem; polynomial approximation; quadrature formulas; iterative methods; textbook

Demailly, Jean-Pierre

L^2 vanishing theorems for positive line bundles and adjunction theory. (English)

Zbl 0883.14005

Catanese, F. (ed.) et al., Transcendental methods in algebraic geometry. Lectures given at the 3rd session of the Centro Internazionale Matematico Estivo (CIME), Cetraro, Italy, July 4–12, 1994. Cetraro: Springer. Lect. Notes Math. 1646, 1-97 (1996).

Main goal of the paper is to describe a few analytic tools which are useful to study questions such as linear series and vanishing theorems for algebraic vector bundles. Also, algebraic and analytic proofs of some results are compared. One of the first applications of the analytic method in algebraic geometry is Kodaira's use of the Bochner technique (1950-60) to relate cohomology and curvature via harmonic forms. Well known is the Akizuki-Kodaira-Nakano theorem (1954): If X is a nonsingular projective algebraic variety and L is a holomorphic line bundle on X with positive curvature, then $H^q(X, \Omega^p_X \otimes L) = 0$ for $p + q > \dim X$. Hörmander (1965) used a refinement of this technique to obtain a fundamental L^2 estimate, concerning solutions of the Cauchy-Riemann operator. Except vanishing theorems, more precise quantative information about solutions of $\bar{\partial}$-equations was obtained. Main tools to relate analytic and algebraic geometry are the multiplier ideal sheaf $I(\phi)$ and positive currents. $I(\phi)$ is defined as a sheaf of germs of holomorphic functions f such that $|f|^2 e^{-2\phi}$ is locally summable, where ϕ is a (locally defined) plurisubharmonic function. Since $I(\phi)$ is a coherent algebraic sheaf over X, we have a direct correspondence between analytic and algebraic objects which takes into account singularities efficiently. Currents, introduced by Lelong (1957), play the role of algebraic cycles, and many classical results of intersection theory apply to currents. Also an analytic interpretation of the Seshadri constant of a line bundle is given and it represents a measure of local positivity. One of the motivations for this work was the conjecture of Fujita: If L is an ample (i.e. positive) line bundle on a projective n-dimensional algebraic variety X then $K_X + (n+2)L$ is very ample. Reider (1988) gave a proof of the Fujita conjecture in the case of surfaces.

Using an analytic approach, in the paper under review it is shown that $2K_X + L$ is very ample under suitable numerical conditions for L. The first part of the proof is to choose an appropriate metric using a complex Monge-Ampère equation and the Aubin-Calabi-Yau theorem. Solution ϕ of the equation assumes logarithmic poles and they are controlled using the intersection theory of currents. Detailed relations to the existing algebraic proofs of similar results are given (Ein-Lazarsfeld, Fujita, Siu). In the last section, a proof of the effective Matsusaka big theorem obtained by *Y.-T. Siu* [Ann. Inst. Fourier 43, No. 5, 1387-1405; Zbl 0803.32017)] is presented. Siu's proof is based on the very ampleness of $2K_X + mL$ together with the theory of holomorphic Morse inequalities [*J.-P. Demailly*, Ann. Inst. Fourier 35, No. 4, 189-229 (1985; Zbl 0565.58017)]. Long and detailed preliminary sections dedicated to the basic facts of complex differential geometry are included which make the main ideas of the paper easier to understand.

For the entire collection see [Zbl 0855.00017].

Reviewer: N.Blažić (Beograd)

MSC:

14F17 Vanishing theorems
32L05 Holomorphic fiber bundles and generalizations
14F43 Other algebro-geometric (co)homologies
14F05 Sheaves, derived categories of sheaves, etc.
32L20 Vanishing theorems (analytic spaces)
32C30 Integration on analytic sets and spaces, currents
32W20 Complex Monge-Ampère operators

Keywords:

positive line bundle; linear series; vanishing theorems; Lelong number; intersection theory; Bochner technique; L^2 estimates; Seshadri constant; numerically effective line bundle; Fujita conjecture; Monge-Ampere equation; very ample line bundle; algebraic vector bundles; effective Matsusaka big theorem

Bertin, José; **Demailly, Jean-Pierre**; **Illusie, Luc**; **Peters, Chris**
Introduction to Hodge theory. (Introduction à la théorie de Hodge.) (French. English summary)
Zbl 0849.14002
Panoramas et Synthèses. 3. Paris: Société Mathématique de France. vi, 272 p. (1996).

The origin of what is currently meant by the notion of Hodge theory can be traced back to *W. V. D. Hodge*'s fundamental work accomplished in the 1930s. In modern terminology, Hodge prepared the ground for describing the De Rham cohomology algebra of a Riemannian manifold in terms of its harmonic differential forms. In the following two decades, Hodge's decomposition principle has been extended to the (then) new sheaf-theoretic and cohomological framework of Hermitean differential geometry, complex-analytic geometry, and transcendental algebraic geometry. The names of G. De Rham, A. Weil, K. Kodaira, and many others stand for the tremendous progress achieved during this period, in particular with regard to deformation and classification theory in these areas. The special algebraic structures (Hodge structures) arising from Hodge decompositions and their generalizations have led to a rather independent field of research in geometry, precisely to the so-called Hodge theory, which represents a powerful and indispensible toolkit for contemporary complex geometry, general algebraic geometry, and – nowadays – also for mathematical physics. The vast activity in Hodge theory and its related areas, especially during the recent twenty years, is not reflected in the current textbook literature, at least not comprehensively or in an updated form compiling the various recent aspects and applications, so that a panoramic overview of the present state of art must be regarded as a highly welcome (and needed) service to the mathematical community.

A conference on the present state of Hodge theory, serving exactly that purpose, took place at the University of Grenoble (France) in November 1994. The book under review grew out of the series of lectures which the authors delivered at this meeting. The aim of the text is to develop a number of fundamental concepts and results of classical and modern Hodge theory, and in this the book is prepared for students and non-expert researchers in the field, who wish to get acquainted in depth with the subject, and obtain a profound up-to-date knowledge of its present level of development. – The material is divided into three main parts, each of which is written by different authors and devoted to various central and complementary aspects of the theory.

Part I, written by *J.-P. Demailly*, is entitled "L^2-Hodge theory and vanishing theorems". The author discusses in detail two fundamental applications of Hilbert L^2-space methods to complex analysis and algebraic geometry, respectively. This part adopts basically the analytic viewpoint and consists, on its side, of two chapters. Chapter 1 provides an introduction to standard complex Hodge theory, including the basics on Hermitean and Kähler geometry, differential operators on vector bundles, Hodge decomposition, Hodge degeneration, the spectral sequence of Hodge-Frölicher, Gauss-Manin connexion, and the deformation behavior of the Hodge groups (after Kodaira). Chapter 2 is devoted to L^2-estimates for the $\overline{\partial}$-operator and the resulting vanishing theorems for cohomology groups of Kähler manifolds and projective varieties. The main topics here are the classical methods of Oka, Bochner, and Hörmander in pseudo-convex analysis, their consequences for cohomology vanishing, as well as the more recent but already well-known fundamental contributions by the author himself towards the interpretation of the great vanishing theorems of A. Nadel and of Kawamata-Viehweg. – The concluding two sections of this chapter deal with the property of very-ampleness of line bundles on projective varieties. The first central result discussed here is the author's analytic approach to the famous conjecture of Fujita, culminating in an improvement of *Y.-T. Siu*'s very recent theorem on an effective bound for very-ampleness [cf. "Effective very ampleness", Invent. Math. 124, No. 1-3, 563-571 (1996)]. The second central result is an effective version of the classical "Big embedding theorem of Matsusaka", whose surprisingly simple proof is due to the author himself (1996), based on some foregoing work of *Y.-T. Siu* [Ann. Inst. Fourier 43, No. 5, 1387-1405 (1993; Zbl 0803.32017)], and methodically related to the effective bound for very-ampleness discussed before. These two last sections provide a particularly up-to-data account on the newest developments in analytical Hodge theory and its (algebraic) applications.

Part II of the text, written by *L. Illusie*, is entitled "Frobenius and Hodge degeneration". These notes aim at introducing non-specialists to those methods and techniques of algebraic geometry over a field of characteristic $p > 0$, which have been used by P. Deligne and the author to give an algebraic proof of the Hodge degeneration and the Kodaira-Akizuki-Nakano vanishing theorem for smooth projective varieties in characteristic zero. Basically, this part of the book is a careful, detailed introduction to the important work "Relèvements modulo p^2 et décomposition du complexe de De Rham" [Invent. Math. 89, 247-270 (1987; Zbl 0632.14017)] by *P. Deligne* and *L. Illusie*. Here the reader is assumed to bring along some basic knowledge of the theory of algebraic schemes and of homological algebra (in categories). After recalling the basics on schemes, differentials and the algebraic De Rham complex in characteristic $p > 0$, the author discusses the following topics: smoothness and coverings, the Frobenius morphism and the Cartier isomorphism, derived categories and spectral sequences, decomposition theorems, vanishing theorems in characteristic p, degeneration theorems, the standard techniques for passing from characteristic p to characteristic zero, and the proof of the above mentioned degeneration and vanishing theorems. The concluding section of this part points to some recent developments and open problems concerning Hodge theory in characteristic p.

Also this part is essentially self-contained, and most proofs are given in detail. Some proofs are – quite naturally – at least outlined, assuming the reader to follow the precise hints to the related textbook literature (mostly EGA) and original papers.

Part III of the book, written by *J. Bertin* and *C. Peters*, is entitled "Variations of Hodge structures, Calabi-Yau manifolds, and mirror symmetry". It consists again of two main chapters, whose interrelation is beautifully explained in a comprehensive introduction. – Chapter I is devoted to the comparatively elementary part of the theory of variation of Hodge structures and its applications in complex algebraic geometry. This includes detailed descriptions of the Hodge bundles, the Hodge filtrations, the De Rham cohomology sheaves, the Gauss-Manin connexion in its general setting (after Katz and Oda) and with its transversality property (due to Griffiths), variations and infinitesimal variations of Hodge structures, the Griffiths period domains for polarized Hodge structures, mixed Hodge structures, limits of Hodge structures (after Deligne), the Picard-Lefschetz theory and the local monodromy theorem, Deligne's degeneration criteria for Hodge spectral sequences, and a brief discussion of the method of vanishing cycles. At the end, the authors give a sketch of the use of Higgs bundles for the construction of variations of Hodge structures, mainly by following Simpson's approach [cf.: *C. T. Simpson*, Proc. Int. Congr. Math., Kyoto 1990, Vol. I, 747-756 (1991; Zbl 0765.14005)], as well as some comments on M. Saito's work on Hodge modules, intersection cohomology, and $\mathcal{D}$-modules in algebraic analysis. – Chapter II reflects the fact that Calabi-Yau manifolds, their Hodge theory, and their mirror symmetry have recently gained enormous significance in both algebraic geometry and theoretical physics, particularly in constructing two-dimensional conformal quantum field theories. The material presented here covers the fundamental facts on Calabi-Yau manifolds, their construction and deformation theory, and their mirror properties. After a digression on the cohomology of hypersurfaces (after Griffiths and Dimca), which is used for the description of the link between the Picard-Fuchs equation and the variation of Calabi-Yau structures, the variation of Hodge structures for families of Calabi-Yau threefolds, their Yukawa couplings, and their mirror symmetries are explained in more depth. The interested reader can find a very complete and comprehensive account on this subject in the recent monography "Symétrie miroir" by *C. Voisin* [Panoramas et Synthèses, No. 2 (1996; see the preceding review)]. In a concluding section, the authors discuss (following an idea of P. Deligne) a possible approach to mirror symmetry via a certain duality between variations of Hodge structures for Calabi-Yau threefolds. A rich bibliography enhances this very systematic and lucid treatise.

Altogether, the present book, in all its three parts, which consistently refer to each other, may be regarded as a masterly introduction to Hodge theory in its classical and very recent, analytic and algebraic aspects. Aimed to students and non-specialists, it is by far much more than only an introduction to the subject. The material leads the reader to the forefront of research in many areas related to Hodge theory, and that in a detailed and highly self-contained manner. As such, this text is also a valuable source for active researchers and teachers in the field, in particular due to the utmost carefully arranged index at the end of the book.

Reviewer: W.Kleinert (Berlin)

MSC:

14C30	Transcendental methods, Hodge theory, Hodge conjecture	Cited in **1** Review
14F17	Vanishing theorems	Cited in **2** Documents
14-02	Research monographs (algebraic geometry)	
14D07	Variation of Hodge structures	
13A35	Characteristic p methods; tight closure	
58A14	Hodge theory (global analysis)	
14D05	Structure of families	
81T30	String and superstring theories	
14J32	Calabi-Yau manifolds	

Keywords:

characteristic p; De Rham cohomology algebra; Hodge theory; vanishing theorems; very-ampleness of line bundles; Hodge degeneration; De Rham complex; Frobenius morphism; Cartier isomorphism; variation of Hodge structures; Gauss-Manin connexion; period domains; Picard-Lefschetz theory; Higgs bundles; Calabi-Yau manifolds; two-dimensional conformal quantum field theories; Picard-Fuchs equation; Yukawa couplings; mirror symmetries

Demailly, Jean-Pierre; **Peternell, Thomas**; **Schneider, Michael**

Compact Kähler manifolds with Hermitian semipositive anticanonical bundle. (English)

Zbl 1008.32008

Compos. Math. 101, No.2, 217-224 (1996).

Summary: This note states a structure theorem for compact Kähler manifolds with semipositive Ricci curvature: Any such manifold has a finite étale covering possessing a de Rham decomposition as a product of irreducible compact Kähler manifolds, each one being either Ricci flat (torus, symplectic or Calabi-Yau manifold) or Ricci semipositive without nontrivial holomorphic forms. Related questions and conjectures concerning the latter case are discussed.

MSC:

32J27	Compact Kähler manifolds: generalizations, classification	Cited in **1** Review
53C55	Hermitian and Kählerian manifolds (global differential geometry)	Cited in **9** Documents

Full Text: Numdam EuDML

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Demailly, Jean-Pierre

Effective bounds for very ample line bundles. (English) Zbl 0862.14004

Invent. Math. 124, No.1-3, 243-261 (1996).

Let L be an ample line bundle on a nonsingular projective n-fold X. A well-known conjecture of T. Fujita asserts that $K_X+(n+1)L$ is generated by global sections and $K_X+(n+2)L$ is very ample. For $n=2$ this follows from I. Reider's theorem and the global generation part of the conjecture was proved for $n=3$ by *L. Ein* and *R. Lazarsfeld* [J. Am. Math. Soc. 6, No. 4, 875-903 (1993; Zbl 0803.14004)]. The present paper is mainly concerned with the very ampleness part of the conjecture. In a previous paper [J. Differ. Geom. 37, No. 2, 323-374 (1993; Zbl 0783.32013)] the author proved that $2K_X+12n^nL$ is very ample, using an analytic method based on the solution of a Monge-Ampère equation. In the present paper, improving a method of *Y.-T. Siu* [Invent. Math. 124, No. 1-3, 563-571 (1996; Zbl 0853.32034)] based on a combination of the Riemann-Roch formula with the vanishing theorem of *A. M. Nadel* [Ann. Math., II. Ser. 132, No. 3, 549-596 (1990; Zbl 0731.53063)] the author proves that $2K_X+mL$ is very ample for $m\geq 2+\binom{3n+1}{n}$ and that $m(K_X+(n+2)L)$ is very ample for $m\geq\binom{3n+1}{n}-2n$. The method of proof gives, as a byproduct, the well-known fact that $K_X+(n+1)L$ is numerically effective (a result originally proved as a consequence of Mori theory). The paper also contains a refinement of a method developed by *Y.-T. Siu* [Ann. Inst. Fourier 43, No. 5, 1387-1405 (1993; Zbl 0803.32017)] which enables the author to obtain a better effective Matsusaka big theorem.

Reviewer: I.Coandă (Bucureşti)

MSC:

14C20 Divisors, linear systems, invertible sheaves
14F05 Sheaves, derived categories of sheaves, etc.
14F17 Vanishing theorems
32C20 Normal analytic spaces

Cited in **7** Documents

Keywords:

Hermitian metrics on line bundles; Fujita conjecture; ample line bundle; very ampleness; vanishing theorem; Mori theory; Matsusaka big theorem

Full Text: DOI

Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael
Holomorphic line bundles with partially vanishing cohomology. (English) Zbl 0859.14005
Teicher, Mina (ed.), Proceedings of the Hirzebruch 65 conference on algebraic geometry, Bar-Ilan University, Ramat Gan, Israel, May 2-7, 1993. Ramat-Gan: Bar-Ilan University, Isr. Math. Conf. Proc. 9, 165-198 (1996).

Let X denote a complex manifold of dimension n. The authors study holomorphic line bundles L on X with partially vanishing cohomology (or having metrics with positive eigenvalues of curvature). They define $\sigma_+(L)$ to be the smallest integer q with the following property: There exists an ample divisor D on X and a constant $c > 0$ such that $H^j(X, mL - pD) = 0$ for all $j > q$ and $mp \geq 0$, $m \geq c(p+1)$. Note that $\sigma_+(L) = 0$ if and only if L is ample while $\sigma_+(L) = n$ if and only if $c_1(L^*)$ is in the closure of the cone of effective divisors. An ample q-flag is defined as a sequence $Y_q \subset Y_{q+1} \subset \cdots \subset Y_n = X$ of subvarieties Y_k of X such that $\dim Y_k = k$ and Y_k is the image of an ample Cartier divisor in the normalization of Y_{k+1}. Then a line bundle L is called q-flag positive if for some ample q-flag, $L \mid Y_q$ is positive.

Vanishing theorem: If $L \in \mathrm{Pic} X$ is q-flag positive then $\sigma_+(L) \leq n - q$.

The converse of this theorem is not true in general. A counter example and a positive result (of converse) for ${}_{n-1}$ bundles over a curve are given. The structure of projective 3-folds with $\sigma_+(-K_X) = 1$, $K_X =$ canonical bundle, is investigated. One has $\sigma_+(-K_X) = 0$ if and only if X is Fano and $\sigma_+(-K_X) \leq 2$ if and only if $\kappa(X) = -\infty$. The authors also study various cones in $NX(X)\otimes$, $NX(X)$ being Néron-Severi group, i.e. the group of divisors modulo numerical equivalence. All these cones coincide for surfaces.

For the entire collection see [Zbl 0828.00035].

Reviewer: U.N.Bhosle (Bombay)

MSC:

14F17 Vanishing theorems
32L20 Vanishing theorems (analytic spaces)
14F05 Sheaves, derived categories of sheaves, etc.
14C22 Picard groups

Keywords:

flag; holomorphic line bundles; vanishing cohomology

Demailly, Jean-Pierre
Compact complex manifolds whose tangent bundles satisfy numerical effectivity properties (joint work with Thomas Peternell and Michael Schneider). (English) Zbl 0880.14003
Geometry and analysis. Papers presented at the Bombay colloquium, India, January 6–14, 1992. Oxford: Oxford University Press. Stud. Math., Tata Inst. Fundam. Res. 13, 67-86 (1995).

A compact Riemann surface always has a hermitian metric with constant curvature, in particular the curvature sign can be taken to be constant: the negative sign corresponds to curves of general type (genus ≥ 2), while the case of zero curvature corresponds to elliptic curves (genus 1), positive curvature being obtained only for ^{1} (genus 0). In higher dimensions the situation is much more subtle and it has been a long standing conjecture due to Frankel to characterize ${}_n$ as the only compact Kähler manifold with positive holomorphic bisectional curvature. Hartshorne strengthened Frankel's conjecture and asserted that ${}_n$ is the only compact complex manifold with ample tangent bundle. In his famous paper in Ann. Math., II. Ser. 110, 593-606 (1979; Zbl 0423.14006), *S. Mori* solved Hartshorne's conjecture by using characteristic p methods. Around the same time *Y.-T. Siu* and *S.-T. Yau* [Invent. Math. 59, 189-204 (1980; Zbl 0442.53056)] gave an analytic proof of the Frankel conjecture. Combining algebraic and analytic tools Mok classified all compact Kähler manifolds with semi-positive holomorphic bisectional curvature. – From the point of view of algebraic geometry, it is natural to consider the class of projective manifolds X whose tangent bundle is numerically effective (nef). This has been done by Campana and Peternell and – in case of dimension 3 – by Zheng. In particular, a complete classification is obtained for dimension at most three. The main purpose of this work is to investigate compact (most often Kähler) manifolds with nef tangent or anticanonical bundles in arbitrary dimension. We first discuss some basic properties of nef vector bundles which will be needed in the sequel in the general context of compact complex manifolds.

We refer to papers by *J.-P. Demailly*, *T. Peternell* and *M. Schneider* [Compos. Math. 89, No. 2, 217-240 (1993) and J. Algebr. Geom. 3, No. 2, 295-345 (1994; Zbl 0827.14027)] for detailed proofs. Instead, we put here the emphasis on some unsolved questions.

For the entire collection see [Zbl 0868.00030].

MSC:

14C20 Divisors, linear systems, invertible sheaves
32J27 Compact Kähler manifolds: generalizations, classification
14F05 Sheaves, derived categories of sheaves, etc.

Keywords:

nef tangent bundles; nef anticanonical bundle; compact Riemann surface

Demailly, Jean-Pierre
L^2-methods and effective results in algebraic geometry. (English) Zbl 0845.14004
Chatterji, S. D. (ed.), Proceedings of the international congress of mathematicians, ICM '94, August 3-11, 1994, Zürich, Switzerland. Vol. II. Basel: Birkhäuser. 817-827 (1995).

Given an ample line bundle L on a projective n-fold, it is an important question to find an integer m_0 such that mL is ample for $m \geq m_0$. The example of curves shows that no universal bound (depending only on n) exists. However T. Fujita has conjectured that if L is an ample line bundle on a projective n-fold then $K_X + (n+2)L$ is very ample, where K_X is the canonical line bundle. Here the author explains how analytic methods lead to a universal bound $m_0 = 2 + \binom{3n+1}{n}$ such that $2K_X + mL$ is very ample for $m \geq m_0$.

For the entire collection see [Zbl 0829.00015].

Reviewer: F.Kirwan (Oxford)

MSC:

14C20 Divisors, linear systems, invertible sheaves
14F05 Sheaves, derived categories of sheaves, etc.

Cited in **1** Document

Keywords:

ample line bundle

Demailly, Jean-Pierre; Passare, Mikael
Residual currents and fundamental class. (Courants résiduels et classe fondamentale.) (French) Zbl 0851.32013
Bull. Sci. Math. 119, No.1, 85-94 (1995).

Let Y be the complex subspace of a complex manifold X defined by a coherent ideal I, which is a locally complete intersection. The authors introduce the notion of the cohomology with supports in the infinitesimal neighbourhood of first order of Y and then, they prove that the residual current R_Y is intrinsically identified to a canonical element of the infinitesimal cohomology of first order with supports in Y and with values in the sheaf of sections of the determinant of the determinant of the conormal bundle to Y.

Reviewer: Vasile Brînzănescu (Bucureşti)

MSC:

32C30 Integration on analytic sets and spaces, currents
32C36 Local cohomology of analytic spaces
58A25 Currents (global analysis)
32C15 Complex spaces

Cited in **1** Review

Keywords:

current; analytic spaces; cohomology with supports

Demailly, Jean-Pierre

Semicontinuity properties of cohomology and of Kodaira-Iitaka dimension. (Propriétés de semi-continuité de la cohomologie et de la dimension de Kodaira-Iitaka.) (French. Abridged English version) Zbl 0851.32015

C. R. Acad. Sci., Paris, Sér. I 320, No.3, 341-346 (1995).

Let $X \to S$ be a proper and flat morphism of complex spaces and let (X_t) be the fibres. Given a sheaf E over X of locally free $\mathcal{O}_X$-modules, inducing on the fibres a family of sheaves $(E_t \to X_t)$, the author shows that the cohomology group dimension $h^q(t) = h^q(X_t, E_t)$ satisfy the following semicontinuity property: for every $q \geq 0$, the sum $h^q(t)-h^{q-1}(t)+\cdots+(-1)^q h^0(t)$ is upper semicontinuous for the Zariski topology. Then, some applications to the Kodaira-Iitaka dimension are given.

Reviewer: Vasile Brînzănescu (Bucureşti)

MSC:

32C35 Analytic sheaves and cohomology groups

35G05 General theory of linear higher-order PDE

Cited in **1** Document

Keywords:

proper morphisms of complex spaces; semicontinuity; Kodaira-Iitaka dimension

Demailly, Jean-Pierre

Ordinary differential equations. Theoretical and numerical aspects. (Gewöhnliche Differentialgleichungen. Theoretische und numerische Aspekte. Aus d. Franz. übers. von Mathias Heckele.) (German) Zbl 0869.65042

Wiesbaden: Vieweg. x, 318 p. (1994).

Die Besonderheit des vorliegenden Buches ist eine integrierte Darstellung der theoretischen Grundlagen und der numerischen Behandlung von Anfangswertaufgaben gewöhnlicher Differentialgleichungen. Der Numerikteil greift dabei thematisch noch weiter aus, indem Rundungsfehler, Polynomapproximation, Quadraturformeln und iterative Verfahren behandelt werden, mit Ausnahme des etwas knapp geratenen Kapitels Iteration sogar ziemlich ausführlich. Ein - was den integrierten Differentialgleichungsteil betrifft - ähnlich aufgebautes Lehrbuch ist von *H. Werner* und *W. Arndt* [Gewöhnliche Differentialgleichungen. Eine Einführung in Theorie und Praxis (1986; MR 88b.34002)] verfaßt worden.

Das vorliegende Lehrbuch besticht durch seine präzise Darstellung der behandelten Sachverhalte und die damit einhergehende Sorgfalt und Eleganz in der Behandlung der mathematischen Aspekte. Mancher Leser würde sich vielleicht eine stärkere Betonung numerischer Gesichtspunkte wünschen, was der Titel des Buches aber auch nicht verspricht. Mit seiner speziellen thematischen Ausrichtung und der inhaltlichen Qualität hat das Buch einen eigenen Platz in der vorliegenden umfangreichen Numerik-Lehrbuchliteratur, und es wird hoffentlich genügend viele Leser finden, die davon profitieren.

Reviewer: R.D.Grigorieff (Berlin)

MSC:

65L05 Initial value problems for ODE (numerical methods)

65L06 Multistep, Runge-Kutta, and extrapolation methods

65D32 Quadrature and cubature formulas (numerical methods)

65H10 Systems of nonlinear equations (numerical methods)

65-01 Textbooks (numerical analysis)

34-01 Textbooks (ordinary differential equations)

Cited in **2** Reviews

Keywords:

ordinary differential equation; initial value problem; roundoff errors; polynomial approximation; quadrature formulas; iterative methods; textbook

Demailly, Jean-Pierre

Regularization of closed positive currents of type (1,1) by the flow of a Chern connection.
(English) Zbl 0824.53064

Skoda, Henri (ed.) et al., Contributions to complex analysis and analytic geometry. Based on a colloquium dedicated to Pierre Dolbeault, Paris, France, June 23-26, 1992. Braunschweig: Vieweg. Aspects Math. E 26, 105-126 (1994).

Let X be a compact complex manifold, and T a closed positive current of (1,1) type. Some questions addressed in this article are related to the approximation of T by smooth closed "positive" currents. It is easy to see a necessary condition for this approximation, namely the cohomology class $\{T\}$ should satisfy $\int_Y \{T\}^p \geq 0$ for every p-dimensional subvariety $Y \subset X$. Thus, in general case, one concerns the approximation of T only by closed "almost positive" currents, as the following principal result shows.

Let γ be a continuous real (1,1) form such that $T \geq \gamma$, u some continuous nonnegative (1,1) form, and ω a (smooth) Hermitian metric on T_X. Then under a certain curvature condition, T can be approximated by closed "almost positive" (1,1) currents T_ε with the following properties: (i) $T_\varepsilon \geq \gamma - \lambda_\varepsilon u - \delta_\varepsilon \omega$; (ii) $\lambda_\varepsilon(x)$ is an increasing family of continuous functions such that for all $x \in X$, $\lim_{\varepsilon \to 0} \lambda_\varepsilon(x) = \nu(T, x)$ (Lelong number of T at x); (iii) the constants $\delta_\varepsilon \to 0$ as $\varepsilon \to 0$, and $\delta_\varepsilon > 0$, $\forall \varepsilon$. For the curvature condition above, we require $(\Theta(T_X) + u \otimes \mathrm{Id}_{T_X})(\theta \otimes \xi, \theta \otimes \xi) \geq 0$ for all θ, ξ of T_X, with $\langle \theta, \xi \rangle = 0$. Moreover if put $T = \alpha + \frac{i}{\pi} \partial\overline{\partial}\psi$, for α a smooth (1,1) form in the same $\partial\overline{\partial}$-cohomology class as T, and ψ an almost plurisubharmonic function, then we have the representation: $T_\varepsilon = \alpha + \frac{i}{\pi} \partial\overline{\partial}\psi_\varepsilon$ such that ψ_ε is smooth over X and increasingly converges to ψ, as $\varepsilon \to 0$. It can be shown that the representation of the above T involving a quasi-psh ψ (i.e. locally the sum of a psh function and a smooth function) is always possible.

Similar results as to the regularization of closed positive currents are treated elsewhere, e.g. [the author, J. Algebr. Geom. 1, No. 3, 361-409 (1992; Zbl 0777.32016)], where a numerical hypothesis rather than a curvature hypothesis is assumed: $c_1(\mathcal{O}_{T_X}(1)) + \pi^* u$ is nef on the total space of (dual) projectivized tangent bundles. This numerical condition does not seem to be directly related to the partial semipositivity curvature condition; for instance, the author remarks that for the curve case the partial semipositivity hypothesis is void. By using the present curvature hypothesis, the author felt it perhaps easier to extend to currents of higher bidegrees.

For the entire collection see [Zbl 0811.00006].

Reviewer: I-Hsun Tsai (Taipei)

MSC:

53C55	Hermitian and Kählerian manifolds (global differential geometry)	Cited in **1** Review
32C30	Integration on analytic sets and spaces, currents	Cited in **10** Documents

Keywords:

closed positive currents; semipositive curvature; $\partial\overline{\partial}$-cohomology; plurisubharmonic function

Demailly, Jean-Pierre; Lempert, László; Shiffman, Bernard

Algebraic approximations of holomorphic maps from Stein domains to projective manifolds.
(English) Zbl 0861.32006

Duke Math. J. 76, No.2, 333-363 (1994).

Let Y, Z be quasi-projective algebraic varieties and let Ω be an open subset of Y. A map $F : \Omega \to Z$ is said to be Nash algebraic if f is holomorphic and the graph of f is contained in an algebraic subvariety of $Y \times Z$ of dimension equal to $\dim Y$.

One of the main results in the paper is the following theorem concerning the approximation of holomorphic maps by Nash algebraic maps:

Theorem 1.1. Let Ω be a Runge domain in an affine algebraic variety S and let $f : \Omega \to X$ be a holomorphic map into a quasi-projective algebraic manifold X. Then for every relatively compact domain $\Omega_0 \Subset \Omega$, there is a sequence of Nash algebraic maps $f_\nu : \Omega_0 \to X$ such that $f_\nu \to f$ uniformly on Ω_0.

As important applications of Theorem 1.1 the authors obtain that the Kobayashi-Royden pseudometric and the Kobayashi pseudodistance on projective algebraic manifolds can be approximated in terms of algebraic curves. It is proved that a type of algebraic approximation is also possible in the case of locally free sheaves.

Using the methods developed in the paper the authors give a more precise form of a result concerning the description of equivalent Nash algebraic vector bundle, obtained by *T. Tancredi* and *A. Tognoli* [Bull. Sci. Math., II. Ser. 117, No. 2, 173-183 (1993; Zbl 0798.32010)]. A result of *E. L. Stout* [Contemp. Math. 32, 259-266 (1984; Zbl 0584.32027)] on the exhaustion of Stein manifolds by Runge domains in affine algebraic manifolds is proved by substantially different methods.

Reviewer: I.Serb (Cluj-Napoca)

MSC:

32E10 Stein spaces, Stein manifolds
14P20 Nash functions and manifolds
32F45 Invariant metrics and pseudodistances

Keywords:

approximation of holomorphic maps; Nash algebraic maps; quasi-projective algebraic manifold; Stein manifolds; Runge domains

Full Text: DOI

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Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael

Compact complex manifolds with numerically effective tangent bundles. (English)

Zbl 0827.14027

J. Algebr. Geom. 3, No.2, 295-345 (1994).

The main result of this fundamental article is: Let X be a compact Kähler manifold with nef tangent bundle T_X. Moreover, let $\widetilde{X}$ be a finite étale cover of X of maximum irregularity $q = q(\widetilde{X}) = h^1(\widetilde{X}, \mathcal{O}_{\widetilde{X}})$. Then: $\pi_1(\widetilde{X}) \cong^{2q}$.

The albanese map $\alpha : \widetilde{X} \to A(\widetilde{X})$ is a smooth fibration over a q-dimensional torus with nef relative tangent bundle.

The fibres of α are Fano manifolds with nef tangent bundles.

Here a line bundle L on a compact complex manifold X with a fixed hermitian metric ω is nef if, for every $\varepsilon > 0$, there exists a smooth hermitian metric h_ε on L such that the curvature satisfies $\Theta_{h_\varepsilon} \geq -\varepsilon\omega$. A bundle E on X is nef if the line bundle $\mathcal{O}_E(1)$ on (E) is nef. – Many other interesting and important results are contained in the article. It is proved that:

Let E be a vector bundle on a compact Kähler manifold X.

If E and E^* are nef, then E admits a filtration whose graded pieces are hermitian flat.

If E is nef, then E is numerically semi-positive.

Moreover, algebraic proofs are given for the result:

Any Moisheson manifold with nef tangent bundle is projective.

A compact Kähler n-fold with T_X nef and $c_1(X)^n > 0$ is Fano.

Further the two following classification results are given:

The smooth non-algebraic compact complex surfaces with nef tangent bundles are:

non-algebraic tori; Kodaira surfaces; Hopf surfaces.

Let X be a non-algebraic three-dimensional compact Kähler manifold. Then T_X is nef if and only if X, up to a finite étale cover, is either a torus or of the form (E), where E and E^* are nef rank-2 vector bundles over a two-dimensional torus.

Reviewer: D.Laksov (Stockholm)

MSC:

14J30	Algebraic threefolds
14C20	Divisors, linear systems, invertible sheaves
32J17	Compact 3-folds (analytic spaces)
14F35	Homotopy theory; fundamental groups (algebraic geometry)
53C55	Hermitian and Kählerian manifolds (global differential geometry)
14E20	Coverings, fundamental group (mappings)

Keywords:

fundamental group; nef line bundle; Albanese map; Moisheson manifold; three-dimensional compact Kähler manifold

Demailly, Jean-Pierre; **Peternell, Thomas**; **Schneider, Michael**
Kähler manifolds with numerically effective Ricci class. (English) Zbl 0884.32023
Compos. Math. 89, No.2, 217-240 (1993).

The purpose of this paper is to contribute to the solution of the following conjectures: Let X be a compact Kähler manifold with numerically effective (nef) anticanonical bundle $-K_X$; then:

Conjecture 1: The fundamental group $\pi_1(X)$ of X has polynomial growth.

Conjecture 2: The Albanese map $\alpha : X \to \mathrm{Alb}(X)$ is surjective.

Section 1 is devoted to proving the following theorem, which is the main contribution to Conjecture 1.

Theorem 1: Let X be a compact Kähler manifold with nef anticanonical bundle; then $\pi_1(X)$ has subexponential growth.

The main tools used in order to prove Theorem 1 are the solution of the Calabi conjecture and volume bounds for geodesic balls due to Bishop and Gage. It should be mentioned that from the proof of Theorem 1 it follows that Conjecture 1 holds in the case $-K_X$ is Hermitian semipositive (Theorem 2).

In Section 2 the following theorem concerning Conjecture 2 is proved.

Theorem 3: Let X be an n-dimensional compact Kähler manifold such that $-K_X$ is nef. Then the Albanese map α is surjective as soon as $\dim \alpha(X)$ is 0,1 or n, and, if X is projective, also for $n-1$; moreover, if X is projective and if the generic fibre F of α has $-K_F$ big, then α is surjective.

Finally, Section 3 is devoted to the study of the structure of projective 3-folds with nef anticanonical bundles; in particular Conjecture 2 is proved in dimension 3 with purely algebraic methods, except in one very special case.

Reviewer: Antonella Nannicini (MR 95b:32044)

MSC:

32J27 Compact Kähler manifolds: generalizations, classification
14J40 Algebraic n-folds ($n > 4$)
32Q15 Kähler manifolds
53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords:

numerically effective Ricci class; compact Kähler manifold; Albanese map; nef anticanonical bundles

Full Text: Numdam EuDML

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Demailly, Jean-Pierre
A numerical criterion for very ample line bundles. (English) Zbl 0783.32013
J. Differ. Geom. 37, No.2, 323-374 (1993).

Let X be a projective algebraic manifold of dimension n and let L be an ample line bundle over X. We give a numerical criterion ensuring that the adjoint bundle $K_X + L$ is very ample. The sufficient conditions are expressed in terms of lower bounds for the intersection numbers $L^p \cdot Y$ over subvarieties Y of X. In the case of surfaces, our criterion gives universal bounds and is only slightly weaker than *I. Reider*'s criterion [Ann. Math., II. Ser. 127, No. 2, 309-316 (1988; Zbl 0663.14010)]. When $\dim X \geq 3$ and $\mathrm{codim} Y \geq 2$, the lower bounds for $L^p \cdot Y$ involve a numerical constant which depends on the geometry of X. By means of an iteration process, it is finally shown that $2K_X + mL$ is very ample for $m \geq 12n^n$. Our approach is mostly analytic and based on a combination of Hörmander's L^2 estimates for the operator $\overline{\partial}$, Lelong number theory and the Aubin-Calabi-Yau theorem.

Reviewer: J.-P.Demailly (Saint-Martin d'Hères)

MSC:

32J15	Compact surfaces (analytic spaces)
32L10	Sections of holomorphic vector bundles
32C30	Integration on analytic sets and spaces, currents

Keywords:

very ample line bundle; plurisubharmonic function; closed positive current; Monge-Ampère equation; intersection theory; numerical criterion; Lelong number; Aubin-Calabi-Yau theorem

Full Text: DOI

Demailly, Jean-Pierre
Monge-Ampère operators, Lelong numbers and intersection theory. (English) Zbl 0792.32006
Ancona, Vincenzo (ed.) et al., Complex analysis and geometry. New York: Plenum Press. The University Series in Mathematics. 115-193 (1993).

This article is a survey on the theory of Lelong numbers, viewed as a tool for studying intersection theory by complex differential geometry. The paper contains earlier works of the author [Mém. Soc. Math. Fr., Nouv. Sér. 19, 124 p. (1985; Zbl 0579.32012) and Acta Math. 159, 153- 169 (1987; Zbl 0629.32011)] and of *Y. T. Siu* [Invent. Math. 27, 53- 156 (1974; Zbl 0289.32003)]. Many results are given with complete proofs, which are shorter and simpler than the original ones. The references contain 37 items on these topics.

For the entire collection see [Zbl 0772.00007].

Reviewer: E.Outerelo (Madrid)

MSC:

32C30	Integration on analytic sets and spaces, currents
32W20	Complex Monge-Ampère operators
32U05	Plurisubharmonic functions and generalizations

Keywords:

Monge-Ampère operators; current; plurisubharmonic functions; Lelong numbers; intersection theory

Demailly, Jean-Pierre

Holomorphic Morse inequalities on q-convex manifolds. (English) Zbl 0771.32011

Several complex variables, Proc. Mittag-Leffler Inst., Stockholm/Swed. 1987-88, Math. Notes 38, 245-257 (1993).

[For the entire collection see Zbl 0759.00008.]

This paper is a nice report ("high level propaganda for a very interesting result and for very interesting tools") of at that time recent work [*T. Bouche*, Ann. Sci. Ec. Norm. Super., IV. Ser. 22, No. 4, 501-513 (1989; Zbl 0693.32016)] which extends previous work by Demailly to the case of strongly q-convex manifolds. The main result of Bouche's paper is the following one.

Theorem A: Let X be a strongly q-convex complex manifold with $n := \dim(X)$, E a rank r vector bundle on X and L a line bundle on X with hermitian metric such that the curvature form $ic(L)$ has at least $n-p+1$ eigenvalues ≥ 0 outside a compact subset of X; set $X(m,L) := \{x \in X : ic(L)$ is non degenerate at x and with exactly m negative eigenvalues$\}$, $X(\leq m, L) := \bigcup_{t\leq m} X(t,L)$ and $X(\geq m, L) := \bigcup_{t\geq m} X(t,L)$. Then for all $m \geq p+q-1$ the following asymptotic inequalities hold:

(a_m) Weak Morse inequalities

$$\dim H^m(X, E\otimes L^k) \leq r\frac{k^n}{n!}\int_{X(m,L)} (-1)^m \left(\frac{i}{2\pi}c(L)\right)^n + o(k^n)$$

(b_m) Strong Morse inequalities:

$$\sum_{m\leq t\leq n} (-1)^{t-m} \dim H^t(X, E\otimes L^k) \leq r\frac{k^n}{n!}\int_{X(\geq m,L)} (-1)^m \left(\frac{i}{2\pi}c(L)\right)^n + o(k^n)$$

This note contains a sketch of the proof of this theorem. Here the main recent and very powerful techniques are explained and used (e.g. Witten's complex); the main tool for the proof of Morse inequalities is a spectral theorem for Schrödinger operators which describes very precisely the asymptotic distribution of eigenvalues for a suitable quadratic form. This report contains the statement, the history and the motivation of two important applications of Theorem A: a very general a priori estimate for Monge-Ampère operator $(id'd'')^n$ on q-convex manifolds and the following stronger form of Grauert-Riemenschneider conjecture:

Theorem B: Let X be a connected n-dimensional compact manifold; if X has a hermitian line bundle L such that $\int_{X(\leq 1,L)} (ic(L)^n > 0$, then X is Moishezon.

Reviewer: E.Ballico (Povo)

MSC:

32F10 q-convexity, q-concavity
32C35 Analytic sheaves and cohomology groups
32J99 Compact analytic spaces
32W20 Complex Monge-Ampère operators

Keywords:

q-convex manifold; strongly q-convex manifold; Morse inequalities; Monge-Ampère operator; Moishezon manifold; curvature form; line bundle; asymptotic extimates for cohomology

Demailly, Jean-Pierre

Regularization of closed positive currents and intersection theory. (English) Zbl 0777.32016

J. Algebr. Geom. 1, No.3, 361-409 (1992).

Let X be a compact complex manifold and let $T = i\partial\overline{\partial}\psi$ be a closed positive current of bidegree (1,1) on X. Under some hypothesis on a lower bound for the Chern curvature of the tangent bundle TX, the current T is proved to be the weak limit of closed currents $T_k = \frac{i}{\pi}\partial\overline{\partial}\psi_k$ with controlled negative parts; the functions ψ_k decrease to ψ as $k \to \infty$ and can be chosen smooth on X. However the presence of positive Lelong numbers of T results in some loss of positivity of T_k.

This regularization is applied to relations between effective and numerically effective divisors, and to some problems of intersection theory.

Reviewer: A.Yu.Rashkovsky (Khar'kov)

MSC:

32J25 Transcendental methods of algebraic geometry
32C30 Integration on analytic sets and spaces, currents
32S60 Stratifications; constructible sheaves; intersection cohomology (analytic spaces)

Cited in **7** Reviews
Cited in **115** Documents

Keywords:

Lelong number; closed positive current; intersection theory

Demailly, Jean-Pierre
Positive currents and intersection theory. (Courants positifs et théorie de l'intersection.) (French) Zbl 0771.32010
Gaz. Math., Soc. Math. Fr. 53, 131-159 (1992).

This is a nice tool for spreading mathematical culture and ideas among mathematicians. It starts with the notion of current (after de Rham) and ends with the use in the subject of top level research and new extremely powerful methods of the author (around 1991). In the middle it is shown how to use the notion of positive current to define and work (via the integration current of the fundamental class of a subvariety) in Intersection Theory (key words: Lelong numbers and multiplicities). The tools come from Analysis and bring (and often solve) with them several interesting problems which cannot be formulated in a purely algebraic way inside Algebraic Geometry. But these methods are very, very strong competitors even on natural very important algebraic problems. Of course, in this paper most of the proofs are omitted, but ideas and difficulties are not skipped. It is pleasant reading and even specialists in not too far fields can find here some ideas/tools useful for their job; everybody can find some recent deep idea (mostly from Demailly brain).

Reviewer: E.Ballico (Povo)

MSC:

32C30 Integration on analytic sets and spaces, currents
58A25 Currents (global analysis)
32J25 Transcendental methods of algebraic geometry
14C17 Intersection theory, etc.

Cited in **4** Documents

Keywords:

positive currents; intersection theory; Lelong numbers; intersection multiplicity; integration current; equimultiplicity

Demailly, Jean-Pierre
Singular Hermitian metrics on positive line bundles. (English) Zbl 0784.32024
Complex algebraic varieties, Proc. Conf., Bayreuth/Ger. 1990, Lect. Notes Math. 1507, 87-104 (1992).

[For the entire collection see Zbl 0745.00049.]

We quote the author's abstract: "The notion of a singular Hermitian metric on a holomorphic line bundle is introduced as a tool for the study of various algebraic questions. One of the main interests of such metrics is the corresponding L^2 vanishing theorem for $\overline{\partial}$ cohomology, which gives a useful criterion for the existence of sections. In this context, numerically effective line bundles and line bundles with maximum Kodaira dimension are characterized by means of positivity properties of the curvature in the sense of currents. The coefficients of isolated logarithmic poles of a plurisubharmonic singular metric are shown to have a simple interpretation in terms of the constant ε of Seshadri's ampleness criterion. Finally, we use singular metrics and approximations of the curvature current to prove a new asymptotic estimate for the dimension of cohomology groups with values in high multiples $\mathcal{O}(kL)$ of a line bundle L with maximum Kodaira dimension".

Reviewer: E.J.Straube (College Station)

MSC:

32L05 Holomorphic fiber bundles and generalizations

Cited in **13** Reviews
Cited in **60** Documents

Keywords:

plurisubharmonic weights; singular Hermitian metric; holomorphic line bundle; vanishing theorem; cohomology

Demailly, J.-P.

Transcendental proof of a generalized Kawamata-Viehweg vanishing theorem. (English) Zbl 1112.32303

Berenstein, Carlos A. (ed.) et al., Geometrical and algebraical aspects in several complex variables. Papers from the conference, Cetraro, Italy, June 1989. Rende: Editoria Elettronica. Semin. Conf. 8, 81-94 (1991).

For the entire collection see [Zbl 0969.00052].

MSC:

32L20 Vanishing theorems (analytic spaces)
14F17 Vanishing theorems
32L10 Sections of holomorphic vector bundles

Cited in **5** Documents

Demailly, Jean Pierre

Holomorphic Morse inequalities. (English) Zbl 0755.32008

Several complex variables and complex geometry, Proc. Summer Res. Inst., Santa Cruz/CA (USA) 1989, Proc. Symp. Pure Math. 52, Part 2, 93-114 (1991).

[For the entire collection see Zbl 0732.00008.]

In this paper the complex analogues of the Morse inequalities for $\overline{\partial}$-cohomology groups with values in holomorphic vector bundles are explained, and some applications of that theory are presented.

Reviewer: B.Nowak (Łódź)

MSC:

32C35 Analytic sheaves and cohomology groups
58E05 Abstract critical point theory
32L10 Sections of holomorphic vector bundles
53C07 Special connections and metrics on vector bundles (Hermite-Einstein-Yang-Mills)

Cited in **1** Review
Cited in **10** Documents

Keywords:

$\overline{\partial}$-cohomology groups; Morse inequalities; holomorphic vector bundles

Blel, Mongi; Demailly, Jean-Pierre; Mouzali, Mokhtar
Sur l'existence du cône tangent à un courant positif fermé. (About the existence of the tangent cone with positive closed current). (French) Zbl 0724.32005
Ark. Mat. 28, No.2, 231-248 (1990).

Let T be a positive closed current of degree p on an open neighborhood Ω of 0 in ^{n}. For $a \in {}^*$ let h_a denote the homothety given by a and $h_a^* T$ the lifted current. If the weak limit $\lim_{|a|\to 0} h_a^* T$ exists it is called the tangent cone of T in 0. The authors show:

Theorem: If for small $r_0 > 0$ one of the following conditions a) or b) is satisfied then the tangent cone of T exists:

$$a) \quad \int_0^{r_0} [(\sqrt{v_T(r) - v_T(r/2)})/r] dr < \infty, \quad b) \quad \int_0^{r_0} [(v_T(r) - v_T(0))/r] dr < \infty.$$

$v_T(r)$ denotes the projective mass of T. -

The authors show that condition b) is optimal in a sense.

Theorem: If T is the current of an analytic subset of pure dimension p in Ω then

$$v_T(r) - v_T(0) \leq C r^\epsilon$$

for small $r > 0$ and suitable numbers $C, \epsilon > 0$. -

A conclusion of these theorems is a result of Thie and King on the existence of a tangent cone for a current induced by an analytic set.

Reviewer: H.-J.Reiffen (Osnabrück)

MSC:

32C30 Integration on analytic sets and spaces, currents
32B15 Analytic subsets of affine space

Keywords:

positive closed current; tangent cone; current induced by an analytic set

Full Text: DOI

References:

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Demailly, Jean-Pierre

Cohomology of q-convex spaces in top degrees. (English) Zbl 0682.32017

Math. Z. 204, No.2, 283-295 (1990).

It is shown that every strongly q-complete subvariety of a complex analytic space has a fundamental system of strongly q-complete neighborhoods. As a consequence, we find a simple proof of Ohsawa's result that every non compact irreducible n-dimensional analytic space is strongly n-convex. An elementary proof of the existence of Hodge decomposition in top degrees for absolutely q-convex manifolds is also given.

Reviewer: J.P.Demailly

MSC:

32F10 q-convexity, q-concavity

Keywords:

strongly q-complete subvariety; strongly q-complete neighborhoods; strongly n-convex; existence of Hodge decomposition

Full Text: DOI EuDML

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Ba Barlet, D.: Convexit\'e de l'espace des cycles. Bull. Soc. Math. Fr.106, 373–397 (1978) · Zbl 0395.32009

G-R Grauert, H., Riemenschneider, O.: K\"ahlersche Mannigfaltigkeiten mit hype-q-konvexem Rand. Problems in analysis: a Symposium in honor of Salomon Bochner. Princeton, Princeton University Press 1970

G-W Greene, R. E., Wu, H.: Embedding of open riemannian manifolds by harmonic functions. Ann. Inst. Fourier25, 215–235 (1975) · Zbl 0307.31003

Ma Malgrange, B.: Existence et approximation des solutions des \'equations aux d\'eriv\'ees partielles et des \'equations de convolution. Ann. Inst. Fourier6, 271–355 (1955/1956) · Zbl 0071.09002

Oh1 Ohsawa, T.: A reduction theorem for cohomology groups of very stronglyq-convex K\"ahler manifolds. Invent. Math.63, 335–354 (1981)/66, 391–393 (1982) · Zbl 0457.32007 · doi:10.1007/BF01393882

Oh2 Ohsawa, T.: Completeness of noncompact analytic spaces. Publ. R.I.M.S., Kyoto Univ.20, 683–692 (1984) · Zbl 0568.32008 · doi:10.2977/prims/1195181418

O-T Ohsawa, T., Takegoshi, K.: Hodge spectral sequence on pseudoconvex domains. Math. Z.197, 1–12 (1988) · Zbl 0638.32016 · doi:10.1007/BF01161626

S1 Siu, Y. T.: Analytic sheaf cohomology groups of dimensionn ofn-dimensional noncompact complex manifolds. Pac. J. Math.28, 407–411 (1969)

S2 Siu, Y. T.: Analytic sheaf cohomology groups of dimensionn ofn-dimensional complex spaces. Trans. Am. Math. Soc.143, 77–94 (1969)

S3 Siu, Y.T.: Every Stein subvariety has a Stein neighborhood. Invent. Math.38, 89–100 (1976) · Zbl 0343.32014 · doi:10.1007/BF01390170

Bedford, Eric; **Demailly, Jean-Pierre**

Two counterexamples concerning the pluricomplex Green function in ^{n}. (English) Zbl 0681.32014

Indiana Univ. Math. J. 37, No.4, 865-867 (1988).

Given a domain Ω in ^{n} and a point $z \in \Omega$, the pluricomplex Green function on Ω with logarithmic pole at z is given by $u_z(\zeta) = \sup\{v(\zeta)$: v is plurisubharmonic on Ω, $v < 0$, and $v(\zeta) \leq \log|\zeta - z| + O(1)\}$. In "Capacities in complex analysis" (1988; Zbl 0655.32001), *U. Cegrell* raised the following questions:

1. Is $u_z \in C^2(\bar{\Omega} - \{z\})$?

2. Is u_z symmetric, i.e., $u_z(\zeta) = u_\zeta(z)$?

L. Lempert [Bull. Soc. Math. Fr. 109, 427-474 (1981; Zbl 0492.32025)] has shown that if Ω is strictly convex and smoothly bounded, then the answer to both of these questions is "Yes". In the paper, the authors provide counterexamples to show that for strongly pseudoconvex domains, the answer to both these questions is "No".

Reviewer: M.Stoll

MSC:

32U05	Plurisubharmonic functions and generalizations	Cited in **5** Documents
32T99	Pseudoconvex domains	
31C10	Pluriharmonic and plurisubharmonic functions	

Keywords:

pluricomplex Green function; strongly pseudoconvex domains

Full Text: DOI

Demailly, Jean-Pierre

Vanishing theorems for tensor powers of a positive vector bundle. (English) Zbl 0651.32019

Proc. 21st Int. Taniguchi Symp., Katata/Japan, Conf., Kyoto/Japan 1987, Lect. Notes Math. 1339, 86-105 (1988).

[For the entire collection see Zbl 0638.00022.]

Let E be a holomorphic vector bundle of rank r over a compact complex manifold X of dimension n, and suppose that E is positive in the sense of Griffiths and that $p + q \geq n + 1$. Let L be a semipositive line bundle and Γ aE an irreducible tensor power representation of GL(E) of highest weight $a = (a_1, ..., a_r)$ with $a_1 \geq a_2 \geq ... \geq a_h > a_{h+1} = ... = a_r = 0$. The author shows that $H^{p,q}(X, \Gamma$ aE$\otimes(\det E)^\ell \otimes L)$ vanishes for $\ell \geq h + A(n, p, q)$, where A(n,p,q) is a certain rational function of n,p,. The best possible value for A(n,p,q) is not known, but an example of *Th. Peternell, J. Le Potier* and *M. Schneider* [Invent. Math. 87, 573-586 (1987; Zbl 0618.14023)] shows, even when $\Gamma\ aE = S$ kE and $p = n$, one requires at least $\ell \geq 1$. The method of proof is to represent Γ aE as the direct image of a positive line bundle over a suitable flag manifold of E and to apply a generalization of Le Potier's isomorphism theorem to this situation. In order to overcome a difficulty arising from the fact that, when $p < n$, the generalized Borel-Le Potier spectral sequence does not degenerate at the E_1 level, the author obtains a new curvature estimate for the bundle of X-relative differential forms on the flag manifold of E.

Reviewer: P.E.Newstead

MSC:

32L20	Vanishing theorems (analytic spaces)	Cited in **2** Documents
32L05	Holomorphic fiber bundles and generalizations	

Keywords:

vanishing theorem; tensor power; positive line bundle

Demailly, Jean-Pierre

Vanishing theorems for tensor powers of an ample vector bundle. (English) Zbl 0647.14005

Invent. Math. 91, No.1, 203-220 (1988).

Let X be a compact complex manifold of dimension n and E resp. L an ample holomorphic vector bundle of rank r resp. an ample line bundle on X. The paper gives generalizations of Griffiths' vanishing theorem $H^{n,q}(X, S^k E \otimes \det(E \otimes L)) = 0$ for $q \geq 1$ [*P. A. Griffiths*, Global Analysis, papers in Honor of K. Kodaira, 185-251 (1969; Zbl 0201.240)] which shall not be repeated here and Le Potier's vanishing theorem $H^{p,q}(X, E) = 0$ for $p + q \geq n + r$ [*J. Le Potier*, Math. Ann. 218, 35-53 (1975; Zbl 0313.32037)] saying that $H^{p,q}(X, E^{\otimes k} \otimes (\det(E))^{\ell} \otimes L) = 0$ for $p + q \geq n + 1$, $k \geq 1$ and $\ell \geq n - p + r - 1$.

The proof rests on a generalization of the Borel-Le Potier spectral sequence and the Kodaira-Akizuki-Nakano vanishing theorem for line bundles. Moreover it is shown that there is a canonical homomorphism $H^{p,q}(X, \wedge^2 E \otimes L) \to H^{p+1,q+1}(X, S^2 E \otimes L)$ which is bijective under some additional hypotheses. Using this the author gives a counterexample to a conjecture of *J. A. Sommese* in Math. Ann. 233, 229-256 (1978; Zbl 0381.14007).

Reviewer: H.Lange

MSC:

14F05 Sheaves, derived categories of sheaves, etc.
32L20 Vanishing theorems (analytic spaces)

Cited in **3** Reviews
Cited in **9** Documents

Keywords:

tensor power of ample vector bundle; vanishing theorem

Full Text: DOI EuDML

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Demailly, Jean-Pierre

Sur les théorèmes d'annulation et de finitude de T. Ohsawa et O. Abdelkader. (On vanishing and finiteness theorems of T. Ohsawa and O. Abdelkader). (French) Zbl 0691.32009

Sémin. d'analyse P. Lelong - P. Dolbeault - H. Skoda, Paris, 1985/86, Lect. Notes Math. 1295, 48-58 (1987).

[For the entire collection see Zbl 0623.00006.]

L'objet de cette note est de donner une démonstration aussi simple que possible des théorèmes d'annulation et de finitude dus à *T. Ohsawa* [Publ. Res. Inst. Math. Sci. 15, 853-870 (1979; Zbl 0434.32014), Publ. Res. Inst. Math. Sci. 17, 113-126 (1981; Zbl 0465.32007)], et des généralisations de ces théorèmes obtenues par *O. Abdelkader* [C. R. Acad. Sci., Paris, Ser. A 290, 75-78 (1980; Zbl 0442.32008) et "Théorèmes de finitude pour la cohomologie d'une variété faiblement 1-complète à valeurs dans un fibré en droites semi-positif", Thèse Doct. d'Etat à l'Univ. Paris VI (1985)].

Reviewer: Résumé

MSC:

32L20 Vanishing theorems (analytic spaces)
32H35 Proper mappings, finiteness theorems
32C35 Analytic sheaves and cohomology groups
32F10 q-convexity, q-concavity
53C55 Hermitian and Kählerian manifolds (global differential geometry)

Cited in **1** Document

Keywords:

vanishing theorem; finiteness theorem; Kähler manifold; weakly 1- complete; hermitian vector bundle

Demailly, Jean-Pierre; Laurent-Thiebaut, Christine

Formules intégrales pour les formes différentielles de type (p,q) dans les variétés de Stein. (Integral formulas for differential forms of type (p,q) in Stein manifolds). (French)

Zbl 0632.32004

Ann. Sci. Éc. Norm. Supér. (4) 20, No. 4, 579-598 (1987).

The Cauchy-Green integral formula for a domain $D \subset\subset$ with piecewise C^1-boundary and for $f \in C^1(\bar{D})$ is

$$(!) \quad f(z) = (1/2\pi i)\int_{\partial D} f(\zeta)d\zeta/(\zeta - z) \quad + \quad (1/2\pi i)\int_D \bar{\partial} f d\zeta \Lambda d\bar{\zeta}/(\zeta - z).$$

Over the past half-century, various integral formulas have been gradually developed that generalise (!) for several complex variables. An excellent, systematic exposition of this work and of some problems that can be solved by such methods is *G. M. Khenkin* and *J. Leiterer*, Theory of functions on complex manifolds (1984; Zbl 0573.32001). The present paper contains the construction of the relevant kernels and integrals for differential forms of type (p,q) on Stein manifolds: §§ 1 and 3 develop and extend the methods of Khenkin and Leiterer concerning the Bochner-Martinelli kernel; §§ 2 and 4 generalise the Koppelman formula and the Koppelman-Leray formula, respectively.

Reviewer: E.J.Akutowicz

MSC:

32A30 Generalizations of function theory to several variables
30E20 Integration, integrals of Cauchy type, etc. (one complex variable)
32A25 Integral representation; canonical kernels (several complex variables)
32E10 Stein spaces, Stein manifolds

Cited in **4** Reviews
Cited in **3** Documents

Keywords:

(p,q)-differential forms on Stein manifolds; tangent and cotangent fiber spaces; Bochner-Martinelli kernel; Koppelman formula; Koppelman-Leray formula

Full Text: DOI Numdam EuDML

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Demailly, Jean-Pierre

Une preuve simple de la conjecture de Grauert-Riemenschneider. (A simple proof of the Grauert-Riemenschneider conjecture). (French) Zbl 0629.32026

Sémin. d'analyse P. Lelong - P. Dolbeault - H. Skoda, Paris 1985/86, Lect. Notes Math. 1295, 24-47 (1987).

[For the entire collection see Zbl 0623.00006.]

The author's abstract: "Let E be a hermitian holomorphic line bundle over a compact complex manifold X. We give an asymptotic upper bound for the dimension of cohomology groups of high tensor powers E^k. This bound is invariantly expressed in terms of an integral of the bundle curvature form. As an application, we find a simple proof of the Grauert- Riemenschneider conjecture, recently solved by Siu: if X possesses a quasi-positive line bundle E, then X is a Moishezon space; furthermore the quasipositivity hypothesis can be weakened here in an integral condition which does not require the bundle E to be pointwise semi- positive."

Reviewer: D.Barlet

MSC:

32J25 Transcendental methods of algebraic geometry
32J99 Compact analytic spaces
32L15 Bundle convexity
32L20 Vanishing theorems (analytic spaces)

Keywords:

hermitian holomorphic line bundle over a compact complex manifold; bundle curvature; Grauert-Riemenschneider conjecture; quasi-positive line bundle; Moishezon space

Demailly, Jean-Pierre

Nombres de Lelong généralisés, théorèmes d'intégralité et d'analyticité. (Generalized Lelong numbers, integrability and analyticity theorems). (French) Zbl 0629.32011

Acta Math. 159, 153-169 (1987).

Let X be a complex Stein space, T a closed positive current of bidimension (p,p) on X and $\phi : X \to [-\infty, +\infty[$ an exhaustive plurisubharmonic function. The author's generalized Lelong number ν (T,ϕ) is defined as the mass of the measure $T \wedge (dd^c\phi)^p$ carried by the polar set $\phi^{-1}(-\infty)$ and is obtained by means of the Monge-Ampère operator of *E. Bedford* and *B. A. Taylor* [ibid. 149, 1-40 (1982; Zbl 0547.32012)]. ν (T,ϕ) generalizes the classical *P. Lelong* ["Plurisubharmonic functions and positive differential forms" (1969; Zbl 0195.116)] and C. O. Kiselman's numbers. The author establishes that ν (T,ϕ) depends only on the behaviour of ϕ in a neighbourhood of the poles. The use of ν (T,ϕ) allows him to obtain very simple proofs of classical results on Lelong numbers, e.g. that these numbers are invariant with respect to local coordinate transformations [cf. *Y. T. Siu*, Invent. Math. 27, 53-156 (1974; Zbl 0289.32003)] and also on *P. Thie*'s [Math. Ann. 172, 269-312 (1967; Zbl 0158.328)] theorem showing that the Lelong number of an analytic set X coincides to the algebraic multiplicity of Y at x. Finally, the author obtains a generalization of Siu's theorem on the analyticity of the level sets associated to Lelong numbers, his result containing as a particular case a recent theorem of C. O. Kiselman on directional Lelong numbers.

Reviewer: P.Caraman

MSC:

32E10	Stein spaces, Stein manifolds
32U05	Plurisubharmonic functions and generalizations
32C30	Integration on analytic sets and spaces, currents
31C10	Pluriharmonic and plurisubharmonic functions
31C15	Generalizations of potentials and capacities

Keywords:
integrability; potential; complex Stein space; current; plurisubharmonic function; generalized Lelong number; analyticity

Full Text: DOI

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Demailly, Jean-Pierre
Théorèmes d'annulation pour la cohomologie des puissances tensorielles d'un fibré vectoriel positif. (Vanishing theorems for cohomology groups of tensor powers of a positive vector bundle). (French) Zbl 0627.32022
C. R. Acad. Sci., Paris, Sér. I 305, 419-422 (1987).

The author cleverly combines the Kodaira-Akizuki-Nakano vanishing theorem with some standard geometrical constructions, in order to prove the following result: Let E, L be holomorphic vector bundles over a compact complex manifold X. Assume $rk(L) = 1$ and $E > 0$, $L \geq 0$ or $E \geq 0$, $L > 0$. Then there is an integer A(n,p,q), so that

$$(1) \quad H^{p,q}(X, Sym^k(E) \otimes (\det E)^1 \otimes L) = 0,$$

whenever $1 \geq A(n,p,q)$ and $p+q \geq n+1$. The constant A(n,p,q) is explicitly given, and it is shown that it is optimal for $p = n$. Relation (1) still holds for $E^{\otimes k}$ (with a different constant).

These vanishing theorems strengthen similar celebrated results of Griffith and Le Potier.

Reviewer: M.Putinar

MSC:

32L20 Vanishing theorems (analytic spaces)
32M10 Homogeneous complex manifolds

Cited in **3** Documents

Keywords:

positive bundle; curvature; Dolbeault cohomology; vanishing theorems

Demailly, Jean-Pierre
Mesures de Monge-Ampère et mesures pluriharmoniques. (Monge-Ampère measures and pluriharmonic measures). (French) Zbl 0595.32006
Math. Z. 194, 519-564 (1987).

Let Ω be a relatively compact open subset in a Stein manifold, and $n = \dim \Omega$. Assume that Ω is hyperconvex, i.e. that there exists a bounded psh (plurisubharmonic) exhaustion function on Ω. A "pluricomplex Green function" u_Ω is then naturally defined on $\Omega \times \Omega$: For all $z \in \Omega$, $u_z(\zeta) := u_\Omega(z, \zeta)$ is the solution of the Dirichlet problem for the complex Monge-Ampère equation $(dd^c u_z)^n = 0$ on $\Omega \setminus \{z\}$ such that $u_z(\zeta) = \log |\zeta - z| + O(1)$ at $\zeta = z$; u_Ω is shown to be continuous outside the diagonal and invariant under biholomorphisms. Bedford and Taylor's Monge-Ampère operators are used in conjunction with a general Lelong-Jensen formula previously found by the author [Mem. Soc. Math. Fr., Nouv. Ser. 19, 124 p. (1985; Zbl 0579.32012)] in order to construct an invariant pluricomplex Poisson kernel $d\mu_z(\zeta) := (2\pi)^{-n}(dd^c u_z(\zeta))^{n-1} \wedge d^c u_z(\zeta)|_{\partial\Omega}$, $(z, \zeta) \in \Omega \times \partial\Omega$. Each measure μ_z on $\partial\Omega$ is such that $\mu_z(V) = V(z)$ for every function V pluriharmonic on Ω and continuous on $\bar{\Omega}$; furthermore, μ_z is carried by the set of strictly pseudoconvex points of $\partial\Omega$ if $\bar{\Omega}$ has a C^2 psh defining function. The principal part of the singularity of $d\mu_z(\zeta)$ on the diagonal of $\partial\Omega$ is then computed explicitly when Ω is strictly pseudoconvex, using an osculation of $\partial\Omega$ by balls. Through a complexification process, it is finally shown that Monge-Ampère measures provide an explicit formula representing every point of a convex compact subset $K \subset {}^n$ as a barycenter of the extremal points of K.

MSC:

32A25	Integral representation; canonical kernels (several complex variables)	Cited in **7** Reviews
32C30	Integration on analytic sets and spaces, currents	Cited in **74** Documents
32F45	Invariant metrics and pseudodistances	
32U05	Plurisubharmonic functions and generalizations	
31C10	Pluriharmonic and plurisubharmonic functions	
32E10	Stein spaces, Stein manifolds	

Keywords:

pluricomplex Green function; Lelong-Jensen formula; Monge-Ampère measures; plurisubharmonic exhaustion function; hyperconvex domain; pluriharmonic measures; Choquet's theorem; barycentric representation

Full Text: DOI EuDML

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Demailly, J.-P.

Mesures de Monge-Ampère et mesures pluriharmoniques. (Monge-Ampère measures and pluriharmonic measures). (French) Zbl 0602.31006

Sémin., Équations Dériv. Partielles 1985-1986, Exposé No.19, 15 p. (1986).

This work develops the potential theory of several complex variables in the form introduced by *E. Bedford* and *B. A. Taylor* [Invent. Math. 37, 1-44 (1976; Zbl 0315.31007); Acta Math. 149, 1-40 (1982; Zbl 0547.32012)]. In particular the author introduces a pluricomplex Green function for every bounded hyperconvex domain Ω in a Stein manifold. The Green function, with pole $z \in \Omega$, is a solution u_z of the Dirichlet problem for the complex Monge-Ampère operator with logarithmic pole at z and boundary values 0 on $\partial\Omega$. Using the functions u_z, the author also constructs pluriharmonic measures μ_z on $\partial\Omega$ which have properties analogous to those of harmonic measure.

The paper concludes with an application to the geometry of convex sets: if K is a compact convex subset of R^n, then through a complexification procedure, the measures μ_z provide a formula which allows every point of K to be represented as a barycentre of the extremal points of K.

Another paper of the author, with the same title [Math. Z. (to appear; Zbl 0595.32006)], gives a more detailed account of this work.

Reviewer: D.Armitage

MSC:

31C10 Pluriharmonic and plurisubharmonic functions
32E10 Stein spaces, Stein manifolds
32U05 Plurisubharmonic functions and generalizations

Cited in **2** Documents

Keywords:
several complex variables; pluricomplex Green function; hyperconvex domain; Stein manifold; Dirichlet problem; complex Monge-Ampère operator; logarithmic pole; pluriharmonic measures

Full Text: Numdam EuDML