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Demailly, Jean-Pierre (ed.); Dinh, Tien-Cuong (ed.); Hai, Le Mau (ed.); Hiep, Pham Hoang (ed.); Khoai, Ha Huy (ed.); Ma, Xiaonan (ed.); Marinescu, George (ed.); Peternell, Thomas (ed.); Sibony, Nessim (ed.)

Preface. (English) Zbl 1433.00044

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From the text,: This issue of Acta Mathematica Vietnamica is dedicated to the proceedings of the Conference "Nevanlinna theory and complex geometry" in Honor of Le Van Thiem's Centenary (Hanoi, 26/02/2018–02/03/2018).

MSC:
00B25 Proceedings of conferences of miscellaneous specific interest
32-06 Proceedings, conferences, collections, etc. pertaining to several complex variables and analytic spaces

Preface

Le Van Thiem received his doctorate in 1945 from Göttingen and he got his Doctorat d'État at the École Normale Supérieure de Paris in 1949.

In 1949, Le Van Thiem returned to Vietnam in the middle of a fierce resistance war for independence of Vietnam. He had a great contribution in building the University in the headquarters of the Resistance, and became a teacher of the first Vietnamese mathematicians. For the next dozen years, Vietnamese mathematicians were either his students or students of his students. We can say that Le Van Thiem is the founder of Mathematics in Vietnam.

In the first stage of his career, Le Van Thiem made a great contribution in the inverse problem of the value distribution theory of meromorphic functions (Nevanlinna theory). Later, he turned to applied mathematics, contributing to solving problems raised in Vietnamese practice such as oriented explosion, ground water under irrigation schemes, and the problem of calculating petroleum reserves.

Le Van Thiem himself is a part of the history of Mathematics in Vietnam. His name is given to a street in Hanoi and some schools throughout the country.

This issue of Acta Mathematica Vietnamica is dedicated to the proceedings of the Conference "Nevanlinna theory and Complex Geometry" in Honor of Le Van Thiem's Centenary (Hanoi, 26/02/2018 – 02/03/2018). We thank the Institute of Mathematics of Vietnam and its staff for their help to make the Conference possible.

Selected Papers of Le Van Thiem

References:

[1] Thiem, L.-V.: Beitrag sum Typenproblem der Riemannschen Flachen. Comment. Math. Helv. 20, 270–287 (1947)

[2] Thiem, L.-V.: Uber das Umkehrproblem der Werterteilungslehre. Comment. Math. Helv. 23, 26–49 (1949)

[3] Thiem, L.-V.: Le degré de ramification d'une surface de Riemann et la croissance de la caractéristique de la fonction uniformisante. C. R. Acad. Sc. Paris 228, 1192–1195 (1949)

[4] Thiem, L.-V.: Un problème de type généralisé. C. R. Acad. Sc. Paris 228, 1270–1272 (1949)

[5] Thiem, L.-V.: Sur un problème d'inversion dans la théorie des fonctions méromorphes. Ann. Sci. Ecole Normale Sup. 67, 51–98 (1950)

[6] Thiem, L.-V.: Sur un problème d'infiltration à travers un sol à deux couchés. Acta Sci. Vietnam., Sectio Sci. Math. et Phys. 1, 3–9 (1964)

Demailly, Jean-Pierre

Recent results on the Kobayashi and Green-Griffiths-Lang conjectures. (English) Zbl 1436.32086

Jpn. J. Math. (3) 15, No. 1, 1-120 (2020).

Summary: The study of entire holomorphic curves contained in projective algebraic varieties is intimately related to fascinating questions of geometry and number theory – especially through the concepts of curvature and positivity which are central themes in Kodaira's contributions to mathematics. The aim of these lectures is to present recent results concerning the geometric side of the problem. The Green-Griffiths-Lang conjecture stipulates that for every projective variety X of general type over $\mathbb{C}$, there exists a proper algebraic subvariety Y of X containing all entire curves $f : \mathbb{C} \to X$. Using the formalism of directed varieties and jet bundles, we show that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet-semi-stability property of the tangent bundle T_X. It is possible to exploit similar techniques to investigate a famous conjecture of Shoshichi Kobayashi (1970), according to which a generic algebraic hypersurface of dimension nn and of sufficiently large degree $d \geq d_n$ in the complex projective space $\mathbb{P}^{n+1}$ is hyperbolic: in the early 2000's, Yum-Tong Siu proposed a strategy that led in 2015 to a proof based on a clever use of slanted vector fields on jet spaces, combined with Nevanlinna theory arguments. In 2016, the conjecture has been settled in a different way by Damian Brotbek, making a more direct use of Wronskian differential operators and associated multiplier ideals; shortly afterwards, Ya Deng showed how the proof could be modified to yield an explicit value of dndn. We give here a short proof based on a substantial simplification of their ideas, producing a bound very similar to Deng's original estimate, namely $d_n = \lfloor \frac{1}{3}(en)^{2n+2} \rfloor$.

MSC:
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32H30 Value distribution theory in higher dimensions

32L10 Sheaves and cohomology of sections of holomorphic vector bundles, general results
53C55 Global differential geometry of Hermitian and Kählerian manifolds
14J40 n-folds ($n > 4$)

Keywords: Kobayashi hyperbolic variety; directed manifold; genus of a curve; jet bundle; jet differential; jet metric; Chern connection and curvature; negativity of jet curvature; variety of general type; Kobayashi conjecture; Green-Griffiths conjecture; Lang conjecture

References:

[1] Acta Soc. Sci. Fennicae. Nova Ser. A.194134

[2] Arrondo, E.; Sols, I.; Speiser, R., Global moduli for contacts, Ark. Mat., 35, 1-57 (1997) – Zbl 0930.14035

[3] Azukawa, K.; Suzuki, M., Some examples of algebraic degeneracy and hyperbolic manifolds, Rocky Mountain J. Math., 10, 655-659 (1980) – Zbl 0416.32012

[4] G. Bérczi, Towards the Green-Griffiths-Lang conjecture via equivariant localisation, preprint, arXiv:1509.03406. – Zbl 1420.32013

[5] G. Bérczi, Thom polynomials and the Green-Griffiths-Lang conjecture for hypersurfaces with polynomial degree, Int. Math. Res. Not. IMRN, rnx332 (2018). – Zbl 1431.32006

[6] Bérczi, G.; Kirwan, F., A geometric construction for invariant jet differentials, Surv. Differ. Geom., 17, 79-125 (2012) – Zbl 1331.58007

[7] Bérczi, G.; Szenes, A., Thom polynomials of Morin singularities, Ann. of Math., 175, 567-629 (2012) – Zbl 1247.58021

[8] Bloch, A., Sur les systèmes de fonctions uniformes satisfaisant à l'équation d'une variété algébrique dont l'irrégularité dépasse la dimension, J. Math. Pures Appl., 5, 19-66 (1926) – JFM 52.0373.04

[9] Bloch, A., Sur les systèmes de fonctions holomorphes à variétés linéaires lacunaires, Ann. Sci. École Norm. Sup., 43, 309-362 (1926) – JFM 52.0326.01

[10] Bogomolov, Fa, Families of curves on a surface of general type, Soviet Math. Dokl., 236, 1294-1297 (1977) – Zbl 0415.14013

[11] Bogomolov, Fa, Holomorphic tensors and vector bundles on projective manifolds, Math. USSR-Izv., 13, 499-555 (1979) – Zbl 0439.14002

[12] Bonavero, L., Inégalités de Morse holomorphes singulières, C. R. Acad. Sci. Paris Sér. I Math., 317, 1163-1166 (1993) – Zbl 0799.32023

[13] Brody, R., Compact manifolds and hyperbolicity, Trans. Amer. Math. Soc., 235, 213-219 (1978) – Zbl 0416.32013

[14] Brody, R.; Green, M., A family of smooth hyperbolic hypersurfaces in $\mathbb{P}_3$, Duke Math. J., 44, 873-874 (1977) – Zbl 0383.32009

[15] Brotbek, D., On the hyperbolicity of general hypersurfaces, Publ. Math. Inst. Hautes Études Sci., 126, 1-34 (2017) – Zbl 06827883

[16] Brotbek, D.; Darondeau, L., Complete intersection varieties with ample cotangent bundles, Invent. Math., 212, 913-940 (2018) – Zbl 1401.14203

[17] Brückmann, P.; Rackwitz, H-G, T-symmetrical tensor forms on complete intersections, Math. Ann., 288, 627-635 (1990) – Zbl 0724.14032

[18] Brunella, M., Courbes entières dans les surfaces algébriques complexes (d'après McQuillan, Demailly-El Goul, . . .), Séminaire Bourbaki, Astérisque, 282, 39-61 (2002)

[19] Brunella, M., Plurisubharmonic variation of the leafwise Poincaré metric, Internat. J. Math., 14, 139-151 (2003) – Zbl 1052.32027

[20] Brunella, M., On the plurisubharmonicity of the leafwise Poincaré metric on projective manifolds, J. Math. Kyoto Univ., 45, 381-390 (2005) – Zbl 1101.32015

[21] Brunella, M., A positivity property for foliations on compact Kähler manifolds, Internat. J. Math., 17, 35-43 (2006) – Zbl 1097.37041

[22] Cantat, S., Deux exemples concernant une conjecture de Serge Lang, C. R. Acad. Sci. Paris Sér. I Math., 330, 581-586 (2000) – Zbl 0957.32007

[23] Carlson, Ja, Some degeneracy theorems for entire functions with values in an algebraic variety, Trans. Amer. Math. Soc., 168, 273-301 (1972) – Zbl 0246.32023

[24] Carlson, Ja; Griffiths, Pa, A defect relation for equidimensional holomorphic mappings between algebraic varieties, Ann. of Math., 95, 557-584 (1972) – Zbl 0248.32018

[25] Cartan, H., Sur les systèmes de fonctions holomorphes à variétés linéaires lacunaires et leurs applications, Thèse, Paris, Ann. Sci. École Norm. Sup., 45, 255-346 (1928) – JFM 54.0357.06

[26] C. Ciliberto, F. Flamini and M. Zaidenberg, A remark on the intersection of plane curves, preprint, arXiv:1704.00320. – Zbl 1443.14035

[27] Clemens, H., Curves on generic hypersurfaces, Ann. Sci. École Norm. Sup., 19, 629-636 (1986) – Zbl 0611.14024

[28] H. Clemens, J. Kollár and S. Mori, Higher Dimensional Complex Geometry, Astérisque, 166, Soc. Math. France, 1988. – Zbl 0689.14016

[29] Clemens, H.; Ran, Z., Twisted genus bounds for subvarieties of generic hypersurfaces, Amer. J. Math., 126, 89-120 (2004) – Zbl 1050.14035

[30] Colley, Sj; Kennedy, G., The enumeration of simultaneous higher-order contacts between plane curves, Compositio Math., 93, 171-209 (1994) – Zbl 0820.14038

[31] Collino, A., Evidence for a conjecture of Ellingsrud and Strømme on the Chow ring of $\mathrm{Hilb}_d(\mathbb{P}^2)$, Illinois J. Math., 32, 171-210 (1988) – Zbl 0706.14001

[32] Cowen, M.; Griffiths, Pa, Holomorphic curves and metrics of negative curvature, J. Analyse Math., 29, 93-153 (1976) – Zbl 0352.32014

[33] L. Darondeau, Effective algebraic degeneracy of entire curves in complements of smooth projective hypersurfaces, preprint, arXiv:1402.1396.

[34] Darondeau, L., Fiber integration on the Demailly tower, Ann. Inst. Fourier (Grenoble), 66, 29-54 (2016) – Zbl 1386.14032

[35] Darondeau, L., On the logarithmic Green-Griffiths conjecture, Int. Math. Res. Not. IMRN, 2016, 1871-1923 (2016) – Zbl 1338.32023

[36] Debarre, O.; Pacienza, G.; Păun, M., Non-deformability of entire curves in projective hypersurfaces of high degree, Ann. Inst. Fourier (Grenoble), 56, 247-253 (2006) – Zbl 1096.32010

[37] Demailly, J-P, Champs magnétiques et inégalités de Morse pour la d''-cohomologie, Ann. Inst. Fourier (Grenoble), 35, 189-229 (1985) – Zbl 0565.58017

[38] Demailly, J-P, Cohomology of q-convex spaces in top degrees, Math. Z., 203, 283-295 (1990) – Zbl 0682.32017

[39] J.-P. Demailly, Singular Hermitian metrics on positive line bundles, In: Proceedings of the Bayreuth conference, Complex Algebraic Varieties, April 2-6, 1990, (eds. K. Hulek, T. Peternell, M. Schneider and F. Schreyer), Lecture Notes in Math., 1507, Springer-Verlag, 1992, pp. 87-104.

[40] Demailly, J-P, Regularization of closed positive currents and intersection theory, J. Algebraic Geom., 1, 361-409 (1992) – Zbl 0777.32016

[41] J.-P. Demailly, L^2 vanishing theorems for positive line bundles and adjunction theory, In: Transcendental Methods in Algebraic Geometry, Cetraro, Italy, July, 1994, (eds. F. Catanese and C. Ciliberto), Lecture Notes in Math., 1646, Fond. CIME/CIME Found. Subser., Springer-Verlag, 1996, pp. 1-97.

[42] Demailly, J-P; Kollár, J.; Lazarsfeld, R., Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials, AMS Summer School on Algebraic Geometry, Santa Cruz, 1995, 285-360 (1997), Providence, RI: Amer. Math. Soc., Providence, RI – Zbl 0919.32014

[43] Demailly, J-P, Variétés hyperboliques et équations différentielles algébriques, Gaz. Math., 73, 3-23 (1997) – Zbl 0901.32019

[44] J.-P. Demailly, Structure of jet differential rings and holomorphic Morse inequalities, talk at the CRM Workshop, The Geometry of Holomorphic and Algebraic Curves in Complex Algebraic Varieties, Montréal, May, 2007.

[45] J.-P. Demailly, On the algebraic structure of the ring of jet differential operators, talk at the conference, Effective Aspects of Complex Hyperbolic Varieties, Aber Wrac'h, France, September 10-14, 2007.

[46] Demailly, J-P, Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture, Pure Appl. Math. Q., 7, 1165-1208 (2011) – Zbl 1316.32014

[47] Demailly, J-P, Hyperbolic algebraic varieties and holomorphic differential equations, Acta Math. Vietnam., 37, 441-512 (2012) – Zbl 1264.32022

[48] J.-P. Demailly, Towards the Green-Griffiths-Lang conjecture, In: Analysis and Geometry, in honor of Mohammed Salah Baouendi, Tunis, March, 2014, (eds. A. Baklouti, A. El Kacimi, S. Kallel and N. Mir), Springer Proc. Math. Stat., 127, Springer-Verlag, 2015, pp. 141-159.

[49] J.-P. Demailly, Proof of the Kobayashi conjecture on the hyperbolicity of very general hypersurfaces, preprint, arXiv:1501.07625.

[50] Demailly, J-P; El Goul, J., Connexions méromorphes projectives et variétés algébriques hyperboliques, C. R. Acad. Sci. Paris Sér. I Math., 324, 1385-1390 (1997) – Zbl 0898.32016

[51] Demailly, J-P; El Goul, J., Hyperbolicity of generic surfaces of high degree in projective 3-space, Amer. J. Math., 122, 515-546 (2000) – Zbl 0966.32014

[52] Demailly, J-P; Lempert, L.; Shiffman, B., Algebraic approximation of holomorphic maps from Stein domains to projective manifolds, Duke Math. J., 76, 333-363 (1994) – Zbl 0861.32006

[53] Demailly, J-P; Peternell, T.; Schneider, M., Compact complex manifolds with numerically effective tangent bundles, J. Algebraic Geom., 3, 295-345 (1994) – Zbl 0827.14027

[54] Y. Deng, Effectivity in the hyperbolicity-related problems, Chap. 4 of the Ph.D. memoir "Generalized Okounkov Bodies, Hyperbolicity-Related and Direct Image Problems" defended on June 26, 2017 at Univ. Grenoble Alpes, Institut Fourier, preprint, arXiv:1606.03831.

[55] Y. Deng, On the Diverio-Trapani conjecture, preprint, arXiv:1703.07560.

[56] G. Dethloff and H. Grauert, On the infinitesimal deformation of simply connected domains in one complex variable, In: International Symposium in Memory of Hua Loo Keng. Vol. II, Beijing, 1988, Springer-Verlag, 1991, pp. 57-88. – Zbl 0864.32012

[57] Dethloff, G.; Lu, Ss-Y, Logarithmic jet bundles and applications, Osaka J. Math., 38, 185-237 (2001) – Zbl 0982.32022

[58] Diverio, S., Differential equations on complex projective hypersurfaces of low dimension, Compos. Math., 144, 920-932 (2008) – Zbl 1193.32013

[59] Diverio, S., Existence of global invariant jet differentials on projective hypersurfaces of high degree, Math. Ann., 344, 293-315 (2009) – Zbl 1166.32013

[60] Diverio, S.; Merker, J.; Rousseau, E., Effective algebraic degeneracy, Invent. Math., 180, 161-223 (2010) – Zbl 1192.32014

[61] S. Diverio and E. Rousseau, A Survey on Hyperbolicity of Projective Hypersurfaces, Publ. Mat. IMPA, Inst. Nac. Mat. Pura Apl., Rio de Janeiro, 2011. – Zbl 1250.32001

[62] Diverio, S.; Rousseau, E., The exceptional set and the Green-Griffiths locus do not always coincide, Enseign. Math., 61, 417-452 (2015) – Zbl 1359.32020

[63] Diverio, S.; Trapani, S., A remark on the codimension of the Green-Griffiths locus of generic projective hypersurfaces of high degree, J. Reine Angew. Math., 649, 55-61 (2010) – Zbl 1211.32015

[64] I. Dolgachev, Weighted projective varieties, In: Proceedings of a Polish-North Amer. Sem. on Group Actions and Vector Fields, Vancouver, 1981, (ed. J.B. Carrels), Lecture Notes in Math., 956, Springer-Verlag, 1982, pp. 34-71.

[65] Duval, J., Une sextique hyperbolique dans $\mathbb{P}^3(\mathbb{C})$, Math. Ann., 330, 473-476 (2004) – Zbl 1071.14045

[66] Duval, J., Sur le lemme de Brody, Invent. Math., 173, 305-314 (2008) – Zbl 1155.32017

[67] Ein, L., Subvarieties of generic complete intersections, Invent. Math., 94, 163-169 (1988) – Zbl 0701.14002

[68] Ein, L., Subvarieties of generic complete intersections. II, Math. Ann., 289, 465-471 (1991) – Zbl 0746.14019

[69] El Goul, J., Algebraic families of smooth hyperbolic surfaces of low degree in $\mathbb{P}^3_{\mathbb{C}}$, Manuscripta Math., 90, 521-532 (1996) – Zbl 0887.32007

[70] J. El Goul, Propriétés de négativité de courbure des variétés algébriques hyperboliques, Thèse de Doctorat, Univ. de Grenoble I, 1997.

[71] Fujimoto, H., On holomorphic maps into a taut complex space, Nagoya Math. J., 46, 49-61 (1972) – Zbl 0214.09004

[72] Fujimoto, H., A family of hyperbolic hypersurfaces in the complex projective space, Complex Variables Theory Appl., 43, 273-283 (2001) – Zbl 1026.32052

[73] Fujita, T., Approximating Zariski decomposition of big line bundles, Kodai Math. J., 17, 1-3 (1994) – Zbl 0814.14006

[74] Gherardelli, G., Sul modello minimo della varietà degli elementi differenziali del 2° ordine del piano projettivo, Atti Accad. Italia. Rend. Cl. Sci. Fis. Mat. Nat., 2, 821-828 (1941) – Zbl 0026.15003

[75] Grauert, H., Jetmetriken und hyperbolische Geometrie, Math. Z., 200, 149-168 (1989) – Zbl 0664.32020

[76] Grauert, H.; Reckziegel, H., Hermitesche Metriken und normale Familien holomorpher Abbildungen, Math. Z., 89, 108-125 (1965) – Zbl 0135.12503

[77] M. Green and P.A. Griffiths, Two applications of algebraic geometry to entire holomorphic mappings, In: The Chern Symposium 1979, Proc. Internat. Sympos., Berkeley, CA, 1979, Springer-Verlag, 1980, pp. 41-74.

[78] Griffiths, Pa, Holomorphic mapping into canonical algebraic varieties, Ann. of Math., 98, 439-458 (1971) – Zbl 0214.48601

[79] R. Hartshorne, Algebraic Geometry, Grad. Texts in Math., 52, Springer-Verlag, 1977.

[80] Hironaka, H., Resolution of singularities of an algebraic variety over a field of characteristic zero, Ann. of Math., 79, 109-326 (1964) – Zbl 0122.38603

[81] Huynh, Dt, Examples of hyperbolic hypersurfaces of low degree in projective spaces, Int. Math. Res. Not. IMRN, 2016, 5518-5558 (2016) – Zbl 1404.32046

[82] Huynh, Dt, Construction of hyperbolic hypersurfaces of low degree in $\mathbb{P}^n(\mathbb{C})$, Internat. J. Math., 27, 1650059 (2016) – Zbl 1352.14037

[83] D.T. Huynh, D.-V. Vu and S.-Y. Xie, Entire holomorphic curves into projective spaces intersecting a generic hypersurface of high degree, preprint, arXiv:1704.03358. – Zbl 1432.32016

[84] Kawamata, Y., On Bloch's conjecture, Invent. Math., 57, 97-100 (1980) – Zbl 0569.32012

[85] Kobayashi, R., Holomorphic curves into algebraic subvarieties of an abelian variety, Internat. J. Math., 2, 711-724 (1991) – Zbl 0767.32015

[86] Kobayashi, S., Hyperbolic Manifolds and Holomorphic Mappings (1970), New York, NY: Marcel Dekker, New York, NY – Zbl 0207.37902

[87] Kobayashi, S., Negative vector bundles and complex Finsler structures, Nagoya Math. J., 57, 153-166 (1975) – Zbl 0326.32016

[88] Kobayashi, S., Intrinsic distances, measures and geometric function theory, Bull. Amer. Math. Soc., 82, 357-416 (1976) – Zbl 0346.32031

[89] Kobayashi, S., The first Chern class and holomorphic tensor fields, J. Math. Soc. Japan, 32, 325-329 (1980) – Zbl 0447.53055

[90] Kobayashi, S., Recent results in complex differential geometry, Jahresber. Deutsch. Math.-Verein., 83, 147-158 (1981) – Zbl 0467.53030

[91] S. Kobayashi, Hyperbolic Complex Spaces, Grundlehren Math. Wiss., 318, Springer-Verlag, 1998. – Zbl 0917.32019

[92] Kobayashi, S.; Ochiai, T., Mappings into compact manifolds with negative first Chern class, J. Math. Soc. Japan, 23, 137-148 (1971) – Zbl 0203.39101

[93] Kobayashi, S.; Ochiai, T., Meromorphic mappings onto compact complex spaces of general type, Invent. Math., 31, 7-16 (1975) – Zbl 0331.32020

[94] Laksov, D.; Thorup, A., These are the differentials of order n, Trans. Amer. Math. Soc., 351, 1293-1353 (1999) – Zbl 0920.13023

[95] Lang, S., Hyperbolic and Diophantine analysis, Bull. Amer. Math. Soc. (N.S.), 14, 159-205 (1986) – Zbl 0602.14019

[96] S. Lang, Introduction to Complex Hyperbolic Spaces, Springer-Verlag, 1987. – Zbl 0628.32001

[97] S.S.-Y. Lu, On hyperbolicity and the Green-Griffiths conjecture for surfaces, In: Geometric Complex Analysis, (eds. J. Noguchi et al.), World Sci. Publ. Co., 1996, pp. 401-408. – Zbl 0941.32024

[98] Lu, Ss-Y; Miyaoka, Y., Bounding curves in algebraic surfaces by genus and Chern numbers, Math. Res. Lett., 2, 663-676 (1995) – Zbl 0870.14020

[99] Lu, Ss-Y; Miyaoka, Y., Bounding codimension-one subvarieties and a general inequality between Chern numbers, Amer. J. Math., 119, 487-502 (1997) – Zbl 0890.14028

[100] Lu, Ssy; Winkelmann, J., Quasiprojective varieties admitting Zariski dense entire holomorphic curves, Forum Math., 24, 399-418 (2012) – Zbl 1273.32023

[101] Lu, Ss-Y; Yau, S-T, Holomorphic curves in surfaces of general type, Proc. Nat. Acad. Sci. U.S.A., 87, 80-82 (1990) – Zbl 0702.32015

[102] Masuda, K.; Noguchi, J., A construction of hyperbolic hypersurface of $\mathbb{P}^n(\mathbb{C})$, Math. Ann., 304, 339-362 (1996) – Zbl 0844.32018

[103] Mcquillan, M., A new proof of the Bloch conjecture, J. Algebraic Geom., 5, 107-117 (1996) – Zbl 0862.14027

[104] Mcquillan, M., Diophantine approximation and foliations, Inst. Hautes Études Sci. Publ. Math., 87, 121-174 (1998) – Zbl 1006.32020

[105] Mcquillan, M., Holomorphic curves on hyperplane sections of 3-folds, Geom. Funct. Anal., 9, 370-392 (1999) – Zbl 0951.14014

[106] Merker, J., Jets de Demailly-Semple d'ordres 4 et 5 en dimension 2, Int. J. Contemp. Math. Sci., 3, 861-933 (2008) – Zbl 1161.13002

[107] Merker, J., Low pole order frames on vertical jets of the universal hypersurface, Ann. Inst. Fourier (Grenoble), 59, 1077-1104 (2009) – Zbl 1172.32005

[108] Merker, J., Application of computational invariant theory to Kobayashi hyperbolicity and to Green-Griffiths algebraic degeneracy, J. Symbolic Comput., 45, 986-1074 (2010) – Zbl 1202.32020

[109] J. Merker, Complex projective hypersurfaces of general type: towards a conjecture of Green and Griffiths, preprint, arXiv:1005.0405; Algebraic differential equations for entire holomorphic curves in projective hypersurfaces of general type: optimal lower degree bound, In: Geometry and Analysis on Manifolds. In Memory of Professor Shoshichi Kobayashi, (eds. T. Ochiai, T. Mabuchi, Y. Maeda, J. Noguchi and A. Weinstein), Progr. Math., 308, Birkhäuser, 2015, pp. 41-142. – Zbl 1333.32033

[110] J. Merker and T.-A. Ta, Degrees $d \geq (\sqrt{n}\,\log n)^n$ and $d \geq (n \log n)^n$ in the conjectures of Green-Griffiths and of Kobayashi, preprint, arXiv:1901.04042.

[111] Meyer, P-A, Qu'est ce qu'une différentielle d'ordre n ?, Exposition. Math., 7, 249-264 (1989) – Zbl 0688.60041

[112] Y. Miyaoka, Algebraic surfaces with positive indices, In: Classification of Algebraic and Analytic Manifolds, Katata Symp. Proc., 1982, Progr. Math., 39, Birkhäuser, 1983, pp. 281-301.

[113] S. Mori and S. Mukai, The uniruledness of the moduli space of curves of genus 11, In: Algebraic Geometry, Tokyo-Kyoto, 1982, Lecture Notes in Math., 1016, Springer-Verlag, pp. 334-353.

[114] Nadel, Am, Hyperbolic surfaces in $\mathbb{P}^3$, Duke Math. J., 58, 749-771 (1989) – Zbl 0686.32015

[115] Noguchi, J., Holomorphic curves in algebraic varieties, Hiroshima Math. J., 7, 833-853 (1977) – Zbl 0412.32025

[116] Noguchi, J., Meromorphic mappings into a compact complex space, Hiroshima Math. J., 7, 411-425 (1977) – Zbl 0365.32017

[117] Noguchi, J., A higher-dimensional analogue of Mordell's conjecture over function fields, Math. Ann., 258, 207-212 (1981) – Zbl 0459.14002

[118] Noguchi, J., Lemma on logarithmic derivatives and holomorphic curves in algebraic varieties, Nagoya Math. J., 83, 213-233 (1981) – Zbl 0429.32003

[119] J. Noguchi, Logarithmic jet spaces and extensions of de Franchis' theorem, In: Contributions to Several Complex Variables, Aspects Math., E9, Friedr. Vieweg, Braunschweig, 1986, pp. 227-249.

[120] Noguchi, J., Hyperbolic manifolds and Diophantine geometry, Sugaku Expositions, 4, 63-81 (1991) – Zbl 0731.14016

[121] J. Noguchi, Chronicle of Bloch's conjecture, private communication.

[122] Noguchi, J., On holomorphic curves in semi-abelian varieties, Math. Z., 228, 713-721 (1998) – Zbl 0949.32011

[123] J. Noguchi and T. Ochiai, Geometric Function Theory in Several Complex Variables. Japanese ed., Iwanami, Tokyo, 1984; English translation, Transl. Math. Monogr., 80, Amer. Math. Soc., Providence, RI, 1990. – Zbl 0713.32001

[124] J. Noguchi and J. Winkelmann, Nevanlinna Theory in Several Complex Variables and Diophantine Approximation, Grundlehren Math. Wiss., 350, Springer-Verlag, 2014. – Zbl 1337.32004

[125] Noguchi, J.; Winkelmann, J.; Yamanoi, K., Degeneracy of holomorphic curves into algebraic varieties, J. Math. Pures Appl., 88, 293-306 (2007) – Zbl 1135.32018

[126] Noguchi, J.; Winkelmann, J.; Yamanoi, K., Degeneracy of holomorphic curves into algebraic varieties. II, Vietnam J. Math., 41, 519-525 (2013) – Zbl 1293.32022

[127] Ochiai, T., On holomorphic curves in algebraic varieties with ample irregularity, Invent. Math., 43, 83-96 (1977) – Zbl 0374.32006

[128] Pacienza, G., Subvarieties of general type on a general projective hypersurface, Trans. Amer. Math. Soc., 356, 2649-2661 (2004) – Zbl 1056.14060

[129] Pacienza, G.; Rousseau, E., On the logarithmic Kobayashi conjecture, J. Reine Angew. Math., 611, 221-235 (2007) – Zbl 1133.32015

[130] Păun, M., Vector fields on the total space of hypersurfaces in the projective space and hyperbolicity, Math. Ann., 340, 875-892 (2008) – Zbl 1137.32010

[131] Riedl E. and Yang D., Applications of a Grassmannian technique in hypersurfaces, preprint, arXiv:1806.02364.

[132] Rousseau, E., Étude des jets de Demailly-Semple en dimension 3, Ann. Inst. Fourier (Grenoble), 56, 397-421 (2006) – Zbl 1092.58003

[133] Royden, H.L., Remarks on the Kobayashi metric, In: Several Complex Variables. II, Proc. Maryland Conference, Lecture Notes in Math., 185, Springer-Verlag, 1971, pp. 125-137. – Zbl 0218.32012

[134] Royden, H.L., The extension of regular holomorphic maps, Proc. Amer. Math. Soc., 43, 306-310 (1974) – Zbl 0292.32019

[135] Seidenberg, A., Reduction of singularities of the differential equation Ady = Bdx, Amer. J. Math., 90, 248-269 (1968) – Zbl 0159.33303

[136] Semple, J.G., Some investigations in the geometry of curve and surface elements, Proc. London Math. Soc., 4, 24-49 (1954) – Zbl 0055.14505

[137] Shiffman, B.; Zaidenberg, M., Hyperbolic hypersurfaces in $\mathbb{P}^n$ of Fermat-Waring type, Proc. Amer. Math. Soc., 130, 2031-2035 (2002) – Zbl 0997.32020

[138] Shirosaki, M., On some hypersurfaces and holomorphic mappings, Kodai Math. J., 21, 29-34 (1998) – Zbl 0942.32018

[139] Siu, Y-T, Every Stein subvariety admits a Stein neighborhood, Invent. Math., 38, 89-100 (1976) – Zbl 0343.32014

[140] Siu, Y-T, Defect relations for holomorphic maps between spaces of different dimensions, Duke Math. J., 55, 213-251 (1987) – Zbl 0623.32018

[141] Siu, Y-T, An effective Matsusaka big theorem, Ann. Inst. Fourier (Grenoble), 43, 1387-1405 (1993) – Zbl 0803.32017

[142] Y.-T. Siu, A proof of the general Schwarz lemma using the logarithmic derivative lemma, personal communication, April, 1997.

[143] Siu, Y-T, Some recent transcendental techniques in algebraic and complex geometry, 439-448 (2002), Beijing: Higher Ed. Press, Beijing – Zbl 1028.32012

[144] Y.-T. Siu, Hyperbolicity in complex geometry, In: The Legacy of Niels Henrik Abel, Springer-Verlag, 2004, pp. 543-566. – Zbl 1076.32011

[145] Siu, Y-T, Hyperbolicity of generic high-degree hypersurfaces in complex projective spaces, Invent. Math., 202, 1069-1166 (2015) – Zbl 1333.32020

[146] Siu, Y-T; Yeung, S-K, Hyperbolicity of the complement of a generic smooth curve of high degree in the complex projective plane, Invent. Math., 124, 573-618 (1996) – Zbl 0856.32017

[147] Siu, Y-T; Yeung, S-K, A generalized Bloch's theorem and the hyperbolicity of the complement of an ample divisor in an abelian variety, Math. Ann., 306, 743-758 (1996) – Zbl 0882.32009

[148] Siu, Y-T; Yeung, S-K, Defects for ample divisors of abelian varieties, Schwarz lemma, and hyperbolic hypersurfaces of low degrees, Amer. J. Math., 119, 1139-1172 (1997) – Zbl 0947.32012

[149] Trapani, S., Numerical criteria for the positivity of the difference of ample divisors, Math. Z., 219, 387-401 (1995) – Zbl 0828.14002

[150] Tsuji, H., Stability of tangent bundles of minimal algebraic varieties, Topology, 27, 429-442 (1988) – Zbl 0698.14008

[151] Venturini, S., The Kobayashi metric on complex spaces, Math. Ann., 305, 25-44 (1996) – Zbl 0859.32011

[152] Voisin, C., On a conjecture of Clemens on rational curves on hypersurfaces, J. Differential Geom., 44, 200-213 (1996) – Zbl 0883.14022

[153] P. Vojta, Diophantine Approximations and Value Distribution Theory, Lecture Notes in Math., 1239, Springer-Verlag, 1987. – Zbl 0609.14011

[154] Winkelmann, J., On Brody and entire curves, Bull. Soc. Math. France, 135, 25-46 (2007) – Zbl 1159.32015

[155] Xie, S-Y, On the ampleness of the cotangent bundles of complete intersections, Invent. Math., 212, 941-996 (2018) – Zbl 1405.14119

[156] Xu, G., Subvarieties of general hypersurfaces in projective space, J. Differential Geom., 39, 139-172 (1994) – Zbl 0823.14030

[157] Zaidenberg, Mg, The complement of a general hypersurface of degree 2n in $\mathbb{CP}^n$ is not hyperbolic, Siberian Math. J., 28, 425-432 (1988)

[158] M.G. Zaidenberg, Hyperbolicity in projective spaces, In: International Symposium on Holomorphic Mappings, Diophantine Geometry and Related Topics, Kyoto, 1992, Sūrikaisekikenkyūsho Kōkyūroku, 819, Kyoto Univ., 1993, pp. 136-156.

Campana, Frédéric; Demailly, Jean-Pierre; Peternell, Thomas

The algebraic dimension of compact complex threefolds with vanishing second Betti number. (English) Zbl 1436.32071

Compos. Math. 156, No. 4, 679-696 (2020).

Summary: We study compact complex three-dimensional manifolds with vanishing second Betti number. In particular, we show that a compact complex manifold homeomorphic to the six-dimensional sphere does carry any non-constant meromorphic function.

MSC:
32J17 Compact complex 33-folds
32J10 Algebraic dependence theorems

Keywords: compact complex threefold; algebraic dimension; algebraic reduction

References:

[1] Barth, W. P., Hulek, K., Peters, C. A. M. and Van De Ven, A., Compact complex surfaces, , second edition (Springer, Berlin, 2004). – Zbl 1036.14016

[2] Campana, F., Demailly, J. P. and Peternell, Th., The algebraic dimension of compact complex threefolds with vanishing second Betti number, Comp. Math.112 (1998), 77-91. – Zbl 0910.32032

[3] Etesi, G., Complex structure on the six dimensional sphere from a spontaneous symmetry breaking, J. Math. Phys.56 (2015), 043508; Erratum ibid. 099901. – Zbl 1322.81079

[4] Gellhaus, Ch. and Heinzner, P., Komplexe Flächen mit holomorphen Vektorfeldern, Abh. Math. Sem. Univ. Hambg.60 (1990), 37-46. – Zbl 0734.32017

[5] Hartshorne, R., Algebraic geometry, (Springer, Berlin, 1977). – Zbl 0367.14001

[6] Inoue, M., On surfaces of type VII_0, Invent. Math.24 (1974), 269-310. – Zbl 0283.32019

[7] Kunz, E. and Waldi, R., Der Führer einer Gorensteinvarietät, J. Reine Angew. Math.388 (1988), 106-115. – Zbl 0666.14020

[8] Lehn, C., Rollenske, S. and Schinko, C., The complex geometry of a hypothetical complex structure on S^6, Differential Geom. Appl.57 (2018), 121-137. – Zbl 1381.53008

[9] Mori, S., Threefolds whose canonical bundles are not numerically effective, Ann. of Math. (2)116 (1982), 133-176. – Zbl 0557.14021

[10] Ueno, K., Classification theory of algebraic varieties and compact complex spaces, (Springer, Berlin, 1975). – Zbl 0299.14007

Demailly, Jean-Pierre; Rahmati, Mohammad Reza

Morse cohomology estimates for jet differential operators. (English) Zbl 1420.32011

Boll. Unione Mat. Ital. 12, No. 1-2, 145-164 (2019).

Summary: We provide detailed holomorphic Morse estimates for the cohomology of sheaves of jet differentials and their dual sheaves. These estimates apply on arbitrary directed varieties, and a special attention has been given to the analysis of the singular situation. As a consequence, we obtain existence results for global jet differentials and global differential operators under positivity conditions for the canonical or anticanonical sheaf of the directed structure.

MSC:
32H30 Value distribution theory in higher dimensions
32L10 Sheaves and cohomology of sections of holomorphic vector bundles, general results
14J17 Singularities of surfaces or higher-dimensional varieties
14J40 n-folds ($n > 4$)
53C55 Global differential geometry of Hermitian and Kähler geometry

Keywords: directed variety; jet bundle; jet differential; jet metric; holomorphic Morse inequalities; canonical sheaf

References:

[1] Bloch, A., Sur les systèmes de fonctions uniformes satisfaisant à l'équation d'une variété algébrique dont l'irrégularité dépasse la dimension, J. de Math., 5, 19-66, (1926) JFM 52.0373.04

[2] Bloch, A., Sur les systèmes de fonctions holomorphes à variétés linéaires lacunaires, Ann. Ecole Normale, 43, 309-362, (1926) · JFM 52.0326.01

[3] Bonavero, L., Inégalités de Morse holomorphes singulières, C. R. Acad. Sci. Paris Sér. I Math., 317, 1163-1166, (1993) · Zbl 0799.32023

[4] Demailly, J-P, Champs magnétiques et inégalités de Morse pour la d''-cohomologie, Ann. Inst. Fourier (Grenoble), 35, 189-229, (1985) · Zbl 0565.58017

[5] Demailly, J.-P.: Singular hermitian metrics on positive line bundles. In: Hulek, K., Peternell, T., Schneider, M., Schreyer, F. (eds.) Lecture Notes in Math. Proceedings of the Bayreuth conference "Complex algebraic varieties", April 2-6, 1990, n° 1507, pp. 87-104. Springer (1992) · Zbl 0784.32024

[6] Demailly, J.-P.: Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. In: Kollár, J., Lazarsfeld, R. (eds.) Algebraic Geometry-Santa Cruz 1995, Proceedings Symposia in Pure Math, vol 62, pp. 285-360. American Mathematical Society Providence, RI (1997)

[7] Demailly, J-P, Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture, Pure Appl. Math. Q., 7, 1165-1208, (2011) · Zbl 1316.32014

[8] Demailly, J-P, Hyperbolic algebraic varieties and holomorphic differential equations. Expanded version of the lectures given at the annual meeting of VIASM, Acta Math. Vietnam, 37, 441-512, (2012)

[9] Demailly, J.-P.: Towards the Green-Griffiths-Lang conjecture. In: Baklouti, A., El Kacimi, A., Kallel, S., Mir N. (eds) Conference "Analysis and Geometry", Tunis, March 2014, in honor of Mohammed Salah Baouendi, pp. 141-159. Springer (2015) · Zbl 1327.14048

[10] Demailly, J.-P.: Recent results on the Kobayashi and Green-Griffiths-Lang conjectures. Contribution to the 16th Takagi lectures in celebration of the 100th anniversary of K.Kodaira's birth, November 2015, to appear in the Japanese Journal of Mathematics. arXiv: Math.AG/1801.04765

[11] Green, M., Griffiths, P.: Two applications of algebraic geometry to entire holomorphic mappings. In: The Chern symposium, 1979, proc. internal. sympos. Berkeley, CA, 1979, pp. 41-74. Springer, New York, (1980)

[12] Lang, S., Hyperbolic and diophantine analysis, Bull. Am. Math. Soc., 14, 159-205, (1986) · Zbl 0602.14019

[13] Merker, J., Low pole order frames on vertical jets of the universal hypersurface, Ann. Inst. Fourier (Grenoble), 59, 1077-1104, (2009) · Zbl 1172.32005

[14] Păun, M., Vector fields on the total space of hypersurfaces in the projective space and hyperbolicity, Math. Ann., 340, 875-892, (2008) · Zbl 1137.32010

[15] Siu, Y.T.: Hyperbolicity in complex geometry. The legacy of Niels Henrik Abel, pp. 543-566. Springer, Berlin (2004)

Demailly, Jean-Pierre

Extension of holomorphic functions and cohomology classes from non reduced analytic subvarieties. (English) Zbl 1404.32036

Byun, Jisoo (ed.) et al., Geometric complex analysis. In honor of Kang-Tae Kim's 60th birthday, Gyeongju, Korea, 2017. Selected papers based on the presentations at the 11th and 12th Korean conferences on several complex variables, KSCV 11 symposium and KSCV 12 symposium, July 4–8, 2016 and July 3–7, 2017. Singapore:

Springer (ISBN 978-981-13-1671-5/hbk; 978-981-13-1672-2/ebook). Springer Proceedings in Mathematics & Statistics 246, 97-113 (2018).

Summary: The goal of this survey is to describe some recent results concerning the L^2 extension of holomorphic sections or cohomology classes with values in vector bundles satisfying weak semi-positivity properties. The results presented here are generalized versions of the Ohsawa-Takegoshi extension theorem, and borrow many techniques from the long series of papers by T. Ohsawa. The recent achievement that we want to point out is that the surjectivity property holds true for restriction morphisms to non necessarily reduced subvarieties, provided these are defined as zero varieties of multiplier ideal sheaves. The new idea involved to approach the existence problem is to make use of L^2 approximation in the Bochner-Kodaira technique. The extension results hold under curvature conditions that look pretty optimal. However, a major unsolved problem is to obtain natural (and hopefully best possible) L^2 estimates for the extension in the case of non reduced subvarieties – the case when Y has singularities or several irreducible components is also a substantial issue.

For the entire collection see [Zbl 1402.30001].

MSC:
32L10 Sheaves and cohomology of sections of holomorphic vector bundles, general results
32D15 Continuation of analytic objects in several complex variables
32E05 Holomorphically convex complex spaces, reduction theory

Keywords: compact Kähler manifold; singular Hermitian metric; coherent sheaf cohomology; Dolbeault cohomology; plurisubharmonic function; L^2 estimates; Ohsawa-Takegoshi extension theorem; multiplier ideal sheaf

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Demailly, Jean-Pierre

Fano manifolds with nef tangent bundles are weakly almost Kähler-Einstein. (English) Zbl 06890455

Asian J. Math. 22, No. 2, 285-290 (2018).

Summary: The goal of this short note is to point out that every Fano manifold with a nef tangent bundle possesses an almost Kähler-Einstein metric, in a weak sense. The technique relies on a regularization theorem for closed positive $(1,1)$-currents. We also discuss related semistability questions and Chern inequalities.

MSC:
14J45 Fano varieties
14M17 Homogeneous spaces and generalizations
32C30 Integration on analytic sets and spaces, currents
32Q10 Positive curvature manifolds
32Q20 Kähler-Einstein manifolds

Keywords: Fano manifold; numerically effective vector bundle; rational homogeneous manifold; Campana-Peternell conjecture; Kähler-Einstein metric; closed positive current; regularization of currents; Schauder fixed point theorem

Demailly, Jean-Pierre

Precise error estimate of the Brent-McMillan algorithm for Euler's constant. (English) Zbl 07321502

Mosc. J. Comb. Number Theory 7, No. 4, 3-38 (2017).

Summary: R. P. Brent and E. M. McMillan [Math. Comput. 34, 305–312 (1980; Zbl 0442.10002)] introduced in 1980 a new algorithm for the computation of Euler's constant γ, based on the use of the Bessel functions $I_0(x)$ and $K_0(x)$. It is the fastest known algorithm for the computation of γ. The time complexity can still be improved by evaluating a certain divergent asymptotic expansion up to its minimal term. Brent-McMillan conjectured in 1980 that the error is of the same magnitude as the last computed term, and R. P. Brent and F. Johansson [Math. Comput. 84, No. 295, 2351–2359 (2015; Zbl 1320.33007)] partially proved it in 2015. They also gave some numerical evidence for a more precise estimate of the error term. We find here an explicit expression of that optimal estimate, along with a complete self-contained formal proof and an even more precise error bound.

MSC:
33C10 Bessel and Airy functions, cylinder functions, ${}_0F_1$
11Y60 Evaluation of number-theoretic constants

Keywords: Euler's constant; Bessel functions; elliptic integral; integration by parts; asymptotic expansion; Euler-Maclaurin formula

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Demailly, Jean-Pierre; Gaussier, Hervé

Algebraic embeddings of smooth almost complex structures. (English) Zbl 06802929

J. Eur. Math. Soc. (JEMS) 19, No. 11, 3391-3419 (2017).

Summary: The goal of this work is to prove an embedding theorem for compact almost complex manifolds into complex algebraic varieties. It is shown that every almost complex structure can be realized by the transverse structure to an algebraic distribution on an affine algebraic variety, namely an algebraic subbundle of the tangent bundle. In fact, there even exist universal embedding spaces for this problem, and their dimensions grow quadratically with respect to the dimension of the almost complex manifold to embed. We give precise variation formulas for the induced almost complex structures and study the related versality conditions. At the end, we discuss the original question raised by F. Bogomolov: can one embed every compact complex manifold as a $\mathcal{C}^\infty$ smooth subvariety that is transverse to an algebraic foliation on a complex projective algebraic variety?

MSC:
32Q60 Almost complex manifolds
32Q40 Embedding theorems
32G05 Deformations of complex structures
53C12 Foliations (differential geometry)

Keywords: deformation of complex structures; almost complex manifolds; complex projective variety; Nijenhuis tensor; transverse embedding; Nash algebraic map

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Demailly, Jean-Pierre

Variational approach for complex Monge-Ampère equations and geometric applications. (English) Zbl 06784933

Séminaire Bourbaki. Volume 2015/2016. Exposés 1104–1119. Avec table par noms d'auteurs de 1948/49 à 2015/16. Paris: Société Mathématique de France (SMF) (ISBN 978-2-85629-855-8/pbk). Astérisque 390, 245-275, Exp. No. 1112 (2017). For the entire collection see [Zbl 1370.00002].

MSC:
32W20 Complex Monge-Ampère operators
32Q25 Calabi-Yau theory 53C55 Hermitian and Kählerian manifolds (global differential geometry)

Cao, JunYan; Demailly, Jean-Pierre; Matsumura, Shin-ichi

A general extension theorem for cohomology classes on non reduced analytic subspaces. (English) Zbl 1379.32017

Sci. China, Math. 60, No. 6, 949-962 (2017).

The authors generalize the Ohsawa-Takegoshi extension theorem with the goal to prove it with the weakest possible hypothesis. The main theorem goes as follows: Let E be a holomorphic line bundle over a holomorphically convex Kähler manifold X. Let h be a (possibly) singular Hermitian metric on E, ψ a quasi-plurisubharmonic function with neat analytic singularities on X. If there exists a continuous function $\delta > 0$ on X such that

$$\Theta_{E,h} + (1 + \alpha\delta) i\partial\bar{\partial}\psi \geq 0$$

in the sense of currents for all $\alpha \in [0, 1]$, then the morphism induced by the natural inclusion $\mathcal{I}(he^{-\psi}) \to \mathcal{I}(h)$, namely

$$H^q(X, K_X \otimes E \otimes \mathcal{I}(he^{-\psi})) \to H^q(X, K_X \otimes E \otimes \mathcal{I}(h)),$$

is injective for every $q \geq 0$. It is noted that most of the argument carries on to the case when X is only weakly pseudoconvex, yet at one place there is a problem and hence this case remains open. An alternative proof using an idea of the third author is presented.

Reviewer: Zywomir Dinew (Kraków)

MSC:
32L10 Sections of holomorphic vector bundles
32Q15 Kähler manifolds
32E05 Holomorphically convex complex spaces, reduction theory

Keywords: holomorphically convex Kähler manifold; Ohsawa-Takegoshi extension theorem; singular Hermitian metric; multiplier ideal sheaf

References:

[1] Dem · Zbl 0507.32021 · DOI: 10.24033/asens.1434

[2] Demailly J-P. Complex Analytic and Differential Geometry. https://www-fourier.ujf-grenoble.fr/ demailly/manu scripts/agbook.pdf, 2009

[3] Demailly J-P. Analytic Methods in Algebraic Geometry. Somerville: International Press, 2012; Beijing: Higher Education Press, 2012

[4] Demailly J-P. On the cohomology of pseudoeffective line bundles. In: Fornaess J, Irgens M, Wold E, eds. Complex Geometry and Dynamics. Abel Symposia, vol. 10. Cham: Springer, 2015, 51–99 · Zbl 1337.32030

[5] Demailly J-P. Extension of holomorphic functions defined on non reduced analytic subvarieties. In: The Legacy of Bernhard Riemann after One Hundred and Fifty Years. Advanced Lectures in Mathematics, vol. 35.1. ArXiv:1510.05230v1, 2015

[6] Demailly J-P, Peternell T, Schneider M. Pseudo-effective line bundles on compact Kähler manifolds. Internat J Math, 2001, 6: 689–741 · Zbl 1111.32302 · DOI: 10.1142/S0129167X01000861

[7] Do · Zbl 0532.58027 · DOI: 10.2307/2006983

[8] Folland G B, Kohn J J. The Neumann Problem for the Cauchy-Riemann Complex. Princeton: Princeton University Press, 1972; Tokyo: University of Tokyo Press, 1972 · Zbl 0247.35093

[9] Fujino O. A transcendental approach to Kollár's injectivity theorem II. J Reine Angew Math, 2013, 681: 149–174 · Zbl 1285.32009

[10] Fujino O, Matsumura S. Injectivity theorem for pseudo-effective line bundles and its applications. ArXiv:1605.02284v1, 2016

[11] Guan Q, Zhou X. A proof of Demailly's strong openness conjecture. Ann of Math, 2015, 182: 605–616 · Zbl 1329.32016 · DOI: 10.4007/annals.2015.182.2.5

[12] Hiêp P H. The weighted log canonical threshold. C R Math Acad Sci Paris, 2014, 352: 283–288 · Zbl 1296.32013 · DOI: 10.1016/j.crma.2014.02.010

[13] · Zbl 0581.32036 · DOI: 10.2977/prims/1195181609

[14] Lempert L. Modules of square integrable holomorphic germs. ArXiv:1404.0407v2, 2014

[15] Matsumura S. An injectivity theorem with multiplier ideal sheaves of singular metrics with transcendental singularities. J Algebraic Geom, in press, arXiv:1308.2033v4, 2013

[16] Matsumura S. An injectivity theorem with multiplier ideal sheaves for higher direct images under Kähler morphisms. ArXiv:1607.05554v1, 2016

[17] Ohsawa T. On a curvature condition that implies a cohomology injectivity theorem of Kollár-Skoda type. Publ Res Inst Math Sci, 2005, 41: 565–577 · Zbl 1103.32005 · DOI: 10.2977/prims/1145475223

[18] Ohsawa T, Takegoshi · Zbl 0625.32011 · DOI: 10.1007/BF01166457

Demailly, Jean-Pierre

Extension of holomorphic functions defined on non reduced analytic subvarieties. (English) Zbl 1360.14025

Ji, Lizhen (ed.) et al., The legacy of Bernhard Riemann after one hundred and fifty years. Volume I. Somerville, MA: International Press; Beijing: Higher Education Press (ISBN 978-1-57146-318-0/pbk; 978-1-57146-316-6/set). Advanced Lectures in Mathematics (ALM) 35, 1, 191-222 (2016).

The article generalizes results and methods of *T. Ohsawa* and *K. Takegoshi* theorem from [Math. Z. 195, 197–204 (1987; Zbl 0625.32011)] to the case when the subvariety Y is not necessarily reduced, by using a multiplier ideal sheaf and jumping numbers. For a holomorphic vector bundle E on a complex manifold X one can discuss the existence of global holomorphic extensions $F \in H^0(X, E)$ of a section $f \in H^0(Y, E|_Y)$ together with L^2 approximations. The article considers this problem when X is a weakly pseudoconvex Kähler manifold with Kähler metric ω and when the holomorphic vector bundle E is equipped with a (possibly singular) hermitian metric $h = e^{-\varphi}$. Let ψ denote a quasi-psh function on X with neat analytic singularities and with log canonical singularities along a analytic subvariety $Y = V(\mathcal{I}(\psi))$ (so that Y is reduced). If the Chern curvature tensor $\Theta_{E,h}$ has the property that $i\Theta_{E,h} + \alpha i\partial\overline{\partial} \otimes Id_E$ is Nakano semipositive for all $\alpha \in [1, 1+\delta]$ and some $\delta > 0$, then for every section $f \in H^0(Y^0, (K_X \otimes E)|_{Y^0})$ on $Y^0 = Y_{\mathrm{reg}}$ such that

$$\int_{Y_0} |f|^2_{\omega,h} dV_{Y^0,\omega}[\psi] < +\infty$$

there exists an extension $F \in H^0(X, K_X \otimes E)$ whose restriction to Y^0 is equal to f, such that

$$\int_X \gamma(\delta\psi)|F|^2_{\omega,h} e^{-\psi} dV_{X,\omega} < \frac{34}{\delta} \int_{Y_0} |f|^2_{\omega,h} dV_{Y^0,\omega}[\psi]$$

The remark states that if F is a $(n, 0)$-form then the product $|F|^2_{\omega,h} dV_{X,\omega}$ does not depend on ω. The author claims that the constant $\frac{34}{\delta}$ in the inequality is not optimal. The concept of the multiplier ideal sheaf used in the proof is parallel yet more general than the one presented by *D. Popovici* [Nagoya Math. J. 180, 1–34 (2005; Zbl 1116.32017)]. For the entire collection see [Zbl 1343.01006].

Reviewer: Malgorzata Marciniak (Flushing)

Cited in 1 Document

MSC:
14C30 Transcendental methods, Hodge theory, Hodge conjecture
14F05 Sheaves, derived categories of sheaves, etc.
32C35 Analytic sheaves and cohomology groups

Keywords: holomorphic function; plurisubharmonic function; multiplier ideal sheaf; L^2 extension theorem; Ohsawa-Takegoshi theorem; log canonical singularities; non reduced subvariety Kähler metric; multiplier ideal sheaf; jumping numbers

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Demailly, Jean-Pierre

Numerical analysis and differential equations. 4th edition. (Analyse numérique et équations différentielles.) (French) Zbl 1362.65001

Grenoble Sciences. Les Ulis: EDP Sciences (ISBN 978-2-7598-1926-3/pbk; 978-2-7598-2004-7/ebook). vii, 368 p. (2016).

Publisher's description: Cet ouvrage est la quatrième édition d'un livre devenu aujourd'hui un classique sur la théorie des équations différentielles ordinaires. Le cours théorique de base est accompagné d'un exposé détaillé des méthodes numériques qui permettent de résoudre ces équations en pratique.

De multiples techniques de l'analyse numérique sont présentées : interpolation polynomiale, intégration numérique, méthodes itératives pour la résolution d'équations. Suit un exposé rigoureux des résultats sur l'existence, l'unicité et la régularité des solutions des équations différentielles, avec étude détaillée des équations du premier et du second ordre, des équations et systèmes linéaires à coefficients constants. Enfin, sont décrites les méthodes numériques à un pas ou multi-pas, avec étude comparative de la stabilité et du coût en temps de calcul. De nombreux exemples concrets, des exercices et problèmes d'application en fin de chapitre facilitent l'apprentissage.

Plusieurs améliorations ont été apportées dans cette dernière version. De nouveaux problèmes ou exercices ont été introduits dans presque tous les chapitres. La principale nouveauté est que l'ouvrage est maintenant un pap-ebook : le site compagnon en accès libre propose au lecteur des compléments théoriques et pratiques, ainsi que la correction d'un grand nombre d'exercices.

Cet ouvrage accessible aux L3, M1 et M2 de mathématiques est très utilisé pour la préparation aux concours de l'enseignement. Il constitue un outil de référence pour les enseignants, chercheurs et scientifiques d'autres disciplines. For the previous edition see [Zbl 0869.65041]. See the review of the German edition in [Zbl 0869.65042].

MSC:
65-01 Textbooks (numerical analysis) 34-01 Textbooks (ordinary differential equations) 65L05 Initial value problems for ODE (numerical methods) 65L06 Multistep, Runge-Kutta, and extrapolation methods 65D32 Quadrature and cubature formulas (numerical methods) 65H10 Systems of nonlinear equations (numerical methods)

Boman, Jan (ed.); Sigurdsson, Ragnar (ed.); Lerner, Nicolas; Demailly, Jean-Pierre; Atiyah, Michael; Treves, François; Helgason, Sigurdur; Grubb, Gerd; Bony, Jean-Michel; Kiselman, Christer O.; Broström, Sofia

To the memory of Lars Hörmander (1931–2012). (English) Zbl 1338.35005

Notices Am. Math. Soc. 62, No. 8, 890-907 (2015).

MSC:
35-03 Historical (partial differential equations) 46-03 Historical (functional analysis) 47-03 Historical (operator theory) 01A70 Biographies, obituaries, personalia, bibliographies

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Demailly, Jean-Pierre

On the cohomology of pseudoeffective line bundles. (English) Zbl 1337.32030

Fornæss, John Erik (ed.) et al., Complex geometry and dynamics. The Abel symposium 2013, Trondheim, Norway, July 2–5, 2013. Cham: Springer (ISBN 978-3-319-20336-2/hbk; 978-3-319-20337-9/ebook). Abel Symposia 10, 51-99 (2015).

Summary: The goal of this survey is to present various results concerning the cohomology of pseudoeffective line bundles on compact Kähler manifolds, and related properties of their multiplier ideal sheaves. In case the curvature is strictly positive, the prototype is the well known Nadel vanishing theorem, which is itself a generalized analytic version of the fundamental Kawamata-Viehweg vanishing theorem of algebraic geometry. We are interested here in the case where the curvature is merely semipositive in the sense of currents, and the base manifold is not necessarily projective. In this situation, one can still obtain interesting information on cohomology, e.g. a Hard Lefschetz theorem with pseudoeffective coefficients, in the form of a surjectivity statement for the Lefschetz map. More recently, Junyan Cao, in his PhD thesis defended in Grenoble, obtained a general Kähler vanishing theorem that depends on the concept of numerical dimension of a given pseudoeffective line bundle. The proof of these results depends in a crucial way on a general approximation result for closed $(1,1)$-currents, based on the use of Bergman kernels, and the related intersection theory of currents. Another important ingredient is the recent proof by Guan and Zhou of the strong openness conjecture. As an application, we discuss a structure theorem for compact Kähler threefolds without nontrivial subvarieties, following a joint work with F. Campana and M. Verbitsky. We hope that these notes will serve as a useful guide to the more detailed and more technical papers in the literature; in some cases, we provide here substantially simplified proofs and unifying viewpoints. For the entire collection see [Zbl 1336.32001].

Cited in 1 Review

Cited in 6 Documents

MSC:
32J27 Compact Kähler manifolds: generalizations, classification

Keywords: pseudoeffective line bundles on Kähler manifolds; multiplier ideal sheaves

References:

[1]
[2] · Zbl 1213.32025 · DOI: 10.1007/s11511-010-0054-7
[3]
[4] · Zbl 1146.32017 · DOI: 10.2977/prims/1210167334
[5]
[6] · Zbl 0685.32007 · DOI: 10.1090/S0894-0347-1989-1001853-2
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[10] · Zbl 0315.31007 · DOI: 10.1007/BF01418826
[11] · Zbl 0445.32021 · DOI: 10.1007/BF01420277
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[21] · Zbl 0777.32016
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[25] · Zbl 1067.14013
[26] · Zbl 1189.14044 · DOI: 10.1215/00127094-2010-008
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[29] · Zbl 1298.14006 · DOI: 10.1007/s11511-014-0107-4
[30] · Zbl 0827.14027
[31] · Zbl 1111.32302 · DOI: 10.1142/S0129167X01000861
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[43] · Zbl 1314.32047 · DOI: 10.1017/S1474748013000091

[44] · Zbl 1295.32042 · DOI: 10.1016/j.crma.2013.10.024

[45] · Zbl 0827.32016

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[49] · Zbl 0865.32019

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[51] · Zbl 0625.32011 · DOI: 10.1007/BF01166457

[52] · Zbl 0606.32018 · DOI: 10.1007/BF01459143

[53] · Zbl 0945.14020

[54] · Zbl 1296.32013 · DOI: 10.1016/j.crma.2014.02.010

[55]

[56] · Zbl 0153.15401 · DOI: 10.1007/BF02063212

[57] · Zbl 0289.32003 · DOI: 10.1007/BF01389965

[58] · Zbl 0577.32031

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[61] · Zbl 0246.32009

[62] · Zbl 0895.32008

[63]

[64] · Zbl 0369.53059 · DOI: 10.1002/cpa.3160310304

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.

Demailly, Jean-Pierre (ed.); van der Geer, Gerard (ed.); Hacon, Christopher (ed.); Kawamata, Yujiro (ed.); Kobayashi, Toshiyuki (ed.); Miyaoka, Yoichi (ed.); Schmid, Wilfried (ed.)

Foreword. (English) Zbl 1332.00108

J. Math. Sci., Tokyo 22, No. 1, iii-iv (2015).

From the text: Professor Kunihiko Kodaira is one of the greatest mathematicians of the twentieth century, and this issue is dedicated to him to commemorate his 100th birthday. The authors of the articles included in this issue belong to various generations.

MSC:
00B30 Festschriften
14-06 Proceedings of conferences (algebraic geometry)
32-06 Proceedings of conferences (several complex variables)

Demailly, Jean-Pierre

Towards the Green-Griffiths-Lang conjecture. (English) Zbl 1327.14048

Baklouti, Ali (ed.) et al., Analysis and geometry. MIMS-GGTM, Tunis, Tunisia, March 24–27, 2014. Proceedings of the international conference. In honour of Mohammed Salah Baouendi. Cham: Springer (ISBN 978-3-319-17442-6/hbk; 978-3-319-17443-3/ebook). Springer Proceedings in Mathematics & Statistics 127, 141-159 (2015).

Summary: The Green-Griffiths-Lang conjecture stipulates that for every projective variety X of general type over $\mathbb{C}$, there exists a proper algebraic subvariety of X containing all non constant entire curves $f : \mathbb{C} \to \mathbb{X}$. Using the formalism of directed varieties, we prove here that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet semistability property of the tangent bundle T_X. We then give a sufficient criterion for the Kobayashi hyperbolicity of an arbitrary directed variety (X, V). For the entire collection see [Zbl 1320.00044].

MSC:
14C30 Transcendental methods, Hodge theory, Hodge conjecture
32J25 Transcendental methods of algebraic geometry
14C20 Divisors, linear systems, invertible sheaves

Keywords: projective algebraic variety; variety of general type; entire curve; jet bundle; semple tower; Green-Griffiths-Lang conjecture; holomorphic morse inequality; semi-stable vector bundle; Kobayashi hyperbolic

References:

[1] J.-P. Demailly, Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. AMS Summer School on Algebraic Geometry, Santa Cruz 1995, in Proceedings Symposia in Pure Mathematics, ed. by J. Kollár, R. Lazarsfeld, Am. Math. Soc. Providence, RI, 285-360 (1997)

[2] J.-P. Demailly, Variétés hyperboliques et équations différentielles algébriques. Gaz. Math. 73, 3-23 (juillet 1997).

[3] J.-P. Demailly, J. El Goul, Hyperbolicity of generic surfaces of high degree in projective 3-space. Am. J. Math. 122, 515-546 (2000) · Zbl 0966.32014 · DOI: 10.1353/ajm.2000.0019

[4] J.-P. Demailly, Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture. Pure App. Math. Q. 7, 1165-1208 (2011). November 2010, arxiv:math.AG/ 1011.3636, dedicated to the memory of Eckart Viehweg · Zbl 1316.32014

[5] S. Diverio, J. Merker, E. Rousseau, Effective algebraic degeneracy. Invent. Math. 180, 161-223 (2010) · Zbl 1192.32014 · DOI: 10.1007/s00222-010-0232-4

[6] S. Diverio, E. Rousseau, The exceptional set and the Green-Griffiths locus do not always coincide. arxiv:math.AG/1302.4756 (v2) · Zbl 1359.32020

[7] M. Green, P. Griffiths, Two applications of algebraic geometry to entire holomorphic mappings, in The Chern Symposium, Proceedings of the International Symposium Berkeley, CA, 1979 (Springer, New York, 1980), pp. 41-74

[8] S. Kobayashi, Hyperbolic Manifolds and Holomorphic Mappings, Pure and Applied Mathematics, vol. 2 (Marcel Dekker Inc., New York, 1970)

[9] S. Kobayashi, Hyperbolic complex spaces, Grundlehren der Mathematischen Wissenschaften, vol. 318 (Springer, Berlin, 1998)

[10] S. Lang, Hyperbolic and Diophantine analysis. Bull. Am. Math. Soc. 14, 159-205 (1986) · Zbl 0602.14019 · DOI: 10.1090/S0273-0979-1986-15426-1

[11] M. McQuillan, Diophantine approximation and foliations. Inst. Hautes Études Sci. Publ. Math. 87, 121-174 (1998) · Zbl 1006.32020 · DOI: 10.1007/BF02698862

[12] M. McQuillan, Holomorphic curves on hyperplane sections of · Zbl 0951.14014 · DOI: 10.1007/s000390050091

[13] M. Păun, Vector fields on the total space of hypersurfaces in the projective space and hyperbolicity. Math. Ann. 340, 875-892 (2008) · Zbl 1137.32010 · DOI: 10.1007/s00208-007-0172-5

[14] Y.T. Siu, Some recent transcendental techniques in algebraic and complex geometry, in Proceedings of the International Congress of Mathematicians, Vol. I, (Higher Ed. Press, Beijing, 2002), pp. 439-448 · Zbl 1028.32012

[15] Y.T. Siu, Hyperbolicity in Complex Geometry, The legacy of Niels Henrik Abel (Springer, Berlin, 2004), pp. 543-566 · Zbl 1076.32011

[16] Y.T. Siu, S.K. Yeung, Hyperbolicity of the complement of a generic smooth curve of high degree in the complex projective plane. Invent. Math. 124, 573-618 (1996) · Zbl 0856.32017 · DOI: 10.1007/s002220050064

Demailly, Jean-Pierre

Structure theorems for compact Kähler manifolds with nef anticanonical bundles. (English) Zbl 1326.32007

Bracci, Filippo (ed.) et al., Complex analysis and geometry. KSCV 10. Proceddings of the 10th symposium, Gyeongju, Korea, August 7–11, 2014. Tokyo: Springer (ISBN 978-4-431-55743-2/hbk; 978-4-431-55744-9/ebook). Springer Proceedings in Mathematics & Statistics 144, 119-133 (2015).

Summary: This survey presents various results concerning the geometry of compact Kähler manifolds with numerically effective first Chern class: structure of the Albanese morphism of such manifolds, relations tying semipositivity of the Ricci curvature with rational connectedness, positivity properties of the Harder-Narasimhan filtration of the tangent bundle. For the entire collection see [Zbl 1328.32001].

Cited in 2 Documents

MSC:
32-02 Research monographs (several complex variables)
32J27 Compact Kähler manifolds: generalizations, classification
32L05 Holomorphic fiber bundles and generalizations

Keywords: compact Kähler manifold; anticanonical bundle; semipositive Ricci curvature; Ricci flat manifold; rationally connected variety; holonomy principle

References:

[1] Boucksom, S., Demailly, J.-P., Paun, M., Peternell, T.: The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension. · Zbl 1267.32017

[2] Demailly, J.-P., Peternell, T., Schneider, M.: Pseudo-effective line bundles on compact Kähler manifolds. Internat. J. Math. 12, 689-741 (2001) · Zbl 1111.32302 · DOI: 10.1142/S0129167X01000861

[3] Graber, T., Harris, J., Starr, J.: Families of rationally connected varieties. J. Amer. Math. Soc. 16, 57-67 (2003) · Zbl 1092.14063 · DOI: 10.1090/S0894-0347-02-00402-2

[4] Păun, M.: On the Albanese map of compact Kähler manifolds with numerically effective Ricci curvature. Comm. Anal. Geom. 9, 35-60 (2001) · Zbl 0980.53091

[5] Campana, F., Peternell, Th., Zhang, Q.: On the Albanese maps of compact Kähler manifolds. Proc. Amer. Math. Soc. 131, 549-553 (2003) · Zbl 1017.32018

[6] Zhang, Q.: On projective varieties with nef anticanonical divisors. Math. Ann. 332, 697-703 (2005) · Zbl 1083.14010 · DOI: 10.1007/s00208-005-0649-z

[7] Peternell, Th: Kodaira dimension of subvarieties II. Intl. J. Math. 17, 619-631 (2006) · Zbl 1126.14019 · DOI: 10.1142/S0129167X0600362X

[8] Brunella, M.: On Kähler surfaces with semipositive Ricci curvature. Riv. Math. Univ. Parma (N.S.), 1, 441-450 (2010) · Zbl 1226.32011

[9] Păun, M.: Relative adjoint transcendental classes and the Albanese map of compact Kähler manifolds with nef Ricci classes.

[10] Cao, J.: A remark on compact Kähler manifolds with nef anticanonical bundles and its applications.

[11] Cao, J.: Vanishing theorems and structure theorems on compact Kähler manifolds. PhD thesis, Université de Grenoble, defended at institut Fourier on 18 Sep 2013.

[12] Cao, J., Höring, A.: Manifolds with nef anticanonical bundle. · Zbl 06695049

[13] de Rham, G.: Sur la reductibilité d'un espace de Riemann. Comment. Math. Helv. 26, 328-344 (1952) · Zbl 0048.15701 · DOI: 10.1007/BF02564308

[14] Bochner, S., Yano, K.: Curvature and Betti Numbers. Annals of Mathematics Studies, No. 32, pp. ix · Zbl 0051.39402

[15] Kodaira, K.: On Kähler varieties of restricted type. Ann. of Math. 60, 28-48 (1954) · Zbl 0057.14102 · DOI: 10.2307/1969701

[16] Berger, M.: Sur les groupes d'holonomie des variétés à connexion affine des variétés riemanniennes. Bull. Soc. Math. Fr. 83, 279-330 (1955) · Zbl 0068.36002

[17] Bishop, R.: A relation between volume, mean curvature and diameter. Amer. Math. Soc. Not. 10, 364 (1963)

[18] Lichnerowicz, A.: Variétés kähleriennes et première classe de Chern. J. Diff. Geom. 1, 195-224 (1967) · Zbl 0167.20004

[19] Lichnerowicz, A.: Variétés Kählériennes à première classe de Chern non négative et variétés riemanniennes à courbure de Ricci généralisée non négative. J. Diff. Geom. 6, 47-94 (1971) · Zbl 0231.53063

[20] Cheeger, J., Gromoll, D.: The splitting theorem for manifolds of nonnegative Ricci curvature. J. Diff. Geom. 6, 119-128 (1971) · Zbl 0223.53033

[21] Cheeger, J., Gromoll, D.: On the structure of complete manifolds of nonnegative curvature. Ann. Math. 96, 413-443 (1972) · Zbl 0246.53049 · DOI: 10.2307/1970819

[22] Bogomolov, F.A.: On the decomposition of Kähler manifolds with trivial canonical class. Math. USSR Sbornik 22, 580-583 (1974) · Zbl 0304.32016 · DOI: 10.1070/SM1974v022n04ABEH001706

[23] Bogomolov, F.A.: Kähler manifolds with trivial canonical class. Izvestija Akad. Nauk 38, 11-21 (1974) · Zbl 0292.32020

[24] Arnol'd, V.I.: Bifurcations of invariant manifolds of differential equations, and normal forms of neighborhoods of elliptic curves. Funct. Anal. Appl. 10, 249-259 (1976). English translation 1977

[25] Aubin, T.: Equations du type Monge-Ampère sur les variétés kähleriennes compactes. C. R. Acad. Sci. Paris Ser. A 283, 119-121 (1976); Bull. Sci. Math. 102, 63-95 (1978) · Zbl 0333.53040

[26] Gauduchon, P.: Le théorème de l'excentricité nulle. C. R. Acad. Sci. Paris 285, 387-390 (1977) · Zbl 0362.53024

[27] Yau, S.T.: On the Ricci curvature of a complex Kähler manifold and the complex Monge-Ampère equation I. Comm. Pure Appl. Math. 31, 339-411 (1978) · Zbl 0369.53059 · DOI: 10.1002/cpa.3160310304

[28] Gromov, M.: Structures métriques pour les variétés riemanniennes. Cours rédigé par J. Lafontaine et P. Pansu, Textes Mathématiques, 1, vol. VII, p. 152. Paris, Cedic/Fernand Nathan (1981)

[29] Gromov, M.: Groups of polynomial growth and expanding maps, Appendix by J. Tits. Publ. I.H.E.S. 53, 53-78 (1981) · Zbl 0474.20018

[30] Kobayashi, S.: Recent results in complex differential geometry. Jber. dt. Math.-Verein. 83, 147-158 (1981) · Zbl 0467.53030

[31] Ueda, T.: On the neighborhood of a compact complex curve with topologically trivial normal bundle. J. Math. Kyoto Univ. 22, 583-607 (1982/83) · Zbl 0519.32019

[32] Kobayashi, S.: Topics in complex differential geometry. In: DMV Seminar, vol. 3. Birkhäuser (1983) · Zbl 0506.53029

[33] Beauville, A.: Variétés kähleriennes dont la première classe de Chern est nulle. J. Diff. Geom. 18, 775-782 (1983) · Zbl 0537.53056

[34] Kollár, J., Miyaoka, Y., Mori, S.: Rationally connected varieties. J. Alg. 1, 429-448 (1992) · Zbl 0780.14026

[35] Campana, F.: Connexité rationnelle des variétés de Fano. Ann. Sci. Ec. Norm. Sup. 25, 539-545 (1992) · Zbl 0783.14022

[36] Demailly, J.-P., Peternell, T., Schneider, M.: Kähler manifolds with numerically effective Ricci class. Compositio Math. 89, 217-240 (1993) · Zbl 0884.32023

[37] Bando, S., Siu, Y.-T.: Stable sheaves and Einstein-Hermitian metrics. In: Mabuchi, T., Noguchi, J., Ochiai, T. (eds.) Geometry and Analysis on Complex Manifolds, pp. 39-50. World Scientific, River Edge (1994) · Zbl 0880.32004

[38] Demailly, J.-P., Peternell, T., Schneider, M.: Compact complex manifolds with numerically effective tangent bundles. J. Alg. Geom. 3, 295-345 (1994) · Zbl 0827.14027

[39] Campana, F.: Fundamental group and positivity of cotangent bundles of compact Kähler manifolds. J. Alg. Geom. 4, 487-502 (1995) · Zbl 0845.32027

[40] Demailly, J.-P., Peternell, T., Schneider, M.: Compact Kähler manifolds with hermitian semipositive anticanonical bundle. Compositio Math. 101, 217-224 (1996) · Zbl 1008.32008

[41] Kollár, J.: Rational Curves on Algebraic Varieties. Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, Band 32, Springer (1996) · Zbl 0877.14012

[42] Zhang, Q.: On projective manifolds with nef anticanonical bundles. J. Reine Angew. Math. 478, 57-60 (1996) · Zbl 0855.14007

[43] Cheeger, J., Colding, T.H.: Lower bounds on Ricci curvature and almost rigidity of warped products. Ann. Math. 144, 189-237 (1996) · Zbl 0865.53037 · DOI: 10.2307/2118589

[44] Păun, M.: Sur le groupe fondamental des variétés kählériennes compactes à classe de Ricci numériquement effective. C. R. Acad. Sci. Paris Sér. I Math. 324, 1249-254 (1997)

[45] Cheeger J., Colding T.H.: On the structure of spaces with Ricci curvature bounded below. J. Differ. Geom., part I: 46, 406-480 (1997), part II: 54, 13-35 (2000), part III: 54, 37-74 (2000) · Zbl 0902.53034

[46] Păun, M.: Sur les variétés kählériennes compactes classe de Ricci numériquement effective. Bull. Sci. Math. 122, 83-92 (1998) · Zbl 0946.53037 · DOI: 10.1016/S0007-4497(98)80078-X

[47] Peternell, Th, Serrano, F.: Threefolds with anti canonical bundles. Coll. Math. 49, 465-517 (1998) · Zbl 0980.14028

Skoda, Henri (ed.); Demailly, Jean-Pierre; Siu, Yum-Tong

In memory of Pierre Lelong. (English) Zbl 1338.01059

Notices Am. Math. Soc. 61, No. 6, 586-595 (2014).

MSC:
01A70 Biographies, obituaries, personalia, bibliographies
32-03 Historical (several complex variables and analytic spaces)

Huckleberry, Alan (ed.); Peternell, Thomas (ed.); Siu, Yum-Tong; Ohsawa, Takeo; Demailly, Jean-Pierre; Barlet, Daniel; Trautmann, Günther; Lieb, Ingo

A tribute to Hans Grauert. (English) Zbl 1338.01041

Notices Am. Math. Soc. 61, No. 5, 472-483 (2014).

MSC:
01A70 Biographies, obituaries, personalia, bibliographies

Campana, F.; Demailly, J.-P.; Peternell, T.

Rationally connected manifolds and semipositivity of the Ricci curvature. (English) Zbl 1369.53052

Hacon, Christopher D. (ed.) et al., Recent advances in algebraic geometry. A volume in honor of Rob Lazarsfeld's 60th birthday. Based on the conference, Ann Arbor, MI, USA, May 16–19, 2013. Cambridge: Cambridge University Press (ISBN 978-1-107-64755-8/pbk; 978-1-107-41600-0/ebook). London Mathematical Society Lecture Note Series 417, 71-91 (2014).

Summary: This paper establishes a structure theorem for compact Kähler manifolds with semipositive anticanonical bundle. Up to finite étale cover, it is proved that such manifolds split holomorphically and isometrically as a product of Ricci flat varieties and of rationally connected manifolds. The proof is based on a characterization of rationally connected manifolds through the nonexistence of certain twisted contravariant tensor products of the tangent bundle, along with a generalized holonomy principle for pseudoeffective line bundles. A crucial ingredient for this is the characterization of uniruledness by the property that the anticanonical bundle is not pseudoeffective. For the entire collection see [Zbl 1318.14002].

Cited in 1 Review

Cited in 5 Documents

MSC:
53C55 Hermitian and Kählerian manifolds (global differential geometry)
14M22 Rationally connected varieties

Demailly, Jean-Pierre; Phạm, Hoàng Hiệp

A sharp lower bound for the log canonical threshold. (English) Zbl 1298.14006

Acta Math. 212, No. 1, 1-9 (2014).

The log canonical threshold of a plurisubharmonic function φ with an isolated singularity at 0 in an open subset of $\mathbb{C}^n$ is the the supremum of $c > 0$ such that $\exp(-2c\varphi)$ is integrable on a neighborhood of the origin. The main result is the sharp lower bound $c(\phi) \geq \sum_{j=0}^{n-1} e_j(\varphi)/e_{j+1}(\varphi)$, where the intersection numbers $e_j(\varphi)$ are the Lelong numbers of $(dd^c\varphi)^j$ at 0.

Reviewer: Jan Stevens (Göteborg)

Cited in 2 Reviews

Cited in 6 Documents

MSC:
14B05 Singularities (algebraic geometry)
32U05 Plurisubharmonic functions and generalizations
32U25 Lelong numbers

14C20 Divisors, linear systems, invertible sheaves

Keywords: log canonical threshold; plurisubharmonic functions; Lelong number

References:

[1] Bedford E., Taylor B. A.: The Dirichlet problem for a complex Monge–Ampère equation. Invent. Math. 37, 1–44 (1976) · Zbl 0315.31007 · DOI: 10.1007/BF01418826

[2] Bedford E, Taylor B.A: A new capacity for plurisubharmonic functions. Acta Math. 149, 1–40 (1982) · Zbl 0547.32012 · DOI: 10.1007/BF02392348

[3] Cegrell U.: The general definition of the complex Monge–Ampère operator. Ann. Inst. Fourier (Grenoble) 54, 159–179 (2004) · Zbl 1065.32020 · DOI: 10.5802/aif.2014

[4] Chel'tsov, I. A., Birationally rigid Fano varieties. Uspekhi Mat. Nauk, 60:5 (2005), 71–160 (Russian); English translation in Russian Math. Surveys, 60 (2005), 875– 965.

[5] Corti A.: Factoring birational maps of threefolds after Sarkisov. J. Algebraic Geom. 4, 223–254 (1995) · Zbl 0866.14007

[6] Corti, A., Singularities of linear systems and 3-fold birational geometry, in Explicit Birational Geometry of 3-folds, London Math. Soc. Lecture Note Ser., 281, pp. 259–312. Cambridge Univ. Press, Cambridge, 2000. · Zbl 0960.14017

[7] Demailly J.-P.: Nombres de Lelong généralisés, théorèmes d'intégralité et d'analyticité. Acta Math. 159, 153–169 (1987) · Zbl 0629.32011 · DOI: 10.1007/BF02392558

[8] Demailly J.-P.: Regularization of closed positive currents and intersection theory. J. Algebraic Geom. 1, 361–409 (1992) · Zbl 0777.32016

[9] Demailly, J.-P., Monge–Ampère operators, Lelong numbers and intersection theory, in Complex Analysis and Geometry, Univ. Ser. Math., pp. 115–193. Plenum, New York, 1993. · Zbl 0792.32006

[10] Demailly, J.-P., Estimates on Monge–Ampère operators derived from a local algebra inequality, in Complex Analysis and Digital Geometry, Acta Univ. Upsaliensis Skr. Uppsala Univ. C Organ. Hist., 86, pp. 131–143. Uppsala Universitet, Uppsala, 2009. · Zbl 1209.32024

[11] Demailly J.-P., Kollár J.: Semi-continuity of complex singularity exponents and Kähler–Einstein metrics on Fano orbifolds. Ann. Sci. École Norm. Sup, 34, 525–556 (2001) · Zbl 0994.32021 · DOI: 10.1016/S0012-9593(01)01069-2

[12] Eisenbud, D., Commutative Algebra. Graduate Texts in Mathematics, 150. Springer, New York, 1995. · Zbl 0819.13001

[13] de Fernex T., Ein L., Mustaţă M.: Bounds for log canonical thresholds with applications to birational rigidity. Math. Res. Lett, 10, 219–236 (2003) · Zbl 1067.14013 · DOI: 10.4310/MRL.2003.v10.n2.a9

[14] de Fernex T., Ein L., Mustaţă M.: Multiplicities and log canonical threshold. J. Algebraic Geom. 13, 603–615 (2004) · Zbl 1068.14006 · DOI: 10.1090/S1056-3911-04-00346-7

[15] Howald J.A.: Multiplier ideals of monomial ideals. Trans. Amer. Math. Soc, 353, 2665–2671 (2001) · Zbl 0979.13026 · DOI: 10.1090/S0002-9947-01-02720-9

[16] Iskovskikh, V.A., Birational rigidity of Fano hypersurfaces in the framework of Mori theory. Uspekhi Mat. Nauk, 56:2 (2001), 3–86 (Russian); English translation in Russian Math. Surveys, 56 (2001), 207–291. · Zbl 0991.14010

[17] Iskovskikh, V. A. & Manin, J. I., Three-dimensional quartics and counterexamples to the L"uroth problem. Mat. Sb., 86(128) (1971), 140–166 (Russian); English translation in Math. USSR–Sb., 15 (1971), 141–166. · Zbl 0222.14009

[18] Kiselman, C.O., Un nombre de Lelong raffiné, in Séminaire d'Analyse Complexe et Géométrie 1985–87, pp. 61–70. Faculté des Sciences de Tunis & Faculté des Sciences et Techniques de Monastir, Monastir, 1987.

[19] Kiselman C.O.: Attenuating the singularities of plurisubharmonic functions. Ann. Polon. Math, 60, 173–197 (1994) · Zbl 0827.32016

[20] Ohsawa T., Takegoshi K.: On the extension of L^2 holomorphic functions. Math. Z, 195, 197–204 (1987) · Zbl 0625.32011 · DOI: 10.1007/BF01166457

[21] Pukhlikov A. V.: Birational isomorphisms of four-dimensional quintics. Invent. Math. 87, 303–329 (1987) · Zbl 0613.14011 · DOI: 10.1007/BF01389417

[22] Pukhlikov, A. V., Birationally rigid Fano hypersurfaces. Iz*v*. Ross. Akad. Nauk Ser. Mat., 66:6 (2002), 159–186 (Russian); English translation in Iz*v*. Math., 66 (2002), 1243–1269. · Zbl 1083.14012

[23] Skoda H.: Sous-ensembles analytiques d'ordre fini ou infini dans $\mathbb{C}^n$. Bull. Soc. Math. France, 100, 353–408 (1972) · Zbl 0246.32009

**Demailly, Jean-Pierre; Dinew, Sławomir; Guedj, Vincent;
Hiep, Pham Hoang; Kołodziej, Sławomir; Zeriahi, Ahmed**

Hölder continuous solutions to Monge-Ampère equations. (English) Zbl 1296.32012

J. Eur. Math. Soc. (JEMS) 16, No. 4, 619-647 (2014).

Let (X, ω) be a compact Kähler manifold of dimension n. The authors study the following complex Monge-Ampère operator

$$MA \qquad \mathrm{MA(u)} := \frac{1}{\mathrm{V}_\omega}(\omega + \mathrm{dd^c u})^{\mathrm{n}}, \quad \text{where} \quad \mathrm{V}_\omega = \int_{\mathrm{X}} \omega^{\mathrm{n}},$$

acting on ω-plurisubharmonic functions, $u \in \mathrm{PSH(X,}\omega)$, which are Hölder continuous, $u \in \mathrm{Hlder(X,}\mathbb{R})$. The following theorem gives a better information about the Hölder exponent of the solution to the complex Monge-Ampère equation than theorems proved recently by *P. Eyssidieux* et al. [J. Am. Math. Soc. 22, No. 3, 607–639 (2009; Zbl 1215.32017)] and *S. Dinew* [J. Inst. Math. Jussieu 9, No. 4, 705–718 (2010; Zbl

1207.32034)].
Theorem A. Let $\mu = f\omega^n = \mathrm{MA(u)}$ be a probability measure absolutely continuous with respect to the Lebesgue measure with density $f \in L^p$, $p > 1$. Then u is Hölder continuous with exponent arbitrary close to $\frac{2}{1+nq}$, where $\frac{1}{p} + \frac{1}{q} = 1$. The optimal value of the Hölder exponent in Theorem A is still unknown, but it cannot be better than $\frac{2}{nq}$, see [*S. Pliś*, Ann. Pol. Math. 86, No. 2, 171–175 (2005; Zbl 1136.32306)] or [*V. Guedj* et al., Bull. Lond. Math. Soc. 40, No. 6, 1070–1080 (2008; Zbl 1157.32033)]. Moreover, Theorem A is generalized from the Kähler case to the case of big cohomology classes. The rest of the paper is devoted to the study of the range

$$\mathrm{MAH(X,\omega) = MA\big(PSH(X,\omega) \cap Hlder(X,\mathbb{R})\big)}.$$

A complete characterization of the set $\mathrm{MAH}(X,\omega)$ is unknown, but some of its properties are proved in the following theorem.
Theorem B. The set $\mathrm{MAH}(X,\omega)$ has the L^p property, i.e. if $\mu \in \mathrm{MAH}(X,\omega)$, $f \geq 0$, $f \in L^p(\mu)$ with $||f||_p = 1$, then $f\mu \in \mathrm{MAH}(X,\omega)$. In particular $\mathrm{MAH}(X,\omega)$ is a convex set. *T.-C. Dinh* et al. [J. Differ. Geom. 84, No. 3, 465–488 (2010; Zbl 1211.32021)] observed that the measures $\mu \in \mathrm{MAH}(X,\omega)$ have the following property

$$DNS \qquad \exp\big(-\epsilon \mathrm{PSH}(X,\omega)\big) \subset \mathrm{L}^1(\mu), \quad \text{forsome} \quad \epsilon > 0.$$

Condition (DNS) gives a full description of the range $\mathrm{MAH}(X,\omega)$ for $n = 1$, see [*T.-C. Dinh* and *N. Sibony*, Comment. Math. Helv. 81, No. 1, 221–258 (2006; Zbl 1094.32005)]. In the article under review it is proved that, for some special class of measures, this characterization is true in higher dimensions.
Theorem C. Let μ be a probability measure with finitely many isolated singularities which is radial or toric. Then $\mu \in \mathrm{MAH}(X,\omega)$ if and only if condition (DNS) is satisfied.

Reviewer: Rafał Czyz (Krakow)

Cited in 7 Documents

MSC:
32U05 Plurisubharmonic functions and generalizations
32U40 Currents 53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords: Monge-Ampère operator; Kähler manifold; pluripotential theory; Hölder continuity

References:

[1] Bedford, E., Taylor, B. A.: The Dirichlet problem for the complex Monge-Ampère oper- ator. Invent. Math. 37, 1-44 (1976) · Zbl 0315.31007 · DOI: 10.1007/BF01418826 · EUDML: 142425

[2] Bedford, E., Taylor, B. A.: A new capacity for plurisubharmonic functions. Acta Math. 149, 1-40 (1982) · Zbl 0547.32012 · DOI: 10.1007/BF02392348

[3] Benelkourchi, S., Guedj, V., Zeriahi, A.: A priori estimates for weak solutions of complex Monge-Ampère equations. Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5) 7, 81-96 (2008) · Zbl 1150.32011

[4] Berman, R.: A thermodynamical formalism for Monge-Ampère equations, Moser-Trudinger inequalities and Kähler-Einstein metrics. Adv. Math. 248, 1254-1297 (2013) · Zbl 1286.58010 · DOI: 10.1016/j.aim.2013.08.024

[5] Berman, R., Demailly, J.-P.: Regularity of plurisubharmonic upper envelopes in big coho- mology classes. In: Perspectives in Analysis, Geometry, and Topology in honor of Oleg Viro (Stockholm, 2008), I. Itenberg et al. (eds.), Progr. Math. 296, Birkhäuser/Springer, Boston, 39-66 (2012) · Zbl 1258.32010 · DOI: 10.1007/978-0-8176-8277-4_3 · arxiv:0905.1246

[6] Birkar, C., Cascini, P., Hacon, C., McKernan, J.: Existence of minimal models for va- rieties of log general type. J. Amer. Math. Soc. 23, 405-468 (2010) · Zbl 1210.14019 · DOI: 10.1090/S0894-0347-09-00649-3 · arxiv:math/0610203

[7] Boucksom, S., Eyssidieux, P., Guedj, V., Zeriahi, A.: Monge-Ampère equations in big cohomology classes. Acta Math. 205, 199-262 (2010) · Zbl 1213.32025 · DOI: 10.1007/s11511-010-0054-7

[8] Demailly, J.-P.: Estimations L^2 pour l'opérateur $\overline{\partial}$ d'un fibré vectoriel holomorphe semi- positif au-dessus d'une variété kählérienne complète. Ann. Sci. École Norm. Sup. 15, 457-511 (1982) · Zbl 0507.32021 · NUMDAM: ASENS_1982_4_15_3_457_0 · EUDML: 82103

[9] Demailly, J.-P.: Regularization of closed positive currents and intersection theory. J. Al- gebraic Geom. 1, 361-409 (1992) · Zbl 0777.32016

[10] Demailly, J.-P.: Monge-Ampère operators, Lelong numbers and intersection theory. In: Complex Analysis and Geometry, Univ. Ser. Math., Plenum, New York, 115-193 (1993) · Zbl 0792.32006

[11] Demailly, J.-P.: Regularization of closed positive currents of type (1.1) by the flow of a Chern connection. In: Contributions to Complex Analysis and Analytic Geometry (Paris, 1992), Aspects Math. E26, Vieweg, 105-126 (1994) · Zbl 0824.53064

[12] Demailly, J.-P., Pali, N.: Degenerate complex Monge-Ampère equations over compact Kähler manifolds. Int. J. Math. 21, 357-405 (2010) · Zbl 1191.53029 · DOI: 10.1142/S0129167X10006070 · arxiv:0710.5109

[13] Dinew, S.: Hölder continuous potentials on manifolds with partially positive curvature. J. Inst. Math. Jussieu 9, 705-718 (2010) · Zbl 1207.32034 · DOI: 10.1017/S1474748010000113

[14] Dinew, S., Zhang, Z.: On stability and continuity of bounded solutions of degenerate complex Monge-Ampère equations over compact Kähler manifolds. Adv. Math. 225, 367-388 (2010) · Zbl 1210.32020 · DOI: 10.1016/j.aim.2010.03.001

[15] Dinh, T. C., Nguyen, V. A., Sibony, N.: Exponential estimates for plurisubharmonic functions and stochastic dynamics. J. Differential Geom. 84, 465-488 (2010) · Zbl 1211.32021 · euclid:jdg/1279114298

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Campana, Frédéric; Demailly, Jean-Pierre; Verbitsky, Misha

Compact Kähler 3-manifolds without nontrivial subvarieties. (English) Zbl 1293.32028

Algebr. Geom. 1, No. 2, 131-139 (2014).

This is a very interesting paper. The authors prove that compact Kähler threefolds without nontrivial subvariety are tori. We describe below the proof in four steps: Step 1 (Corollary 4.3): If X is a compact simple Kähler threefold, then K_X is pseudo-effective. That is the Chern class of the canonical line bundle has a positive $(1,1)$-current. X is simple if there is an intersection A of a set of accountable dense Zariski open sets such that each point in A is not contained in any nontrivial subvariety. The description on page 131 of the "simple" is a little bit confusing. The authors use the celebrated result of Brunella
Theorem 4.1. They notice that $h^{2,0} > 0$, otherwise X is projective. Step 2 (Corollary 2.6): If X is a compact Kähler manifold without nontrivial subvariety, then K_X is nef but not big. This comes from Theorem 2.5. Step 3 (Corollary 3.3): $\mathcal{X}(\mathcal{O}) = 0$. This comes from a manipulation of the Riemann-Roch formula with a version of the Hard Lefschetz Theorem 3.1. Step 4 (Lemma 1.4.): Another application of the Riemann-Roch formula noticing that $h^{3,0} \leq 1$. See also the authors' summary.

Reviewer: Daniel Guan (Riverside)

MSC:
32J17 Compact 3-folds (analytic spaces)
32J27 Compact Kähler manifolds: generalizations, classification
32J25 Transcendental methods of algebraic geometry

Keywords: compact Kähler threefolds; simple manifolds; holomorphic foliations; complex torus; hyperkähler manifolds

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Demailly, Jean-Pierre

Pierre Lelong: a fundamental work in complex analysis and analytical geometry. (Pierre Lelong: une œuvre fondatrice en analyse complexe et en géométrie analytique.) (French) Zbl 1296.01027

Gaz. Math., Soc. Math. Fr. 135, 63-66 (2013).

Cited in 1 Document

MSC:
01A70 Biographies, obituaries, personalia, bibliographies
32-03 Historical (several complex variables and analytic spaces)

Demailly, Jean-Pierre

Episciences: a publishing platform for open archive overlay journals. (English) Zbl 1290.01031

Eur. Math. Soc. Newsl. 87, 31-32 (2013).

MSC:
01A80 Sociology (and profession) of mathematics

Demailly, Jean-Pierre; Hacon, Christopher D.; Păun, Mihai

Extension theorems, non-vanishing and the existence of good minimal models. (English) Zbl 1278.14022

Acta Math. 210, No. 2, 203-259 (2013).

Let X be a complex projective manifold (or normal complex projective variety with mild singularities). The aim of the minimal model program is to construct a birational model $X \dashrightarrow X'$ such that either X' admits a fibration with general fibre a Fano variety or X' is a good minimal model, that is some positive multiple of the canonical divisor $K_{X'}$ defines a morphism. If X is covered by rational curves or X is of general type (that is some positive multiple of K_X defines a birational map) the minimal model program is completed in the landmark paper by *C. Birkar* et al., [J. Am. Math. Soc. 23, No. 2, 405–468 (2010; Zbl 1210.14019)]. Thus the main challenge is now to study projective manifolds X that are not covered by rational curves and not of general type. By a fundamental result of *S. Boucksom* et al. [J. Algebr. Geom. 22, No. 2, 201–248 (2013; Zbl 1267.32017)], the canonical divisor K_X is then pseudoeffective, that is K_X is a limit of effective divisor. However the nonvanishing conjecture claims that some positive multiple of the canonical divisor is actually effective. Once we know that there exists at least one effective pluricanonical divisor D one can hope to establish the existence of good minimal models inductively by proving that the restriction morphism

$$H^0(X, \mathcal{O}_X(mK_X)) \to H^0(D, \mathcal{O}_D(mK_X))$$

is surjective for $m \gg 0$. A similar extension result played a crucial role in the proof of the existence of flips by *C. D. Hacon* and *J. McKernan* [J. Am. Math. Soc. 23, No. 2, 469–490 (2010; Zbl 1210.14021)]. In the paper under review the authors realise an important step of this strategy by proving the following "plt" extension theorem:
Let X be a projective manifold and $S + B$ a $\mathbb{Q}$-divisor with simple normal crossings such that
1) $(X, S+B)$ is plt (i.e. S is a prime divisor with $\mathrm{mult_S(S+B)} = 1$ and $\lfloor B \rfloor = 0$), and
2) there exists an effective $\mathbb{Q}$-divisor $D \sim_{\mathbb{Q}} K_X + S + B$ such that

$$S \subset \mathrm{Supp(D)} \subset \mathrm{Supp(S+B)},$$

and
3) for any ample divisor A and any rational number $\epsilon > 0$, there is an effective $\mathbb{Q}$-divisor

$D \sim_{\mathbb{Q}} K_X + S + B + \epsilon A$ whose support does not contain S).
Consider $\pi : \tilde{X} \to X$ a log-resolution of $(X, S + B)$, so that we have

$$K_{\tilde{X}} + \tilde{S} + \tilde{B} = \pi^*(K_X + S + B) + \tilde{E}$$

where $\tilde{S}$ is the strict transform of S. Let m be an integer, such that $m(K_X + S + B)$ is Cartier, and let u be a section of $m(K_X + S + B)|_S$, such that

$$Z_{\pi^*(u)} + m\tilde{E}|_{\tilde{S}} \geq m\Xi,$$

where $Z_{\pi^*(u)}$ is the zero divisor of the section $\pi^*(u)$ and Ξ the extension obstruction divisor (cf. [Zbl 1210.14021]). Then u extends to X.
The main achievement of this theorem compared to earlier extension results is that one does not assume B to be strictly positive (i.e. ample or big). The authors conjecture that their statement also holds under the weaker assumption that the pair $(X, S + B)$ is dlt. This stronger extension result would then reduce the minimal model conjecture to the nonvanishing problem. More precisely the authors prove the following theorem: Suppose that the "dlt" extension theorem holds in dimension n. Suppose also that the non-vanishing conjecture holds for semi-log-canonical pairs of dimension n. Then every n-dimensional projective manifold that is not covered by rational curves has a good minimal model.

Reviewer: Andreas Höring (Nice)

Cited in 2 Reviews

Cited in 9 Documents

MSC:
14E30 Minimal model program (Mori theory, extremal rays)
14J40 Algebraic n-folds ($n > 4$)
32J25 Transcendental methods of algebraic geometry

Keywords: extension theorem; minimal model; MMP; abundance conjecture; nonvanishing conjecture

References:

[1] Ambro F.: Nef dimension of minimal models. Math. Ann., 330, 309–322 (2004) · Zbl 1081.14074 · DOI: 10.1007/s00208-004-0550-1

[2] Ambro F.: The moduli b-divisor of an lc-trivial fibration. Compos. Math., 141, 385–403 (2005) · Zbl 1094.14025 · DOI: 10.1112/S0010437X04001071

[3] Berndtsson, B., The extension theorem of Ohsawa–Takegoshi and the theorem of Donnelly–Fefferman. Ann. Inst. Fourier (Grenoble), 46 (1996), 1083–1094. · Zbl 0853.32024

[4] Berndtsson, B. & Păun, M., Quantitative extensions of pluricanonical forms and closed positive currents. Nagoya Math. J., 205 (2012), 25–65. · Zbl 1248.32012

[5] Birkar C.: Ascending chain condition for log canonical thresholds and termination of log flips. Duke Math. J., 136, 173–180 (2007) · Zbl 1109.14018 · DOI: 10.1215/S0012-7094-07-13615-9

[6] Birkar C.: On existence of log minimal models II. J. Reine Angew. Math., 658, 99–113 (2011) · Zbl 1226.14021

[7] Birkar, C., Cascini, P., Hacon, C.D. & McKernan, J., Existence of minimal models for varieties of log general type. J. Amer. Math. Soc., 23 (2010), 405–468. · Zbl 1210.14019

[8] Claudon, B., Invariance for multiples of the twisted canonical bundle. Ann. Inst. Fourier (Grenoble), 57 (2007), 289–300. · Zbl 1122.32013

[9] Corti, A. & Lazić, V., New outlook on the minimal model program, II. Preprint, 2010. arXiv:1005.0614 [math.AG]. · Zbl 1273.14033

[10] Demailly, J.-P., Singular Hermitian metrics on positive line bundles, in Complex Algebraic Varieties (Bayreuth, 1990), Lecture Notes in Math., 1507, pp. 87–104. Springer, Berlin–Heidelberg, 1992.

[11] Demailly, J.-P. On the Ohsawa–Takegoshi–Manivel L^2 extension theorem, in Complex Analysis and Geometry (Paris, 1997), Progr. Math., 188, pp. 47–82. Birkhäuser, Basel, 2000. · Zbl 0959.32019

[12] Demailly, J.-P. Analytic Methods in Algebraic Geometry. Surveys of Modern Mathematics, 1. Int. Press, Somerville, MA, 2012. · Zbl 1271.14001

[13] Ein, L. & Popa,M., Extension of sections via adjoint ideals. Math. Ann., 352 (2012), 373–408. · Zbl 1248.14007

[14] de Fernex, T. & Hacon, C.D., Deformations of canonical pairs and Fano varieties. J. Reine Angew. Math., 651 (2011), 97–126. · Zbl 1220.14026

[15] Fujino, O., Abundance theorem for semi log canonical threefolds. Duke Math. J., 102 (2000), 513–532. · Zbl 0986.14007

[16] Fujino, O. Special termination and reduction to pl flips, in Flips for 3-folds and 4-folds, Oxford Lecture Ser. Math. Appl., 35, pp. 63–75. Oxford Univ. Press, Oxford, 2007. · Zbl 1286.14025

[17] Fujino, O. Fundamental theorems for the log minimal model program. Publ. Res. Inst. Math. Sci., 47 (2011), 727–789. · Zbl 1234.14013

[18] Fukuda, S., Tsuji's numerically trivial fibrations and abundance. Far East J. Math. Sci. (FJMS), 5 (2002), 247–257. · Zbl 1076.14506

[19] Gongyo, Y., Remarks on the non-vanishing conjecture. Preprint, 2012. arXiv: 1201.1128 [math.AG]. · Zbl 1246.14026

[20] Gongyo, Y. & Lehmann, B., Reduction maps and minimal model theory. Preprint, 2011. arXiv:1103.1605 [math.AG]. · Zbl 1264.14025

[21] Hacon, C.D. & Kovács, S. J., Classification of Higher Dimensional Algebraic Varieties. Oberwolfach Seminars, 41. Birkhäuser, Basel, 2010. · Zbl 1204.14001

[22] Hacon, C. D. & McKernan, J., Boundedness of pluricanonical maps of varieties of general type. Invent. Math., 166 (2006), 1–25. · Zbl 1121.14011

[23] Hacon, C. D. & McKernan, J. Existence of minimal models for varieties of log general type. II. J. Amer. Math. Soc., 23 (2010), 469–490. · Zbl 1210.14021

[24] Hacon, C. D., McKernan, J. & Xu, C., ACC for log canonical thresholds. Preprint, 2012. arXiv:1208.4150 [math.AG]. · Zbl 1320.14023

[25] Kawamata, Y., Pluricanonical systems on minimal algebraic varieties. Invent. Math., 79 (1985), 567–588. · Zbl 0593.14010

[26] Kawamata, Y The Zariski decomposition of log-canonical divisors, in Algebraic Geometry, Bowdoin, 1985 (Brunswick, ME, 1985), Proc. Sympos. Pure Math., 46, pp. 425–433. Amer. Math. Soc., Providence, RI, 1987.

[27] Kawamata, Y Abundance theorem for minimal threefolds. Invent. Math., 108 (1992), 229–246. · Zbl 0777.14011

[28] Kawamata, Y On the cone of divisors of Calabi–Yau fiber spaces. Internat. J. Math., 8 (1997), 665–687. · Zbl 0931.14022

[29] Keel, S., Matsuki, K. & McKernan, J., Log abundance theorem for threefolds. Duke Math. J., 75 (1994), 99–119. · Zbl 0818.14007

[30] Klimek, M., Pluripotential Theory. London Mathematical Society Monographs, 6. Oxford University Press, New York, 1991.

[31] Kollár, J. & Mori, S., Birational Geometry of Algebraic Varieties. Cambridge Tracts in Mathematics, 134. Cambridge University Press, Cambridge, 1998. · Zbl 0926.14003

[32] Kollár, J. (ed.), Flips and Abundance for Algebraic Threefolds (Salt Lake City, UT, 1991). Astérisque, 211. Société Mathématique de France, Paris, 1992.

[33] Lai, C.-J., Varieties fibered by good minimal models. Math. Ann., 350 (2011), 533– 547. · Zbl 1221.14018

[34] Lelong, P., Fonctions plurisousharmoniques et formes différentielles positives. Gordon & Breach, Paris, 1968. · Zbl 0195.11603

[35] Lelong, P. Éléments extrémaux dans le côone des courants positifs fermés de type (1, 1) et fonctions plurisousharmoniques extrémales. C. R. Acad. Sci. Paris Sér. A-B, 273 (1971), A665–A667.

[36] Manivel, L., Un théorème de prolongement L^2 de sections holomorphes d'un fibré hermitien. Math. Z., 212 (1993), 107–122.

[37] McNeal, J. D. & Varolin, D., Analytic inversion of adjunction: L^2 extension theorems with gain. Ann. Inst. Fourier (Grenoble), 57 (2007), 703–718. · Zbl 1208.32011

[38] Miyaoka, Y., Abundance conjecture for 3-folds: case v = 1. Compos. Math., 68 (1988), 203–220. · Zbl 0681.14019

[39] Nakayama, N., Zariski-Decomposition and Abundance. MSJ Memoirs, 14. Mathematical Society of Japan, Tokyo, 2004. · Zbl 1061.14018

[40] Ohsawa, T., On the extension of L^2 holomorphic functions. VI. A limiting case, in Explorations in Complex and Riemannian Geometry, Contemp. Math., 332, pp. 235–239. Amer. Math. Soc., Providence, RI, 2003. · Zbl 1049.32010

[41] Ohsawa, T. Generalization of a precise L^2 division theorem, in Complex Analysis in Several Variables, Adv. Stud. Pure Math., 42, pp. 249–261. Math. Soc. Japan, Tokyo, 2004. · Zbl 1078.32004

[42] Ohsawa, T. & Takegoshi, K., On the extension of L^2 holomorphic functions. Math. Z., 195 (1987), 197–204. · Zbl 0625.32011

[43] Păun, M., Siu's invariance of plurigenera: a one-tower proof. J. Differential Geom., 76 (2007), 485–493. · Zbl 1122.32014

[44] Păun, M. Relative critical exponents, non-vanishing and metrics with minimal singularities. Invent. Math., 187 (2012), 195–258. · Zbl 1251.32018

[45] Shokurov, V. V., Letters of a bi-rationalist. VII. Ordered termination. Tr. Mat. Inst. Steklova, 264 (2009), 184–208 (Russian); English translation in Proc. Steklov Inst. Math., 264 (2009), 178–200. · Zbl 1312.14041

[46] Siu, Y.-T., Analyticity of sets associated to Lelong numbers and the extension of closed positive currents. Invent. Math., 27 (1974), 53–156. · Zbl 0289.32003

[47] Siu, Y.-T. Every Stein subvariety admits a Stein neighborhood. Invent. Math., 38 (1976/77), 89–100. · Zbl 0343.32014

[48] Siu, Y.-T. Invariance of plurigenera. Invent. Math., 134 (1998), 661–673. · Zbl 0955.32017

[49] Siu, Y.-T. Extension of twisted pluricanonical sections with plurisubharmonic weight and invariance of semipositively twisted plurigenera for manifolds not necessarily of general type, in Complex Geometry (Göttingen, 2000), pp. 223–277. Springer, Berlin–Heidelberg, 2002. · Zbl 1007.32010

[50] Siu, Y.-T. Finite generation of canonical ring by analytic method. Sci. China Ser. A, 51 (2008), 481–502. · Zbl 1153.32021

[51] Siu, Y.-T. Abundance conjecture, in Geometry and Analysis. No. 2, Adv. Lect. Math., 18, pp. 271–317. Int. Press, Somerville, MA, 2011. · Zbl 1263.14018

[52] Skoda, H., Application des techniques L^2à la théorie des idéaux d'une algèbre de fonctions holomorphes avec poids. Ann. Sci. Éc. Norm. Super., 5 (1972), 545–579. · Zbl 0254.32017

[53] Takayama S.: Pluricanonical systems on algebraic varieties of general type. Invent. Math., 165, 551–587 (2006) · Zbl 1108.14031 · DOI: 10.1007/s00222-006-0503-2

[54] Takayama S.: On the invariance and the lower semi-continuity of plurigenera of algebraic varieties. J. Algebraic Geom., 16, 1–18 (2007) · Zbl 1113.14028 · DOI: 10.1090/S1056-3911-06-00455-3

[55] Tian G.: On Kähler–Einstein metrics on certain Kähler manifolds with C1(M) > 0. Invent. Math., 89, 225–246 (1987) · Zbl 0599.53046 · DOI: 10.1007/BF01389077

[56] Tsuji, H., Extension of log pluricanonical forms from subvarieties. Preprint, 2007. arXiv:0709.2710 [math.AG].

[57] Varolin D., Varolin D.: Takayama-type extension theorem. Compos. Math., 144, 522–540 (2008) · Zbl 1163.32008 · DOI: 10.1112/S0010437X07002989

Demailly, Jean-Pierre

Applications of pluripotential theory to algebraic geometry. (English) Zbl 1271.32037

Bracci, Filippo (ed.) et al., Pluripotential theory. Lectures of the CIME course, Cetraro, Italy, 2011. Berlin: Springer; Florence: Fondazione CIME (ISBN 978-3-642-36420-4/pbk; 978-3-642-36421-1/ebook). Lecture Notes in Mathematics 2075. CIME Foundation Subseries, 143-263 (2013).

Summary: These lectures are devoted to the study of various contemporary problems of algebraic geometry, using fundamental tools from complex potential theory, namely plurisubharmonic functions, positive currents and Monge-Ampère operators. Since their inception by Oka and Lelong in the mid 1940s, plurisubharmonic functions have been used extensively in many areas of algebraic and analytic geometry, as they are the function theoretic counterpart of pseudoconvexity, the complexified version of convexity. One such application is the theory of L^2 estimates via the Bochner-Kodaira-Hörmander technique, which provides very strong existence theorems for sections of holomorphic vector bundles with positive curvature. One can mention here the foundational work achieved by Bochner, Kodaira, Nakano, Morrey, Kohn, Andreotti-Vesentini, Grauert, Hörmander, Bombieri, Skoda and Ohsawa-Takegoshi in the course of more than four decades. Another development is the theory of holomorphic Morse inequalities (1985), which relate certain curvature integrals with the asymptotic cohomology of large tensor powers of line or vector bundles, and bring a useful complement to the Riemann-Roch formula.We describe here the main techniques involved in the proof of holomorphic Morse inequalities (Sect. 1) and their link with Monge-Ampère operators and intersection theory. Section 2, especially, gives a fundamental approximation theorem for closed (1, 1)-currents, using a Bergman kernel technique in combination with the Ohsawa-Takegoshi theorem. As an application, we study the geometric properties of positives cones of an algebraic variety (nef and pseudo-effective cone), and derive from there some results about asymptotic cohomology functionals in Sect. 3. The last Sect. 4 provides an application to the study of the Green-Griffiths-Lang conjecture. The latter conjecture asserts that every entire curve drawn on a projective variety of general type should satisfy a global algebraic equation; via a probabilistic curvature estimate, holomorphic Morse inequalities imply that entire curves must at least satisfy a global algebraic differential equation. For the entire collection see [Zbl 1266.31001].

MSC:
32U05 Plurisubharmonic functions and generalizations
32W20 Complex Monge-Ampère operators
32U40 Currents

Keywords: plurisubharmonic functions; positive currents; Monge-Ampère operator; holomorphic Morse inequalities; Ohsawa-Takegoshi theorem

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Boucksom, Sébastien; Demailly, Jean-Pierre; Păun, Mihai; Peternell, Thomas

The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension. (English) Zbl 1267.32017

J. Algebr. Geom. 22, No. 2, 201-248 (2013).

Summary: We prove that a holomorphic line bundle on a projective manifold is pseudo-effective if and only if its degree on any member of a covering family of curves is non-negative. This is a consequence of a duality statement between the cone of pseudo-effective divisors and the cone of "movable curves", which is obtained from a general theory of movable intersections and approximate Zariski decomposition for closed positive $(1, 1)$-currents. As a corollary, a projective manifold has a pseudo-effective canonical bundle if and only if it is not uniruled. We also prove that a 4-fold with a canonical bundle which is pseudo-effective and of numerical class zero in restriction to curves of a good covering family, has non-negative Kodaira dimension.

Cited in 7 Reviews

Cited in 71 Documents

MSC:
32Q15 Kähler manifolds
32J27 Compact Kähler manifolds: generalizations, classification

Keywords: compact Kähler manifold; projective Kähler manifold; Kodaira dimension; uniruled manifold

References:

[1] Florin Ambro, Nef dimension of minimal models, Math. Ann. 330 (2004), no. 2, 309 – 322. · Zbl 1081.14074 · DOI: 10.1007/s00208-004-0550-1 · https://doi.org/10.1007%2Fs00208-004-0550-1

[2] Sébastien Boucksom, Charles Favre, and Mattias Jonsson, Differentiability of volumes of divisors and a problem of Teissier, J. Algebraic Geom. 18 (2009), no. 2, 279 – 308. · Zbl 1162.14003

[3] Sébastien Boucksom, On the volume of a line bundle, Internat. J. Math. 13 (2002), no. 10, 1043 – 1063. · Zbl 1101.14008 · DOI: 10.1142/S0129167X02001575 · https://doi.org/10.1142%2FS0129167X02001575

[4] Boucksom, S.: Cônes positifs des variétés complexes compactes, Thesis, Grenoble 2002.

[5] Marco Brunella, A positivity property for foliations on compact Kähler manifolds, Internat. J. Math. 17 (2006), no. 1, 35 – 43. · Zbl 1097.37041 · DOI: 10.1142/S0129167X06003333 · https://doi.org/10.1142%2FS0129167X06003333

[6] F. Campana, Coréduction algébrique d'un espace analytique faiblement kählérien compact, Invent. Math. 63 (1981), no. 2, 187 – 223 (French). · Zbl 0436.32024 · DOI: 10.1007/BF01393876 · https://doi.org/10.1007%2FBF01393876

[7] Frédéric Campana, Remarques sur le revêtement universel des variétés kählériennes compactes, Bull. Soc. Math. France 122 (1994), no. 2, 255 – 284 (French, with English and French summaries).

[8] Frédéric Campana, Orbifolds, special varieties and classification theory: an appendix, Ann. Inst. Fourier (Grenoble) 54 (2004), no. 3, 631 – 665 (English, with English and French summaries). · Zbl 1062.14015

[9] Frédéric Campana and Thomas Peternell, Algebraicity of the ample cone of projective varieties, J. Reine Angew. Math. 407 (1990), 160 – 166. · Zbl 0728.14004

[10] Frédéric Campana and Thomas Peternell, Appendix to the article of T. Peternell: ["Toward a Mori theory and compact Kähler threefolds. III", Bull. Soc. Math. France 129 (2001), no. 3, 339 – 356; MR1881199 (2003b:32024)] the Kodaira dimension of Kummer threefolds, Bull. Soc. Math. France 129 (2001), no. 3, 357 – 359 (English, with English and French summaries).

[11] Frédéric Campana and Thomas Peternell, Appendix to the article of T. Peternell: ["Toward a Mori theory and compact Kähler threefolds. III", Bull. Soc. Math. France 129 (2001), no. 3, 339 – 356; MR1881199 (2003b:32024)] the Kodaira dimension of Kummer threefolds, Bull. Soc. Math. France 129 (2001), no. 3, 357 – 359 (English, with English and French summaries).

[12] Jean-Louis Colliot-Thélène, Arithmétique des variétés rationnelles et problèmes birationnels, Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Berkeley, Calif., 1986) Amer. Math. Soc., Providence, RI, 1987, pp. 641 – 653 (French).

[13] Olivier Debarre, Classes de cohomologie positives dans les variétés kählériennes compactes (d'après Boucksom, Demailly, Nakayama, Păun, Peternell et al.), Astérisque 307 (2006), Exp. No. 943, viii, 199 – 228 (French, with French summary). Séminaire Bourbaki. Vol. 2004/2005.

[14] Jean-Pierre Demailly, Champs magnétiques et inégalités de Morse pour la d''-cohomologie, Ann. Inst. Fourier (Grenoble) 35 (1985), no. 4, 189 – 229 (French, with English summary). · Zbl 0565.58017

[15] Jean-Pierre Demailly, Singular Hermitian metrics on positive line bundles, Complex algebraic varieties (Bayreuth, 1990) Lecture Notes in Math., vol. 1507, Springer, Berlin, 1992, pp. 87 – 104. · Zbl 0784.32024 · DOI: 10.1007/BFb0094512 · https://doi.org/10.1007%2FBFb0094512

[16] Jean-Pierre Demailly, Regularization of closed positive currents and intersection theory, J. Algebraic Geom. 1 (1992), no. 3, 361 – 409. · Zbl 0777.32016

[17] Demailly, J.-P., Vanishing theorems and effective results in algebraic geometry, School on algebraic Geometry held at ICTP in January 2000, Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2001.

[18] Jean-Pierre Demailly, Kähler manifolds and transcendental techniques in algebraic geometry, International Congress of Mathematicians. Vol. I, Eur. Math. Soc., Zürich, 2007, pp. 153 – 186. · Zbl 1141.14007 · DOI: 10.4171/022-1/8 · https://doi.org/10.4171%2F022-1%2F8

[19] Demailly, J.-P., Holomorphic Morse inequalities and asymptotic cohomology groups: a tribute to Bernhard Riemann, arXiv:1003.5067, math.CV, to appear in Proceedings of the Riemann International School of Mathematics, Advances in Number Theory and Geometry, held at Verbania (Italy), April 2009.

[20] Jean-Pierre Demailly, Lawrence Ein, and Robert Lazarsfeld, A subadditivity property of multiplier ideals, Michigan Math. J. 48 (2000), 137 – 156. Dedicated to William Fulton on the occasion of his 60th birthday. · Zbl 1077.14516 · DOI: 10.1307/mmj/1030132712 · https://doi.org/10.1307%2Fmmj%2F1030132712

[21] Jean-Pierre Demailly and Mihai Paun, Numerical characterization of the Kähler cone of a compact Kähler manifold, Ann. of Math. (2) 159 (2004), no. 3, 1247 – 1274. · Zbl 1064.32019 · DOI: 10.4007/annals.2004.159.1247 · https://doi.org/10.4007%2Fannals.2004.159.1247

[22] Jean-Pierre Demailly, Thomas Peternell, and Michael Schneider, Compact complex manifolds with numerically effective tangent bundles, J. Algebraic Geom. 3 (1994), no. 2, 295 – 345. · Zbl 0827.14027

[23] Jean-Pierre Demailly, Thomas Peternell, and Michael Schneider, Holomorphic line bundles with partially vanishing cohomology, Proceedings of the Hirzebruch 65 Conference on Algebraic Geometry (Ramat Gan, 1993) Israel Math. Conf. Proc., vol. 9, Bar-Ilan Univ., Ramat Gan, 1996, pp. 165 – 198. · Zbl 0859.14005

[24] Jean-Pierre Demailly, Thomas Peternell, and Michael Schneider, Pseudo-effective line bundles on compact Kähler manifolds, Internat. J. Math. 12 (2001), no. 6, 689 – 741. · Zbl 1111.32302 · DOI: 10.1142/S0129167X01000861 · https://doi.org/10.1142%2FS0129167X01000861

[25] Thomas Eckl, Tsuji's numerical trivial fibrations, J. Algebraic Geom. 13 (2004), no. 4, 617 – 639. · Zbl 1065.14009

[26] Takao Fujita, Approximating Zariski decomposition of big line bundles, Kodai Math. J. 17 (1994), no. 1, 1 – 3. · Zbl 0814.14006 · DOI: 10.2996/kmj/1138039894 · https://doi.org/10.2996%2Fkmj%2F1138039894

[27] Tom Graber, Joe Harris, and Jason Starr, Families of rationally connected varieties, J. Amer. Math. Soc. 16 (2003), no. 1, 57 – 67. · Zbl 1092.14063

[28] Robin Hartshorne, Ample subvarieties of algebraic varieties, Lecture Notes in Mathematics, Vol. 156, Springer-Verlag, Berlin-New York, 1970. Notes written in collaboration with C. Musili. · Zbl 0208.48901

[29] Christopher D. Hacon and James Mckernan, On Shokurov's rational connectedness conjecture, Duke Math. J. 138 (2007), no. 1, 119 – 136. · Zbl 1128.14028 · DOI: 10.1215/S0012-7094-07-13813-4 · https://doi.org/10.1215%2FS0012-7094-07-13813-4

[30] Daniel Huybrechts, Birational symplectic manifolds and their deformations, J. Differential Geom. 45 (1997), no. 3, 488 – 513. · Zbl 0917.53010

[31] Kawamata, Y., Matsuki, K., Matsuda, K., Introduction to the minimal model program, Adv. Stud. Pure Math. 10 (1987), 283-360. · Zbl 0672.14006

[32] János Kollár and Shigefumi Mori, Classification of three-dimensional flips, J. Amer. Math. Soc. 5 (1992), no. 3, 533 – 703. · Zbl 0773.14004

[33] János Kollár, Higher direct images of dualizing sheaves. II, Ann. of Math. (2) 124 (1986), no. 1, 171 – 202. · Zbl 0605.14014 · DOI: 10.2307/1971390 · https://doi.org/10.2307%2F1971390

[34] Shoshichi Kobayashi, Differential geometry of complex vector bundles, Publications of the Mathematical Society of Japan, vol. 15, Princeton University Press, Princeton, NJ; Princeton University Press, Princeton, NJ, 1987. Kanô Memorial Lectures, 5. · Zbl 0708.53002

[35] Kollař́, J., et al., Flips and abundance for algebraic 3-folds, Astérisque 211, Soc. Math. France, 1992.

[36] Lazarsfeld, R., Ampleness modulo the branch locus of the bundle associated to a branched covering, Comm. Alg. 28 (2000), 5598-5599.

[37] Robert Lazarsfeld, Positivity in algebraic geometry. I, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 48, Springer-Verlag, Berlin, 2004. Classical setting: line bundles and linear series. Robert Lazarsfeld, Positivity in algebraic geometry. II, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 49, Springer-Verlag, Berlin, 2004. Positivity for vector bundles, and multiplier ideals.

[38] Yoichi Miyaoka, The Chern classes and Kodaira dimension of a minimal variety, Algebraic geometry, Sendai, 1985, Adv. Stud. Pure Math., vol. 10, North-Holland, Amsterdam, 1987, pp. 449 – 476.

[39] Yoichi Miyaoka, On the Kodaira dimension of minimal threefolds, Math. Ann. 281 (1988), no. 2, 325 – 332. · Zbl 0625.14023 · DOI: 10.1007/BF01458437 · https://doi.org/10.1007%2FBF01458437

[40] Yoichi Miyaoka, Abundance conjecture for 3-folds: case $\nu = 1$, Compositio Math. 68 (1988), no. 2, 203 – 220. · Zbl 0681.14019

[41] Yoichi Miyaoka and Shigefumi Mori, A numerical criterion for uniruledness, Ann. of Math. (2) 124 (1986), no. 1, 65 – 69. · Zbl 0606.14030 · DOI: 10.2307/1971387 · https://doi.org/10.2307%2F1971387

[42] Shigefumi Mori, Classification of higher-dimensional varieties, Algebraic geometry, Bowdoin, 1985 (Brunswick, Maine, 1985) Proc. Sympos. Pure Math., vol. 46, Amer. Math. Soc., Providence, RI, 1987, pp. 269 – 331. · Zbl 1103.14301

[43] Shigefumi Mori, Flip theorem and the existence of minimal models for 3-folds, J. Amer. Math. Soc. 1 (1988), no. 1, 117 – 253. · Zbl 0649.14023

[44] Noboru Nakayama, On Weierstrass models, Algebraic geometry and commutative algebra, Vol. II, Kinokuniya, Tokyo, 1988, pp. 405 – 431.

[45] Noboru Nakayama, Zariski-decomposition and abundance, MSJ Memoirs, vol. 14, Mathematical Society of Japan, Tokyo, 2004.

[46] Mihai Paun, Sur l'effectivité numérique des images inverses de fibrés en droites, Math. Ann. 310 (1998), no. 3, 411 – 421 (French). · Zbl 1023.32014 · DOI: 10.1007/s002080050154 · https://doi.org/10.1007%2Fs002080050154

[47] Thomas Peternell, Towards a Mori theory on compact Kähler threefolds. III, Bull. Soc. Math. France 129 (2001), no. 3, 339 – 356 (English, with English and French summaries). · Zbl 0994.32017

[48] Thomas Peternell, Kodaira dimension of subvarieties. II, Internat. J. Math. 17 (2006), no. 5, 619 – 631. · Zbl 1126.14019 · DOI: 10.1142/S0129167X0600362X · https://doi.org/10.1142%2FS0129167X0600362X

[49] Thomas Peternell, Michael Schneider, and Andrew J. Sommese, Kodaira dimension of subvarieties, Internat. J. Math. 10 (1999), no. 8, 1065 – 1079. · Zbl 1077.14515 · DOI: 10.1142/S0129167X9900046X · https://doi.org/10.1142%2FS0129167X9900046X

[50] Thomas Peternell and Andrew J. Sommese, Ample vector bundles and branched coverings. II, The Fano Conference, Univ. Torino, Turin, 2004, pp. 625 – 645. · Zbl 1071.14018

[51] Shepherd-Barron, N., Miyaoka's theorems on the generic semi-negativity of T_X, Astérisque 211 (1992), 103-114. · Zbl 0809.14034

[52] Tsuji, H., Numerically trivial fibrations., math.AG/0001023 (2000).

[53] Thomas Bauer, Frédéric Campana, Thomas Eckl, Stefan Kebekus, Thomas Peternell, Sławomir Rams, Tomasz Szemberg, and Lorenz Wotzlaw, A reduction map for nef line bundles, Complex geometry (Göttingen, 2000) Springer, Berlin, 2002, pp. 27 – 36.

[54] Shing Tung Yau, On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation. I, Comm. Pure Appl. Math. 31 (1978), no. 3, 339 – 411. · Zbl 0369.53059 · DOI: 10.1002/cpa.3160310304 · https://doi.org/10.1002%2Fcpa.3160310304

Demailly, Jean-Pierre (ed.); Hulek, Klaus (ed.); Peternell, Thomas (ed.)

Complex analysis. Abstracts from the workshop held September 2–8, 2012. (Komplexe Analysis.) (English) Zbl 1349.00135

Oberwolfach Rep. 9, No. 3, 2597-2656 (2012).

Summary: The aim of this workshop was to discuss recent developments in several complex variables and complex geometry. Special emphasis was put on the interaction of analytic and algebraic methods. Topics included Kähler geometry, Ricci-flat manifolds, moduli theory and themes related to the minimal model program.

MSC:
00B05 Collections of abstracts of lectures 00B25 Proceedings of conferences of miscellaneous specific interest
32-06 Proceedings of conferences (several complex variables)
14-06 Proceedings of conferences (algebraic geometry)

References:

[1]

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.

Demailly, Jean-Pierre

Henri Cartan and multivariate holomorphic functions. (Henri Cartan et les fonctions holomorphes de plusieurs variables.) (French) Zbl 1294.32002

Harinck, Pascale (ed.) et al., Henri Cartan et André Weil. Mathématiciens du XX^{e} siècle. Journées mathématiques X-UPS, Palaiseau, France, May 3–4, 2012. Palaiseau: Les Éditions de l'École Polytechnique (ISBN 978-2-7302-1610-4/pbk). 99-168 (2012).

The paper is a review of some fundamental results on holomorphic functions of several variables, and of the role of Henri Cartan in the development of this theory, in particular in the theory of coherent sheaves which he developed and which stands now as one of the most fundamental tools in complex geometry and in algebraic geometry. The exposition is concise but self-contained and the stress is on the essential facts. A few historical remarks are useful for understanding the motivations behind the ideas. All this makes the text much more attractive than many other texts written on the subject. The bibliographical references organized in sections are also very useful. For the entire collection see [Zbl 1270.01009].

Reviewer: Athanase Papadopoulos (MR3014194)

MSC:
32-03 Historical (several complex variables and analytic spaces)
32A10 Holomorphic functions (several variables)

32B10 Germs of analytic sets, local parametrization
32C35 Analytic sheaves and cohomology groups
32H99 Holomorphic mappings on analytic spaces

Keywords: holomorphic functions of several variables; coherent sheaves; complex geometry; algebraic geometry

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Demailly, Jean-Pierre

Hyperbolic algebraic varieties and holomorphic differential equations. (English) Zbl 1264.32022

Acta Math. Vietnam. 37, No. 4, 441-512 (2012).

The long and exhaustive article reviews and explains recent methods and results in the theory of hyperbolic algebraic varieties, including several new results of the author which are focussed on some of the most challenging conjectures about hyperbolicity of algebraic manifolds of general type, the conjectures of Green-Griffiths and Lang. The conjectures of Green-Griffiths state that a projective algebraic variety X is of general type if and only if X is measure hyperbolic, and that X then contains a proper subvariety Y such that $f(\mathbb{C}) \subset \mathbb{Y}$ for every non-constant holomorphic map $f : \mathbb{C} \to \mathbb{X}$. An affirmative solution for these conjectures would imply that every very generic hypersurface in $\mathbb{P}^{\ltimes+\rtimes}$, $n \geq 3$, of degree $d \geq 2n+1$ is hyperbolic, see [*L. Ein*, Invent. Math. 94, No. 1, 163–169 (1988; Zbl 0701.14002); Math. Ann. 289, No. 3, 465–471 (1991; Zbl 0746.14019)] and [*C. Voisin*, J. Differ. Geom. 44, No. 1, 200–213 (1996; Zbl 0883.14022)]. The author studies hyperbolicity problems in a more general setting. He works in the class of directed manifolds and transfers classical hyperbolicity concepts for complex varieties to this category. Special emphasis lies on the application of the Ahlfors-Schwarz lemma, jets of curves, Semple jet bundles, k-jet bundles, holomorphic Morse inequalities, jet differentials and k-jet metrics with negative curvature. A (compact resp. projective) directed manifold is understood as a pair (X, V) of a (compact resp. projective) connected complex manifold X and an irreducible closed analytic subspace V of the holomorphic tangent bundle T_X such that $V \cap T_{X,x}$ is a linear subspace of $T_{X,x}$ for every $x \in X$. Based on the Brody criterion it turns out that a compact directed manifold (X, V) is hyperbolic iff every holomorphic map $f : \mathbb{C} \to \mathbb{X}$ with $f'(\mathbb{C}) \subset \mathbb{V}$ is constant. A projective directed manifold (X, V) is by definition algebraic hyperbolic if X admits an Hermitian metric with fundamental form ω such that for some $\epsilon > 0$ the inequality $-\chi(\overline{C}) \geq \epsilon \int_C \omega$ is satisfied for every irreducible closed algebraic curve $C \subset X$ tangent to V and normalized by $\overline{C}$. Hyperbolic projective directed manifolds are algebraic hyperbolic, but the converse is not known. The main results of the paper are partial answers to a generalized version of the conjectures mentioned above. Assume that (X, V) is a projective directed manifold and that the canonical bundle of V is big. The generalized Green-Griffiths-Lang conjecture claims the existence of a proper subvariety Y of X with $f(\mathbb{C}) \subset \mathbb{Y}$ for every non-constant holomorphic $f : \mathbb{C} \to \mathbb{X}$ with $f'(\mathbb{C}) \subset \mathbb{V}$. Under the assumption that X is of general type the author proves the existence of global algebraic differential operators P on X with $P(f, f', \ldots, f^{(k)}) = 0$ for every such f. Another interesting result of the author is related to the hyperbolicity of generic algebraic hypersurfaces in $\mathbb{P}^{\ltimes+\rtimes}$ of sufficiently high degree. *Y.-T. Siu* ["Hyperbolicity of generic high-degree hypersurfaces in complex projective spaces", Preprint, arXiv:1209.2723] has shown that there exists a sequence (d_n) in $\mathbb{N}$ with the property that a generic algebraic hypersurface in $\mathbb{P}^{\ltimes+\nVdash}$ of degree $d \geq d_n$, $n \geq 2$, is hyperbolic. *S. Diverio, J. Merker* and *E. Rousseau* [Invent. Math. 180, No. 1, 161–223 (2010; Zbl 1192.32014)] proved that $d_n := 2^{n^5}$ does the job. The paper under review yields a considerable improvement of these estimates. The theorem of Siu is fulfilled for $d_2 := 286$, $d_3 := 7316$ and $d_n := \lfloor \frac{n^4}{3}(n\log(n\log(24n)))^n \rfloor$ for $n \geq 4$.

Reviewer: Eberhard Oeljeklaus (Bremen)

Cited in 2 Documents

MSC:
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32L10 Sections of holomorphic vector bundles 53C55 Hermitian and Kählerian manifolds (global differential geometry)
14J40 Algebraic n-folds ($n > 4$)

Keywords: Kobayashi hyperbolic variety; directed manifold; jet bundle; Chern connection; variety of general type; Green-Griffiths conjecture; Lang conjecture

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Berman, Robert; Demailly, Jean-Pierre

Regularity of plurisubharmonic upper envelopes in big cohomology classes. (English) Zbl 1258.32010

Itenberg, Ilia (ed.) et al., Perspectives in analysis, geometry, and topology. On the occasion of the 60th birthday of Oleg Viro. Based on the Marcus Wallenberg symposium on perspectives in analysis, geometry, and topology, Stockholm, Sweden, May 19–25, 2008. Basel: Birkhäuser (ISBN 978-0-8176-8276-7/hbk; 978-0-8176-8277-4/ebook). Progress in Mathematics 296, 39-66 (2012).

The authors deal with the regularity of certain quasiplurisubharmonic upper envelopes. The main theorem says that if X is a compact complex manifold of the Fujiki class $\mathcal{C}$ (these are the smooth varieties that are bimeromorphic to compact Kähler manifolds, or equivalently, they carry a cohomology class $\{\alpha\} \in H^{1,1}(X,\mathbb{R})$ which is big), then let $\{\alpha\}$ be the aforementioned big class and let T_0 be the current obtained from α by adding the dd^c of a quasiplurisubharmonic function ψ_0 with only analytic singularities, such that it dominates some positive multiple of the fixed Hermitian metric on X. Then the following function (called the upper envelope)

$$\varphi = \sup\left\{\psi \leq 0 : \psi \text{is} \alpha\text{−plurisubharmonic}\right\}$$

is itself quasiplurisubharmonic and has locally bounded second order derivatives outside the analytic set given by the "$-\infty$"-set of ψ_0. The order of blow up near this set is also estimated. The main theorem is then applied to obtain a priori inequalities for the solution of the Dirichlet problem for a degenerate Monge-Ampère operator, to the study of geodesics in the space of Kähler potentials and finally to obtain a logarithmic modulus of continuity for Tsuji's supercanonical metrics. For the entire collection see [Zbl 1230.00045].

Reviewer: Zywomir Dinew (Kraków)

Cited in 1 Review

Cited in 16 Documents

MSC:
32U05 Plurisubharmonic functions and generalizations

Keywords: plurisubharmonic function; quasiplurisubharmonic function; upper envelope; Monge-Ampère operator

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Demailly, Jean-Pierre

Analytic methods in algebraic geometry. (English) Zbl 1271.14001

Surveys of Modern Mathematics 1. Somerville, MA: International Press; Beijing: Higher Education Press (ISBN 978-1-57146-234-3/pbk). 231 p. (2012).

The book under review is based upon a series of lectures given by Jean-Pierre Demailly at the Park City Mathematics Institute in 2008, it was partly published in [IAS/Park City Mathematics Series 17, 295–370 (2010; Zbl 1222.32043)]. The main aim is to give a presentation of analytic techniques especially as it relates to positivity of vector bundles. The book begins by briefly reviewing the concepts of sheaf cohomology, plurisubharmonic functions, currents and other topics. In Chapter 4, it moves on to the Bochner technique and applications to the Akizuki-Nakano-Kodaira vanishing theorem. Chapter 5 covers L^2 estimates and multiplier ideal sheaves. Chapter 6 covers pseudo effective and nef line bundles and Kawamata-Viehweg vanishing and applications. The next chapter covers applications of these ideas, results towards Fujita's conjecture, such as Reider's Theorem and work of Siu. Chapter 8 covers Holomorphic Morse inequalities as introduced by Demailly. Chapter 9 covers effective versions of Matsusaka's big theorem. Chapter 11 covers the question of surjectivity of global sections for maps of vector bundles and an application to the Briancon-Skoda Theorem. Chapter 12 covers the Ohsawa-Takegoshi L^2 Extension Theorem and Skoda's division theorem. Chapter 13 focuses on approximation of positive currents and plurisubharmoic functions and relations to the Hodge Conjecture. The remainder of the book (chapters 15 through 20), cover various topics in higher dimensional algebraic geometry such as Subadditivity of multiplier ideals, invariance of plurigenera, the Kähler cone and Pseudo-efective cone, abundance, and many other topics.

Reviewer: Karl Schwede (University Park)

Cited in 1 Review

Cited in 12 Documents

MSC:
14-02 Research monographs (algebraic geometry)
14F18 Multiplier ideals
14C20 Divisors, linear systems, invertible sheaves
14J40 Algebraic n-folds ($n > 4$)

32C30 Integration on analytic sets and spaces, currents
32U40 Currents

Keywords: positivity; vanishing theorem; multiplier ideal; current; extension theorem; line bundles

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Demailly, Jean-Pierre

Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture. (English) Zbl 1316.32014

Pure Appl. Math. Q. 7, No. 4, 1165-1207 (2011).

Summary: The goal of this work is to study the existence and properties of non constant entire curves f drawn in a complex irreducible n-dimensional variety X, and more specifically to show that they must satisfy certain global algebraic or differential equations as soon as X is projective of general type. By means of holomorphic Morse inequalities and a probabilistic analysis of the cohomology of jet spaces, we are able to reach a significant step towards a generalized version of the Green-Griffiths-Lang conjecture.

Cited in 2 Reviews

Cited in 11 Documents

MSC:
32L20 Vanishing theorems (analytic spaces)
14C30 Transcendental methods, Hodge theory, Hodge conjecture
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds

Demailly, Jean-Pierre

A converse to the Andreotti-Grauert theorem.
(English. French summary) Zbl 1228.32020

Ann. Fac. Sci. Toulouse, Math. (6) 20, Spec. Issue, 123-135 (2011).

Summary: The goal of this paper is to show that there are strong relations between certain Monge-Ampère integrals appearing in holomorphic Morse inequalities, and asymptotic cohomology estimates for tensor powers of holomorphic line bundles. Especially, we prove that these relations hold without restriction for projective surfaces, and in the special case of the volume, i.e. of asymptotic 0-cohomology, for all projective manifolds. These results can be seen as a partial converse to the Andreotti-Grauert vanishing theorem.

Reviewer: Viorel Vâjâitu (Bucureşti)

MSC:
32J25 Transcendental methods of algebraic geometry
32L10 Sections of holomorphic vector bundles
14C20 Divisors, linear systems, invertible sheaves
14F99 Homology and cohomology theory (algebraic geometry)

Keywords: asymptotic cohomology functions; holomorphic Morse inequalities; volume of a line bundle

References:

[1] Andreotti (A.), Grauert (H.).– Théorèmes de finitude pour la cohomologie des espaces complexes, Bull. Soc. Math. France 90, p. 193-259 (1962). Zbl0106.05501 MR150342 · Zbl 0106.05501 · NUMDAM: BSMF_1962__90__193_0 · EUDML: 87019

[2] Angelini (F.).– An algebraic version of Demailly's asymptotic Morse inequalities; arXiv: alg-geom/9503005, Proc. Amer. Math. Soc. 124 p. 3265-3269 (1996). Zbl0860.14019 MR1389502 · Zbl 0860.14019 · DOI: 10.1090/S0002-9939-96-03829-4 · arxiv:alg-geom/9503005

[3] Boucksom (S.).– On the volume of a line bundle, Internat. J. Math. 13, p. 1043-1063 (2002). Zbl1101.14008 MR1945706 · Zbl 1101.14008 · DOI: 10.1142/S0129167X02001575

[4] Boucksom (S.), Demailly (J.-P.), Păun (M.), Peternell (Th.).– The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension, arXiv: math.AG/0405285, see also Proceedings of the ICM 2006 in Madrid. MR1351504

[5] Demailly (J.-P.).– Estimations L^2 pour l'opérateur $\overline{\partial}$ d'un fibré vectoriel holomorphe semi-positif au dessus d'une variété kählérienne complète, Ann. Sci. École Norm. Sup. 15, p. 457-511 (1982). Zbl0507.32021 MR690650 · Zbl 0507.32021 · NUMDAM: ASENS_1982_4_15_3_457_0 · EUDML: 82103

[6] Demailly (J.-P.).– Champs magnétiques et inégalités de Morse pour la d''-cohomologie, Ann. Inst. Fourier (Grenoble) 35, p. 189-229 (1985). Zbl0565.58017 MR812325 · Zbl 0565.58017 · DOI: 10.5802/aif.1034 · NUMDAM: AIF_1985__35_4_189_0 · EUDML: 74695

[7] Demailly (J.-P.).– Holomorphic Morse inequalities, Lectures given at the AMS Summer Institute on Complex Analysis held in Santa Cruz, July 1989, Proceedings of Symposia in Pure Mathematics, Vol. 52, Part 2, p. 93-114 (1991). Zbl0755.32008 MR1128538 · Zbl 0755.32008

[8] Demailly (J.-P.).– Regularization of closed positive currents and Intersection Theory, J. Alg. Geom. 1, p. 361-409 (1992). Zbl0777.32016 MR1158622 · Zbl 0777.32016

[9] Demailly (J.-P.).– Holomorphic Morse inequalities and asymptotic cohomology groups: a tribute to Bernhard Riemann, Milan Journal of Mathematics 78, p. 265-277 (2010). Zbl1205.32017 MR2684780 · Zbl 1205.32017 · DOI: 10.1007/s00032-010-0118-3

[10] Demailly (J.-P.), Ein (L.) and Lazarsfeld (R.).– A subadditivity property of multiplier ideals, Michigan Math. J. 48, p. 137-156 (2000). Zbl1077.14516 MR1786484 · Zbl 1077.14516 · DOI: 10.1307/mmj/1030132712 · arxiv:math/0002035

[11] Demailly (J.-P.), Păun (M.).– Numerical characterization of the Kähler cone of a compact Kähler manifold, arXiv: math.AG/0105176 ; Annals of Math. 159, p. 1247-1274 (2004). Zbl1064.32019 MR2113021 · Zbl 1064.32019 · DOI: 10.4007/annals.2004.159.1247

[12] de Fernex (T.), Küronya (A.), Lazarsfeld (R.).– Higher cohomology of divisors on a projective variety, Math. Ann. 337, p. 443-455 (2007). Zbl1127.14012 MR2262793 · Zbl 1127.14012 · DOI: 10.1007/s00208-006-0044-4

[13] Fujita (T.).– Approximating Zariski decomposition of big line bundles, Kodai Math. J. 17, p. 1-3 (1994). Zbl0814.14006 MR1262949 · Zbl 0814.14006 · DOI: 10.2996/kmj/1138039894

[14] Hironaka (H.).– Resolution of singularities of an algebraic variety over a field of characteristic zero, Ann. of Math. 79, p. 109-326 (1964). Zbl0122.38603 MR199184 · Zbl 0122.38603 · DOI: 10.2307/1970486

[15] Küronya (A.).– Asymptotic cohomological functions on projective varieties, Amer. J. Math. 128, p. 1475-1519 (2006). Zbl1114.14005 MR2275909 · Zbl 1114.14005 · DOI: 10.1353/ajm.2006.0044 · http://muse.jhu.edu/journals/american_journal_of_mathematics/v128/128.6kuronya.pdf

[16] Lazarsfeld (R.).– Positivity in Algebraic Geometry I.-II, Ergebnisse der Mathematik und ihrer Grenzgebiete, Vols. 48-49, Springer Verlag, Berlin, 2004. Zbl1093.14500 MR2095471 · Zbl 1093.14500

[17] Totaro (B.).– Line bundles with partially vanishing cohomology, July 2010, arXiv: math.AG/1007.3955.

Demailly, Jean-Pierre

Structure theorems for projective and Kähler varieties. (English) Zbl 1222.32043

McNeal, Jeffery (ed.) et al., Analytic and algebraic geometry. Common problems, different methods. Lecture notes from the Park City Mathematics Institute (PCMI) graduate summer school on analytic and algebraic geometry, Park City, UT, USA, Summer 2008. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-4908-8/hbk). IAS/Park City Mathematics Series 17, 295-370 (2010).

The main purpose of these notes is to describe some basic structure theorems for projective or compact Kähler varieties and their cohomology, using recent techniques of complex analysis and potential theory. One central unifying concept is that one of positivity, which can be viewed either in algebraic terms (positivity of divisors and algebraic cycles) or in more analytic terms (plurisubharmonicity, positive currents, Hermitian connections with positive curvature). In the 20th century, powerful L^2 techniques have emerged, giving rise to an incredible amount of geometric consequences. Here the author refers to these results and points out several topics: algebro-analytic characterizations of the Kaehler cone, the pseudo-effective cone of divisors, concepts of volume and mobile intersections, super-canonical metrics, non vanishing theorem, finiteness of the canonical ring. For the entire collection see [Zbl 1202.00101].

Reviewer: Gabriela Paola Ovando (Rosario)

Cited in 1 Review

Cited in 1 Document

MSC:
32Q15 Kähler manifolds

Keywords: projective varieties; Kähler varieties; positivity

Demailly, Jean-Pierre (ed.); Hulek, Klaus (ed.); Peternell, Thomas (ed.)

Complex analysis. Abstracts from the workshop held August 29th – September 4th, 2010. (Komplexe analysis.) (English) Zbl 1209.00048

Oberwolfach Rep. 7, No. 3, 2283-2333 (2010).

Summary: The aim of this workshop was to discuss recent developments in several complex variables and complex geometry. Special emphasis was put on the interaction between model theory and the classification theory of complex manifolds. Other topics included Kähler geometry, foliations, complex symplectic manifolds and moduli theory.

MSC:
00B05 Collections of abstracts of lectures
32-06 Proceedings of conferences (several complex variables)
14-06 Proceedings of conferences (algebraic geometry)
32Qxx Complex manifolds
14Jxx Surfaces and higher-dimensional varieties
14Dxx Families, fibrations

Demailly, Jean-Pierre

Holomorphic Morse inequalities and asymptotic cohomology groups: a tribute to Bernhard Riemann. (English. French summary) Zbl 1205.32017

Milan J. Math. 78, 265-277 (2010).

Summary: The goal of this note is to present the potential relationships between certain Monge-Ampère integrals appearing in holomorphic Morse inequalities, and asymptotic cohomology estimates for tensor powers of line bundles, as recently introduced by algebraic geometers. The expected most general statements, which are still conjectural, certainly owe a debt to Riemann's pioneering work, which led to the concept of Hilbert polynomials and to the Hirzebruch-Riemann-Roch formula during the XX-th century.

Cited in 2 Documents

MSC:
32L10 Sections of holomorphic vector bundles
14B05 Singularities (algebraic geometry)
14C17 Intersection theory, etc.

Keywords: holomorphic Morse inequalities; Monge-Ampère integrals; Dolbeault cohomology; asymptotic cohomology groups; Riemann-Roch formula; Hermitian metrics; Chern curvature tensor; plurisuharmonic approximation

References:

[1] Andreotti A., Grauert H.: Théorèmes de finitude pour la cohomologie des espaces complexes. Bull. Soc. Math. France 90, 193–259 (1962) · Zbl 0106.05501

[2] Birkenhage Ch., Lange H.: Complex Abelian Varieties; Second augmented edition, Grundlehren der Math. Wissenschaften, Springer Heidelberg (2004)

[3] Boucksom S.: On the volume of a line bundle. Internat. J. Math. 13, 1043–1063 (2002) · Zbl 1101.14008 · DOI: 10.1142/S0129167X02001575

[4] Berman, R., Demailly, J.-P.: Regularity of plurisubharmonic upper envelopes in big cohomology classes; arXiv: math.CV/0905.1246, to appear in: Perspectives in Analysis, Geometry and Topology, In honor of Oleg Y. Viro, ed. by B. Juhl- Jöricke, I. Itenberg and M. Passare, Birkhaüser.

[5] Boucksom, S., Demailly, J.-P., Păun, M., Peternell, Th.: The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension; arXiv: math.AG/0405285, see also Proceedings of the ICM 2006 in Madrid. · Zbl 1267.32017

[6] Demailly J.-P.: Champs magnétiques et inégalités de Morse pour la d”- cohomologie. Ann. Inst. Fourier (Grenoble) 35, 189–229 (1985) · Zbl 0565.58017

[7] Demailly, J.-P.: Holomorphic Morse inequalities; Lectures given at the AMS Summer Institute on Complex Analysis held in Santa Cruz, July 1989, Proceedings of Symposia in Pure Mathematics, Vol. 52, Part 2 (1991), 93–114.

[8] Demailly, J.-P.: Singular hermitian metrics on positive line bundles; Proceedings of the Bayreuth conference Complex algebraic varieties, April 2-6, 1990, ed. by K. Hulek, T. Peternell, M. Schneider, F. Schreyer, Lecture Notes in Math. ndeg 1507, Springer Verlag, 1992.

[9] Demailly J.-P.: Regularization of closed positive currents and Intersection Theory. J. Alg. Geom. 1, 361–409 (1992) · Zbl 0777.32016

[10] Demailly J.-P., Ein L., Lazarsfeld R.: A subadditivity property of multiplier ideals. Michigan Math. J. 48, 137–156 (2000) · Zbl 1077.14516 · DOI: 10.1307/mmj/1030132712

[11] Demailly J.-P., Păun M.: Numerical characterization of the Kähler cone of a compact Kähler manifold; arXiv: math.AG/0105176; Annals of Math. 159, 1247–1274 (2004) · Zbl 1064.32019

[12] de Fernex T., Küronya A., Lazarsfeld R.: Higher cohomology of divisors on a projective variety. Math. Ann. 337, 443–455 (2007) · Zbl 1127.14012

[13] Fujita T.: Approximating Zariski decomposition of big line bundles. Kodai Math. J. 17, 1–3 (1994) · Zbl 0814.14006 · DOI: 10.2996/kmj/1138039894

[14] Hirzebruch F.: Arithmetic genera and the theorem of Riemann-Roch for algebraic varieties. Proc. Nat. Acad. Sci. U.S.A. 40, 110–114 (1954) · Zbl 0055.38803 · DOI: 10.1073/pnas.40.2.110

[15] Hirzebruch, F.: Neue topologische Methoden in der algebraischen Geometrie; Ergebnisse der Mathematik und ihrer Grenzgebiete (N.F.), Heft 9. Springer Verlag, Berlin-Göttingen-Heidelberg, 1956. English translation: Topological methods in algebraic geometry; Springer Verlag, Berlin, 1966. · Zbl 0070.16302

[16] Küronya A.: Asymptotic cohomological functions on projective varieties. Amer. J. Math. 128, 1475–1519 (2006) · Zbl 1114.14005 · DOI: 10.1353/ajm.2006.0044

[17] Laeng, L.: Estimations spectrales asymptotiques en géométrie hermitienne; Thèse de Doctorat de l'Université de Grenoble I, octobre 2002,
http://www-fourier.ujf-grenoble.fr/THESE/ps/laeng.ps.gz and
http://tel.archives-ouvertes.fr/tel-00002098/en/

[18] Lazarsfeld R.: Positivity in Algebraic Geometry I, II ; Ergebnisse der Mathematik und ihrer Grenzgebiete, Vols. 48–49. Springer Verlag, Berlin (2004)

[19] Riemann, B.: Theorie der Abel'schen Functionen; J. für Math. 54 (1857), also in Gesammelte mathematische Werke (1990), 120–144. · ERAM 054.1427cj

[20] Roch G.: Über die Anzahl der willkurlichen Constanten in algebraischen Functionen. J. für Math. 64, 372–376 (1865) · ERAM 064.1685cj

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Demailly, Jean-Pierre; Kobayashi, Shoshichi; Narasimhan, Raghavan; Siu, Yum-Tong

Cartan and complex analytic geometry. (English) Zbl 1195.01062

Notices Am. Math. Soc. 57, No. 8, 952-960 (2010).

MSC:
01A70 Biographies, obituaries, personalia, bibliographies
32-03 Historical (several complex variables and analytic spaces)

Demailly, Jean-Pierre; Pali, Nefton

Degenerate complex Monge-Ampère equations over compact Kähler manifolds. (English) Zbl 1191.53029

Int. J. Math. 21, No. 3, 357-405 (2010).

The Calabi conjecture was solved by *S.-T. Yau* [Commun. Pure Appl. Math. 31, 339–411 (1978; Zbl 0362.53049)]. *A. Bedford* and *B. A. Taylor* [Invent. Math. 37, 1–44 (1976; Zbl 0315.31007)] initiated a new method for the study of degenerate complex Monge-Ampére equations. In this paper, the authors prove existence and uniqueness of the solution of some very general type of degenerate complex Monge-Ampére equations, and investigate their regularity. Also the existence and fine regularity properties of the solutions of complex Monge-Ampére equations with respect to a given degenerate metric are proved.

Reviewer: Constantin Călin (Iaşi)

Cited in 18 Documents

MSC:
53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.) 53C55 Hermitian and Kählerian manifolds (global differential geometry)
32J15 Compact surfaces (analytic spaces)

Keywords: complex Monge-Ampère equations; Kähler-Einstein metrics; closed positive currents; plurisubharmonic functions; capacities; Orlicz spaces

References:

[1] DOI: 10.1007/978-1-4612-5734-9 · DOI: 10.1007/978-1-4612-5734-9

[2] DOI: 10.1007/BF01418826 · DOI: 10.1007/BF01418826

[3] DOI: 10.1512/iumj.2003.52.2346 · Zbl 1054.32024 · DOI: 10.1512/iumj.2003.52.2346

[4] DOI: 10.1007/s00209-002-0483-x · Zbl 1076.32036 · DOI: 10.1007/s00209-002-0483-x

[5] DOI: 10.1090/S0002-9939-07-08858-2 · Zbl 1116.32024 · DOI: 10.1090/S0002-9939-07-08858-2

[6] Demailly J.-P., Ann. Sci. Ecole Norm. Sup. (4) 15 pp 457–

[7] Demailly J.-P., J. Alg. Geom. 1 pp 361–

[8] DOI: 10.4007/annals.2004.159.1247 · Zbl 1064.32019 · DOI: 10.4007/annals.2004.159.1247

[9] DOI: 10.1007/BF02922247 · Zbl 1087.32020 · DOI: 10.1007/BF02922247

[10] Gilbarg D., Elliptic Partial Differential Equations of Second Order (2001) · Zbl 1042.35002

[11] DOI: 10.2307/1970486 · Zbl 0122.38603 · DOI: 10.2307/1970486

[12] Hörmander L., An Introduction to Complex Analysis in Several Variables 7 (1990)

[13] Iwaniec T., Geometric Function Theory and Non-linear Analysis (2001) · Zbl 1045.30011

[14] DOI: 10.1007/978-3-642-66282-9 · DOI: 10.1007/978-3-642-66282-9

[15] DOI: 10.1007/BF01388524 · Zbl 0593.14010 · DOI: 10.1007/BF01388524

[16] DOI: 10.1007/BF02392879 · Zbl 0913.35043 · DOI: 10.1007/BF02392879

[17] DOI: 10.1512/iumj.2003.52.2220 · Zbl 1039.32050 · DOI: 10.1512/iumj.2003.52.2220

[18] Moishezon B. G., Amer. Math. Soc. Transl. 63 pp 51– · Zbl 0186.26204 · DOI: 10.1090/trans2/063/02

[19] Pali N., Indiana Univ. Math. J.

[20] DOI: 10.1007/BF01459143 · Zbl 0606.32018 · DOI: 10.1007/BF01459143

[21] Peternell Th., Bayreuth. Math. Schr. 54 pp 1–

[22] Rao M. M., Pure and Applied Math 146, in: Theory of Orlicz Spaces (1991) · Zbl 0724.46032

[23] Siciak J., Extremal Plurisubharmonic Functions and Capacities in ℂn (1982) · Zbl 0579.32025

[24] Skoda H., Bull. Soc. Math. France 100 pp 353–

[25] DOI: 10.1007/BF01389077 · Zbl 0599.53046 · DOI: 10.1007/BF01389077

[26] Tian G., Acta Math. Sinica (N.S.) 4 pp 250–

[27] DOI: 10.1007/s11401-005-0533-x · Zbl 1102.53047 · DOI: 10.1007/s11401-005-0533-x

[28] DOI: 10.1007/BF02392630 · Zbl 1036.53052 · DOI: 10.1007/BF02392630

[29] DOI: 10.1090/S0894-0347-06-00552-2 · Zbl 1185.53078 · DOI: 10.1090/S0894-0347-06-00552-2

[30] DOI: 10.1007/BF01449219 · Zbl 0631.53051 · DOI: 10.1007/BF01449219

[31] DOI: 10.1002/cpa.3160310304 · Zbl 0369.53059 · DOI: 10.1002/cpa.3160310304

[32] Zhang Z., Int. Math. Res. Not. pp 18–

[33] Dinh T.-C., Acta Math

[34] DOI: 10.1007/BF01446636 · Zbl 0832.32010 · DOI: 10.1007/BF01446636

[35] DOI: 10.1215/S0012-7094-85-05210-X · Zbl 0578.32023 · DOI: 10.1215/S0012-7094-85-05210-X

Demailly, Jean-Pierre

Estimates on Monge-Ampère operators derived from a local algebra inequality. (English) Zbl 1209.32024

Passare, Mikael (ed.), Complex analysis and digital geometry. Proceedings from the Kiselmanfest, Uppsala, Sweden, May 2006 on the occasion of Christer Kiselman's retirement. Uppsala: Univ. Uppsala (ISBN 978-91-554-7672-4/pbk). 131-143 (2009).

Author's abstract: The goal of this short note is to relate the integrability property of the exponential $e^{-2\varphi}$ of a plurisubharmonic function φ with isolated or compactly

supported singularities to a priori bounds for the Monge-Ampère mass of $(dd^c\varphi)^n$. The inequality is valid locally or globally on an arbitrary open subset Ω in $\mathbb{C}^{\ltimes}$. We show that $\int_\Omega (dd\varphi)^n < n^n$ implies $\int_K e^{-2\varphi} < +\infty$ for every compact subset K in Ω, while functions of the form $\varphi(z) = n \log |z - z_0|$, $z_0 \in \Omega$, appear as limit cases. The result is derived from an inequality of pure local algebra, which turns out a posteriori to be equivalent to it, proved by A. Corti in dimension $n = 2$, and later extended by L. Ein, T. De Fernex and M. Mustaţă to arbitrary dimensions. For the entire collection see [Zbl 1192.00078].

Reviewer: Daniel Barlet (Nancy)

Cited in 2 Reviews

Cited in 2 Documents

MSC:
32W20 Complex Monge-Ampère operators
32U10 Plurisubharmonic exhaustion functions
32S05 Local singularities (analytic spaces)
14B05 Singularities (algebraic geometry)
14C17 Intersection theory, etc.

Keywords: Monge-Ampère operator; local algebra; monomial ideal; Hilbert-Samuel multiplicity; log-canonical threshold; plurisubharmonic function; Ohsawa-Takegoshi L^2 extension theorem; approximation of singularities; birational rigidity

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Demailly, Jean-Pierre (ed.); Hulek, Klaus (ed.); Mok, Ngaiming (ed.); Peternell, Thomas (ed.)

Complex analysis. Abstracts from the workshop held August 24–30, 2008. (Komplexe Analysis.) (English) Zbl 1177.14011

Oberwolfach Rep. 5, No. 3, 2165-2218 (2008).

Introduction: The workshop *Komplexe Analysis*, organised by Jean-Pierre Demailly (Grenoble), Klaus Hulek (Hannover), Ngaiming Mok (Hong Kong) and Thomas Peternell (Bayreuth) was held August 24th–August 30, 2008. This meeting was well attended with 46 participants from Europe, US, and the Far East. The participants included several leaders in the field as well as many young (non-tenured) researchers. The aim of the meeting was to present recent important results in several complex variables and complex geometry with particular emphasis on topics linking different areas of the field, as well as to discuss new directions and open problems. Altogether there were nineteen talks of 60 minutes each, a programme which left sufficient time for informal discussions and joint work on research projects. One of the topics at the center of the conference was the classification theory of higher dimensional varieties. Y. Kawamata lectured on the connections between the minimal model programme and derived categories; A. Corti discussed an approach to the finite generation of the canonical ring without minimal models, but still in connection with the seminal work which was presented by J. McKernan in the last Complex Analysis meeting in Oberwolfach 2006, where the finite generation of the canonical ring of varieties of general type was announced. Extension theorems, non vanishing and positivity result for certain direct image sheaves play a role in the global classification of complex manifolds. This was largely discussed by M. Paun and B. Berndtsson. In their work analytic methods are central, whereas the talks by Kawamata and Corti were more of an algebraic nature. Also very much on the analytic side and connected to Berndtsson's talk, H. Tsuji lectured on generalised Kähler-Einstein metrics. Families of projective manifolds over higher-dimensional base spaces were considered in the talk by S. Kebekus. Direct images of coherent sheaves also play a central role in this context. About five years ago, Campana introduced new variations on the concept of "orbifolds"; they were already the suject of talks in past sessions and have turned out to be of increasing interest – in the present session, new results on the hyperbolicity of orbifolds were presented in the talk by E. Rousseau. As to varieties with special geometry, K. Oguiso spoke on non-algebraic hyperkähler manifolds and, with a rather different flavour, F. Catanese on complex and real threefolds fibered by rational curves, with a special emphasis on real algebraic geometry. J. Chen discussed the influence of terminal singularities in three-dimensional geometry, a more algebraic topic. On the analytic side, A. Teleman reported on recent progress in the classification of non-Kähler surfaces in the so called Kodaira class VII, using gauge-theoretical methods, and S. K. Yeung lectured on new results on fake projective planes. Group actions and envelopes of holomorphy were the topics of the talk by X. Zhou. S. Boucksom discussed equidistribution of Fekete points on complex manifolds, in relation with energy functionals for Monge-Ampère operators. R. Lazarsfeld presented a very interesting new approach to study properties of linear systems and line bundles via convex geometry. Overall, moduli spaces appeared to be a central theme in the workshop, and were discussed extensively in at least four talks: V. Gritsenko considered moduli spaces of K3-surfaces; S. Grushevsky spoke on intersection numbers of divisor on the moduli space of curves, and K. Ludwig and G. Farkas lectured on the moduli spaces of spin and Prym curves, their singularities, Kodaira dimension and enumerative geometry.

MSC:
14-06 Proceedings of conferences (algebraic geometry)
14Jxx Surfaces and higher-dimensional varieties
32-06 Proceedings of conferences (several complex variables)
32Qxx Complex manifolds 00B05 Collections of abstracts of lectures

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Demailly, Jean-Pierre; Hwang, Jun-Muk; Peternell, Thomas

Compact manifolds covered by a torus. (English) Zbl 1144.14035

J. Geom. Anal. 18, No. 2, 324-340 (2008).

Let X be a compact complex manifold that is the image of a complex torus by a surjective holomorphic map $A \to X$. The main theorem of this paper states that X is a Kähler manifold and that, up to taking a finite étale cover, X is a product of projective spaces and a torus. This very nice statement generalises similar results for projective manifolds by *O. Debarre* [C. R. Acad. Sci., Paris, Sér. I 309, No. 2, 119–122 (1989; Zbl 0699.14050)] and *J.-M. Hwang* and *N. Mok* [Math. Z. 238, No. 1, 89–100 (2001; Zbl

1076.14021)]. Technically speaking, the proof falls into two independent parts: In the first part, the authors observe that the morphism $A \to X$ is equidimensional. Since a complex torus is Kähler, a difficult theorem of *J. Varouchas* [Math. Ann. 283, No. 1, 13–52 (1989; Zbl 0632.53059)] then implies that X is also Kähler. Note that the authors give a rather short and self-contained proof of Varouchas' theorem in the appendix. For the second part, we observe that the main theorem implies a-posteriori that the tangent bundle of X is nef. The strategy of the proof is now to show that many of the tools used in the study of manifolds with nef tangent bundle by *J.-P. Demailly, T. Peternell* and *M. Schneider* [J. Algebr. Geom. 3, No. 2, 295–345 (1994; Zbl 0827.14027)] are still available under the hypothesis that X is covered by a torus. For example it is shown that the Albanese map of X is a surjective submersion with connected fibres, and the fundamental group is almost abelian. Furthermore the anticanonical bundle is semi-ample and induces an equidimensional fibration. The main theorem is then established by comparing the Albanese morphism and the anticanonical fibration.

Reviewer: Andreas Höring (Paris)

Cited in 1 Document

MSC:
14J40 Algebraic n-folds ($n > 4$)
14C30 Transcendental methods, Hodge theory, Hodge conjecture
32J25 Transcendental methods of algebraic geometry

Keywords: complex torus; abelian variety; projective space; Kähler manifold; Albanese morphism; fundamental group; étale cover; nef divisor; nef tangent bundle; anticanonical bundle; numerically flat vector bundle

References:

[1] Barlet, D.: Espace analytique réduit des cycles analytiques complexes compacts d'un espace analytique complexe de dimension finie. In: Séminaire F. Norguet: Fonctions de plusieurs variables complexes, 1974/75. Lecture Notes in Math., vol. 482, pp. 1–158. Springer, Berlin (1975)

[2] Birkenhake, C., Lange, H.: Complex Abelian Varieties, 2nd augmented edn. Grundlehren der Mathematischen Wissenschaften, vol. 302. Springer, Berlin (2004) · Zbl 1056.14063

[3] Boucksom, S., Demailly, J.-P., Păun, M., Peternell, Th.: The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension. math.AG/0405285 (2004)

[4] Campana, F.: Coréduction algébrique d'un espace analytique faiblement kählérien compact. Invent. Math. 63(2), 187–223 (1981) · Zbl 0447.32009 · DOI: 10.1007/BF01393876

[5] Campana, F., Peternell, Th.: Cycle spaces. In: Several complex variables, VII, Encyclopaedia Math. Sci., vol. 74, pp. 319–349. Springer, Berlin (1994) · Zbl 0811.32020

[6] Debarre, O.: Images lisses d'une variété abélienne simple. C. R. Acad. Sci. Paris 309, 119–122 (1989) · Zbl 0699.14050

[7] Demailly, J.-P.: Estimations L^2 pour l'opérateur $\overline{\partial}$ d'un fibré vectoriel holomorphe semi-positif au-dessus d'une variété kählérienne complète. Ann. Sci. École Norm. Suppl. 4e Sér. 15, 457–511 (1982)

[8] Demailly, J.-P.: Regularization of closed positive currents and intersection theory. J. Algebraic Geom. 1, 361–409 (1992) · Zbl 0777.32016

[9] Demailly, J.-P.: Monge-Ampère operators, Lelong numbers and intersection theory. In: Ancona, V., Silva, A. (eds.) Complex Analysis and Geometry, Univ. Series in Math., pp. 115–193. Plenum, New York (1993) · Zbl 0792.32006

[10] Demailly, J.-P., Păun, M.: Numerical characterization of the Kähler cone of a compact Kähler manifold. Ann. Math. 159(3), 1247–1274 (2004) · Zbl 1064.32019 · DOI: 10.4007/annals.2004.159.1247

[11] Demailly, J.-P., Peternell, Th., Schneider, M.: Compact complex manifolds with numerically effective tangent bundles. J. Algebraic Geom. 3, 295–345 (1994) · Zbl 0827.14027

[12] Hwang, J.M., Mok, N.: Projective manifolds dominated by abelian varieties. Math. Z. 238, 89–100 (2001) · Zbl 1076.14021 · DOI: 10.1007/PL00004902

[13] Kawamata, Y.: Pluricanonical systems on minimal algebraic varieties. Invent. Math. 79, 567–588 (1985) · Zbl 0593.14010 · DOI: 10.1007/BF01388524

[14] Moishezon, B.G.: On n-dimensional compact varieties with n algebraically indepedent meromorphic functions. Am. Math. Soc. Transl. II. Ser. 63, 51–177 (1967) · Zbl 0186.26204

[15] Nakayama, N.: The lower semi-continuity of the plurigenera of complex varieties. Adv. Stud. Pure Math. 10, 551–590 (1987) · Zbl 0649.14003

[16] Okonek, Ch., Schneider, M., Spindler, H.: Vector Bundles on Complex Projective Spaces, Progress in Mathematics, vol. 3. Birkhäuser, Boston (1980) · Zbl 0438.32016

[17] Păun, M.: Sur l'effectivité numérique des images inverses de fibrés en droites. Math. Ann. 310, 411–421 (1998) · Zbl 1023.32014 · DOI: 10.1007/s002080050154

[18] Richberg, R.: Stetige Streng pseudokonvexe Funktionen. Math. Ann. 175, 257–286 (1968) · Zbl 0153.15401 · DOI: 10.1007/BF02063212

[19] Ueno, K.: Classification theory of algebraic varieties and compact complex spaces. Lecture Notes in Math., vol. 439. Springer, Berlin (1975) · Zbl 0299.14007

[20] Varouchas, J.: Stabilité de la classe des variétés kählériennes par certains morphismes propres. Invent. Math. 77(1), 117–127 (1984) · Zbl 0529.53049 · DOI: 10.1007/BF01389138

[21] Varouchas, J.: Kähler spaces and proper open morphisms. Math. Ann. 283, 13–52 (1989) · Zbl 0632.53059 · DOI: 10.1007/BF01457500

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Demailly, Jean-Pierre; Kosarew, Siegmund; Malgrange, Bernard

Adrien Douady and Banach analytic spaces. (Adrien Douady et les espaces analytiques banachiques.) (French) Zbl 1168.01321

Gaz. Math., Soc. Math. Fr. 113, 35-38 (2007).

MSC:
01A70 Biographies, obituaries, personalia, bibliographies 01A60 Mathematics in the 20th century 46-03 Historical (functional analysis)

Demailly, Jean-Pierre

Kähler manifolds and transcendental techniques in algebraic geometry. (English) Zbl 1141.14007

Sanz-Solé, Marta (ed.) et al., Proceedings of the international congress of mathematicians (ICM), Madrid, Spain, August 22–30, 2006. Volume I: Plenary lectures and ceremonies. Zürich: European Mathematical Society (EMS) (ISBN 978-3-03719-022-7/hbk). 153-186 (2007).

This paper surveys some of the main recent advances in the study of the geometry of projective or compact Kähler manifolds obtained by using local and global complex analytic methods. After a short introduction, section 2 presents well-known definitions and results, necessary for the paper. Section 3 describes the main results obtained by *J.-P. Demailly* and *M. Păun* [Ann. Math. 159, 1247–1274 (2004; Zbl 1064.32019)], namely:

Theorem 3.1. Let X be a compact Kähler manifold. Let $\mathcal{P}$ be the set of real $(1,1)$ cohomology classes $\{\alpha\}$ which are numerically positive on analytic cycles, i.e. such that $\int_Y \alpha^p > 0$, for every irreducible analytic set Y in X, $p = \dim Y$. Then the Kähler cone $\mathcal{K}$ of X is one of the connected components of $\mathcal{P}$.

Corollary 3.2. If X is projective algebraic, then $\mathcal{K} = \mathcal{P}$.

These results (which are new even in the projective case) can be seen as a generalization of the well-known Nakai–Moishezon criterion. Sketches of the proofs are given. The section 3 ends with a consequence about the dual of the cone $\mathcal{K}$ in $H_{\mathbb{R}}^{n-1,n-1}(X)$. Section 4 is devoted to deformations of compact Kähler manifolds. Kodaira showed in the 60s that every Kähler surface X is a limit by deformations of algebraic surfaces. The long-standing question whether a similar property holds in higher dimensions was shown in negative by C. Voisin:

Theorem 4.1. (C. Voisin)
(i) In any dimension ≥ 4, there exist compact Kähler manifolds which do not have the homotopy type (or even the homology ring) of a complex projective manifold.

(ii) In any dimension ≥ 8, there exist compact Kähler manifolds X such that no compact bimeromorphic model X' of X has the homotopy type of a complex projective manifold.

Then, the behaviour of the Kähler cone of X_t as t approaches the "bad strata" (X_t in a deformation of Kähler manifolds) is given (this is a result of the above paper by Demailly - Păun). Section 5 presents results on positive cones in $H^{n-1,n-1}(X)$ and Serre duality. We shall give only two results: Theorem 5.3. (Demailly - Păun) If X is Kähler, then the cones $\bar{\mathcal{K}} \subset H^{1,1}(X, \mathbb{R})$ and $\mathcal{N} \subset H^{n-1,n-1}_{\mathbb{R}}(X)$ are dual by Poincaré duality, and $\mathcal{N}$ is the closed convex cone generated by classes $[Y] \wedge \omega^{p-1}$, where $Y \subset X$ ranges over p-dimensional analytic subsets, $p = 1, 2, \ldots, n$, and ω ranges over Kähler forms. The next result is from the paper of *S. Boucksom, J.-P. Demailly, M. Păun* and *Th. Peternell* [The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension, arXiv:math/0405285]: Theorem 5.14. If X is projective, then a class $\alpha \in NS_{\mathbb{R}}(X)$ is pseudo-effective if (and only if) it is in the dual cone of the cone $SME(X)$ of strongly movable curves. The sections ends with some applications and conjectures. The final section 6 presents new results around the invariance of plurigenera. We give only one result which is a special case of a result of Păun: Corollary 6.3. (Siu) For any projective family $t \mapsto X_t$ of algebraic varieties, the plurigenera $p_m(X_t) = h^\circ(X_t, mK_{X_t})$ do not depend on t. For the entire collection see [Zbl 1111.00009].

Reviewer: Vasile Brînzănescu (Bucureşti)

Cited in 4 Documents

MSC:
14C30 Transcendental methods, Hodge theory, Hodge conjecture
32C30 Integration on analytic sets and spaces, currents
32L20 Vanishing theorems (analytic spaces)

Keywords: projective variety; Kähler manifold; Hodge theory; positive current

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Demailly, Jean-Pierre

Towards a revaluation of mathematics and science teaching: GRIP initiatives and SLECC network classes. (Vers une réévaluation de l'enseignement des mathématiques et des sciences: initiatives du GRIP et réseau de classes SLECC.) (French) Zbl 1343.00016

Gaz. Math., Soc. Math. Fr. 110, 61-64 (2006).

MSC:
00A35 Methodology of mathematics, didactics 97B40 Higher education 97B70 Syllabuses, educational standards

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Demailly, Jean-Pierre (ed.); Hulek, Klaus (ed.); Peternell, Thomas (ed.)

Report 40/2006: Komplexe Analysis (August 27th – September 2nd, 2006). (English) Zbl 1109.14301

Oberwolfach Rep. 3, No. 3, 2399-2446 (2006).

Abstract: The main aim of this workshop was to discuss recent developments in several complex variables and complex geometry. The topics included: classification of higher dimensional varieties, mirror symmetry, hyperbolicity, Kähler geometry and classical geometric questions. Contributions:

MSC:
14-XX Algebraic geometry
32-XX Several complex variables and analytic spaces 00B05 Collections of abstracts of lectures

Demailly, Jean-Pierre; Eckl, Thomas; Peternell, Thomas

Line bundles on complex tori and a conjecture of Kodaira. (English) Zbl 1078.32014

Comment. Math. Helv. 80, No. 2, 229-242 (2005).

In 1963 Kodaira proved that every smooth compact Kähler surface is almost algebraic in the sense that it can be realized as a deformation of a projective surface. Until recently it was an open problem whether it is true in general that a compact Kähler manifold is almost algebraic. An affirmative answer would have implied that every compact Kähler manifold has the homotopy type of a projective algebraic manifold and is projective if it is rigid. But in 2004 *C. Voisin* [Invent. Math. 157, No. 2, 329–343 (2004; Zbl 1065.32010)] constructed for every dimension greater than three compact Kähler manifolds with homotopy type different from the homotopy type of a projective manifold. Her examples are built from compact complex tori by blowing up processes. In the paper under review the authors are equally interested in giving an affirmative answer to the above question for a subclass of compact Kähler manifolds and in finding new counterexamples. They discuss different strategies for $\mathbb{P}(\mathbb{V})$-bundles on complex tori. These considerations are based on the fact that the structure of $\mathbb{P}(\mathbb{V})$-bundles or $\mathbb{P}_\diagdown$-bundles over a compact complex manifold survives under deformation (Theorem 8). The authors consider the following situation: Let A be a three-dimensional compact complex torus and L_1, L_2, L_3 holomorphic line bundles on A representing three linear independent elements in the Néron-Severi group $\mathrm{NS}(A)$. The manifold $Y = \mathbb{P}(\mathcal{O}_\mathbb{A} \oplus \mathbb{L}_{\nVdash}) \times_\mathbb{A} \mathbb{P}(\mathcal{O}_\mathbb{A} \oplus \mathbb{L}_{\nvDash}) \times_\mathbb{A} \mathbb{P}(\mathcal{O}_\mathbb{A} \oplus \mathbb{L}_{\nVdash})$ is a holomorphic $\mathbb{P}^{\nVdash}_{\nvDash}$-bundle over A with a natural holomorphic section Z given by the direct summand $\mathcal{O}_A$ in every factor. The main result of the paper (Theorem 4) asserts that Y is algebraically approximable by projective Albanese bundles $Y_n \to A_n$ with $Y_n = \mathbb{P}(\mathcal{O}_{\mathbb{A}_\ltimes} \oplus \mathbb{L}_{\nVdash}) \times_{\mathbb{A}_\ltimes} \mathbb{P}(\mathcal{O}_{\mathbb{A}_\ltimes} \oplus \mathbb{L}_{\nvDash}) \times_{\mathbb{A}_\ltimes} \mathbb{P}(\mathcal{O}_{\mathbb{A}_\ltimes} \oplus \mathbb{L}_{\nVdash})$ and $\lim_{n\to\infty} A_n = A$ in the sense of deformation theory. The proof uses an explicit description of $\mathrm{NS}(A)$ in terms of skew-symmetric integer 6×6 matrices and calculations with support of the computer algebra program Macauley 2 [see *D. Eisenbud* et al., Algorithms and Computation in Mathematics. 8. (Berlin: Springer) (2002; Zbl 0973.00017)]. Blowing up in the bundle Y every fiber F in the point $F \cap Z$ gives a compact Kähler manifold X, a holomorphic fiber bundle over A with projective rational fiber. Under the additional assumption that not all of the line bundles L_1, L_2, L_3 remain holomorphic under small deformations of A, the manifold X is rigid (Proposition 3) and could a priori be a counterexample. But the assumption on the L_i forces $A_n = A$ in Theorem 4, hence X is already projective. The authors explain their ideas how modifications of their construction and more general settings could eventually lead to new counter-examples.

Reviewer: Eberhard Oeljeklaus (Bremen)

Cited in 3 Documents

MSC:
32J27 Compact Kähler manifolds: generalizations, classification
32G05 Deformations of complex structures
32Q15 Kähler manifolds

Keywords: compact Kähler manifold; almost algebraic; algebraically approximable; Kodaira conjecture Software: Macaulay2

Demailly, Jean-Pierre (ed.); Hulek, Klaus (ed.); Peternell, Thomas (ed.)

Workshop: Complex analysis. (Komplexe Analysis.) (English) Zbl 1078.30501

Oberwolfach Rep. 1, No. 3, Report 42, 2171-2215 (2004).

Contributions:

– Daniel Barlet, Application of complex analysis to oscillating integrals p.2171

– Ingrid C.Bauer (joint with F.Catanese and F.Grunewald), Beauville surfaces without real structures and group theory p.2171

– Michel Brion, Extension of equivariant vector bundles p.2174

– Ciprian S.Borcea, Polygon spaces, tangents to quadrics and special Lagrangians p.2177

– Bertrand Deroin

– Immersed Levi-flat hypersurfaces into non negatively curved complex surfaces p.2179

– Wolfgang Ebeling (joint with Sabir M.Gusein-Zade and José Seade), Indices of 1-forms on singular varieties p.2182

– Akira Fujiki (joint with Massimiliano Pontecorvo), Anti-self-dual hermitian metrics on Inoue surfaces p.2184

– Samuel Grushevsky, Addition formulas for theta functions, and linear systems on abelian varieties p.2186

– Peter Heinzner (joint with Gerald Schwarz), Cartan decomposition of the moment map p.2188

– Jun-Muk Hwang, Bound on the number of curves of a given degree through a general point of a projective variety p.2191

– Priska Jahnke (joint with Thomas Peternell and Ivo Radloff), Threefolds with big and nef anticanonical bundles p.2193

– Shigeyuki Kondo, A complex ball uniformization for the moduli spaces of del Pezzo surfaces via periods of K3 surfaces p.2195

– Michael Lönne, Braid monodromy of hypersurface singularities p.2197

– Laurent Meersseman (joint with Alberto Verjovsky), A foliation of S5 by complex surfaces and its moduli space p.2199

– Keiji Oguiso, Automorphisms of hyperkähler manifolds p.2201

– Edoardo Sernesi (joint with A.Bruno), The non-Petri locus for pencils p.2203

– Andrew J.Sommese (joint with Jan Verschelde and Charles W.Wampler), Numerically decomposing the intersection of algebraic varieties p.2203

– Andrei Teleman (joint with Matei Toma), Complex geometric applications of Gauge Theory p.2204

MSC:
30-06 Proceedings of conferences (functions of one complex variable)
32-06 Proceedings of conferences (several complex variables) 00B05 Collections of abstracts of lectures

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Demailly, Jean-Pierre; Paun, Mihai

Numerical characterization of the Kähler cone of a compact Kähler manifold. (English) Zbl 1064.32019

Ann. Math. (2) 159, No. 3, 1247-1274 (2004).

The Kähler cone of a compact Kähler manifold is the set of cohomology classes of smooth positive definite clossed $(1,1)$-forms. The authors show that this cone depends only on the intersection product of the cohomology ring, the Hodge structure and the homology classes of analytic cycles: if X is a compact Kähler manifold, the Kähler cone $\mathcal{K}$ of X is one of the connected components of the set $\mathcal{P}$ of real $(1,1)$-cohomology classes $\{\alpha\}$ which are numerically positive on the analytic cycles, i.e. such that $\int_Y \alpha^p > 0$ for every irreducible analytic set in X, $p = \dim Y$. This result can be considered as a generalization of the Nakai-Moishezon criterion, which provide a necessary and sufficient criterion for a line bundle to be ample. If X is projective then $\mathcal{K} = \mathcal{P}$. If X is a compact Kähler manifold, the $(1,1)$-cohomology class α is nef (numerically effective free) if and only if there exists a Kähler metric ω on X such that $\int_Y \alpha^k \wedge \omega^{p-k} \geq 0$ for all irreducible analytic sets Y and all $k = 1, 2, \ldots, p = \dim Y$. A $(1,1)$-cohomology class $\{\alpha\}$ on X is nef if and only if for every irreducible analytic set Y in X, $p = \dim Y$, and for every Kähler metric ω on X, one has $\int_Y \alpha \wedge \omega^{p-1} \geq 0$. First, the authors obtain a sufficient condition for a nef class to contain a Kähler current. Then the main result is obtained by an induction on the dimension. The obtained result has an important application to the deformation theory of compact Kähler manifolds: consider $\mathcal{X} \to S$ a deformation of compact Kähler manifolds over an irreducible base S. There exists a countable union $S' = \bigcup S_\nu$ of analytic subsets $S_\nu \subset S$, such that the Kähler cones $\mathcal{K}_t \subset H^{1,1}(X_t, \mathbb{C})$ are invariant over $S \smallsetminus S'$ under parallel transport with respect to the $(1,1)$-projection $\nabla^{1,1}$ of the Gauss-Manin connection.

Reviewer: Vasile Oproiu (Iaşi)

Cited in 8 Reviews

Cited in 68 Documents

MSC:
32Q15 Kähler manifolds
32Q25 Calabi-Yau theory 53C55 Hermitian and Kählerian manifolds (global differential geometry)
32J27 Compact Kähler manifolds: generalizations, classification

Keywords: Kähler manifolds; Kähler cone; nef cohomology classes; Kähler currents

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Demailly, Jean-Pierre

On the geometry of positive cones of projective and Kähler varieties. (English) Zbl 1071.14013

Collino, Alberto (ed.) et al., The Fano conference. Papers of the conference organized to commemorate the 50th anniversary of the death of Gino Fano (1871–1952), Torino, Italy, September 29–October 5, 2002. Torino: Università di Torino, Dipartimento di Matematica. 395-422 (2004).

Summary: The goal of these notes is to give a short introduction to several works by Sébastien Boucksom, Mihai Paun, Thomas Peternell and myself on the geometry of positive cones of projective or Kähler manifolds. Mori theory has shown that the structure of projective algebraic manifolds is – up to a large extent – governed by the geometry of its cones of divisors or curves. In the case of divisors, two cones are of primary importance: the cone of ample divisors and the cone of effective divisors (and the closure of these cones as well). We introduce here the analogous transcendental cones for arbitrary compact Kähler manifolds, and show that these cones depend only on analytic cycles and on the Hodge structure of the base manifold. Also, we obtain new very precise duality statements connecting the cones of curves and divisors via Serre duality. As a consequence, we are able to prove one of the basic conjectures in the classification of projective algebraic varieties – a subject which Gino Fano contributed to in many ways: a projective algebraic manifold X is uniruled (i.e. covered by rational curves) if and only if its canonical class $c_1(K_X)$ does not lie in the closure of the cone spanned by effective divisors. For the entire collection see [Zbl 1051.00013].

Cited in 1 Document

MSC:
14C30 Transcendental methods, Hodge theory, Hodge conjecture
14C20 Divisors, linear systems, invertible sheaves
32J27 Compact Kähler manifolds: generalizations, classification

Keywords: transcendental cones; analytic cycles; Hodge structure; duality

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Demailly, Jean-Pierre; Peternell, Thomas

A Kawamata-Viehweg vanishing theorem on compact Kähler manifolds. (English) Zbl 1077.32504

J. Differ. Geom. 63, No. 2, 231-277 (2003).

This paper appeared earlier under the same title in the proceedings of a conference. See *J.-P. Demailly* and *T. Peternell*, Surv. Differ. Geom. 8, 139–169 (2003; Zbl 1053.32011).

Reviewer: Imre Patyi (Atlanta)

Cited in 1 Review

Cited in 8 Documents

MSC:
32L20 Vanishing theorems (analytic spaces)
32J27 Compact Kähler manifolds: generalizations, classification

Keywords: Kawamata-Viehweg vanishing theorem; compact Kähler spaces; abundance for threefolds

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Demailly, Jean-Pierre; Peternell, Thomas

A Kawamata-Viehweg vanishing theorem on compact Kähler manifolds. (English) Zbl 1053.32011

Yau, S.-T. (ed.), Surveys in differential geometry. Lectures on geometry and topology held in honor of Calabi, Lawson, Siu, and Uhlenbeck at Harvard University, Cambridge, MA, USA, May 3–5, 2002. Somerville, MA: International Press (ISBN 1-57146-114-0/hbk). Surv. Differ. Geom. 8, 139-169 (2003).

This article by well-known experts in complex geometry adds results to the ongoing effort to extend some important parts of Mori's theory of complex projective varieties to the case of compact Kähler manifolds and spaces. It appeared in the proceedings of a prestigious conference, and as a journal paper in [J. Differ. Geom. 63, No. 2, 231–277 (2003; Zbl 1077.32504)] under the same title. The main results of this long and involved paper are as follows. In claim 0.1 the authors obtain a Kawamata-Viehweg vanishing theorem for the cohomology group $H^q(X, K_X + L) = 0$, $q \geq n-1$, where X is a normal compact Kähler space of dimension n, and $L \to X$ is a nef line bundle with $L^2 \neq 0$. The proof of claim 0.1 is via demonstrating that the natural coefficient map induces zero in cohomology $H^{n-1}(X, K_X \otimes L \otimes \mathcal{J}) \to H^{n-1}(X, K_X \otimes L)$, where $\mathcal{J}$ is a suitable multiplier ideal sheaf corresponding to a singular metric h on L. The latter vanishing is reduced to the study of a divisor D associated to h by Siu decomposition, and consists in showing that $H^0(D, (-L + D)|D) = 0$, done by working with Hodge index inequalities. Then claim 0.1 is applied to abundance for threefolds given in claim 0.3: If X is a $\mathbb{Q}$-Gorenstein Kähler threefold with only terminal singularities and K_X nef, then $\kappa(X) \geq 0$ for the Kodaira dimension. The paper is informative and pleasant to read. For the entire collection see [Zbl 1034.53003].

Reviewer: Imre Patyi (Atlanta)

Cited in 1 Review

Cited in 1 Document

MSC:
32L20 Vanishing theorems (analytic spaces)
32J27 Compact Kähler manifolds: generalizations, classification

Keywords: Kawamata-Viehweg vanishing theorem; compact Kähler spaces; abundance for threefolds

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Demailly, Jean-Pierre

On the Frobenius integrability of certain holomorphic p-forms. (English) Zbl 1011.32019

Bauer, Ingrid (ed.) et al., Complex geometry. Collection of papers dedicated to Hans Grauert on the occasion of his 70th birthday. Berlin: Springer. 93-98 (2002).

Summary: The goal of this note is to exhibit the integrability properties (in the sense of the Frobenius theorem) of holomorphic p-forms with values in certain line bundles with semi-negative curvature on a compact Kähler manifold. There are in fact very strong restrictions, both on the holomorphic form and on the curvature of the semi-negative line bundle. In particular, these observations provide interesting information on the structure of projective manifolds which admit a contact structure: either they are Fano manifolds or, thanks to results of Kebekus-Peternell-Sommese-Wisniewski, they are biholomorphic to the projectivization of the cotangent bundle of another suitable projective manifold. For the entire collection see [Zbl 0989.00069].

Cited in 1 Review

Cited in 7 Documents

MSC:
32Q15 Kähler manifolds
32J25 Transcendental methods of algebraic geometry

Keywords: Frobenius integrability; holomorphic p-forms

Bertin, José; Demailly, Jean-Pierre; Illusie, Luc; Peters, Chris

Introduction to Hodge theory. Transl. from the French by James Lewis and Chris Peters. (English) Zbl 0996.14003

SMF/AMS Texts and Monographs. 8. Providence, RI: American Mathematical Society (AMS). ix, 232 p. (2002).

This book is the English translation of the French original published in 1996 under the title "Introduction à la théorie de Hodge" (1996; Zbl 0849.14002). Back then it appeared as volume 3 in the new series "Panoramas et Synthèses" edited by Société Mathématique de France. Grown out of a set of lectures which the authors had delivered at a conference on the present state of Hodge theory (Grenoble 1994), the aim of the text was to develop a number of fundamental concepts and results of both classical and modern Hodge theory, primarily addressed to graduate students and non-expert researchers in the field. In the English translation of this profound introduction to classical and modern Hodge theory, which discusses the subject in great depth and leads the reader to the forefront of contemporary research in many areas related to Hodge theory, the text has been left entirely unchanged.

Now as five years before, this book provides a masterly guide through Hodge theory and its various applications. It still maintains its unique up-to-date character, within the textbook and survey literature on the subject, as well as its significant role as an indispensible source for active researchers and teachers in the field, together with the additional advantage that its translation into English makes it now accessible to the entire mathematical and physical community worldwide. Without any doubt, this is exactly what both those communities and this excellent book on Hodge theory needed and deserved.

Reviewer: Werner Kleinert (Berlin)

Cited in 5 Documents

MSC:
14C30 Transcendental methods, Hodge theory, Hodge conjecture
14-02 Research monographs (algebraic geometry)
14F17 Vanishing theorems
14D07 Variation of Hodge structures 81T30 String and superstring theories 58A14 Hodge theory (global analysis)
14D05 Structure of families
14J32 Calabi-Yau manifolds

Keywords: characteristic *p*; De Rham cohomology algebra; Hodge degeneration; variation of Hodge structures; Gauss-Manin connexion; period domains; Picard-Lefschetz theory; Calabi-Yau manifolds; Higgs bundles; Hodge theory; vanishing theorems; mirror symmetry

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Campana, Frédéric; Demailly, Jean-Pierre

L^2 -cohomology on the coverings of a compact complex manifold. (French)
Zbl 1066.32012

Ark. Mat. 39, No. 2, 263-282 (2001).

The aim of this paper is to define a natural L^2-cohomology on any unramified covering of a complex analytic space X, with values in the lifting of any coherent analytic sheaf on X. This L^2 cohomology has been constructed independently by *P. Eyssidieux* [Math. Ann. 317, 527–566 (2000; Zbl 0964.32008)]. It is seen that the usual properties of sheaf cohomology such as cohomology exact sequences or spectral sequences hold in this L^2-cohomology on X. If X is projective and non-singular there are L^2 vanishing theorems analogous to those of Kodaira-Serre and Kawamata-Viehweg. When X is compact it is possible to define the Γ-dimension for Galois coverings. This Γ-dimension turns out to be finite in this case. An extension of Atiyah's index theorem is given in this context.

Reviewer: A. Diaz-Cano (Madrid)

Cited in 2 Documents

MSC:
32C35 Analytic sheaves and cohomology groups
32C99 General theory of analytic spaces
32T99 Pseudoconvex domains 58J20 Index theory and related fixed-point theorems (PDE on manifolds)

References:

[1] [AG] Andreotti, A. etGrauert, H., Théorèmes de finitude pour la cohomologie des espaces complexes,Bull. Soc. Math. France 90 (1962), 193–259.

[2] [AV] Andreotti, A. etVesentini, E., Carleman estimates for the Laplace-Beltrami equation in complex manifolds,Inst. Hautes Études Sci. Publ. Math 25 (1965), 81–130. · Zbl 0138.06604 · DOI: 10.1007/BF02684398

[3] [A] Atiyah, M., Elliptic operators, discrete groups and von Neumann algebras, dans Elliptic Operators, Discrete Groups and von Neumann Algebras. Colloque ”Analyse et Topologie” en l’honneur de Henri Cartan (Orsay, 1974), Astérisque32–33, p. 43–72, Soc. Math. France, Paris, 1976.

[4] [C] Campana, F., Fundamental group and positivity of cotangent bundles of compact Kähler manifolds,J. Algebraic Geom. 4 (1995), 487–502. · Zbl 0845.32027

[5] [D] Demailly, J.-P.., Estimations L^2 pour l’opérateur $\bar{\partial}$ d’un fibré vectoriel hermitien semi-positif,Ann. Sci. École Norm. Sup. 15 (1982), 457–511.

[6] [E1] Eyssidieux, P., Théorie de l’adjonction L^2 sur le revêtement universel, Preprint, 1997.

[7] [E2] Eyssidieux, P., Systèmes linéaires adjoints L^2.Ann. Inst. Fourier (Grenoble) 49 (1999),vi, ix-x, 141–176.

[8] [E3] Eyssidieux, P., Invariants de von Neumann des faisceaux analytiques cohérents, Math. Ann. 317 (2000), 527–566. · Zbl 0964.32008 · DOI: 10.1007/PL00004413

[9] [G] Gromov, M., Kähler hyperbolicity and L^2-Hodge theory,J. Differential Geom 33 (1991), 263–291. · Zbl 0719.53042

[10] [H]Hörmander, L.,An Introduction to Complex Analysis in Several Variables. 3rd ed., North-Holland Math. Library7, Elsevier, Amsterdam, 1990. · Zbl 0685.32001

[11] [JZ] Jost, J. etZuo, K., Vanishing theorems for L^2-cohomology on infinite coverings of compact Kähler manifolds and applications in algebraic geometry,Comm. Anal. Geom. 8 (2000), 1–30.

[12] [K] Kollár, J.,Shafarevitch Maps and Automorphic Forms, Princeton Univ. Press, Princeton, N. J., 1995.

[13] [NR] Napier, T. etRamachandran, M.., The $L^2\bar{\partial}$ -method, weak Lefschetz theorems, and the topology of Kähler manifolds,J. Amer. Math. Soc. 11 (1998), 375–396. · Zbl 0897.32007 · DOI: 10.1090/S0894-0347-98-00257-4

[14] [O] Ohsawa, T., A reduction theorem for cohomology groups of very stronglyq-convex Kähler manifolds,Invent. Math. 63 (1981), 335–354; Invent. Math.66 (1982) 391–393. · Zbl 0457.32007 · DOI: 10.1007/BF01393882

[15] [P] Pansu, P., Introduction to L^2 Betti numbers, dansRiemannian Geometry (Waterloo, Ont., 1993) (Lovrić, M., Maung, M.-O. et Wang, M. Y.-K., éds) Fields Inst. Monogr.4, p. 53–86, Amer. Math. Soc., Providence, R. I., 1996.

[16] [W] Weil, A.,Introduction à l’étude des variétés kählériennes, Hermann, Paris, 1958.

Demailly, Jean Pierre

Refoundation of mathematics in France. (Italian) Zbl 1106.01306

Lett. Mat. Pristem 42, 10-14 (2001).

The young and (even more) promising author of the present study, Jean-Piere Demailly, approaches the problem of mathematics refoundation from very (highly) French positions, by proudly asserting that the "nouvelles maths" have been initiated in France and nowhere else. The role played in France by the "commission for the long-sum reflexion on the teaching of mathematics" is emphasized, and the name of its president – Jean-Pierre Kahane – is mentioned. Actually, the study is Demailly's contribution to the round table organized in Paris on "Mathematics and the Teaching of Sciences". It is quite a revolutionary text. Extremely "refreshing" for the reader is the idea of having illustrated the text with coloured, challenging reproductions of Picasso.

Reviewer: Cristina Irimia (Iaşi)

MSC:
01A60 Mathematics in the 20th century

Keywords: mathematics; France; J. P. Kahane; teaching of science

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Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael

Pseudo-effective line bundles on compact Kähler manifolds. (English) Zbl 1111.32302

Int. J. Math. 12, No. 6, 689-741 (2001).

Summary: The goal of this work is to pursue the study of pseudo-effective line bundles and vector bundles. Our first result is a generalization of the Hard-Lefschetz theorem for cohomology with values in a pseudo-effective line bundle. The Lefschetz map is shown to be surjective when (and, in general, only when) the pseudo-effective line bundle is twisted by its multiplier ideal sheaf. This result has several geometric applications, e.g., to the study of compact Kähler manifolds with pseudo-effective canonical or anti-canonical line bundles. Another concern is to understand pseudo-effectivity in more algebraic terms. In this direction, we introduce the concept of an "almost" nef line bundle, and mean by this that the degree of the bundle is nonnegative on sufficiently generic curves. It can be shown that pseudo-effective line bundles are almost nef, and our hope is that the converse also holds. This can be checked in some cases, e.g., for the canonical bundle of a projective 3-fold. From this, we derive some geometric properties of the Albanese map of compact Kähler 3-folds.

Cited in 1 Review

Cited in 34 Documents

MSC:
32J27 Compact Kähler manifolds: generalizations, classification
32Q15 Kähler manifolds
32Q57 Classification theorems

References:

[1] DOI: 10.1112/plms/s3-7.1.414 · Zbl 0084.17305 · DOI: 10.1112/plms/s3-7.1.414

[2] Beauville A., J. Diff. Geom. 18 pp 745– (1983)

[3] DOI: 10.1007/BF01393876 · DOI: 10.1007/BF01393876

[4] Campana F., Ann. Sci. ENS 25 pp 539– (1992)

[5] DOI: 10.1002/mana.19971870104 . Zbl 0889.32027 . DOI: 10.1002/mana.19971870104

[6] Campana F., Holomorphic 2-forms on complex 3-folds, J. Algebraic Geom. 9 pp 223– (2000)

[7] DOI: 10.1007/BF02392558 · Zbl 0629.32011 · DOI: 10.1007/BF02392558

[8] DOI: 10.1007/BFb0094512 · DOI: 10.1007/BFb0094512

[9] Demailly J.-P., J. Algebraic Geom. 1 pp 361– (1992)

[10] Demailly J.-P., Compositio Math. 89 pp 217– (1993)

[11] Demailly J.-P., J. Algebraic Geom. 3 pp 295– (1994)

[12] Demailly J.-P., Israel Math. Conf. Proc. 9 pp 165– (1996)

[13] Demailly J.-P., Compositio Math. 101 pp 217– (1996)

[14] Fischer G., Math. 306 pp 88– (1979)

[15] Kawamata Y., Adv. Stud. Pure Math. 10 pp 283– (1987)

[16] DOI: 10.2307/1971351 · Zbl 0598.14015 · DOI: 10.2307/1971351

[17] Kollár J., Algebraic Geom. 1 pp 429– (1992)

[18] Krasnov V. A., Mat. Zametki 17 pp 119– (1975)

[19] DOI: 10.5802/aif.65 · Zbl 0071.09002 · DOI: 10.5802/aif.65

[20] Miyaoka Y., Comment. Math. University St. Pauli 42 pp 1– (1993)

[21] DOI: 10.2307/2007050 · Zbl 0557.14021 · DOI: 10.2307/2007050

[22] Mori S., J. Amer. Math. Soc. 1 pp 177– (1988)

[23] DOI: 10.2307/1971387 · Zbl 0606.14030 · DOI: 10.2307/1971387

[24] DOI: 10.2977/prims/1195144881 · Zbl 0926.14004 · DOI: 10.2977/prims/1195144881

[25] Bull. Soc. Math. France 127 pp 115– (1999) · Zbl 0939.32020 · DOI: 10.24033/bsmf.2344

[26] DOI: 10.1073/pnas.86.19.7299 · Zbl 0711.53056 · DOI: 10.1073/pnas.86.19.7299

[27] DOI: 10.1007/BF01231450 · Zbl 0861.14036 · DOI: 10.1007/BF01231450

[28] DOI: 10.1007/BF01166457 · Zbl 0625.32011 · DOI: 10.1007/BF01166457

[29] DOI: 10.1016/S0764-4442(99)80408-X · Zbl 0894.53051 · DOI: 10.1016/S0764-4442(99)80408-X

[30] DOI: 10.1016/S0007-4497(98)80078-X · Zbl 0946.53037 · DOI: 10.1016/S0007-4497(98)80078-X

[31] DOI: 10.1007/s002080050154 · Zbl 1023.32014 · DOI: 10.1007/s002080050154

[32] DOI: 10.1007/BF02571950 · Zbl 0815.14009 · DOI: 10.1007/BF02571950

[33] DOI: 10.1007/s002080050207 · Zbl 0919.32016 · DOI: 10.1007/s002080050207

[34] Peternell Th., Collectanea Math. 49 pp 465– (1998)

[35] DOI: 10.1007/BF01406077 · Zbl 0205.25102 · DOI: 10.1007/BF01406077

[36] DOI: 10.1007/BF02063212 · Zbl 0153.15401 · DOI: 10.1007/BF02063212

[37] DOI: 10.1007/BF01389965 · Zbl 0289.32003 · DOI: 10.1007/BF01389965

[38] Skoda H., Bull. Soc. Math. France 100 pp 353– (1972) · Zbl 0246.32009 · DOI: 10.24033/bsmf.1743

[39] Takegoshi K., Osaka J. Math. 34 pp 783– (1997)

[40] DOI: 10.1007/BF01458060 · Zbl 0628.32037 · DOI: 10.1007/BF01458060

[41] Zhang Q., J. Reine Angew. Math. 478 pp 57– (1996)

Demailly, Jean-Pierre

Multiplier ideal sheaves and analytic methods in algebraic geometry. (English) Zbl 1102.14300

Demailly, J.-P. (ed.) et al., School on vanishing theorems and effective results in algebraic geometry. Lecture notes of the school held in Trieste, Italy, April 25–May 12, 2000. Trieste: The Abdus Salam International Centre for Theoretical Physics (ISBN 92-95003-09-8/pbk). ICTP Lect. Notes 6, 1-148 (2001).

The lecture notes under review, based on the author's course at the 2000 Trieste school on vanishing theorems and effective results in algebraic geometry, are an extended version of the author's CIME lectures [in: Transcendental methods in algebraic geometry. Lect. 3rd sess. CIME , Cetraro, Italy, 1994. Lect. Notes Math. 1646, 1–97 (1996; Zbl 0883.14005)]. They give a comprehensive survey on the application of analytic methods to algebraic geometry, especially to vanishing theorems. Aimed at non-specialist (in the author's words, they are "written with the idea of serving as an analytic toolbox for algebraic geometers"), they provide a lot of historical and introductory material on the subject, as well as very advanced topics and recent developments. For a detailed account see the review of [loc. cit.]. The main changes are due to the incorporation of *Y. T. Siu*'s new result on the deformation invariance of plurigenera of varieties of general type [Invent. Math. 134, No.3, 661-673 (1998; Zbl 0955.32017)]. For the entire collection see [Zbl 0986.00053].

Reviewer: Olaf Teschke (Berlin)

Cited in 3 Reviews

Cited in 30 Documents

MSC:
14F17 Vanishing theorems
32J25 Transcendental methods of algebraic geometry
32L10 Sections of holomorphic vector bundles
14C20 Divisors, linear systems, invertible sheaves
32Q15 Kähler manifolds
32L20 Vanishing theorems (analytic spaces)

Demailly, J.-P. (ed.); Göttsche, L. (ed.); Lazarsfeld, R. (ed.)

School on vanishing theorems and effective results in algebraic geometry. Lecture notes of the school held in Trieste, Italy, April 25–May 12, 2000. (English) Zbl 0986.00053

ICTP Lecture Notes. 6. Trieste: The Abdus Salam International Centre for Theoretical Physics. vii, 393 p. (2001).

The articles of this volume will be reviewed individually. Indexed articles: *Demailly, Jean-Pierre*, Multiplier ideal sheaves and analytic methods in algebraic geometry., 1-148 [Zbl 1102.14300] *Smith, Karen E.*, Tight closure and vanishing theorems., 149-213 [Zbl 1079.13500] *Helmke, Stefan*, The base point free theorem and the Fujita conjecture., 215-248 [Zbl 1101.14300] *Viehweg, Eckart*, Positivity of direct image sheaves and applications to families of higher dimensional manifolds., 249-284 [Zbl 1092.14044] *Peternell, Thomas*, Subsheaves in the tangent bundle: Integrability, stability and positivity, 285-334 [Zbl 1027.14009] *Hwang, Jun-Muk*, Geometry of minimal rational curves on Fano manifolds., 335-393 [Zbl 1086.14506]

MSC:
00B25 Proceedings of conferences of miscellaneous specific interest
14-06 Proceedings of conferences (algebraic geometry)

Keywords: School; Vanishing theorems; Proceedings Algebraic geometry; Trieste (Italy)

Demailly, Jean-Pierre; Kollár, János

Semi-continuity of complex singularity exponents and Kähler-Einstein metrics on Fano orbifolds. (English) Zbl 0994.32021

Ann. Sci. Éc. Norm. Supér. (4) 34, No. 4, 525-556 (2001).

Let φ be a plurisubharmonic function on a complex manifold X. The complex singularity exponent $c_K(\varphi)$ of φ on a compact set $K \subset X$ is the supremum over $c \geq 0$ such that $\exp(-2c\varphi)$ is integrable on a neighborhood of K. The notion plays an important role in complex analysis and algebraic geometry, and several other characteristics of singularities for analytic objects (holomorphic functions, coherent ideal sheaves, divisors, currents) are its particular cases.

The main results of the paper is lower semicontinuity of the map $\varphi \mapsto c_K(\varphi)$, which means that if $\varphi_j \to \varphi$ in $L^1_{\text{loc}}(X)$ then $\exp(-2c\varphi_j) \to \exp(-2c\varphi)$ in L^1-norm over a neighborhood of K for all positive $c < c_K(\varphi)$.

As a consequence, a comparatively simple proof is given for the existence of Kähler-Einstein metrics on certain Fano orbifolds. In this way, the authors produce three new examples of rigid del Pezzo surfaces with quotient singularities which admit a Kähler-Einstein metric.

Reviewer: Alexandr Yu.Rashkovsky (Khar'kov)

Cited in 13 Reviews

Cited in 74 Documents

MSC:
32S05 Local singularities (analytic spaces)
14B05 Singularities (algebraic geometry)
14J45 Fano varieties
32U05 Plurisubharmonic functions and generalizations
32U25 Lelong numbers

Keywords: Arnold multiplicity; multiplier ideal sheaf; Lelong number; complex singularity exponent; Kähler-Einstein metrics; Fano orbifolds; del Pezzo surfaces

References:

[1] Angehrn U. , Siu Y.-T. , Effective freeness and point separation for adjoint bundles , Invent. Math. 122 (1995) 291 - 308 . MR 1358978 | Zbl 0847.32035 · Zbl 0847.32035 · DOI: 10.1007/BF01231446 · EUDML: 144322

[2] Andreotti A. , Vesentini E. , Carleman estimates for the Laplace-Beltrami equation in complex manifolds , Publ. Math. I.H.E.S. 25 (1965) 81 - 130 . Numdam | Zbl 0138.06604 · Zbl 0138.06604 · DOI: 10.1007/BF02684398 · NUMDAM: PMIHES_1965__25__81_0 · NUMDAM: PMIHES_1965__27__153_0 · EUDML: 103855

[3] Arnold V.I. , Gusein-Zade S.M. , Varchenko A.N. , Singularities of Differentiable Maps , Progress in Math. , Birkhäuser , 1985 . MR 777682

[4] Aubin T. , Équations du type Monge-Ampère sur les variétés kählériennes compactes , C. R. Acad. Sci. Paris Ser. A 283 (1976) 119 - 121 , Bull. Sci. Math. 102 (1978) 63 - 95 . Zbl 0333.53040 · Zbl 0333.53040

[5] Barlet D. , Développements asymptotiques des fonctions obtenues par intégration sur les fibres , Invent. Math. 68 (1982) 129 - 174 . MR 666639 | Zbl 0508.32003 · Zbl 0508.32003 · DOI: 10.1007/BF01394271 · EUDML: 142929

[6] Bombieri E. , Algebraic values of meromorphic maps , Invent. Math. 10 (1970) 267 - 287 , Addendum, Invent. Math. 11 (1970) 163 - 166 . MR 306201 | Zbl 0214.33702 · Zbl 0214.33702 · DOI: 10.1007/BF01418775 · EUDML: 142035

[7] Boyer C. , Galicki K. , New Sasakian-Einstein 5-manifolds as links of isolated hypersurface singularities , Manuscript , February 2000 .

[8] Demailly J.-P. , Nombres de Lelong généralisés, théorèmes d'intégralité et d'analyticité , Acta Math. 159 (1987) 153 - 169 . MR 908144 | Zbl 0629.32011 · Zbl 0629.32011 · DOI: 10.1007/BF02392558

[9] Demailly J.-P. , Transcendental proof of a generalized Kawamata-Viehweg vanishing theorem , C. R. Acad. Sci. Paris Sér. I Math. 309 (1989) 123 - 126 , in: Berenstein C.A. , Struppa D.C. (Eds.), Proceedings of the Conference "Geometrical and Algebraical Aspects in Several Complex Variables" held at Cetraro (Italy) , 1989 , pp. 81 - 94 . Zbl 01648048 · Zbl 1112.32303

[10] Demailly J.-P. , Singular hermitian metrics on positive line bundles , in: Hulek K. , Peternell T. , Schneider M. , Schreyer F. (Eds.), Proc. Conf. Complex algebraic varieties (Bayreuth, April 2-6, 1990) , Lecture Notes in Math. , 1507 , Springer-Verlag , Berlin , 1992 . Zbl 0784.32024 · Zbl 0784.32024

[11] Demailly J.-P. , Regularization of closed positive currents and intersection theory , J. Alg. Geom. 1 (1992) 361 - 409 . MR 1158622 | Zbl 0777.32016 · Zbl 0777.32016

[12] Demailly J.-P. , Monge-Ampère operators, Lelong numbers and intersection theory , in: Ancona V. , Silva A. (Eds.), Complex Analysis and Geometry , Univ. Series in Math. , Plenum Press , New York , 1993 . Zbl 0792.32006 · Zbl 0792.32006

[13] Demailly J.-P. , A numerical criterion for very ample line bundles , J. Differential Geom. 37 (1993) 323 - 374 . MR 1205448 | Zbl 0783.32013 · Zbl 0783.32013

[14] Demailly J.-P. , L^2 vanishing theorems for positive line bundles and adjunction theory , Lecture Notes of the CIME Session, Transcendental Methods in Algebraic Geometry, Cetraro, Italy , July 1994 , 96 p, Duke e-prints alg-geom/9410022 . arXiv | Zbl 0883.14005 · Zbl 0883.14005 · http://alg-geom/9410022

[15] Dolgachev I. , Weighted projective varieties , in: Group Actions and Vector Fields, Proc. Polish-North Am. Semin., Vancouver 1981 , Lect. Notes in Math. , 956 , Springer-Verlag , 1982 , pp. 34 - 71 . Zbl 0516.14014 · Zbl 0516.14014

[16] Fletcher A.R. , Working with weighted complete intersections , Preprint MPI/89-35, Max-Planck Institut für Mathematik , Bonn , 1989 , Revised version: Iano - Fletcher A.R. , in: Corti A., Reid M. (Eds.), Explicit Birational Geometry of 3-folds , Cambridge Univ. Press, 2000, pp. 101-173. Zbl 0960.14027 · Zbl 0960.14027

[17] Fujiki A. , Kobayashi R. , Lu S.S.Y. , On the fundamental group of certain open normal surfaces , Saitama Math. J. 11 (1993) 15 - 20 . MR 1259272 | Zbl 0798.14009 · Zbl 0798.14009

[18] Futaki A. , An obstruction to the existence of Einstein-Kähler metrics , Invent. Math. 73 (1983) 437 - 443 . Zbl 0506.53030 · Zbl 0506.53030 · DOI: 10.1007/ BF01388438 · EUDML: 143056

[19] Hironaka H. , Resolution of singularities of an algebraic variety over a field of characteristic zero, I, II , Ann. Math. 79 (1964) 109 - 326 . MR 199184 | Zbl 0122.38603 · Zbl 0122.38603 · DOI: 10.2307/1970486

[20] Hörmander L. , An Introduction to Complex Analysis in Several Variables , North-Holland Math. Libr. , 7 , North-Holland , Amsterdam , 1973 . MR 1045639 | Zbl 0271.32001 · Zbl 0271.32001

[21] Johnson J.M. , Kollár J. , Kähler-Einstein metrics on log del Pezzo surfaces in weighted projective 3-spaces , Ann. Inst. Fourier 51 (2001) 69 - 79 . Numdam | Zbl 0974.14023 · Zbl 0974.14023 · DOI: 10.5802/aif.1815 · NUMDAM: AIF_2001__51_1_69_0 · EUDML: 115914

[22] Kawamata Y. , Matsuda K. , Matsuki K. , Introduction to the minimal model problem , Adv. Stud. Pure Math. 10 (1987) 283 - 360 . MR 946243 | Zbl 0672.14006 · Zbl 0672.14006

[23] Kollár J. , (with 14 coauthors) , Flips and Abundance for Algebraic Threefolds , Astérisque , 211 , 1992 . Zbl 0814.14038 · Zbl 0814.14038

[24] Kollár J. , Shafarevich Maps and Automorphic Forms , Princeton Univ. Press , 1995 . MR 1341589 | Zbl 0871.14015 · Zbl 0871.14015

[25] Kollár J. , Singularities of pairs, Algebraic Geometry, Santa Cruz, 1995 , in: Proceedings of Symposia in Pure Math. Vol. 62 , AMS , 1997 , pp. 221 - 287 . MR 1492525 | Zbl 0905.14002 · Zbl 0905.14002

[26] Lelong P. , Intégration sur un ensemble analytique complexe , Bull. Soc. Math. France 85 (1957) 239 - 262 . Numdam | MR 95967 | Zbl 0079.30901 · Zbl 0079.30901 · NUMDAM: BSMF_1957__85__239_0 · EUDML: 86922

[27] Lelong P. , Plurisubharmonic Functions and Positive Differential Forms , Gordon and Breach, New York, and Dunod , Paris , 1969 . Zbl 0195.11604 · Zbl 0195.11604

[28] Lichnerowicz A. , Sur les transformations analytiques des variétés kählériennes , C. R. Acad. Sci. Paris 244 (1957) 3011 - 3014 . MR 94479 | Zbl 0080.37501 · Zbl 0080.37501

[29] Lichtin B. , An upper semicontinuity theorem for some leading poles of | f | 2 s , in: Complex Analytic Singularities , Adv. Stud. Pure Math. , 8 , North-Holland , Amsterdam , 1987 , pp. 241 - 272 . MR 894297 | Zbl 0615.32007 · Zbl 0615.32007

[30] Lichtin B. , Poles of | f (z , w)| 2 s and roots of the B -function , Ark. för Math. 27 (1989) 283 - 304 . MR 1022282 | Zbl 0719.32016 · Zbl 0719.32016 · DOI: 10.1007/BF02386377

[31] Manivel L. , Un théorème de prolongement L^2 de sections holomorphes d'un fibré vectoriel , Math. Z. 212 (1993) 107 - 122 . Article | MR 1200166 | Zbl 0789.32015 · Zbl 0789.32015 · DOI: 10.1007/BF02571643 · EUDML: 174474

[32] Matsushima Y. , Sur la structure du groupe d'homéomorphismes analytiques d'une certaine variété kählérienne , Nagoya Math. J. 11 (1957) 145 - 150 . Article | MR 94478 | Zbl 0091.34803 · Zbl 0091.34803 · http://minidml.mathdoc.fr/cgi-bin/location?id=00079219

[33] Nadel A.M. , Multiplier ideal sheaves and existence of Kähler-Einstein metrics of positive scalar curvature , Proc. Nat. Acad. Sci. USA 86 (1989) 7299 - 7300 . Zbl 0711.53056 · Zbl 0711.53056 · DOI: 10.1073/pnas.86.19.7299

[34] Nadel A.M. , Multiplier ideal sheaves and Kähler-Einstein metrics of positive scalar curvature , Annals of Math. 132 (1990) 549 - 596 . Zbl 0731.53063 · Zbl 0731.53063 · DOI: 10.2307/1971429

[35] Ohsawa T. , Takegoshi K. , On the extension of L^2 holomorphic functions , Math. Z. 195 (1987) 197 - 204 . MR 892051 | Zbl 0625.32011 · Zbl 0625.32011 · DOI: 10.1007/BF01166457 · EUDML: 183686

[36] Ohsawa T. , On the extension of L^2 holomorphic functions, II , Publ. RIMS, Kyoto Univ. 24 (1988) 265 - 275 . Article | MR 944862 | Zbl 0653.32012 · Zbl 0653.32012 · DOI: 10.2977/prims/1195175200 ·
http://minidml.mathdoc.fr/cgi-bin/location?id=00259678

[37] Phong D.H. , Sturm J. , Algebraic estimates, stability of local zeta functions, and uniform estimates for distribution functions , preprint , January 1999 , to appear in Ann. of Math. arXiv | MR 1792297 | Zbl 0995.11065 · Zbl 0995.11065 · DOI: 10.2307/2661384 · http://www.math.princeton.edu/ annals/issues/2000/152_1.html · EUDML: 122075

[38] Phong D.H. , Sturm J. , On a conjecture of Demailly and Kollár , preprint , April 2000 . MR 1803721 · Zbl 0978.32004

[39] Shokurov V. , 3-fold log flips , Izv. Russ. Acad. Nauk Ser. Mat. 56 (1992) 105 - 203 . MR 1162635 · Zbl 0785.14023

[40] Siu Y.T. , Analyticity of sets associated to Lelong numbers and the extension of closed positive currents , Invent. Math. 27 (1974) 53 - 156 . MR 352516 | Zbl 0289.32003 · Zbl 0289.32003 · DOI: 10.1007/BF01389965 · EUDML: 142308

[41] Siu Y.T. , Lectures on Hermitian-Einstein metrics for stable bundles and Kähler-Einstein metrics , DMV Seminar (Band 8) , Birkhäuser-Verlag , Basel , 1987 . Zbl 0631.53004 · Zbl 0631.53004

[42] Siu Y.T. , The existence of Kähler-Einstein metrics on manifolds with positive anticanonical line bundle and a suitable finite symmetry group , Ann. of Math. 127 (1988) 585 - 627 . Zbl 0651.53035 · Zbl 0651.53035 · DOI: 10.2307/2007006

[43] Siu Y.T. , An effective Matsusaka big theorem , Ann. Inst. Fourier. 43 (1993) 1387 - 1405 . Numdam | MR 1275204 | Zbl 0803.32017 · Zbl 0803.32017 · DOI: 10.5802/aif.1378 · NUMDAM: AIF_1993__43_5_1387_0 · EUDML: 75042

[44] Skoda H. , Sous-ensembles analytiques d'ordre fini ou infini dans $\mathbb{C}^n$, Bull. Soc. Math. France 100 (1972) 353 - 408 . Numdam | MR 352517 | Zbl 0246.32009 · Zbl 0246.32009 · NUMDAM: BSMF_1972__100__353_0 · EUDML: 87191

[45] Skoda H. , Estimations L^2 pour l'opérateur $\overline{\partial}$ et applications arithmétiques , in: Séminaire P. Lelong (Analyse), année 1975/76 , Lecture Notes in Math. , 538 , Springer-Verlag , Berlin , 1977 , pp. 314 - 323 . Zbl 0363.32004 · Zbl 0363.32004

[46] Tian G. , On Kähler-Einstein metrics on certain Kähler manifolds with c 1 (M)>0 , Invent. Math. 89 (1987) 225 - 246 . Zbl 0599.53046 · Zbl 0599.53046 · DOI: 10.1007/BF01389077 · EUDML: 143479

[47] Varchenko A.N. , Complex exponents of a singularity do not change along the stratum $\mu =$ constant , Functional Anal. Appl. 16 (1982) 1 - 9 . Zbl 0498.32010 · Zbl 0498.32010 · DOI: 10.1007/BF01081801

[48] Varchenko A.N. , Semi-continuity of the complex singularity index , Functional Anal. Appl. 17 (1983) 307 - 308 . Zbl 0536.32005 · Zbl 0536.32005 · DOI: 10.1007/BF01076724

[49] Varchenko A.N. , Asymptotic Hodge structure . . . , Math. USSR Izv. 18 (1992) 469 - 512 . Zbl 0489.14003 · Zbl 0489.14003 · DOI: 10.1070/IM1982v018n03ABEH001395

[50] Yau S.T. , On the Ricci curvature of a complex Kähler manifold and the complex Monge-Ampère equation I , Comm. Pure and Appl. Math. 31 (1978) 339 - 411 . Zbl 0369.53059 · Zbl 0369.53059 · DOI: 10.1002/cpa.3160310304

Demailly, Jean-Pierre; Ein, Lawrence; Lazarsfeld, Robert

A subadditivity property of multiplier ideals. (English) Zbl 1077.14516

Mich. Math. J. 48, Spec. Vol., 137-156 (2000).

Summary: Given an effective $\mathbb{Q}$-divisor D on a smooth complex variety, one can associate to D its multiplier ideal sheaf $J(D)$, which measures in a somewhat subtle way the singularities of D. Because of their strong vanishing properties, these ideals have come to play an increasingly important role in higher dimensional geometry. We prove that for two effective $\mathbb{Q}$-divisors D and E, one has the “subadditivity” relation: $J(D+E) \subseteq J(D).J(E)$. We also establish several natural variants, including the analogous statement for the analytic multiplier ideals associated to plurisubharmonic functions. As an application, we give a new proof of a theorem of *T. Fujita* [Kodai Math. J. 17, No. 1, 1–3 (1994; Zbl 0814.14006)] concerning the volume of a big linear series on a projective variety. The first section of the paper contains an overview of the construction and basic properties of multiplier ideals from an algebro-geometric perspective, as well as a discussion of the relation between some asymptotic algebraic constructions and their analytic counterparts.

MSC:
14E99 Birational geometry
14J17 Singularities of surfaces

Demailly, Jean-Pierre

On the Ohsawa-Takegoshi-Manivel L^2 extension theorem. (English) Zbl 0959.32019

Dolbeault, P. (ed.) et al., Complex analysis and geometry. Proceedings of the international conference in honor of Pierre Lelong on the occasion of his 85th birthday, Paris, France, September 22-26, 1997. Basel: Birkhäuser. Prog. Math. 188, 47-82 (2000).

The Ohsawa-Takegoshi-Manivel L^2 extension theorem addresses the following basic problem: Let Y be a complex analytic submanifold of a complex manifold X; given a holomorphic function f on Y satisfying suitable L^2 conditions on Y, find a holomorphic extension F of f to X, together with a good L^2 estimate for F on X.

The first satisfactory solution of this problem has been obtained by T. Ohsawa and K. Takegoshi. The author follows here a more geometric approach due to L. Manivel, which provides a more general extension theorem in the framework of vector bundles and higher cohomology groups. The first goal of this note is to simplify further Manivel's approach, as well as to point out a technical difficulty in Manivel's proof. The author uses a simplified and slightly extended version of the original Ohsawa-Takegoshi a priori inequality. Then the Ohsawa-Takegoshi-Manivel extension theorem is applied to solve several important problems of complex analysis or geometry. The first of these is an approximation theorem for plurisubharmonic functions. It is shown that the approximation can be made with a uniform convergence of the Lelong numbers of the holomorphic functions towards those of the given plurisubharmonic function. This result contains as a special case Siu's theorem on the analyticity of Lelong number sublevel sets. By combining some of the results provided by the proof of that approximation theorem with Skoda's L^2 estimates for the division of holomorphic functions, a Briançon-Skoda type theorem for Nadel's multiplier ideal sheaves is obtained. Using this result and some ideas of R. Lazarsfeld, it is obtained a new proof of a recent result of T. Fujita: the growth of the number of sections of multiples of a big line bundle is given by the highest power of the first Chern class of the numerically effective part in the line bundle Zariski decomposition. For the entire collection see [Zbl 0940.00031].

Reviewer: A.V.Chernecky (Odessa)

Cited in 18 Documents

MSC:
32D15 Continuation of analytic objects (several variables)
32U05 Plurisubharmonic functions and generalizations

Keywords: L^2 extension theorem; a priori inequality; L^2 existence theorem; approximation theorem; multiplier ideal sheaves; Zariski decomposition of big line bundles; plurisubharmonic functions

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Demailly, Jean-Pierre; El Goul, Jawher

Hyperbolicity of generic surfaces of high degree in projective 3-space. (English) Zbl 0966.32014

Am. J. Math. 122, No.3, 515-546 (2000).

The main result of this paper is to prove that a very generic surface X in $\mathbb{P}^3$ of degree $d \geq 21$ is Kobayashi hyperbolic, that is there is no nonconstant holomorphic map from

$\mathbb{C} \to X$. As a consequence of the proof, they also prove that the complement of a very generic curve in $\mathbb{P}^2$ is hyperbolic and hyperbolically imbedded for all degrees $d \geq 21$. We note that previously, Siu-Yeung proved the hyperbolicity of the complement of a generic smooth curve of high degree in $\mathbb{P}^2$. The approach roughly is divided into the following steps: First use the Riemann-Roch calculations to prove the existence of suitable jet differentials which vanish on an ample divisor; then use Ahlfors-Schwarz lemma to conclude that the image of f sits in the base locus of the global sections of jet differentials; finally, it is hoped to show, by analysing the base locus carefully, that the base locus actually is a proper subvariety of X.

Reviewer: Min Ru (Houston)

Cited in 5 Reviews

Cited in 22 Documents

MSC:
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32H30 Value distribution theory in higher dimensions

Keywords: hyperbolic; jet differentials; Riemann-Roch; surface of general type; Kobayashi hyperbolic

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Demailly, Jean-Pierre

L^2 methods and effective results in algebraic geometry. (Méthodes L^2 et résultats effectifs en géométrie algébrique.) (French) Zbl 0962.14014

Séminaire Bourbaki. Volume 1998/99. Exposés 850-864. Paris: Société Mathématique de France, Astérisque. 266, 59-90, Exp. No. 852 (2000).

The paper is a review of analytic methods (L^2 Hodge theory) used in algebraic geometry for studying adjoint linear systems, vanishing theorem for algebraic vector bundles and invariance of plurigenera of general type families. Among the topics discussed in the paper are singular metrics, applications to Fujita's conjecture [*T. Fujita* in: Algebraic Geometry, Proc. Symp., Sendai 1985, Adv. Stud. Pure Math. 10, 167–178 (1987; Zbl 0659.14002)] on global generation of adjoint linear systems, and analytic tools in Siu's proof [*Y.-T. Siu*, Invent. Math. 134, No. 3, 661–673 (1998; Zbl 0955.32017)] of invariance of plurigenera for a family of general type. For the entire collection see [Zbl 0939.00019].

Reviewer: Taras E.Panov (Moskva)

MSC:
14F43 Other algebro-geometric (co)homologies
14C30 Transcendental methods, Hodge theory, Hodge conjecture
14J60 Vector bundles on surfaces and higher-order varieties, and their moduli
14F05 Sheaves, derived categories of sheaves, etc.
14C20 Divisors, linear systems, invertible sheaves
32J25 Transcendental methods of algebraic geometry
14N30 Adjunction problems

Keywords: adjoint linear systems; line bundle; Fujita conjecture; variety of general type; invariance of plurigenera; Hodge theory

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Demailly, Jean-Pierre

Pseudoconvex-concave duality and regularization of currents. (English) Zbl 0960.32011

Schneider, Michael (ed.) et al., Several complex variables. Cambridge: Cambridge University Press. Math. Sci. Res. Inst. Publ. 37, 233-271 (1999).

The paper investigates some basic properties of Finsler metrics on holomorphic vector bundles in the perspective of obtaining geometric versions of the Serre duality theorem. A duality framework under which pseudo-convexity and pseudo-concavity properties get exchanged is established. These duality properties are related to several geometric problems, e.g., the conjecture of Hartshorne and Schneider.

Finally, a new shorter and more geometric proof of a basic regularization theorem for closed $(1,1)$-currents is shown. For the entire collection see [Zbl 0933.00014].

Reviewer: Viorel Vâjâitu (Bucureşti)

Cited in 5 Documents

MSC:
32F10 q-convexity, q-concavity
32C30 Integration on analytic sets and spaces, currents
32J25 Transcendental methods of algebraic geometry

Keywords: Hartshorne-Schneider conjecture; plurisubharmonic function; regularization of currents; Finsler metrics; pseudoconvexity; pseudoconcavity

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Campana, Frédéric; Demailly, Jean-Pierre; Peternell, Thomas

The algebraic dimension of compact complex threefolds with vanishing second Betti number. (English) Zbl 0910.32032

Compos. Math. 112, No.1, 77-91 (1998).

The abstract of the authors describes the content of the paper quite precisely. It reads (with very small changes) as follows: "This note investigates compact complex manifolds X of dimension three with second Betti number $b_2 = 0$. If X admits a nonconstant meromorphic function, then the authors prove that either $b_1(X) = 1$ and $b_3(X) = 0$ or that $b_1(X) = 0$ and $b_3(X) = 2$. The main idea is to show that $c_3(X) = 0$ by means of a vanishing theorem for generic line bundles on X. As a consequence a compact complex threefold homeomorphic to the 6-Sphere S^6 cannot admit a non-constant meromorphic function. Furthermore they investigate the structure of threefolds with $b_2 = 0$ and algebraic dimension one, in the case when the algebraic reduction $X \to \mathbb{P}_1$ is holomorphic".

Reviewer: E.Oeljeklaus (Bremen)

Cited in 1 Review

Cited in 4 Documents

MSC:
32J17 Compact 3-folds (analytic spaces)
14C20 Divisors, linear systems, invertible sheaves

Keywords: algebraic reduction; generic vanishing theorem; topological Euler characteristic; algebraic dimension

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Demailly, Jean-Pierre

Hyperbolic projective varieties and algebraic differential equations. (Variétés projectives hyperboliques et équations différentielles algébriques.) (French) Zbl 0937.32012

Hirzebruch, Friedrich et al., Journée en l'honneur de Henri Cartan. Paris: Société Mathématique de France, SMF Journ. Annu. 1997, 3-17 (1997).

From the introduction (translated from the French): "The aim of the text is to offer an introduction, as elementary as possible, to an important result concerning the geometry of the images of holomorphic curves in complex algebraic varieties". For the entire collection see [Zbl 0932.00086].

Cited in 2 Documents

MSC:
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32J10 Algebraic dependence theorems (compact analytic spaces)
32L05 Holomorphic fiber bundles and generalizations
32H30 Value distribution theory in higher dimensions
32-02 Research monographs (several complex variables)

Keywords: Nevanlinna theory; geometry of the images of holomorphic curves; complex algebraic varieties

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Hirzebruch, Friedrich; Demailly, Jean-Pierre; Lannes, Jean

Conference in honor of Henri Cartan. (Journée en l'honneur de Henri Cartan.) (French) Zbl 0932.00086

SMF Journée Annuelle. 1997. Paris: Société Mathématique de France, iv, 27 p. (1997).

The articles of this volume will be reviewed individually. Indexed articles: *Hirzebruch, F.*, Learning complex analysis in Münster–Paris, Zürich and Princeton from 1945 to 1953., 1-2 [Zbl 1071.01500] *Demailly, Jean-Pierre*, Hyperbolic projective varieties and algebraic differential equations, 3-17 [Zbl 0937.32012] *Lannes, Jean*, Diverse aspects of the Steenrod operations, 18-27 [Zbl 0934.55002]

MSC:
00B30 Festschriften 00B15 Collections of articles of miscellaneous specific interest
32-06 Proceedings of conferences (several complex variables)

Keywords: Journée; Honneur; Dedication; Conference

Demailly, Jean-Pierre

Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. (English) Zbl 0919.32014

Kollár, János (ed.) et al., Algebraic geometry. Proceedings of the Summer Research Institute, Santa Cruz, CA, USA, July 9–29, 1995. Providence, RI: American Mathematical Society. Proc. Symp. Pure Math. 62(pt.2), 285-360 (1997).

This are notes of a series of lectures delivered at the Santa Cruz AMS Summer School on Algebraic Geometry. They are mainly devoted to the study of complex varieties through a few geometric questions related to hyperbolicity in the sense of Kobayashi. A convenient framework for this is the category of "directed manifolds", that is, the category of pairs (X, V) where X is a complex manifold and V a holomorphic subbundle of T_X. If X is compact, the pair (X, V) is hyperbolic if and only if there are no nonconstant entire holomorphic curves $f : \mathbb{C} \to X$ tangent to V (Brody's criterion). The author describes a construction of projectivized k-jet bundles $P_k V$, which generalizes a construction made by Semple in 1954 and allows to analyze hyperbolicity in terms of negativity properties of the curvature.

An overview information on the lecture notes is given by their contents.

1. Hyperbolicity concepts and directed manifolds

2. Hyperbolicity and bounds for the genus of curves

3. The Ahlfors-Schwarz lemma for metrics of negative curvature

4. Projectivization of a directed manifold

5. Jets of curves and semple jet bundles

6. Jet differentials

7. k-Jet metrics with negative curvature

8. Algebraic criterion for the negativity of jet curvature

9. Proof of the Bloch theorem

10. Logarithmic jet bundles and a conjecture of Lang

11. Projective meromorphic connections and Wronskians

12. Decomposition of jets in irreducible representations

13. Riemann-Roch calculations and study of the base locus

14. Appendix: A vanishing theorem for holomorphic tensor fields. For the entire collection see [Zbl 0882.00033].

Reviewer: J.Eichhorn (Greifswald)

Cited in 8 Reviews

Cited in 27 Documents

MSC:
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32L10 Sections of holomorphic vector bundles
14J40 Algebraic n-folds ($n > 4$) 53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords: Kobayashi hyperbolicity

Demailly, Jean-Pierre

Hyperbolic varieties and algebraic differential equations. (Variétés hyperboliques et équations différentielles algébriques.) (French) Zbl 0901.32019

Gaz. Math., Soc. Math. Fr. 73, 3-23 (1997).

In this survey article, the author presents the relationship between the existence of entire curves (i.e. holomorphic curves $f : \mathbb{C} \to X$) on an algebraic variety X and global algebraic differential operators on the variety X. We mention that the nonexistence of non constant entire curves is equivalent to the Kobayashi's hyperbolicity.

The author gives a complete proof of the following vanishing result of *M. Green* and *Ph. Griffiths*, presented with an incomplete proof in Proc. Int. Chern Symp., Berkely 1979, 41-74 (1980; Zbl 0508.32010)]: "Let X be a projective algebraic variety and let $f : \mathbb{C} \to X$ be a non constant entire curve. Then $P(f', \dots, f^{(k)}) \equiv 0$ for any algebraic differential operator P with values in the dual L^* of a holomorphic line bundle L on X, with positive curvature". As an application one obtaines explicit examples of hyperbolic algebraic surfaces of small degree by applying the above vanishing result to wronskian operators.

Reviewer: Vasile Brînzănescu (Bucureşti)

Cited in 4 Documents

MSC:
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
32-02 Research monographs (several complex variables)
32H30 Value distribution theory in higher dimensions
32A22 Nevanlinna theory (local); growth estimates; other inequalities (several complex variables)

Keywords: hyperbolic varieties; projective varieties of general type; wronskian

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Demailly, Jean-Pierre; El Goul, Jawher

Meromorphic partial projective connections and hyperbolic projective varieties. (Connexions méromorphes projectives partielles et variétés algébriques hyperboliques.) (French. Abridged English version) Zbl 0898.32016

C. R. Acad. Sci., Paris, Sér. I 324, No. 12, 1385-1390 (1997).

S. Kobayashi conjectured in [Hyperbolic manifolds and holomorphic mappings, Marcel Dekker, NY (1970; Zbl 0207.37902)] that a generic hypersurface of $\mathbb{CP}^n$ of sufficiently high degree d (where the expected bound is $d \geq 2n-1$) is hyperbolic. The conjecture is true for $\mathbb{CP}^2$, but for $n \geq 3$ a few number of examples are known. For $\mathbb{CP}^3$ (where the expected bound is 5) the first example of a smooth hyperbolic surface in $\mathbb{CP}^3$ of any degree $d \geq 50$ was obtained by *R. Brody* and *M. Green* [Duke Math. J. 44, 873-874 (1977; Zbl 0383.32009)] and *A. M. Nadel* [Duke Math. J. 58, No. 3, 749-771 (1989; Zbl 0686.32015)] obtained examples of degree $d \geq 21$ and the second author [Manuscr. Math. 90, No. 4, 521-532 (1996)] gave examples of degree $d \geq 14$. In this paper, following some ideas of *Y. T. Siu* [Duke Math. J. 55, 213-251 (1987; Zbl 0623.32018)] and A. Nadel, the authors introduce the concept of meromorphic connection and construct Wronskian operators acting on jets of holomorphic curves. Then using some results, the authors give examples of hyperbolic algebraic surfaces in $\mathbb{CP}^3$ with arbitrary degree $d \geq 11$.

Reviewer: Raul Ibañez (Bilbão)

Cited in 2 Documents

MSC:
32Q45 Hyperbolic and Kobayashi hyperbolic manifolds
14H10 Families, algebraic moduli (curves)
32A20 Meromorphic functions (several variables)
32C25 Analytic subsets and submanifolds 53A20 Projective differential geometry 53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords: meromorphic connection; Wronskian operator; hyperbolic surfaces; complex projective space

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Demailly, Jean-Pierre

Numerical analysis and differential equations. Nouvelle éd. (Analyse numérique et équations différentielles.) (French) Zbl 0869.65041

Grenoble: Presses Univ. de Grenoble. 309 p. (1996). See the review of the German translation (1994; Zbl 0869.65042).

Cited in 2 Reviews

Cited in 1 Document

MSC:
65L05 Initial value problems for ODE (numerical methods) 65L06 Multistep, Runge-Kutta, and extrapolation methods 65D32 Quadrature and cubature formulas (numerical methods) 65H10 Systems of nonlinear equations (numerical methods) 65-01 Textbooks (numerical analysis) 34-01 Textbooks (ordinary differential equations)

Keywords: ordinary differential equation; initial value; problem; roundoff problem; polynomial approximation; quadrature formulas; iterative methods; textbook

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Demailly, Jean-Pierre

L^2 vanishing theorems for positive line bundles and adjunction theory. (English) Zbl 0883.14005

Catanese, F. (ed.) et al., Transcendental methods in algebraic geometry. Lectures given at the 3rd session of the Centro Internazionale Matematico Estivo (CIME), Cetraro, Italy, July 4–12, 1994. Cetraro: Springer. Lect. Notes Math. 1646, 1-97 (1996).

The main goal of the paper is to describe a few analytic tools which are useful to study questions such as linear series and vanishing theorems for algebraic vector bundles. Also, algebraic and analytic proofs of some results are compared. One of the first applications of the analytic method in algebraic geometry is Kodaira's use of the Bochner technique (1950-60) to relate cohomology and curvature via harmonic forms. Well known is the Akizuki-Kodaira-Nakano theorem (1954): If X is a nonsingular projective algebraic variety and L is a holomorphic line bundle on X with positive curvature, then $H^q(X, \Omega^p_X \otimes L) = 0$ for $p + q > \dim X$. Hörmander (1965) used a refinement of this technique to obtain a fundamental L^2 estimate, concerning solutions of the Cauchy-Riemann operator. Except vanishing theorems, more precise quantative information about solutions of $\bar\partial$-equations was obtained. Main tools to relate analytic and algebraic geometry are the multiplier ideal sheaf $I(\phi)$ and positive currents. $I(\phi)$ is defined as a sheaf of germs of holomorphic functions f such that $|f|^2 e^{-2\phi}$ is locally summable, where ϕ is a (locally defined) plurisubharmonic function. Since $I(\phi)$ is a coherent algebraic sheaf over X, we have a direct correspondence between analytic and algebraic objects which takes into account singularities efficiently. Currents, introduced by Lelong (1957), play the role of algebraic cycles, and many classical results of intersection theory apply to currents. Also an analytic interpretation of the Seshadri constant of a line bundle is given and it represents a measure of local positivity. One of the motivations for this work was the conjecture of Fujita: If L is an ample (i.e. positive) line bundle on a projective n-dimensional algebraic variety X then $K_X + (n + 2)L$ is very ample. Reider (1988) gave a proof of the Fujita conjecture in the case of surfaces.

Using an analytic approach, in the paper under review it is shown that $2K_X + L$ is very ample under suitable numerical conditions for L. The first part of the proof is to choose an appropriate metric using a complex Monge-Ampère equation and the Aubin-Calabi-Yau theorem. Solution ϕ of the equation assumes logarithmic poles and they are controlled using the intersection theory of currents. Detailed relations to the existing algebraic proofs of similar results are given (Ein-Lazarsfeld, Fujita, Siu). In the last section, a proof of the effective Matsusaka big theorem obtained by *Y.-T. Siu* [Ann. Inst. Fourier 43, No. 5, 1387-1405; Zbl 0803.32017)] is presented. Siu's proof is based on the very ampleness of $2K_X + mL$ together with the theory of holomorphic Morse inequalities [*J.-P. Demailly*, Ann. Inst. Fourier 35, No. 4, 189-229 (1985; Zbl 0565.58017)]. Long and detailed preliminary sections dedicated to the basic facts of complex differential geometry are included which make the main ideas of the paper easier to understand. For the entire collection see [Zbl 0855.00017].

Reviewer: N.Blažić (Beograd)

Cited in 1 Review

Cited in 16 Documents

MSC:
14F17 Vanishing theorems
32L05 Holomorphic fiber bundles and generalizations
14F43 Other algebro-geometric (co)homologies
14F05 Sheaves, derived categories of sheaves, etc.
32L20 Vanishing theorems (analytic spaces)
32C30 Integration on analytic sets and spaces, currents
32W20 Complex Monge-Ampère operators

Keywords: positive line bundle; linear series; vanishing theorems; Lelong number; intersection theory; Bochner technique; L^2 estimates; Seshadri constant; numerically effective line bundle; Fujita conjecture; Monge-Ampere equation; very ample line bundle; algebraic vector bundles; effective Matsusaka big theorem

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Bertin, José; Demailly, Jean-Pierre; Illusie, Luc; Peters, Chris

Introduction to Hodge theory. (Introduction à la théorie de Hodge.) (French. English summary) Zbl 0849.14002

Panoramas et Synthèses. 3. Paris: Société Mathématique de France. vi, 272 p. (1996).

The origin of what is currently meant by the notion of Hodge theory can be traced back to *W. V.D. Hodge*'s fundamental work accomplished in the 1930s. In modern terminology, Hodge prepared the ground for describing the De Rham cohomology algebra of a Riemannian manifold in terms of its harmonic differential forms. In the following two decades, Hodge's decomposition principle has been extended to the (then) new sheaf-theoretic and cohomological framework of Hermitean differential geometry, complex-analytic geometry, and transcendental algebraic geometry. The names of G. De Rham, A. Weil, K. Kodaira, and many others stand for the tremendous progress achieved during this period, in particular with regard to deformation and classification theory in these areas. The special algebraic structures (Hodge structures) arising from Hodge decompositions and their generalizations have led to a rather independent field of research in geometry, precisely to the so-called Hodge theory, which represents a powerful and indispensible toolkit for contemporary complex geometry, general algebraic geometry, and – nowadays – also for mathematical physics. The vast activity in Hodge theory and its related areas, especially during the recent twenty years, is not reflected in the current textbook literature, at least not comprehensively or in an updated form compiling the various recent aspects and applications, so that a panoramic overview of the present state of art must be regarded as a highly welcome (and needed) service to the mathematical community.

A conference on the present state of Hodge theory, serving exactly that purpose, took place at the University of Grenoble (France) in November 1994. The book under review grew out of the series of lectures which the authors delivered at this meeting. The aim of the text is to develop a number of fundamental concepts and results of classical and modern Hodge theory, and in this the book is prepared for students and non-expert researchers in the field, who wish to get acquainted in depth with the subject, and obtain a profound up-to-date knowledge of its present level of development. – The material is divided into three main parts, each of which is written by different authors and devoted to various central and complementary aspects of the theory.

Part I, written by *J.-P. Demailly*, is entitled " L^2-Hodge theory and vanishing theorems". The author discusses in detail two fundamental applications of Hilbert L^2-space methods to complex analysis and algebraic geometry, respectively. This part adopts basically the analytic viewpoint and consists, on its side, of two chapters. Chapter 1 provides an introduction to standard complex Hodge theory, including the basics on Hermitean and Kähler geometry, differential operators on vector bundles, Hodge decomposition, Hodge degeneration, the spectral sequence of Hodge-Frölicher, Gauss-Manin connexion, and the deformation behavior of the Hodge groups (after Kodaira). Chapter 2 is devoted to L^2-estimates for the $\overline{\partial}$-operator and the resulting vanishing theorems for cohomology groups of Kähler manifolds and projective varieties. The main topics here are the classical methods of Oka, Bochner, and Hörmander in pseudo-convex analysis, their consequences for cohomology vanishing, as well as the more recent but already well-known fundamental contributions by the author himself towards the interpretation of the great vanishing theorems of A. Nadel and of Kawamata-Viehweg. – The concluding two sections of this chapter deal with the property of very-ampleness of line bundles on projective varieties. The first central result discussed here is the author's analytic approach to the famous conjecture of Fujita, culminating in an improvement of *Y.-T. Siu*'s very recent theorem on an effective bound for very-ampleness [cf. "Effective very ampleness", Invent. Math. 124, No. 1-3, 563-571 (1996)]. The second central result is an effective version of the classical "Big embedding theorem of Matsusaka", whose surprisingly simple proof is due to the author himself (1996), based on some foregoing work of *Y.-T. Siu* [Ann. Inst. Fourier 43, No. 5, 1387-1405 (1993; Zbl 0803.32017)], and methodically related to the effective bound for very-ampleness discussed before. These two last sections provide a particularly up-to-data account on the newest developments in analytical Hodge theory and its (algebraic) applications.

Part II of the text, written by *L. Illusie*, is entitled "Frobenius and Hodge degeneration". These notes aim at introducing non-specialists to those methods and techniques of algebraic geometry over a field of characteristic $p > 0$, which have been used by P. Deligne and the author to give an algebraic proof of the Hodge degeneration and the Kodaira-Akizuki-Nakano vanishing theorem for smooth projective varieties in characteristic zero. Basically, this part of the book is a careful, detailed introduction to the important work "Relèvements modulo p^2 et décomposition du complexe de De Rham" [Invent. Math. 89, 247-270 (1987; Zbl 0632.14017)] by *P. Deligne* and *L. Illusie*. Here the reader is assumed to bring along some basic knowledge of the theory of algebraic schemes and of homological algebra (in categories). After recalling the basics on schemes, differentials and the algebraic De Rham complex in characteristic $p > 0$, the author discusses the following topics: smoothness and coverings, the Frobenius morphism and the Cartier isomorphism, derived categories and spectral sequences, decomposition theorems, vanishing theorems in characteristic p, degeneration theorems, the standard techniques for passing from characteristic p to characteristic zero, and the proof of the above mentioned degeneration and vanishing theorems. The concluding section of this part points to some recent developments and open problems concerning Hodge theory in characteristic p.

Also this part is essentially self-contained, and most proofs are given in detail. Some proofs are – quite naturally – at least outlined, assuming the reader to follow the precise hints to the related textbook literature (mostly EGA) and original papers.

Part III of the book, written by *J. Bertin* and *C. Peters*, is entitled "Variations of Hodge structures, Calabi-Yau manifolds, and mirror symmetry". It consists again of two main chapters, whose interrelation is beautifully explained in a comprehensive introduction. – Chapter I is devoted to the comparatively elementary part of the theory of variation of Hodge structures and its applications in complex algebraic geometry. This includes detailed descriptions of the Hodge bundles, the Hodge filtrations, the De Rham cohomology sheaves, the Gauss-Manin connexion in its general setting (after Katz and Oda) and with its transversality property (due to Griffiths), variations and infinitesimal variations of Hodge structures, the Griffiths period domains for polarized Hodge structures, mixed Hodge structures, limits of Hodge structures (after Deligne), the Picard-Lefschetz theory and the local monodromy theorem, Deligne's degeneration criteria for Hodge spectral sequences, and a brief discussion of the method of vanishing cycles. At the end, the authors give a sketch of the use of Higgs bundles for the construction of variations of Hodge structures, mainly by following Simpson's approach [cf.: *C. T. Simpson*, Proc. Int. Congr. Math., Kyoto 1990, Vol. I, 747-756 (1991; Zbl 0765.14005)], as well as some comments on M. Saito's work on Hodge modules, intersection cohomology, and $\mathcal{D}$-modules in algebraic analysis. – Chapter II reflects the fact that Calabi-Yau manifolds, their Hodge theory, and their mirror symmetry have recently gained enormous significance in both algebraic geometry and theoretical physics, particularly in constructing two-dimensional conformal quantum field theories. The material presented here covers the fundamental facts on Calabi-Yau manifolds, their construction and deformation theory, and their mirror properties. After a digression on the cohomology of hypersurfaces (after Griffiths and Dimca), which is used for the description of the link between the Picard-Fuchs equation and the variation of Calabi-Yau structures, the variation of Hodge structures for families of Calabi-Yau threefolds, their Yukawa couplings, and their mirror symmetries are explained in more depth. The interested reader can find a very complete and comprehensive account on this subject in the recent monography "Symétrie miroir" by *C. Voisin* [Panoramas et Synthèses, No. 2 (1996; see the preceding review)]. In a concluding section, the authors discuss (following an idea of P. Deligne) a possible approach to mirror symmetry via a certain duality between variations of Hodge structures for Calabi-Yau threefolds. A rich bibliography enhances this very systematic and lucid treatise.

Altogether, the present book, in all its three parts, which consistently refer to each other, may be regarded as a masterly introduction to Hodge theory in its classical and very recent, analytic and algebraic aspects. Aimed to students and non-specialists, it is by far much more than only an introduction to the subject. The material leads the reader to the forefront of research in many areas related to Hodge theory, and that in a detailed and highly self-contained manner. As such, this text is also a valuable source for active researchers and teachers in the field, in particular due to the utmost carefully arranged index at the end of the book.

Reviewer: W.Kleinert (Berlin)

MSC:
14C30 Transcendental methods, Hodge theory, Hodge conjecture
14F17 Vanishing theorems
14-02 Research monographs (algebraic geometry)
14D07 Variation of Hodge structures 13A35 Characteristic p methods; tight closure
58A14 Hodge theory (global analysis)
14D05 Structure of families 81T30 String and superstring theories
14J32 Calabi-Yau manifolds

Keywords: characteristic p; De Rham cohomology algebra; Hodge theory; vanishing theorems; very-ampleness of line bundles; Hodge degeneration; De Rham complex; Frobenius morphism; Cartier isomorphism; variation of Hodge structures; Gauss-Manin connexion; period domains; Picard-Lefschetz theory; Higgs bundles; Calabi-Yau manifolds; two-dimensional conformal quantum field theories; Picard-Fuchs equation; Yukawa couplings; mirror symmetries

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Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael

Compact Kähler manifolds with Hermitian semipositive anticanonical bundle. (English) Zbl 1008.32008

Compos. Math. 101, No.2, 217-224 (1996).

Summary: This note states a structure theorem for compact Kähler manifolds with semipositive Ricci curvature: Any such manifold has a finite étale covering possessing a de Rham decomposition as a product of irreducible compact Kähler manifolds, each one being either Ricci flat (torus, symplectic or Calabi-Yau manifold) or Ricci semipositive without nontrivial holomorphic forms. Related questions and conjectures concerning the latter case are discussed.

Cited in 1 Review

Cited in 9 Documents

MSC:
32J27 Compact Kähler manifolds: generalizations, classification 53C55 Hermitian and Kählerian manifolds (global differential geometry)

References:

[1] Aubin, T. : Equations du type Monge-Ampère sur les variétés kähleriennes compactes . C. R. Acad. Sci. Paris Ser. A 283 (1976) 119-121; Bull. Sci. Math. 102 (1978) 63-95. · Zbl 0374.53022

[2] Beauville, A. : Variétés kähleriennes dont la première classe de Chern est nulle . J. Diff. Geom. 18 (1983) 775-782. · Zbl 0537.53056 · DOI: 10.4310/jdg/1214438181

[3] Berger, M. : Sur les groupes d'holonomie des variétés à connexion affine des variétés riemanniennes . Bull. Soc. Math. France 83 (1955) 279-330. · Zbl 0068.36002 · DOI: 10.24033/bsmf.1464 · NUMDAM: BSMF_1955__83__279_0 · EUDML: 86895

[4] Bishop, R. : A relation between volume, mean curvature and diameter . Amer. Math. Soc. Not. 10 (1963) p. 364.

[5] Bogomolov, F.A. : On the decomposition of Kähler manifolds with trivial canonical class . Math. USSR Sbornik 22 (1974) 580-583. · Zbl 0304.32016 · DOI: 10.1070/SM1974v022n04ABEH001706

[6] Bogomolov, F.A. : Kähler manifolds with trivial canonical class . Izvestija Akad. Nauk 38 (1974) 11-21. · Zbl 0299.32022 · DOI: 10.1070/IM1974v008n01ABEH002093

[7] Brückmann, P. and Rackwitz, H.– G.: T-symmetrical tensor forms on complete intersections . Math. Ann. 288 (1990) 627-635. · Zbl 0724.14032 · DOI: 10.1007/BF01444555 · EUDML: 164757

[8] Campana, F. : Fundamental group and positivity of cotangent bundles of compact Kähler manifolds . Preprint 1993. · Zbl 0845.32027

[9] Cheeger, J. and Gromoll, D. : The splitting theorem for manifolds of nonnegative Ricci curvature . J. Diff. Geom. 6 (1971) 119-128. · Zbl 0223.53033 · DOI: 10.4310/jdg/1214430220

[10] Cheeger, J. and Gromoll, D. : On the structure of complete manifolds of nonnegative curvature . Ann. Math. 96 (1972) 413-443. · Zbl 0246.53049 · DOI: 10.2307/1970819

[11] Demailly, J.-P. , Peternell, T. and Schneider, M. : Kähler manifolds with numerically effective Ricci class . Compositio Math. 89 (1993) 217-240. · Zbl 0884.32023 · NUMDAM: CM_1993__89_2_217_0 · EUDML: 90258

[12] Demailly, J.-P. , Peternell, T. and Schneider, M. : Compact complex manifolds with numerically effective tangent bundles . J. Alg. Geom. 3 (1994) 295-345. · Zbl 0827.14027

[13] Kobayashi, S. : Recent results in complex differential geometry . Jber. dt. Math.-Verein. 83 (1981) 147-158. · Zbl 0467.53030

[14] Kobayashi, S. : Topics in complex differential geometry . In DMV Seminar , Vol. 3., Birkhäuser 1983. · Zbl 0506.53029

[15] Lichnerowicz, A. : Variétés kähleriennes et première classe de Chern . J. Diff. Geom. 1 (1967) 195-224. · Zbl 0167.20004 · DOI: 10.4310/jdg/1214428089

[16] Lichnerowicz, A. : Variétés Kählériennes à première classe de Chern non négative et variétés riemanniennes à courbure de Ricci généralisée non négative . J. Diff. Geom. 6 (1971) 47-94. · Zbl 0231.53063 · DOI: 10.4310/jdg/1214430218

[17] Manivel, L. : Birational invariants of algebraic varieties . Preprint Institut Fourier, no. 257 (1993). · Zbl 0811.14008

[18] Ogus, A. : The formal Hodge filtration . Invent. Math. 31 (1976) 193-228. · Zbl 0339.14004 · DOI: 10.1007/BF01403145 · EUDML: 142361

[19] Yau, S.T. : On the Ricci curvature of a complex Kähler manifold and the complex Monge-Ampère equation I . Comm. Pure and Appl. Math. 31 (1978) 339-411. · Zbl 0369.53059 · DOI: 10.1002/cpa.3160310304

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Demailly, Jean-Pierre

Effective bounds for very ample line bundles. (English) Zbl 0862.14004

Invent. Math. 124, No.1-3, 243-261 (1996).

Let L be an ample line bundle on a nonsingular projective n-fold X. A well-known conjecture of T. Fujita asserts that $K_X + (n+1)L$ is generated by global sections and $K_X + (n+2)L$ is very ample. For $n = 2$ this follows from I. Reider's theorem and the global generation part of the conjecture was proved for $n = 3$ by *L. Ein* and *R. Lazarsfeld* [J. Am. Math. Soc. 6, No. 4, 875-903 (1993; Zbl 0803.14004)]. The present paper is mainly concerned with the very ampleness part of the conjecture. In a previous paper [J. Differ. Geom. 37, No. 2, 323-374 (1993; Zbl 0783.32013)] the author proved that $2K_X + 12n^nL$ is very ample, using an analytic method based on the solution of a Monge-Ampère equation. In the present paper, improving a method of *Y.-T. Siu* [Invent. Math. 124, No. 1-3, 563-571 (1996; Zbl 0853.32034)] based on a combination of the Riemann-Roch formula with the vanishing theorem of *A. M. Nadel* [Ann. Math., II. Ser. 132, No. 3, 549-596 (1990; Zbl 0731.53063)] the author proves that $2K_X + mL$ is very ample for $m \geq 2 + \binom{3n+1}{n}$ and that $m(K_X + (n+2)L)$ is very ample for $m \geq \binom{3n+1}{n} - 2n$. The method of proof gives, as a byproduct, the well-known fact that $K_X + (n+1)L$ is numerically effective (a result originally proved as a consequence of Mori theory). The paper also contains a refinement of a method developed by *Y.-T. Siu* [Ann. Inst. Fourier 43, No. 5, 1387-1405 (1993; Zbl 0803.32017)] which enables the author to obtain a better effective Matsusaka big theorem.

Reviewer: I.Coandă (Bucureşti)

Cited in 7 Documents

MSC:
14C20 Divisors, linear systems, invertible sheaves
14F05 Sheaves, derived categories of sheaves, etc.
14F17 Vanishing theorems
32C20 Normal analytic spaces

Keywords: Hermitian metrics on line bundles; Fujita conjecture; ample line bundle; very ampleness; vanishing theorem; Mori theory; Matsusaka big theorem

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Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael

Holomorphic line bundles with partially vanishing cohomology. (English)
Zbl 0859.14005

Teicher, Mina (ed.), Proceedings of the Hirzebruch 65 conference on algebraic geometry, Bar-Ilan University, Ramat Gan, Israel, May 2-7, 1993. Ramat-Gan: Bar-Ilan University, Isr. Math. Conf. Proc. 9, 165-198 (1996).

Let X denote a complex manifold of dimension n. The authors study holomorphic line bundles L on X with partially vanishing cohomology (or having metrics with positive eigenvalues of curvature). They define $\sigma_+(L)$ to be the smallest integer q with the following property: There exists an ample divisor D on X and a constant $c > 0$ such that $H^j(X, mL - pD) = 0$ for all $j > q$ and $mp \geq 0$, $m \geq c(p+1)$. Note that $\sigma_+(L) = 0$ if and only if L is ample while $\sigma_+(L) = n$ if and only if $c_1(L^*)$ is in the closure of the cone of effective divisors. An ample q-flag is defined as a sequence $Y_q \subset Y_{q+1} \subset \cdots \subset Y_n = X$ of subvarieties Y_k of X such that $\dim Y_k = k$ and Y_k is the image of an ample Cartier divisor in the normalization of Y_{k+1}. Then a line bundle L is called q-flag positive if for some ample q-flag, $L \mid Y_q$ is positive.

Vanishing theorem: If $L \in$ PicX is q-flag positive then $\sigma_+(L) \leq n - q$.

The converse of this theorem is not true in general. A counter example and a positive result (of converse) for $\mathbb{P}_{n-1}$ bundles over a curve are given. The structure of projective 3-folds with $\sigma_+(-K_X) = 1$, K_X = canonical bundle, is investigated. One has $\sigma_+(-K_X) = 0$ if and only if X is Fano and $\sigma_+(-K_X) \leq 2$ if and only if $\kappa(X) = -\infty$. The authors also study various cones in $NX(X) \otimes \mathbb{R}$, $NX(X)$ being Néron-Severi group, i.e. the group of divisors modulo numerical equivalence. All these cones coincide for surfaces. For the entire collection see [Zbl 0828.00035].

Reviewer: U.N.Bhosle (Bombay)

Cited in 2 Reviews

Cited in 7 Documents

MSC:
14F17 Vanishing theorems
32L20 Vanishing theorems (analytic spaces)
14F05 Sheaves, derived categories of sheaves, etc.
14C22 Picard groups

Keywords: flag; holomorphic line bundles; vanishing cohomology

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Demailly, Jean-Pierre

Compact complex manifolds whose tangent bundles satisfy numerical effectivity properties (joint work with Thomas Peternell and Michael Schneider). (English)
Zbl 0880.14003

Geometry and analysis. Papers presented at the Bombay colloquium, India, January 6–14, 1992. Oxford: Oxford University Press. Stud. Math., Tata Inst. Fundam. Res. 13, 67-86 (1995).

A compact Riemann surface always has a hermitian metric with constant curvature, in particular the curvature sign can be taken to be constant: the negative sign corresponds to curves of general type (genus ≥ 2), while the case of zero curvature corresponds to elliptic curves (genus 1), positive curvature being obtained only for $\mathbb{P}^1$ (genus 0). In higher dimensions the situation is much more subtle and it has been a long standing conjecture due to Frankel to characterize $\mathbb{P}_n$ as the only compact Kähler manifold with positive holomorphic bisectional curvature. Hartshorne strengthened Frankel's conjecture and asserted that $\mathbb{P}_n$ is the only compact complex manifold with ample tangent bundle. In his famous paper in Ann. Math., II. Ser. 110, 593-606 (1979; Zbl 0423.14006), *S. Mori* solved Hartshorne's conjecture by using characteristic p methods. Around the same time *Y.-T. Siu* and *S.-T. Yau* [Invent. Math. 59, 189-204 (1980; Zbl 0442.53056)] gave an analytic proof of the Frankel conjecture. Combining algebraic and analytic tools Mok classified all compact Kähler manifolds with semi-positive holomorphic bisectional curvature. – From the point of view of algebraic geometry, it is natural to consider the class of projective manifolds X whose tangent bundle is numerically effective (nef). This has been done by Campana and Peternell and – in case of dimension 3 – by Zheng. In particular, a complete classification is obtained for dimension at most three. The main purpose of this work is to investigate compact (most often Kähler) manifolds with nef tangent or anticanonical bundles in arbitrary dimension. We first discuss some basic properties of nef vector bundles which will be needed in the sequel in the general context of compact complex manifolds. We refer to papers by *J.-P. Demailly*, *T. Peternell* and *M. Schneider* [Compos. Math. 89, No. 2, 217-240 (1993) and J. Algebr. Geom. 3, No. 2, 295-345 (1994; Zbl 0827.14027)] for detailed proofs. Instead, we put here the emphasis on some unsolved questions. For the entire collection see [Zbl 0868.00030].

MSC:
14C20 Divisors, linear systems, invertible sheaves
32J27 Compact Kähler manifolds: generalizations, classification
14F05 Sheaves, derived categories of sheaves, etc.

Keywords: nef tangent bundles; nef anticanonical bundle; compact Riemann surface

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Demailly, Jean-Pierre

L^2-methods and effective results in algebraic geometry. (English) Zbl 0845.14004

Chatterji, S. D. (ed.), Proceedings of the international congress of mathematicians, ICM '94, August 3-11, 1994, Zürich, Switzerland. Vol. II. Basel: Birkhäuser. 817-827 (1995).

Given an ample line bundle L on a projective n-fold, it is an important question to find an integer m_0 such that mL is ample for $m \geq m_0$. The example of curves shows that no universal bound (depending only on n) exists. However T. Fujita has conjectured that if L is an ample line bundle on a projective n-fold then $K_X + (n+2)L$ is very ample, where K_X is the canonical line bundle. Here the author explains how analytic methods lead to a universal bound $m_0 = 2 + \binom{3n+1}{n}$ such that $2K_X + mL$ is very ample for $m \geq m_0$. For the entire collection see [Zbl 0829.00015].

Reviewer: F.Kirwan (Oxford)

Cited in 1 Document

MSC:
14C20 Divisors, linear systems, invertible sheaves
14F05 Sheaves, derived categories of sheaves, etc.

Keywords: ample line bundle

Demailly, Jean-Pierre; Passare, Mikael

Residual currents and fundamental class. (Courants résiduels et classe fondamentale.) (French) Zbl 0851.32013

Bull. Sci. Math. 119, No.1, 85-94 (1995).

Let Y be the complex subspace of a complex manifold X defined by a coherent ideal I, which is a locally complete intersection. The authors introduce the notion of the cohomology with supports in the infinitesimal neighbourhood of first order of Y and then, they prove that the residual current R_Y is intrinsically identified to a canonical element of the infinitesimal cohomology of first order with supports in Y and with values in the sheaf of sections of the determinant of the determinant of the conormal bundle to Y.

Reviewer: Vasile Brînzănescu (Bucureşti)

Cited in 1 Review

MSC:
32C30 Integration on analytic sets and spaces, currents
32C36 Local cohomology of analytic spaces 58A25 Currents (global analysis)
32C15 Complex spaces

Keywords: current; analytic spaces; cohomology with supports

Demailly, Jean-Pierre

Semicontinuity properties of cohomology and of Kodaira-Iitaka dimension. (Propriétés de semi-continuité de la cohomologie et de la dimension de Kodaira-Iitaka.) (French. Abridged English version) Zbl 0851.32015

C. R. Acad. Sci., Paris, Sér. I 320, No.3, 341-346 (1995).

Let $X \to S$ be a proper and flat morphism of complex spaces and let (X_t) be the fibres. Given a sheaf E over X of locally free $\mathcal{O}_X$-modules, inducing on the fibres a family of sheaves $(E_t \to X_t)$, the author shows that the cohomology group dimension $h^q(t) = h^q(X_t, E_t)$ satisfy the following semicontinuity property: for every $q \geq 0$, the sum $h^q(t) - h^{q-1}(t) + \cdots + (-1)^q h^0(t)$ is upper semicontinuous for the Zariski topology. Then, some applications to the Kodaira-Iitaka dimension are given.

Reviewer: Vasile Brînzănescu (Bucureşti)

MSC:
32C35 Analytic sheaves and cohomology groups 35G05 General theory of linear higher-order PDE

Keywords: proper morphisms of complex spaces; semicontinuity; Kodaira-Iitaka dimension

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Demailly, Jean-Pierre

Ordinary differential equations. Theoretical and numerical aspects. (Gewöhnliche Differentialgleichungen. Theoretische und numerische Aspekte. Aus d. Franz. übers. von Mathias Heckele.) (German) Zbl 0869.65042

Wiesbaden: Vieweg. x, 318 p. (1994).

Die Besonderheit des vorliegenden Buches ist eine integrierte Darstellung der theoretischen Grundlagen und der numerischen Behandlung von Anfangswertaufgaben gewöhnlicher Differentialgleichungen. Der Numerikteil greift dabei thematisch noch weiter aus, indem Rundungsfehler, Polynomapproximation, Quadraturformeln und iterative Verfahren behandelt werden, mit Ausnahme des etwas knapp geratenen Kapitels Iteration sogar ziemlich ausführlich. Ein - was den integrierten Differentialgleichungsteil betrifft - ähnlich aufgebautes Lehrbuch ist von *H. Werner* und *W. Arndt* [Gewöhnliche Differentialgleichungen. Eine Einführung in Theorie und Praxis (1986; MR 88b.34002)] verfaßt worden.

Das vorliegende Lehrbuch besticht durch seine präzise Darstellung der behandelten Sachverhalte und die damit einhergehende Sorgfalt und Eleganz in der Behandlung der mathematischen Aspekte. Mancher Leser würde sich vielleicht eine stärkere Betonung numerischer Gesichtspunkte wünschen, was der Titel des Buches aber auch nicht verspricht. Mit seiner speziellen thematischen Ausrichtung und der inhaltlichen Qualität hat das Buch einen eigenen Platz in der vorliegenden umfangreichen Numerik-Lehrbuchliteratur, und es wird hoffentlich genügend viele Leser finden, die davon profitieren.

Reviewer: R.D.Grigorieff (Berlin)

Cited in 2 Reviews

MSC:
65L05 Initial value problems for ODE (numerical methods) 65L06 Multistep, Runge-Kutta, and extrapolation methods 65D32 Quadrature and cubature formulas (numerical methods) 65H10 Systems of nonlinear equations (numerical methods) 65-01 Textbooks (numerical analysis) 34-01 Textbooks (ordinary differential equations)

Keywords: ordinary differential equation; initial value problem; roundoff errors; polynomial approximation; quadrature formulas; iterative methods; textbook

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Demailly, Jean-Pierre

Regularization of closed positive currents of type (1,1) by the flow of a Chern connection. (English) Zbl 0824.53064

Skoda, Henri (ed.) et al., Contributions to complex analysis and analytic geometry. Based on a colloquium dedicated to Pierre Dolbeault, Paris, France, June 23-26, 1992. Braunschweig: Vieweg. Aspects Math. E 26, 105-126 (1994).

Let X be a compact complex manifold, and T a closed positive current of (1,1) type. Some questions addressed in this article are related to the approximation of T by smooth closed "positive" currents. It is easy to see a necessary condition for this approximation, namely the cohomology class $\{T\}$ should satisfy $\int_Y \{T\}^p \geq 0$ for every p-dimensional subvariety $Y \subset X$. Thus, in general case, one concerns the approximation of T only by closed "almost positive" currents, as the following principal result shows.

Let γ be a continuous real (1,1) form such that $T \geq \gamma$, u some continuous nonnegative (1,1) form, and ω a (smooth) Hermitian metric on T_X. Then under a certain curvature condition, T can be approximated by closed "almost positive" (1,1) currents T_ε with the following properties: (i) $T_\varepsilon \geq \gamma - \lambda_\varepsilon u - \delta_\varepsilon \omega$; (ii) $\lambda_\varepsilon(x)$ is an increasing family of continuous functions such that for all $x \in X$, $\lim_{\varepsilon \to 0} \lambda_\varepsilon(x) = \nu(T, x)$ (Lelong number of T at x); (iii) the constants $\delta_\varepsilon \to 0$ as $\varepsilon \to 0$, and $\delta_\varepsilon > 0$, $\forall \varepsilon$. For the curvature condition above, we require $(\Theta(T_X) + u \otimes \mathrm{Id}_{\mathrm{T_X}})(\theta \otimes \xi, \theta \otimes \xi) \geq 0$ for all θ, ξ of T_X, with $\langle \theta, \xi \rangle = 0$. Moreover if put $T = \alpha + \frac{i}{\pi} \partial\overline{\partial}\psi$, for α a smooth (1,1) form in the same $\partial\overline{\partial}$-cohomology class as T, and ψ an almost plurisubharmonic function, then we have the representation: $T_\varepsilon = \alpha + \frac{i}{\pi} \partial\overline{\partial}\psi_\varepsilon$ such that ψ_ε is smooth over X and increasingly converges to ψ, as $\varepsilon \to 0$. It can be shown that the representation of the above T involving a quasi-psh ψ (i.e. locally the sum of a psh function and a smooth function) is always possible.

Similar results as to the regularization of closed positive currents are treated elsewhere, e.g. [the author, J. Algebr. Geom. 1, No. 3, 361-409 (1992; Zbl 0777.32016)], where a numerical hypothesis rather than a curvature hypothesis is assumed: $c_1(\mathcal{O}_{T_X}(1)) + \pi^* u$ is nef on the total space of (dual) projectivized tangent bundles. This numerical condition does not seem to be directly related to the partial semipositivity curvature condition; for instance, the author remarks that for the curve case the partial semipositivity hypothesis is void. By using the present curvature hypothesis, the author felt it perhaps easier to extend to currents of higher bidegrees. For the entire collection see [Zbl 0811.00006].

Reviewer: I-Hsun Tsai (Taipei)

Cited in 1 Review

Cited in 8 Documents

MSC:
53C55 Hermitian and Kählerian manifolds (global differential geometry)
32C30 Integration on analytic sets and spaces, currents

Keywords: closed positive currents; semipositive curvature; $\partial\overline{\partial}$-cohomology; plurisubharmonic function

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Demailly, Jean-Pierre; Lempert, László; Shiffman, Bernard

Algebraic approximations of holomorphic maps from Stein domains to projective manifolds. (English) Zbl 0861.32006

Duke Math. J. 76, No.2, 333-363 (1994).

Let Y, Z be quasi-projective algebraic varieties and let Ω be an open subset of Y. A map $F : \Omega \to Z$ is said to be Nash algebraic if f is holomorphic and the graph of f is contained in an algebraic subvariety of $Y \times Z$ of dimension equal to $\dim Y$.

One of the main results in the paper is the following theorem concerning the approximation of holomorphic maps by Nash algebraic maps:

Theorem 1.1. Let Ω be a Runge domain in an affine algebraic variety S and let $f : \Omega \to X$ be a holomorphic map into a quasi-projective algebraic manifold X. Then for every relatively compact domain $\Omega_0 \subset\subset \Omega$, there is a sequence of Nash algebraic maps $f_\nu : \Omega_0 \to X$ such that $f_\nu \to f$ uniformly on Ω_0.

As important applications of Theorem 1.1 the authors obtain that the Kobayashi-Royden pseudometric and the Kobayashi pseudodistance on projective algebraic manifolds can be approximated in terms of algebraic curves. It is proved that a type of algebraic approximation is also possible in the case of locally free sheaves.

Using the methods developed in the paper the authors give a more precise form of a result concerning the description of equivalent Nash algebraic vector bundle, obtained by *T. Tancredi* and *A. Tognoli* [Bull. Sci. Math., II. Ser. 117, No. 2, 173-183 (1993; Zbl 0798.32010)]. A result of *E. L. Stout* [Contemp. Math. 32, 259-266 (1984; Zbl 0584.32027)] on the exhaustion of Stein manifolds by Runge domains in affine algebraic manifolds is proved by substantially different methods.

Reviewer: I.Serb (Cluj-Napoca)

Cited in 1 Review

Cited in 14 Documents

MSC:
32E10 Stein spaces, Stein manifolds
14P20 Nash functions and manifolds
32F45 Invariant metrics and pseudodistances

Keywords: approximation of holomorphic maps; Nash algebraic maps; quasi-projective algebraic manifold; Stein manifolds; Runge domains

References:

[1] A. Andreotti and E. Vesentini, Carleman estimates for the Laplace-Beltrami equation on complex manifolds , Inst. Hautes Études Sci. Publ. Math. (1965), no. 25, 81-130. · Zbl 0138.06604 · DOI: 10.1007/BF02684398 ·
NUMDAM: PMIHES_1965__25__81_0 ·
NUMDAM: PMIHES_1965__27__153_0 · EUDML: 103855

[2] E. Bishop, Mappings of partially analytic spaces , Amer. J. Math. 83 (1961), 209-242. JSTOR: · Zbl 0118.07701 · DOI: 10.2307/2372953 · http://links.jstor.org/sici?sici=0002-9327%28196104%2983%3A2%3C209%3AMOPAS%3E2.0.CO%3B2-2&origin=euclid

[3] M. Cornalba and P. Griffiths, Analytic cycles and vector bundles on non-compact algebraic varieties , Invent. Math. 28 (1975), 1-106. · Zbl 0293.32026 · DOI: 10.1007/BF01389905 · EUDML: 142315

[4] J.-P. Demailly, Cohomology of q-convex spaces in top degrees , Math. Z. 204 (1990), no. 2, 283-295. · Zbl 0682.32017 · DOI: 10.1007/BF02570874 · EUDML: 183771

[5] J.-P. Demailly, Singular Hermitian metrics on positive line bundles , Complex algebraic varieties (Bayreuth, 1990), Lecture Notes in Math., vol. 1507, Springer, Berlin, 1992, pp. 87-104. · Zbl 0784.32024 · DOI: 10.1007/BFb0094512

[6] J.-P. Demailly, Algebraic criteria for the hyperbolicity of projective manifolds , in preparation.

[7] L. van den Dries, A specialization theorem for analytic functions on compact sets , Proc. Kon. Nederl. Akad. Wetensch. 85 (1982), 391-396. · Zbl 0526.30004

[8] D. A. Eisenman, Intrinsic measures on complex manifolds and holomorphic mappings, Memoirs of the American Mathematical Society, No. 96, American Mathematical Society, Providence, R.I., 1970. · Zbl 0197.05901

[9] Y. Eliashberg and M. Gromov, Embeddings of Stein manifolds of dimension n into the affine space of dimension $3n/2 + 1$, Ann. of Math. (2) 136 (1992), no. 1, 123-135. JSTOR: · Zbl 0758.32012 · DOI: 10.2307/2946547 · http://links.jstor.org/sici?sici=0003-486X%28199207%292%3A136%3A1%3C123%3AEOSMOD%3E2.0.CO%3B2-M&origin=euclid

[10] H. Grauert, Analytische Faserungen über holomorph-vollständigen Räumen , Math. Ann. 135 (1958), 263-273. · Zbl 0081.07401 · DOI: 10.1007/BF01351803 · EUDML: 160618

[11] P. Griffiths, Topics in algebraic and analytic geometry , Princeton University Press, Princeton, N.J., 1974. · Zbl 0302.14003

[12] R. C. Gunning and H. Rossi, Analytic functions of several complex variables , Prentice-Hall Inc., Englewood Cliffs, N.J., 1965. · Zbl 0141.08601

[13] H. Hironaka, Resolution of singularities of an algebraic variety over a field of characteristic zero. I, II , Ann. of Math. (2) 79 (1964), 109-203; ibid. (2) 79 (1964), 205-326. JSTOR: · Zbl 0122.38603 · DOI: 10.2307/1970486 · http://links.jstor.org/sici?sici=0003-486X%28196403%292%3A79%3A2%3C205%3AROSOAA%3E2.0.CO%3B2-I&origin=euclid

[14] L. Hörmander, An introduction to complex analysis in several variables , North-Holland Mathematical Library, vol. 7, North-Holland Publishing Co., Amsterdam, 1990. · Zbl 0685.32001

[15] S. Kobayashi, Invariant distances on complex manifolds and holomorphic mappings , J. Math. Soc. Japan 19 (1967), 460-480. · Zbl 0158.33201 · DOI: 10.2969/jmsj/01940460

[16] S. Kobayashi, Hyperbolic manifolds and holomorphic mappings , Pure and Applied Mathematics, vol. 2, Marcel Dekker Inc., New York, 1970. · Zbl 0207.37902

[17] S. Lang, Hyperbolic and Diophantine analysis , Bull. Amer. Math. Soc. (N.S.) 14 (1986), no. 2, 159-205. · Zbl 0602.14019 · DOI: 10.1090/S0273-0979-1986-15426-1

[18] S. Lang, Introduction to complex hyperbolic spaces , Springer-Verlag, New York, 1987. · Zbl 0628.32001

[19] R. Narasimhan, Imbedding of holomorphically complete complex spaces , Amer. J. Math. 82 (1960), 917-934. JSTOR: · Zbl 0104.05402 · DOI: 10.2307/2372949 · http://links.jstor.org/sici?sici=0002-9327%28196010%2982%3A4%3C917%3AIO HCCS%3E2.0.CO%3B2-R&origin=euclid

[20] J. Nash, Real algebraic manifolds , Ann. of Math. (2) 56 (1952), 405-421. JSTOR: · Zbl 0048.38501 · DOI: 10.2307/1969649 · http://links.jstor.org/sici?sici=0003-486X%28195211%292%3A56%3A3%3C405%3ARAM%3E2.0.CO%3B2-B&origin= euclid

[21] J. Noguchi and T. Ochiai, Geometric function theory in several complex variables , Translations of Mathematical Monographs, vol. 80, American Mathematical Society, Providence, RI, 1990. · Zbl 0713.32001

[22] K. Oka, Sur les fonctions de plusieurs variables, II. Domaines d'holomorphie , J. Sci. Hiroshima Univ. 7 (1937), 115-130. · Zbl 0017.12204

[23] R. Richberg, Stetige streng pseudokonvexe Funktionen , Math. Ann. 175 (1968), 257-286. · Zbl 0153.15401 · DOI: 10.1007/BF02063212 · EUDML: 161667

[24] H. L. Royden, Remarks on the Kobayashi metric , Several complex variables, II (Proc. Internat. Conf., Univ. Maryland, College Park, Md., 1970), Springer, Berlin, 1971, 125-137. Lecture Notes in Math., Vol. 185. · Zbl 0218.32012

[25] N. Sibony, Quelques problèmes de prolongement de courants en analyse complexe , Duke Math. J. 52 (1985), no. 1, 157-197. · Zbl 0578.32023 · DOI: 10.1215/S0012-7094-85-05210-X

[26] Y. T. Siu, Every Stein subvariety admits a Stein neighborhood , Invent. Math. 38 (1976/77), no. 1, 89-100. · Zbl 0343.32014 · DOI: 10.1007/BF01390170 · EUDML: 142443

[27] E. L. Stout, Algebraic domains in Stein manifolds , Proceedings of the conference on Banach algebras and several complex variables (New Haven, Conn., 1983), Contemp. Math., vol. 32, Amer. Math. Soc., Providence, RI, 1984, pp. 259-266. · Zbl 0584.32027

[28] A. Tancredi and A. Tognoli, Relative approximation theorems of Stein manifolds by Nash manifolds , Boll. Un. Mat. Ital. A (7) 3 (1989), no. 3, 343-350. · Zbl 0691.32006

[29] A. Tancredi and A. Tognoli, On the extension of Nash functions , Math. Ann. 288 (1990), no. 4, 595-604. · Zbl 0699.32006 · DOI: 10.1007/BF01444552 · EUDML: 164754

[30] A. Tancredi and A. Tognoli, Some remarks on the classification of complex Nash vector bundles , Bull. Sci. Math. 117 (1993), no. 2, 173-183. · Zbl 0798.32010

[31] A. Weil, L'intégrale de Cauchy et les fonctions de plusieurs variables , Math. Ann. 111 (1935), 178-182. · Zbl 0011.12301 · DOI: 10.1007/BF01472212 · EUDML: 159767

[32] H. Whitney, Local properties of analytic varieties , Differential and Combinatorial Topology (A Symposium in Honor of Marston Morse), Princeton Univ. Press, Princeton, N. J., 1965, pp. 205-244. · Zbl 0129.39402

[33] S.-T. Yau, A general Schwarz lemma for Kähler manifolds , Amer. J. Math. 100 (1978), no. 1, 197-203. JSTOR: · Zbl 0424.53040 · DOI: 10.2307/2373880 · http://links.jstor.org/sici?sici=0002-9327%28197802%29100%3A1%3C197%3AAGSL FK%3E2.0.CO%3B2-H&origin=euclid

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Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael

Compact complex manifolds with numerically effective tangent bundles. (English) Zbl 0827.14027

J. Algebr. Geom. 3, No.2, 295-345 (1994).

The main result of this fundamental article is: Let X be a compact Kähler manifold with nef tangent bundle T_X. Moreover, let $\widetilde{X}$ be a finite étale cover of X of maximum irregularity $q = q(\widetilde{X}) = h^1(\widetilde{X}, \mathcal{O}_{\widetilde{X}})$. Then: $\pi_1(\widetilde{X}) \cong \mathbb{Z}^{2q}$.

The albanese map $\alpha : \widetilde{X} \to A(\widetilde{X})$ is a smooth fibration over a q-dimensional torus with nef relative tangent bundle.

The fibres of α are Fano manifolds with nef tangent bundles.

Here a line bundle L on a compact complex manifold X with a fixed hermitian metric ω is nef if, for every $\varepsilon > 0$, there exists a smooth hermitian metric h_ε on L such that the curvature satisfies $\Theta_{h_\varepsilon} \geq -\varepsilon\omega$. A bundle E on X is nef if the line bundle $\mathcal{O}_E(1)$ on $\mathbb{P}(E)$ is nef. – Many other interesting and important results are contained in the article. It is proved that:

Let E be a vector bundle on a compact Kähler manifold X.

If E and E^* are nef, then E admits a filtration whose graded pieces are hermitian flat.

If E is nef, then E is numerically semi-positive.

Moreover, algebraic proofs are given for the result:

Any Moisheson manifold with nef tangent bundle is projective.

A compact Kähler n-fold with T_X nef and $c_1(X)^n > 0$ is Fano.

Further the two following classification results are given:

The smooth non-algebraic compact complex surfaces with nef tangent bundles are:

non-algebraic tori; Kodaira surfaces; Hopf surfaces.

Let X be a non-algebraic three-dimensional compact Kähler manifold. Then T_X is nef if and only if X, up to a finite étale cover, is either a torus or of the form $\mathbb{P}(E)$, where E and E^* are nef rank-2 vector bundles over a two-dimensional torus.

Reviewer: D. Laksov (Stockholm)

Cited in 10 Reviews

Cited in 60 Documents

MSC:
14J30 Algebraic threefolds
14C20 Divisors, linear systems, invertible sheaves
32J17 Compact 3-folds (analytic spaces)
14F35 Homotopy theory; fundamental groups (algebraic geometry) 53C55 Hermitian and Kählerian manifolds (global differential geometry)
14E20 Coverings, fundamental group (mappings)

Keywords: fundamental group; nef line bundle; Albanese map; Moisheson manifold; three-dimensional compact Kähler manifold

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Demailly, Jean-Pierre; Peternell, Thomas; Schneider, Michael

Kähler manifolds with numerically effective Ricci class. (English) Zbl 0884.32023

Compos. Math. 89, No.2, 217-240 (1993).

The purpose of this paper is to contribute to the solution of the following conjectures: Let X be a compact Kähler manifold with numerically effective (nef) anticanonical bundle $-K_X$; then:

Conjecture 1: The fundamental group $\pi_1(X)$ of X has polynomial growth.

Conjecture 2: The Albanese map $\alpha : X \to \mathrm{Alb}(X)$ is surjective.

Section 1 is devoted to proving the following theorem, which is the main contribution to Conjecture 1.

Theorem 1: Let X be a compact Kähler manifold with nef anticanonical bundle; then $\pi_1(X)$ has subexponential growth.

The main tools used in order to prove Theorem 1 are the solution of the Calabi conjecture and volume bounds for geodesic balls due to Bishop and Gage. It should be mentioned that from the proof of Theorem 1 it follows that Conjecture 1 holds in the case $-K_X$ is Hermitian semipositive (Theorem 2).

In Section 2 the following theorem concerning Conjecture 2 is proved.

Theorem 3: Let X be an n-dimensional compact Kähler manifold such that $-K_X$ is nef. Then the Albanese map α is surjective as soon as $\dim \alpha(X)$ is 0,1 or n, and, if X is projective, also for $n-1$; moreover, if X is projective and if the generic fibre F of α has $-K_F$ big, then α is surjective.

Finally, Section 3 is devoted to the study of the structure of projective 3-folds with nef anticanonical bundles; in particular Conjecture 2 is proved in dimension 3 with purely algebraic methods, except in one very special case.

Reviewer: Antonella Nannicini (MR 95b:32044)

Cited in 3 Reviews

Cited in 8 Documents

MSC:
32J27 Compact Kähler manifolds: generalizations, classification
14J40 Algebraic n-folds ($n > 4$)
32Q15 Kähler manifolds 53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords: numerically effective Ricci class; compact Kähler manifold; Albanese map; nef anticanonical bundles

References:

[1] Aubin, T. : Equations du type Monge-Ampère sur les variétés kählériennes compactes , C.R. Acad. Sci Paris Ser. A 283 (1976) 119-121; Bull. Sci. Math. 102 (1978) 63-95. · Zbl 0374.53022

[2] Beauville, A. : Variétés Kählériennes dont la première classe de Chern est nulle , J. Diff. Geom. 18 (1983) 755-782. · Zbl 0537.53056 · DOI: 10.4310/jdg/1214438181

[3] Bishop, R. : A relation between volume, mean curvature and diameter , Amer. Math. Soc. Not. 10 (1963) 364.

[4] Bogomolov, F.A. : On the decomposition of Kähler manifolds with trivial canonical class , Math. USSR Sbornik 22 (1974) 580-583. · Zbl 0304.32016 · DOI: 10.1070/SM1974v022n04ABEH001706

[5] Bogomolov, F.A. : Kähler manifolds with trivial canonical class , Izvestija Akad. Nauk 38 (1974) 11-21 and Math. USSR Izvestija 8 (1974) 9-20. · Zbl 0299.32022 · DOI: 10.1070/IM1974v008n01ABEH002093

[6] Calabi, E. : On Kähler manifolds with vanishing canonical class, Alg. geometry and topology , Symposium in honor of S. Lefschetz, Princeton Univ. Press, Princeton (1957) 78-89. · Zbl 0080.15002

[7] Campana, F. , Peternell, Th. : On the second exterior power of tangent bundles of 3-folds , Comp. Math 83 (1992), 329-346. · Zbl 0824.14037 · NUMDAM: CM_1992__83_3_329_0 · EUDML: 90172

[8] Demailly, J.-P. , Peternell, Th. , Schneider, M. : Compact complex manifolds with numerically effective tangent bundles , Preprint 1991. To appear in J. Alg. Geom. · Zbl 0827.14027

[9] Gage, M.E. : Upper bounds for the first eigenvalue of the Laplace-Beltrami operator , Indiana Univ. J. 29 (1980) 897-912. · Zbl 0465.53031 · DOI: 10.1512/iumj.1980.29.29061

[10] Gromov, M. : Groups of polynomial growth and expanding maps , Appendix by J. Tits, Publ. I.H.E.S. 53 (1981) 53-78. · Zbl 0474.20018 · DOI: 10.1007/BF02698687 · NUMDAM: PMIHES_1981__53__53_0 · EUDML: 103974

[11] Hartshorne, R. : Algebraic Geometry, Graduate Texts in Math ., Springer, Berlin, 1977. · Zbl 0367.14001

[12] Heintze, E. , Karcher, H. : A general comparison theorem with applications to volume estimates for submanifolds , Ann. Scient. Ec. Norm. Sup. 4e Série, 11 (1978) 451-470. · Zbl 0416.53027 · DOI: 10.24033/asens.1354 ·
NUMDAM: ASENS_1978_4_11_4_451_0 · EUDML: 82023

[13] Kawamata, Y. : A generalization of Kodaira-Ramanujam's vanishing theorem , Math. Ann. 261 (1982) 43-46. · Zbl 0476.14007 · DOI: 10.1007/BF01456407 · EUDML: 182862

[14] Kawamata, Y. , Matsuki, K. , Matsuda, K. : Introduction to the minimal model pro-gram , Adv. Studies Pure Math. 10 (1987) 283-360. · Zbl 0672.14006

[15] Kobayashi, S. : On compact Kähler manifolds with positive definite Ricci tensor , Ann. Math 74 (1961) 570-574. · Zbl 0107.16002 · DOI: 10.2307/1970298

[16] Kollár, J. : Higher direct images of dualizing sheaves , Ann. Math. 123 (1986) 11-42. · Zbl 0598.14015 · DOI: 10.2307/1971351

[17] Lichnerowicz, A. : Variétés Kählériennes et première classe de Chern , J. Diff. Geom. 1 (1967) 195-224. · Zbl 0167.20004 · DOI: 10.4310/jdg/1214428089

[18] Lichnerowicz, A. : Variétés Kählériennes à première classe de Chern non négative et variétés riemanniennes à courbure de Ricci généralisée non négative , J. Diff. Geom. 6 (1971) 47-94. · Zbl 0231.53063 · DOI: 10.4310/jdg/1214430218

[19] Lichnerowicz, A. : Variétés Kählériennes à première classe de Chern non négative et situation analogue dans le cas riemannien, Ist. Naz. Alta Mat., Rome, Symposia Math ., vol. 10, Academic Press, New-York (1972) 3-18. · Zbl 0267.53035

[20] Matsushima, Y. : Recent results on holomorphic vector fields , J. Diff. Geom. 3 (1969) 477-480. · Zbl 0201.25902 · DOI: 10.4310/jdg/1214429068

[21] Miyanishi, M. : Algebraic methods in the theory of algebraic threefolds , Adv. Studies in Pure Math. 1 (1983) 69-99. · Zbl 0537.14027

[22] Mori, S. : Threefolds whose canonical bundles are not numerically effective , Ann. Math. 116 (1982) 133-176. · Zbl 0557.14021 · DOI: 10.2307/2007050

[23] Myers, S.B. : Riemannian manifolds with positive mean curvature , Duke Math. J. 8 (1941) 401-404. · Zbl 0025.22704 · DOI: 10.1215/S0012-7094-41-00832-3

[24] Viehweg, E. : Vanishing theorems , J. Reine Angew. Math. 335 (1982) 1-8. · Zbl 0485.32019 · DOI: 10.1515/crll.1982.335.1 · crelle:GDZPPN002199688 · EUDML: 152458

[25] Yau, S.T. : Calabi's conjecture and some new results in algebraic geometry , Proc. Nat. Acad. Sci. USA 74 (1977) 1789-1790. · Zbl 0355.32028 · DOI: 10.1073/pnas.74.5.1798

[26] Yau, S.T. : On the Ricci curvature of a complex Kähler manifold and the complex Monge-Ampère equation I , Comm. Pure and Appl. Math. 31 (1978) 339-411. · Zbl 0369.53059 · DOI: 10.1002/cpa.3160310304

Demailly, Jean-Pierre

A numerical criterion for very ample line bundles. (English) Zbl 0783.32013

J. Differ. Geom. 37, No.2, 323-374 (1993).

Let X be a projective algebraic manifold of dimension n and let L be an ample line bundle over X. We give a numerical criterion ensuring that the adjoint bundle K_X+L is very ample. The sufficient conditions are expressed in terms of lower bounds for the intersection numbers $L^p\cdot Y$ over subvarieties Y of X. In the case of surfaces, our criterion gives universal bounds and is only slightly weaker than *I. Reider*'s criterion [Ann. Math., II. Ser. 127, No. 2, 309-316 (1988; Zbl 0663.14010)]. When $\dim X\geq 3$ and codim$Y\geq 2$, the lower bounds for $L^p\cdot Y$ involve a numerical constant which depends on the geometry of X. By means of an iteration process, it is finally shown that $2K_X+mL$ is very ample for $m\geq 12n^n$. Our approach is mostly analytic and based on a combination of Hörmander's L^2 estimates for the operator $\overline{\partial}$, Lelong number theory and the Aubin-Calabi-Yau theorem.

Reviewer: J.-P. Demailly (Saint-Martin d'Hères)

Cited in 4 Reviews

Cited in 54 Documents

MSC:
32J15 Compact surfaces (analytic spaces)
32L10 Sections of holomorphic vector bundles
32C30 Integration on analytic sets and spaces, currents

Keywords: very ample line bundle; plurisubharmonic function; closed positive current; Monge-Ampère equation; intersection theory; numerical criterion; Lelong number; Aubin-Calabi-Yau theorem

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Demailly, Jean-Pierre

Monge-Ampère operators, Lelong numbers and intersection theory. (English)
Zbl 0792.32006

Ancona, Vincenzo (ed.) et al., Complex analysis and geometry. New York: Plenum Press. The University Series in Mathematics. 115-193 (1993).

This article is a survey on the theory of Lelong numbers, viewed as a tool for studying intersection theory by complex differential geometry. The paper contains earlier works of the author [Mém. Soc. Math. Fr., Nouv. Sér. 19, 124 p. (1985; Zbl 0579.32012) and Acta Math. 159, 153- 169 (1987; Zbl 0629.32011)] and of *Y. T. Siu* [Invent. Math. 27, 53- 156 (1974; Zbl 0289.32003)]. Many results are given with complete proofs, which are shorter and simpler than the original ones. The references contain 37 items on these topics. For the entire collection see [Zbl 0772.00007].

Reviewer: E.Outerelo (Madrid)

Cited in 1 Review

Cited in 71 Documents

MSC:
32C30 Integration on analytic sets and spaces, currents
32W20 Complex Monge-Ampère operators
32U05 Plurisubharmonic functions and generalizations

Keywords: Monge-Ampère operators; current; plurisubharmonic functions; Lelong numbers; intersection theory

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Demailly, Jean-Pierre

Holomorphic Morse inequalities on q-convex manifolds. (English) Zbl 0771.32011

Several complex variables, Proc. Mittag-Leffler Inst., Stockholm/Swed. 1987-88, Math. Notes 38, 245-257 (1993).

[For the entire collection see Zbl 0759.00008.]

This paper is a nice report ("high level propaganda for a very interesting result and for very interesting tools") of at that time recent work [*T. Bouche*, Ann. Sci. Ec. Norm. Super., IV. Ser. 22, No. 4, 501-513 (1989; Zbl 0693.32016)] which extends previous work by Demailly to the case of strongly q-convex manifolds. The main result of Bouche's paper is the following one.

Theorem A: Let X be a strongly q-convex complex manifold with $n := \dim(X)$, E a rank r vector bundle on X and L a line bundle on X with hermitian metric such that the curvature form $ic(L)$ has at least $n-p+1$ eigenvalues ≥ 0 outside a compact subset of X; set

$$X(m,L) := \{x \in X \,:\, ic(L) \text{ is non degenerate at } x \text{ and with exactly } m \text{ negative eigenvalues}\},$$

$$X(\leq m,L) := \bigcup_{t\leq m} X(t,L),$$

$$X(\geq m,L) := \bigcup_{t\geq m} X(t,L).$$

Then for all $m \geq p+q-1$ the following asymptotic inequalities hold:

(a_m) Weak Morse inequalities

$$\dim H^m(X, E \otimes L^k) \leq r \frac{k^n}{n!} \int_{X(m,L)} (-1)^m \left(\frac{i}{2\pi} c(L) \right)^n + o(k^n)$$

(b_m) Strong Morse inequalities:

$$\sum_{m \leq t \leq n} (-1)^{t-m} \dim H^t(X, E \otimes L^k) \leq r \frac{k^n}{n!} \int_{X(\geq m,L)} (-1)^m \left(\frac{i}{2\pi} c(L) \right)^n + o(k^n)$$

This note contains a sketch of the proof of this theorem. Here the main recent and very powerful techniques are explained and used (e.g. Witten's complex); the main tool for the proof of Morse inequalities is a spectral theorem for Schrödinger operators which describes very precisely the asymptotic distribution of eigenvalues for a suitable quadratic form. This report contains the statement, the history and the motivation of two important applications of Theorem A: a very general a priori estimate for Monge-Ampère operator $(id'd'')^n$ on q-convex manifolds and the following stronger form of Grauert-Riemenschneider conjecture:

Theorem B: Let X be a connected n-dimensional compact manifold; if X has a hermitian line bundle L such that $\int_{X(\leq 1,L)} (ic(L)^n > 0$, then X is Moishezon.

Reviewer: E. Ballico (Povo)

MSC:
32F10 q-convexity, q-concavity
32C35 Analytic sheaves and cohomology groups
32J99 Compact analytic spaces
32W20 Complex Monge-Ampère operators

Keywords: q-convex manifold; strongly q-convex manifold; Morse inequalities; Monge-Ampère operator; Moishezon manifold; curvature form; line bundle; asymptotic extimates for cohomology

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Demailly, Jean-Pierre

Regularization of closed positive currents and intersection theory. (English)
Zbl 0777.32016

J. Algebr. Geom. 1, No.3, 361-409 (1992).

Let X be a compact complex manifold and let $T = i\partial\overline{\partial}\psi$ be a closed positive current of bidegree (1,1) on X. Under some hypothesis on a lower bound for the Chern curvature of the tangent bundle TX, the current T is proved to be the weak limit of closed currents $T_k = \frac{i}{\pi}\partial\overline{\partial}\psi_k$ with controlled negative parts; the functions ψ_k decrease to ψ as $k \to \infty$ and can be chosen smooth on X. However the presence of positive Lelong numbers of T results in some loss of positivity of T_k.

This regularization is applied to relations between effective and numerically effective divisors, and to some problems of intersection theory.

Reviewer: A.Yu.Rashkovsky (Khar'kov)

Cited in 7 Reviews

Cited in 108 Documents

MSC:
32J25 Transcendental methods of algebraic geometry
32C30 Integration on analytic sets and spaces, currents
32S60 Stratifications; constructible sheaves; intersection cohomology (analytic spaces)

Keywords: Lelong number; closed positive current; intersection theory

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Demailly, Jean-Pierre

Positive currents and intersection theory. (Courants positifs et théorie de l'intersection.) (French) Zbl 0771.32010

Gaz. Math., Soc. Math. Fr. 53, 131-159 (1992).

This is a nice tool for spreading mathematical culture and ideas among mathematicians. It starts with the notion of current (after de Rham) and ends with the use in the subject of top level research and new extremely powerful methods of the author (around 1991). In the middle it is shown how to use the notion of positive current to define and work (via the integration current of the fundamental class of a subvariety) in Intersection Theory (key words: Lelong numbers and multiplicities). The tools come from Analysis and bring (and often solve) with them several interesting problems which cannot be formulated in a purely algebraic way inside Algebraic Geometry. But these methods are very, very strong competitors even on natural very important algebraic problems. Of course, in this paper most of the proofs are omitted, but ideas and difficulties are not skipped. It is pleasant reading and even specialists in not too far fields can find here some ideas/tools useful for their job; everybody can find some recent deep idea (mostly from Demailly brain).

Reviewer: E.Ballico (Povo)

Cited in 2 Documents

MSC:
32C30 Integration on analytic sets and spaces, currents 58A25 Currents (global analysis)
32J25 Transcendental methods of algebraic geometry
14C17 Intersection theory, etc.

Keywords: positive currents; intersection theory; Lelong numbers; intersection multiplicity; integration current; equimultiplicity

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Demailly, Jean-Pierre

Singular Hermitian metrics on positive line bundles. (English) Zbl 0784.32024

Complex algebraic varieties, Proc. Conf., Bayreuth/Ger. 1990, Lect. Notes Math. 1507, 87-104 (1992).

[For the entire collection see Zbl 0745.00049.]

We quote the author's abstract: "The notion of a singular Hermitian metric on a holomorphic line bundle is introduced as a tool for the study of various algebraic questions. One of the main interests of such metrics is the corresponding L^2 vanishing theorem for $\overline{\partial}$ cohomology, which gives a useful criterion for the existence of sections. In this context, numerically effective line bundles and line bundles with maximum Kodaira dimension are characterized by means of positivity properties of the curvature in the sense of currents. The coefficients of isolated logarithmic poles of a plurisubharmonic singular metric are shown to have a simple interpretation in terms of the constant ε of Seshadri's ampleness criterion. Finally, we use singular metrics and approximations of the curvature current to prove a new asymptotic estimate for the dimension of cohomology groups with values in high multiples $\mathcal{O}(kL)$ of a line bundle L with maximum Kodaira dimension".

Reviewer: E.J.Straube (College Station)

Cited in 13 Reviews

Cited in 56 Documents

MSC:
32L05 Holomorphic fiber bundles and generalizations

Keywords: plurisubharmonic weights; singular Hermitian metric; holomorphic line bundle; vanishing theorem; cohomology

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Demailly, Jean-Pierre

Transcendental proof of a generalized Kawamata-Viehweg vanishing theorem. (English) Zbl 1112.32303

Berenstein, Carlos A. (ed.) et al., Geometrical and algebraical aspects in several complex variables. Papers from the conference, Cetraro, Italy, June 1989. Rende: Editoria Elettronica. Semin. Conf. 8, 81-94 (1991). For the entire collection see [Zbl 0969.00052].

Cited in 5 Documents

MSC:
32L20 Vanishing theorems (analytic spaces)
14F17 Vanishing theorems
32L10 Sections of holomorphic vector bundles

Demailly, Jean Pierre

Holomorphic Morse inequalities. (English) Zbl 0755.32008

Several complex variables and complex geometry, Proc. Summer Res. Inst., Santa Cruz/CA (USA) 1989, Proc. Symp. Pure Math. 52, Part 2, 93-114 (1991).

[For the entire collection see Zbl 0732.00008.]

In this paper the complex analogues of the Morse inequalities for $\overline{\partial}$-cohomology groups with values in holomorphic vector bundles are explained, and some applications of that theory are presented.

Reviewer: B.Nowak (Łódź)

Cited in 1 Review

Cited in 9 Documents

MSC:
32C35 Analytic sheaves and cohomology groups 58E05 Abstract critical point theory 32L10 Sections of holomorphic vector bundles 53C07 Special connections and metrics on vector bundles (Hermite-Einstein-Yang-Mills)

Keywords: $\overline{\partial}$-cohomology groups; Morse inequalities; holomorphic vector bundles

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Blel, Mongi; Demailly, Jean-Pierre; Mouzali, Mokhtar

Sur l'existence du cône tangent à un courant positif fermé. (About the existence of the tangent cone with positive closed current). (French) Zbl 0724.32005

Ark. Mat. 28, No.2, 231-248 (1990).

Let T be a positive closed current of degree p on an open neighborhood Ω of 0 in $\mathbb{C}^n$. For $a \in \mathbb{C}^*$ let h_a denote the homothety given by a and h_a^*T the lifted current. If the weak limit $\lim_{|a|\to 0} h_a^*T$ exists it is called the tangent cone of T in 0. The authors show:

Theorem: If for small $r_0 > 0$ one of the following conditions a) or b) is satisfied then the tangent cone of T exists:

$$a) \quad \int_0^{r_0} [(\sqrt{v_T(r) - v_T(r/2)})/r]dr < \infty, \quad b) \quad \int_0^{r_0} [(v_T(r) - v_T(0))/r]dr < \infty.$$

$v_T(r)$ denotes the projective mass of T.–

The authors show that condition b) is optimal in a sense.

Theorem: If T is the current of an analytic subset of pure dimension p in Ω then

$$v_T(r) - v_T(0) \leq Cr^\epsilon$$

for small $r > 0$ and suitable numbers $C, \epsilon > 0$.–

A conclusion of these theorems is a result of Thie and King on the existence of a tangent cone for a current induced by an analytic set.

Reviewer: H.-J.Reiffen (Osnabrück)

MSC:
32C30 Integration on analytic sets and spaces, currents
32B15 Analytic subsets of affine space

Keywords: positive closed current; tangent cone; current induced by an analytic set

References:

[1] Barlet, D., Développements asymptotiques des fonctions obtenues par intégration sur les fibres,Invent. Math. 68 (1982), 129–174. · Zbl 0508.32003 · DOI: 10.1007/BF01394271

[2] Blel, M., Cône tangent à un courant positif fermé de type (1, 1),C. R. Acad. Sci. Paris Sér. I Math. 309 (1989), 543–546. · Zbl 0688.32013

[3] Blel, M.,Cône tangent à un courant positif fermé, preprint de la Faculté des Sciences de Monastir, 1988.

[4] Federer, H.,Geometric measure theory (Grundlehren der Mathematischen Wissenschaften 158), Springer-Verlag, Berlin, 1969. · Zbl 0176.00801

[5] Harvey, R., Holomorphic chains and their boundaries, in:Proc. Symp. Pure Math. 30–1, pp. 309–382, Am. Math. Soc., Providence, 1977. · Zbl 0374.32002

[6] King, J. R., The currents defined by analytic varieties,Acta Math. 127 (1971), 185–220. · Zbl 0224.32008 · DOI: 10.1007/BF02392053

[7] Kiselman, C. O.,Tangents of plurisubharmonic functions, preprint Uppsala University (Sweden), December 1988. · Zbl 0810.31006

[8] Lelong, P., Fonctions plurisousharmoniques et formes différentielles positives, Dunod, Paris, Gordon & Breach, New York, 1968. · Zbl 0195.11603

[9] Lelong, P. etGruman, L.,Entire functions of several complex variables (Grundlehren der Mathematischen Wissenschaften 282), Springer-Verlag, Berlin, 1982.

[10] Mouzali, M.,Conditions suffisantes pour l'existence du cône tangent à un courant positif fermé, Thèse de 3e Cycle, Université de Grenoble I, mai 1989.

[11] Narasimhan, R.,Introduction to the theory of analytic spaces, Lecture Notes in Mathematics 25, Springer-Verlag, Berlin, 1966. · Zbl 0168.06003

[12] Thie, P., The Lelong number of a point of a complex analytic set,Math. Ann.,172 (1967), 269–312. · Zbl 0158.32804 · DOI: 10.1007/BF01351593

Demailly, Jean-Pierre

Cohomology of q-convex spaces in top degrees. (English) Zbl 0682.32017

Math. Z. 204, No.2, 283-295 (1990).

It is shown that every strongly q-complete subvariety of a complex analytic space has a fundamental system of strongly q-complete neighborhoods. As a consequence, we find a simple proof of Ohsawa's result that every non compact irreducible n-dimensional analytic space is strongly n-convex. An elementary proof of the existence of Hodge decomposition in top degrees for absolutely q-convex manifolds is also given.

Reviewer: J.-P. Demailly

Cited in 3 Reviews

Cited in 34 Documents

MSC:
32F10 q-convexity, q-concavity

Keywords: strongly q-complete subvariety; strongly q-complete neighborhoods; strongly n-convex; existence of Hodge decomposition

References:

[1] [A-G] Andreotti, A., Grauert, H.: Théorèmes de finitude pour la cohomologie des espaces complexes. Bull. Soc. Math. Fr.90, 193–259 (1962) · Zbl 0106.05501

[2] [Ba] Barlet, D.: Convexité de l'espace des cycles. Bull. Soc. Math. Fr.106, 373–397 (1978) · Zbl 0395.32009

[3] [G-R] Grauert, H., Riemenschneider, O.: Kählersche Mannigfaltigkeiten mit hype-q-konvexem Rand. Problems in analysis: a Symposium in honor of Salomon Bochner. Princeton, Princeton University Press 1970

[4] [G-W] Greene, R. E., Wu, H.: Embedding of open riemannian manifolds by harmonic functions. Ann. Inst. Fourier25, 215–235 (1975) · Zbl 0307.31003

[5] [Ma] Malgrange, B.: Existence et approximation des solutions des équations aux dérivées partielles et des équations de convolution. Ann. Inst. Fourier6, 271–355 (1955/1956) · Zbl 0071.09002

[6] [Oh1] Ohsawa, T.: A reduction theorem for cohomology groups of very stronglyq-convex Kähler manifolds. Invent. Math.63, 335–354 (1981)/66, 391–393 (1982) · Zbl 0457.32007 · DOI: 10.1007/BF01393882

[7] [Oh2] Ohsawa, T.: Completeness of noncompact analytic spaces. Publ. R.I.M.S., Kyoto Univ.20, 683–692 (1984) · Zbl 0568.32008 · DOI: 10.2977/prims/1195181418

[8] [O-T] Ohsawa, T., Takegoshi, K.: Hodge spectral sequence on pseudoconvex domains. Math. Z.197, 1–12 (1988) · Zbl 0638.32016 · DOI: 10.1007/BF01161626

[9] [S1] Siu, Y. T.: Analytic sheaf cohomology groups of dimensionn ofn-dimensional noncompact complex manifolds. Pac. J. Math.28, 407–411 (1969)

[10] [S2] Siu, Y. T.: Analytic sheaf cohomology groups of dimensionn ofn-dimensional complex spaces. Trans. Am. Math. Soc.143, 77–94 (1969)

[11] [S3] Siu, Y.T.: Every Stein subvariety has a Stein neighborhood. Invent. Math.38, 89–100 (1976) · Zbl 0343.32014 · DOI: 10.1007/BF01390170

Bedford, Eric; Demailly, Jean-Pierre

Two counterexamples concerning the pluricomplex Green function in $\mathbb{C}^n$. (English) Zbl 0681.32014

Indiana Univ. Math. J. 37, No.4, 865-867 (1988).

Given a domain Ω in $\mathbb{C}^n$ and a point $z \in \Omega$, the pluricomplex Green function on Ω with logarithmic pole at z is given by $u_z(\zeta) = \sup\{v(\zeta)$: v is plurisubharmonic on Ω, $v < 0$, and $v(\zeta) \leq \log|\zeta - z| + O(1)\}$. In "Capacities in complex analysis" (1988; Zbl 0655.32001), *U. Cegrell* raised the following questions:

1. Is $u_z \in C^2(\bar{\Omega} - \{z\})$?

2. Is u_z symmetric, i.e., $u_z(\zeta) = u_\zeta(z)$?

L. Lempert [Bull. Soc. Math. Fr. 109, 427-474 (1981; Zbl 0492.32025)] has shown that if Ω is strictly convex and smoothly bounded, then the answer to both of these questions is "Yes". In the paper, the authors provide counterexamples to show that for strongly pseudoconvex domains, the answer to both these questions is "No".

Reviewer: M.Stoll

Cited in 5 Documents

MSC:
32U05 Plurisubharmonic functions and generalizations
32T99 Pseudoconvex domains 31C10 Pluriharmonic and plurisubharmonic functions

Keywords: pluricomplex Green function; strongly pseudoconvex domains

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Demailly, Jean-Pierre

Vanishing theorems for tensor powers of a positive vector bundle. (English)
Zbl 0651.32019

Proc. 21st Int. Taniguchi Symp., Katata/Japan, Conf., Kyoto/Japan 1987, Lect. Notes Math. 1339, 86-105 (1988).

[For the entire collection see Zbl 0638.00022.]

Let E be a holomorphic vector bundle of rank r over a compact complex manifold X of dimension n, and suppose that E is positive in the sense of Griffiths and that $p+q \geq n+1$. Let L be a semipositive line bundle and $\Gamma^a E$ an irreducible tensor power representation of $\mathrm{GL}(E)$ of highest weight $a=(a_1,...,a_r)$ with $a_1 \geq a_2 \geq ... \geq a_h > a_{h+1} = ... = a_r = 0$. The author shows that $H^{p,q}(X, \Gamma^a E \otimes (\det E)^\ell \otimes L)$ vanishes for $\ell \geq h + A(n,p,q)$, where $A(n,p,q)$ is a certain rational function of n,p,q. The best possible value for $A(n,p,q)$ is not known, but an example of *Th. Peternell*, *J. Le Potier* and *M. Schneider* [Invent. Math. 87, 573-586 (1987; Zbl 0618.14023)] shows, even when $\Gamma^a E = S^k E$ and $p=n$, at least $\ell \geq 1$ is required. The method of proof is to represent $\Gamma^a E$ as the direct image of a positive line bundle over a suitable flag manifold of E and to apply a generalization of Le Potier's isomorphism theorem to this situation. In order to overcome a difficulty arising from the fact that, when $p<n$, the generalized Borel-Le Potier spectral sequence does not degenerate at the E_1 level, the author obtains a new curvature estimate for the bundle of X-relative differential forms on the flag manifold of E.

Reviewer: P.E.Newstead

Cited in 2 Documents

MSC:
32L20 Vanishing theorems (analytic spaces)
32L05 Holomorphic fiber bundles and generalizations

Keywords: vanishing theorem; tensor power; positive line bundle

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Demailly, Jean-Pierre

Vanishing theorems for tensor powers of an ample vector bundle. (English)
Zbl 0647.14005

Invent. Math. 91, No.1, 203-220 (1988).

Let X be a compact complex manifold of dimension n and E resp. L an ample holomorphic vector bundle of rank r, resp. an ample line bundle on X. The paper gives generalizations of Griffiths' vanishing theorem $H^{n,q}(X, S^k E \otimes \det(E \otimes L)) = 0$ for $q \geq 1$ [*P. A. Griffiths*, Global Analysis, papers in Honor of K. Kodaira, 185-251 (1969; Zbl 0201.240)] which shall not be repeated here and Le Potier's vanishing theorem $H^{p,q}(X,E) = 0$ for $p+q \geq n+r$ [*J. Le Potier*, Math. Ann. 218, 35-53 (1975; Zbl 0313.32037)] saying that $H^{p,q}(X, E^{\otimes k} \otimes (\det(E))^\ell \otimes L) = 0$ for $p+q \geq n+1$, $k \geq 1$ and $\ell \geq n-p+r-1$.

The proof rests on a generalization of the Borel-Le Potier spectral sequence and the Kodaira-Akizuki-Nakano vanishing theorem for line bundles. Moreover it is shown that there is a canonical homomorphism $H^{p,q}(X, \wedge^2 E \otimes L) \to H^{p+1,q+1}(X, S^2 E \otimes L)$ which is bijective under some additional hypotheses. Using this the author gives a counterexample to a conjecture of *J. A. Sommese* in Math. Ann. 233, 229-256 (1978; Zbl 0381.14007).

Reviewer: H.Lange

MSC:
14F05 Sheaves, derived categories of sheaves, etc.
32L20 Vanishing theorems (analytic spaces)

Keywords: tensor power of ample vector bundle; vanishing theorem

References:

[1] Akizuki, Y., Nakano, S.: Note on Kodaira-Spencer's proof of Lefschetz theorems. Proc. Jap. Acad.30, 266-272 (1954) · Zbl 0059.14701 · DOI: 10.3792/pja/1195526105

[2] Borel, A., Weil, A.: Représentations linéaires et espaces homogènes kählériens des groupes de Lie compacts. Séminaire Bourbaki (exposé no 100 par J.-P. Serre), (mai 1954)

[3] Bott, R.: Homogeneous vector bundles. Ann. Math.66, 203-248 (1957) · Zbl 0094.35701 · DOI: 10.2307/1969996

[4] Demailly, J.-P.: Théorèmes d'annulation pour la cohomologie des puissances tensorielles d'un fibré positif. C.R. Acad. Sci. Paris Sér. I Math.,305, (1987) (à paraître.) · Zbl 0627.32022

[5] Demailly, J.-P.: Vanishing theorems for tensor powers of a positive vector bundle. (to appear in the Proceedings of the Conference on Geometry and Analysis on Manifolds held in Katata, Japan (August 1987), Lect. Notes Math., Springer, Berlin Heidelberg New York) · Zbl 0627.32022

[6] Demazure, B.: A very simple proof of Bott's theorem. Invent. Math.33, 271-272 (1976) · Zbl 0383.14017 · DOI: 10.1007/BF01404206

[7] Godement, R.: Théorie des faisceaux. Hermann, Paris, 1958 · Zbl 0080.16201

[8] Griffiths, P.A.: Hermitian differential geometry, Chern classes and positive vector bundles. Global Analysis, Papers in honor of K. Kodaira, Princeton Univ. Press, Princeton (1969), pp. 185-251

[9] Hartshorne, R.: Ample vector bundles. Publ. Math. I.H.E.S.29, 63-94 (1966) · Zbl 0173.49003

[10] Kraft, H.: Geometrische Methoden in der Invariantentheorie. Aspekte der Mathematik, Band D 1, Braunschweig, Vieweg Sohn, 1985 · Zbl 0669.14003

[11] Peternell, Th., Le Potier, J., Schneider, M.: Vanishing theorems, linear and quadratic normality. Invent. Math.87, 573-586 (1987) · Zbl 0618.14023 · DOI: 10.1007/BF01389243

[12] Peternell, Th., Le Potier, J., Schneider, M.: Direct images of sheaves of differentials and the Atiyah class. Math. Z.196, 75-85 (1987) · Zbl 0662.14006 · DOI: 10.1007/BF01179269

[13] Le Potier, J.: Annulation de la cohomologie à valeurs dans un fibré vectoriel holomorphe de rang quelconque. Math. Ann.218, 35-53 (1975) · Zbl 0313.32037 · DOI: 10.1007/BF01350066

[14] Schneider, M.: Ein einfacher Beweis des Verschwindungssatzes für positive holomorphe Vektorraumbündel. Manuscr. Math.11, 95-101 (1974) · Zbl 0275.32014 · DOI: 10.1007/BF01189093

[15] Sommese, A.J.: Submanifolds of abelian varieties. Math. Ann.233, 229-256 (1978) · Zbl 0381.14007 · DOI: 10.1007/BF01405353

Demailly, Jean-Pierre

Sur les théorèmes d'annulation et de finitude de T. Ohsawa et O. Abdelkader.
(On vanishing and finiteness theorems of T. Ohsawa and O. Abdelkader). (French) Zbl 0691.32009

Sémin. d'analyse P. Lelong - P. Dolbeault - H. Skoda, Paris, 1985/86, Lect. Notes Math. 1295, 48-58 (1987).

[For the entire collection see Zbl 0623.00006.]

L'objet de cette note est de donner une démonstration aussi simple que possible des théorèmes d'annulation et de finitude dus à *T. Ohsawa* [Publ. Res. Inst. Math. Sci. 15, 853-870 (1979; Zbl 0434.32014), Publ. Res. Inst. Math. Sci. 17, 113-126 (1981; Zbl 0465.32007)], et des généralisations de ces théorèmes obtenues par *O. Abdelkader* [C. R. Acad. Sci., Paris, Ser. A 290, 75-78 (1980; Zbl 0442.32008) et "Théorèmes de finitude pour la cohomologie d'une variété faiblement 1-complète à valeurs dans un fibré en droites semi- positif", Thèse Doct. d'Etat à l'Univ. Paris VI (1985)].

Reviewer: Résumé

Cited in 1 Document

MSC:
32L20 Vanishing theorems (analytic spaces)
32H35 Proper mappings, finiteness theorems
32C35 Analytic sheaves and cohomology groups
32F10 q-convexity, q-concavity 53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords: vanishing theorem; finiteness theorem; Kähler manifold; weakly 1- complete; hermitian vector bundle

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Demailly, Jean-Pierre; Laurent-Thiébaut, Christine

Formules intégrales pour les formes différentielles de type (p,q) dans les variétés de Stein.
(Integral formulas for differential forms of type (p,q) in Stein manifolds). (French) Zbl 0632.32004

Ann. Sci. Éc. Norm. Supér. (4) 20, No. 4, 579-598 (1987).

The Cauchy-Green integral formula for a domain $D \subset\subset \mathbb{C}$ with piecewise C^1-boundary and for $f \in C^1(\bar{D})$ is

$$(!) \quad f(z) = (1/2\pi i) \int_{\partial D} f(\zeta) d\zeta/(\zeta - z) \quad + \quad (1/2\pi i) \int_D \bar{\partial} f d\zeta \Lambda d\bar{\zeta}/(\zeta - z).$$

Over the past half-century, various integral formulas have been gradually developed that generalise (!) for several complex variables. An excellent, systematic exposition of this work and of some problems that can be solved by such methods is *G. M. Khenkin* and *J. Leiterer*, Theory of functions on complex manifolds (1984; Zbl 0573.32001). The present paper contains the construction of the relevant kernels and integrals for differential forms of type (p,q) on Stein manifolds: §§ 1 and 3 develop and extend the methods of Khenkin and Leiterer concerning the Bochner-Martinelli kernel; §§ 2 and 4 generalise the Koppelman formula and the Koppelman-Leray formula, respectively.

Reviewer: E.J.Akutowicz

Cited in 4 Reviews

Cited in 2 Documents

MSC:
32A30 Generalizations of function theory to several variables 30E20 Integration, integrals of Cauchy type, etc. (one complex variable)
32A25 Integral representation; canonical kernels (several complex variables)
32E10 Stein spaces, Stein manifolds

Keywords: (p,q)-differential forms on Stein manifolds; tangent and cotangent fiber spaces; Bochner-Martinelli kernel; Koppelman formula; Koppelman-Leray formula

References:

[1] M. ANDERSSON et B. BERNDTSSON , Henkin-Ramirez Formulas with Weight Factors (Ann. de l'Inst. Fourier, vol. 32, 1982 , p. 91-110). Numdam | MR 84j:32003 | Zbl 0466.32001 · Zbl 0466.32001 · DOI: 10.5802/aif.881 ·
NUMDAM: AIF_1982__32_3_91_0 · EUDML: 74554

[2] G. HENKIN et J. LEITERER , Global Integral Formulas for Solving the $\overline{\partial}$-Equation on Stein Manifolds (Ann. Pol. Math., vol. 39, 1981 , p. 93-116). MR 82k:32016 | Zbl 0477.32020 · Zbl 0477.32020

[3] G. HENKIN et J. LEITERER , Theory of Functions on Complex Manifolds , Birkhaüser, Verlag, 1984 . · Zbl 0726.32001

[4] N. KERZMAN , Hölder and L^p Estimates for Solution of $\overline{\partial} u = f$ in Strongly Pseudoconvex Domains (Comm. Pure Appl. Math., vol. 24, 1971 , p. 301-379). MR 43 #7658 | Zbl 0217.13202 · Zbl 0217.13202 · DOI: 10.1002/cpa.3160240303

[5] Ch. LAURENT-THIEBAUT , Formules intégrales de Koppelman sur une variété de Stein (Proc. Amer. Math. Soc., vol. 90, 1984 , p. 221-225). MR 85d:32008 | Zbl 0587.32005 · Zbl 0587.32005 · DOI: 10.2307/2045344

[6] Ch. LAURENT-THIEBAUT , Transformation de Bochner-Martinelli dans une variété de Stein .

[7] I. LIEB , Die Cauchy-Riemannschen Differentialgleichungen auf streng pseudokonvexen Gebieten (Math. Ann., vol. 190, 1971 , p. 6-44 et 199, 1972 , p. 241-256). Zbl 0199.42702 · Zbl 0199.42702 · DOI: 10.1007/BF01349966 · EUDML: 162090

[8] N. ØVRELID , Integral Representation Formulas and L^p Estimates for the $\overline{\partial}$-Equation (Math. Scand., vol. 29, 1971 , p. 137-160). Zbl 0227.35069 · Zbl 0227.35069 · EUDML: 166200

[9] B. BERNDTSSON , A Formula for Interpolation and Division in $\mathbb{C}^n$ (Math. Ann., vol. 263, 1983 , p. 393-418). Article | MR 85b:32005 | Zbl 0499.32013 · Zbl 0499.32013 · DOI: 10.1007/BF01457051 · EUDML: 163757

[10] P. GRIFFITHS et J. HARRIS , Principles of Algebraic Geometry , Wiley-interscience, New York, 1978 . MR 80b:14001 | Zbl 0408.14001 · Zbl 0408.14001

Demailly, Jean-Pierre

Une preuve simple de la conjecture de Grauert-Riemenschneider. (A simple proof of the Grauert-Riemenschneider conjecture). (French) Zbl 0629.32026

Sémin. d'analyse P. Lelong - P. Dolbeault - H. Skoda, Paris 1985/86, Lect. Notes Math. 1295, 24-47 (1987).

[For the entire collection see Zbl 0623.00006.]

The author's abstract: "Let E be a hermitian holomorphic line bundle over a compact complex manifold X. We give an asymptotic upper bound for the dimension of cohomology groups of high tensor powers E^k. This bound is invariantly expressed in terms of an integral of the bundle curvature form. As an application, we find a simple proof of the Grauert-Riemenschneider conjecture, recently solved by Siu: if X possesses a quasi-positive line bundle E, then X is a Moishezon space; furthermore the quasipositivity hypothesis can be weakened here in an integral condition which does not require the bundle E to be pointwise semi- positive."

Reviewer: D.Barlet

MSC:
32J25 Transcendental methods of algebraic geometry
32J99 Compact analytic spaces
32L15 Bundle convexity
32L20 Vanishing theorems (analytic spaces)

Keywords: hermitian holomorphic line bundle over a compact complex manifold; bundle curvature; Grauert-Riemenschneider conjecture; quasi-positive line bundle; Moishezon space

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Demailly, Jean-Pierre

Nombres de Lelong généralisés, théorèmes d'intégralité et d'analyticité. (Generalized Lelong numbers, integrability and analyticity theorems). (French) Zbl 0629.32011

Acta Math. 159, 153-169 (1987).

Let X be a complex Stein space, T a closed positive current of bidimension (p,p) on X and $\phi : X \to [-\infty, +\infty[$ an exhaustive plurisubharmonic function. The author's generalized Lelong number $\nu(T,\phi)$ is defined as the mass of the measure $T \wedge (dd^c\phi)^p$ carried by the polar set $\phi^{-1}(-\infty)$ and is obtained by means of the Monge-Ampère operator of *E. Bedford* and *B. A. Taylor* [ibid. 149, 1-40 (1982; Zbl 0547.32012)]. ν (T,ϕ) generalizes the classical *P. Lelong* ["Plurisubharmonic functions and positive differential forms" (1969; Zbl 0195.116)] and C. O. Kiselman's numbers. The author establishes that $\nu(T,\phi)$ depends only on the behaviour of ϕ in a neighbourhood of the poles. The use of $\nu(T,\phi)$ allows him to obtain very simple proofs of classical results on Lelong numbers, e.g. that these numbers are invariant with respect to local coordinate transformations [cf. *Y. T. Siu*, Invent. Math. 27, 53-156 (1974; Zbl 0289.32003)] and also on *P. Thie*'s [Math. Ann. 172, 269-312 (1967; Zbl 0158.328)] theorem showing that the Lelong number of an analytic set X coincides to the algebraic multiplicity of Y at x. Finally, the author obtains a generalization of Siu's theorem on the analyticity of the level sets associated to Lelong numbers, his result containing as a particular case a recent theorem of C. O. Kiselman on directional Lelong numbers.

Reviewer: P.Caraman

Cited in 4 Reviews

Cited in 22 Documents

MSC:
32E10 Stein spaces, Stein manifolds
32U05 Plurisubharmonic functions and generalizations
32C30 Integration on analytic sets and spaces, currents 31C10 Pluriharmonic and plurisubharmonic functions 31C15 Generalizations of potentials and capacities

Keywords: integrability; potential; complex Stein space; current; plurisubharmonic function; generalized Lelong number; analyticity

References:

[1] Bedford, E. &Taylor, B. A., A new capacity for plurisubharmonic functions.Acta Math., 149 (1982), 1–41. · Zbl 0547.32012 · DOI: 10.1007/BF02392348

[2] Bombieri, E., Algebraic values of meromorphic maps.Invent. Math., 10 (1970), 267–287, and Addendum,Invent. Math., 11 (1970), 163–166. · Zbl 0214.33702 · DOI: 10.1007/BF01418775

[3] Chern, S. S., Levine, H. I. & Nirenberg, L., Intrinsic norms on a complex manifold.Global Analysis (papers in honor of K. Kodaira), p. 119–139. Univ. of Tokyo Press, Tokyo, 1969. · Zbl 0202.11603

[4] Demailly, J.-P., Sur les nombres de Lelong associés à l'image directe d'un courant positif fermé. Ann. Inst. Fourier (Grenoble), 32 (1982), 37–66. · Zbl 0457.32005

[5] –, Mesures de Monge-Ampère et caractérisation géométrique des variétés algébriques affines. Mém. Soc. Math. France (N. S.), 19 (1985), 1–124.

[6] Hironaka, H., Resolution of singularities of an algebraic variety I, II.Ann. of Math., 79 (1964), 109–326. · Zbl 0122.38603 · DOI: 10.2307/1970486

[7] Hörmander, L.,An introduction to Complex Analysis in several variables. 2nd edition, North-Holland Math. libr., vol. 7, Amsterdam, London, 1973. · Zbl 0271.32001

[8] Kiselman, C. O., The partial Legendre transformation for plurisubharmonic functions.Invent. Math., 49 (1978), 137–148. · Zbl 0388.32009 · DOI: 10.1007/BF01403083

[9] –, Densité des fonctions plurisousharmoniques.Bull. Soc. Math. France, 107 (1979), 295–304. · Zbl 0416.32007

[10] Kiselman, C. O. Un nombre de Lelong raffiné. Communication orale aux Journées Complexes du Sud de la France (mai 1986).

[11] Lelong, P., Intégration sur un ensemble analytique complexe.Bull. Soc. Math. France, 85 (1957), 239–262. · Zbl 0079.30901

[12] –, Fonctions entières (n variables) et fonctions plurisousharmoniques d'ordre fini dans $\mathbb{C}^n$ .J. Analyse Math. Jerusalem, 12 (1964), 365–407. · Zbl 0126.29602 · DOI: 10.1007/BF02807441

[13] –Plurisubharmonic functions and positive differential forms. Gordon and Breach, New-York, and Dunod, Paris, 1969.

[14] Remmert, R., Projectionen analytischer Mengen.Math. Ann., 130 (1956), 410–441. · Zbl 0070.07701 · DOI: 10.1007/BF01343236

[15] –, Holomorphe und meromorphe Abbildungen komplexer Räume.Math. Ann., 133 (1957), 328–370. · Zbl 0079.10201 · DOI: 10.1007/BF01342886

[16] Siu, Y. T., Analyticity of sets associated to Lelong numbers and the extension of closed positive currents.Invent. Math., 27 (1974), 53–156. · Zbl 0289.32003 · DOI: 10.1007/BF01389965

[17] Skoda, H., Sous-ensembles analytiques d'ordre fini ou infini dans $\mathbb{C}^n$ .Bull. Soc. Math. France, 100 (1972), 353–408. · Zbl 0246.32009

[18] Thie, P., The Lelong number of a point of a complex analytic set.Math. Ann., 172 (1967), 269–312. · Zbl 0158.32804 · DOI: 10.1007/BF01351593

Demailly, Jean-Pierre

Théorèmes d'annulation pour la cohomologie des puissances tensorielles d'un fibré vectoriel positif. (Vanishing theorems for cohomology groups of tensor powers of a positive vector bundle). (French) Zbl 0627.32022

C. R. Acad. Sci., Paris, Sér. I 305, 419-422 (1987).

The author cleverly combines the Kodaira-Akizuki-Nakano vanishing theorem with some standard geometrical constructions, in order to prove the following result: Let E, L be holomorphic vector bundles over a compact complex manifold X. Assume $rk(L) = 1$ and $E > 0$, $L \geq 0$ or $E \geq 0$, $L > 0$. Then there is an integer A(n,p,q), so that

$$(1) \quad H^{p,q}(X, Sym^k(E) \otimes (\det E)^1 \otimes L) = 0,$$

whenever $1 \geq A(n,p,q)$ and $p + q \geq n + 1$. The constant A(n,p,q) is explicitly given, and it is shown that it is optimal for $p = n$. Relation (1) still holds for $E^{\otimes k}$ (with a different constant).

These vanishing theorems strengthen similar celebrated results of Griffith and Le Potier.

Reviewer: M. Putinar

Cited in 3 Documents

MSC:
32L20 Vanishing theorems (analytic spaces)
32M10 Homogeneous complex manifolds

Keywords: positive bundle; curvature; Dolbeault cohomology; vanishing theorems

Demailly, Jean-Pierre

Mesures de Monge-Ampère et mesures pluriharmoniques. (Monge-Ampère measures and pluriharmonic measures). (French) Zbl 0595.32006

Math. Z. 194, 519-564 (1987).

Let Ω be a relatively compact open subset in a Stein manifold, and $n = \dim_{\mathbb{C}} \Omega$. Assume that Ω is hyperconvex, i.e. that there exists a bounded psh (plurisubharmonic) exhaustion function on Ω. A "pluricomplex Green function" u_Ω is then naturally defined on $\Omega \times \Omega$: For all $z \in \Omega$, $u_z(\zeta) := u_\Omega(z, \zeta)$ is the solution of the Dirichlet problem for the complex Monge-Ampère equation $(dd^c u_z)^n = 0$ on $\Omega \smallsetminus \{z\}$ such that

$$u_z(\zeta) = \log |\zeta - z| + O(1)$$

at $\zeta = z$; u_Ω is shown to be continuous outside the diagonal and invariant under biholomorphisms. Bedford and Taylor's Monge-Ampère operators are used in conjunction with a general Lelong-Jensen formula previously found by the author [Mem. Soc. Math. Fr., Nouv. Ser. 19, 124 p. (1985; Zbl 0579.32012)] in order to construct an invariant pluricomplex Poisson kernel $d\mu_z(\zeta) := (2\pi)^{-n}(dd^c u_z(\zeta))^{n-1} \wedge d^c u_z(\zeta)|_{\partial\Omega}$, $(z,\zeta) \in \Omega \times \partial\Omega$. Each measure μ_z on $\partial\Omega$ is such that $\mu_z(V) = V(z)$ for every function V pluriharmonic on Ω and continuous on $\bar{\Omega}$; furthermore, μ_z is carried by the set of strictly pseudoconvex points of $\partial\Omega$ if $\bar{\Omega}$ has a C^2 psh defining function. The principal part of the singularity of $d\mu_z(\zeta)$ on the diagonal of $\partial\Omega$ is then computed explicitly when Ω is strictly pseudoconvex, using an osculation of $\partial\Omega$ by balls. Through a complexification process, it is finally shown that Monge-Ampère measures provide an explicit formula representing every point of a convex compact subset $K \subset \mathbb{R}^n$ as a barycenter of the extremal points of K.

MSC:
32A25 Integral representation; canonical kernels (several complex variables)
32C30 Integration on analytic sets and spaces, currents
32F45 Invariant metrics and pseudodistances
32U05 Plurisubharmonic functions and generalizations
32C10 Pluriharmonic and plurisubharmonic functions
32E10 Stein spaces, Stein manifolds

Keywords: pluricomplex Green function; Lelong-Jensen formula; Monge-Ampère measures; plurisubharmonic exhaustion function; hyperconvex domain; pluriharmonic measures; Choquet's theorem; barycentric representation

References:

[1] Bedford, E., Taylor, B.A.: The Dirichlet problem for the complex Monge-Amp?re equation. Invent. Math.37, 1-44 (1976) · Zbl 0325.31013 · DOI: 10.1007/BF01418826

[2] Bedford, E., Taylor, B.A.: A new capacity for plurisubharmonic functions. Acta Math.,149 1-41 (1982) · Zbl 0547.32012 · DOI: 10.1007/BF02392348

[3] Choquet, G.: Existence et unicité des représentations intégrales au moyen des points extrémaux dans les cônes convexes. Sém. Bourbaki, exposé no 139, 15 p. (Déc. 1956)

[4] Demailly, J.-P.: Mesures de Monge-Ampère et caractérisation géométrique des variétés algébriques affines, mémoire (nouvelle série) no 19, Soc. Math. de France, 1985

[5] Diederich, K., Fornaess, J.E.: Pseudoconvex domains: bounded strictly plurisubharmonic functions. Invent. Math.39, 129-141 (1977) · Zbl 0353.32025 · DOI: 10.1007/BF01390105

[6] Fornaess, J.E.: Embedding strictly pseudoconvex domains in convex domains. Am. J. Math.98, 529-569 (1976) · Zbl 0334.32020 · DOI: 10.2307/2373900

[7] Gamelin, T.W., Sibony, N.: Subharmonicity for uniform algebras. J. Funct. Anal.35, 64-108 (1980) · Zbl 0422.46043 · DOI: 10.1016/0022-1236(80)90081-6

[8] Kerzman, N., Rosay, J.-P.: Fonctions plurisousharmoniques d'exhaustion born?es et domaines taut. Math. Ann.257, 171-184 (1981) · Zbl 0461.32006 · DOI: 10.1007/BF01458282

[9] Klimek, M.: Extremal plurisubharmonic functions and invariant pseudodistances. Bull. Soc. Math. France113, 123-142 (1985) · Zbl 0584.32037

[10] Lelong, P.: Fonctionnelles analytiques et fonctions enti?res (n variables). Presses de l'Univ. de Montr?al, S?m. de Math. Sup?rieures, ?t? 1967, no 28, Montr?al, 1968

[11] Lempert, L.: La métrique de Kobayashi et la représentation des domaines sur la boule. Bull. Soc. Math. France109, 427-474 (1981) · Zbl 0492.32025

[12] Lempert, L.: Solving the degenerate Monge-Ampère equation with one concentrated singularity. Math. Ann.263, 515-532 (1983) · Zbl 0531.35020 · DOI: 10.1007/BF01457058

[13] Phelps, R.: Lectures on Choquet's theorem. Van Nostrand Math. Studies no 7, Princeton, New Jersey, 1966 · Zbl 0135.36203

[14] Richberg, R.: Stetige streng pseudokonvexe Funktionen. Math. Ann.175, 257-286 (1968) · Zbl 0153.15401 · DOI: 10.1007/BF02063212

[15] Rudin, W.: Function theory in the unit ball of Cn. Grundlehren der math. Wissenschaften 241. Berlin Heidelberg New York: Springer 1980

[16] Sibony, N.: Remarks on the Kobayashi metric, manuscrit, communication personnelle (juin 1986)

[17] Siciak, J.: On some extremal functions and their applications in the theory of analytic functions of several complex variables. Trans. Am. Math. Soc.105, 322-357 (1962) · Zbl 0111.08102 · DOI: 10.1090/S0002-9947-1962-0143946-5

[18] Siciak, J.: Extremal plurisubharmonic functions in Cn. Ann. Pol. Math.39, 175-211 (1981) · Zbl 0477.32018

[19] Siciak, J.: Extremal plurisubharmonic functions and capacities in Cn. Sophia Kokyuroku in Math., Tokyo, 1982 · Zbl 0579.32025

[20] Stehlé, J.-L.: Fonctions plurisousharmoniques et convexité holomorphe de certains espaces fibrés analytiques. Sém. P. Lelong (Analyse) 1973/74, Lecture Notes in Math. no 474, 155-179 Berlin Heidelberg New York: Springer 1975

[21] Taylor, B.A.: An estimate for an extremal plurisubharmonic function on Cn, S?m. P. Lelong, P. Dolbeault, H. Skoda (Analyse) 1982/83, Lecture Notes in Math. no 1028 318-328. Berlin Heidelberg New York: Springer 1983

[22] Walsh, J.B.: Continuity of envelopes of plurisubharmonic functions. J. Math. Mech.18, 143-148 (1968) · Zbl 0159.16002

[23] Zeriahi, A.: Fonctions plurisousharmoniques extrémales, approximation et croissance des fonctions holomorphes sur des ensembles algébriques. Thèse de Doctorat-ès-Sciences, Univ. de Toulouse, 1986

Demailly, Jean-Pierre

Mesures de Monge-Ampère et mesures pluriharmoniques. (Monge-Ampère measures and pluriharmonic measures). (French) Zbl 0602.31006

Sémin., Équations Dériv. Partielles 1985-1986, Exposé No.19, 15 p. (1986).

This work develops the potential theory of several complex variables in the form introduced by *E. Bedford* and *B. A. Taylor* [Invent. Math. 37, 1-44 (1976; Zbl 0315.31007); Acta Math. 149, 1-40 (1982; Zbl 0547.32012)]. In particular the author introduces a pluricomplex Green function for every bounded hyperconvex domain Ω in a Stein manifold. The Green function, with pole $z \in \Omega$, is a solution u_z of the Dirichlet problem for the complex Monge-Ampère operator with logarithmic pole at z and boundary values 0 on $\partial\Omega$. Using the functions u_z, the author also constructs pluriharmonic measures μ_z on $\partial\Omega$ which have properties analogous to those of harmonic measure.

The paper concludes with an application to the geometry of convex sets: if K is a compact convex subset of R^n, then through a complexification procedure, the measures μ_z provide a formula which allows every point of K to be represented as a barycentre of the extremal points of K.

Another paper of the author, with the same title [Math. Z. (to appear; Zbl 0595.32006)], gives a more detailed account of this work.

Reviewer: D.Armitage

Cited in 2 Documents

MSC:
31C10 Pluriharmonic and plurisubharmonic functions
32E10 Stein spaces, Stein manifolds
32U05 Plurisubharmonic functions and generalizations

Keywords: several complex variables; pluricomplex Green function; hyperconvex domain; Stein manifold; Dirichlet problem; complex Monge-Ampère operator; logarithmic pole; pluriharmonic measures

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Demailly, Jean-Pierre

Sur l'identité de Bochner-Kodaira-Nakano en géométrie hermitienne. (The Bochner-Kodaira-Nakano identity in hermitian geometry). (French) Zbl 0594.32031

Sémin. analyse P. Lelong - P. Dolbeault - H. Skoda, Années 1983/84, Lect. Notes Math. 1198, 88-97 (1986).

[For the entire collection see Zbl 0583.00011.]

It is obtained a generalized Kodaira-Nakano identity, relating the holomorphic and antiholomorphic Laplace-Beltrami operators of a holomorphic hermitian vector bundle over a complex hermitian manifold.

Reviewer: A.Pankov

Cited in 4 Documents

MSC:
32L05 Holomorphic fiber bundles and generalizations
32Q99 Complex manifolds 53C55 Hermitian and Kählerian manifolds (global differential geometry)

Keywords: Bochner-Kodaira-Nakano identity; Laplace-Beltrami operators; holomorphic hermitian vector bundle; complex hermitian manifold

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Demailly, Jean-Pierre

Un exemple de fibré holomorphe non de Stein à fibre ℂ2 au-dessus du disque ou du plan. (An example of a non-Stein holomorphic fiber bundle over the disk or the plane, with fiber $\mathbb{C}^2$). (French) Zbl 0594.32030

Sémin. analyse P. Lelong - P. Dobeault - H. Skoda, Années 1983/84, Lect. Notes Math. 1198, 98-104 (1986).

[For the entire collection see Zbl 0583.00011.]

It is constructed a simple example of a non-Stein holomorphic fiber bundle over the disk with fiber $\mathbb{C}^2$. Moreover, it is shown that all holomorphic functions on the bundle arise from functions on the base.

Reviewer: A.Pankov

Cited in 1 Document

MSC:
32L05 Holomorphic fiber bundles and generalizations
32E10 Stein spaces, Stein manifolds

Keywords: non-Stein holomorphic fiber bundle

Demailly, Jean-Pierre

Fonction de Green pluricomplexe et mesures pluriharmoniques. (French)
Zbl 0900.31004

Séminaire de théorie spectrale et géométrie. Année 1985-1986. Chambéry: Univ. de Savoie, Fac. des Sciences, Service de Math. Sémin. Théor. Spectrale Géom., Chambéry-Grenoble. 4, 131-143 (1986).

From the introduction: "L'objet de cet exposé est de montrer comment à un domaine pseudoconvexe Ω relativement compact dans une variété complexe on peut associer une fonction de Green généralisée, invariante par biholomorphisme. Cette fonction est définie comme la solution $u_z(\zeta)$ du problème de Dirichlet pour l'équation de Monge-Ampère complexe $(dd^c u_z)^n = 0$ sur $\Omega \setminus \{z\}$, ayant un pôle logarithmique au point

z." Le présent exposé est une version condensée de l'article [Math. Z. 194, 519-564 (1987; Zbl 0595.32006)], où le lecteur trouvera des démonstrations détaillées de tous les résultats mentionnés ici. For the entire collection see [Zbl 0825.00039].

MSC:
31C10 Pluriharmonic and plurisubharmonic functions
32T99 Pseudoconvex domains
32A25 Integral representation; canonical kernels (several complex variables)

Keywords: generalized Green function; Monge-Ampère equation; logarithmic pole

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Demailly, Jean-Pierre

Champs magnétiques et inégalités de Morse pour la d''-cohomologie. (Magnetic fields and Morse inequalities for d''-cohomology). (French) Zbl 0595.58014

C. R. Acad. Sci., Paris, Sér. I 301, 119-122 (1985).

This is an announcement of the paper which appeared under the same title in Ann. Inst. Fourier 35, No.4, 189-229 (1985; Zbl 0565.58017). If E is a Hermitian line bundle over a compact complex manifold X, $ic(E)$ the curvature form of the canonical connection of E, F a holomorphic fiber bundle of rank r over X, $X(q)$ the set of points where $ic(E)$ has index q, the author proves that $h^q q_k = \dim H^q(X, E^k \otimes F)$ satisfies as $k \to +\infty$ the asymptotic Morse inequality

$$h_k^q \leq r \frac{k^n}{n!} \int_{X(q)} (-1)^q (ic(E)/2\pi)^n + o(k^n).$$

An analogous inequality holds for $\sum_{j=0}^q (-1)^j h_k^j$. As an application, the author obtains geometric characterizations of Moishezon spaces, improving recent results of Y. T. Siu.

Reviewer: G.Roos

Cited in 2 Reviews

Cited in 3 Documents

MSC:
58E35 Variational inequalities (global problems)
32J25 Transcendental methods of algebraic geometry
32L10 Sections of holomorphic vector bundles

Keywords: $\overline{\partial}$-cohomology; Schrödinger operator; Morse inequality; Moishezon spaces

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Demailly, Jean-Pierre

Mesures de Monge-Ampère et caractérisation géométrique des variétés algébriques affines. (French) Zbl 0579.32012

Mém. Soc. Math. Fr., Nouv. Sér. 19, 124 p. (1985).

Let X be an irreducible n-dimensional Stein space and $\varphi : X \to [-\infty, R[$ a continuous psh (plurisubharmonic) exhaustion function. Each level set $S(r) = \{x \in X\ ;\ \varphi(x) = r\}$, $r < R$, is shown to carry an intrinsic positive measure μ_r; the measure μ_r is given by the $(2n-1)$-form $(dd^c\varphi)^{n-1} \wedge d^c\varphi$ when φ is smooth, and otherwise μ_r is constructed by means of the Monge-Ampère operators introduced by Bedford and Taylor. In this context, a general Lelong-Jensen formula

$$\mu_r(V) = \int_{-\infty}^{r} dt \int_{\varphi<t} dd^cV \wedge (dd^c\varphi)^{n-1} + \int_{\varphi<r} V(dd^c\varphi)^n,$$

is proved and used to study the growth and convexity properties of plurisubharmonic or holomorphic functions. If $(dd^c\varphi)^n = 0$ on $\{\varphi > r_0\}$, the function $r \to \mu_r(V)$ is shown to be convex and increasing in the interval $]r_0, R[$. Furthermore, if the volume $\tau(r) = \int_{\varphi<r}(dd^c\varphi)^n$ has moderate growth, i.e. if $\tau(r) = o(r)$, then bounded holomorphic functions on X are constant; using Siegel's method, we prove also in that case that the ring of holomorphic functions with φ-polynomial growth has a transcendance degree $\leq n$. This last result is then applied in order to obtain a necessary and sufficient geometric criterion characterizing affine algebraic manifolds: X is algebraic iff it has finite Monge-Ampère volume and if the Ricci-curvature of the metric $dd^c(\exp(\varphi))$ is bounded below by $-dd^c\psi$, where $\psi \leq A\varphi + B$.

MSC:
32C30 Integration on analytic sets and spaces, currents
32U05 Plurisubharmonic functions and generalizations
32A22 Nevanlinna theory (local); growth estimates; other inequalities (several complex variables)
32J10 Algebraic dependence theorems (compact analytic spaces)
32J99 Compact analytic spaces

Keywords: closed positive current; plurisubharmonic functions; Siegel's theorem; Monge-Ampère operators; growth; convexity; holomorphic functions; affine algebraic manifolds; Ricci-curvature

References:

[1] E. BISHOP .– Conditions for the analyticity of certain sets ; Michigan Math. Jour. 11 (1964), pp. 289-304. Article | MR 29 #6057 | Zbl 0143.30302 · Zbl 0143.30302 · DOI: 10.1307/mmj/1028999180 · http://minidml.mathdoc.fr/cgi-bin/location?id=00061061

[2] N. BOURBAKI .– Topologie générale , chap. 1 à 4 ; Hermann, Paris, 1971 . Zbl 0249.54001 · Zbl 0249.54001

[3] D. BURNS .– Curvatures of Monge-Ampère foliations and parabolic manifolds ; Ann. of Math. 115 (1982), pp. 349-373. MR 84a:32031 | Zbl 0507.32011 · Zbl 0507.32011 · DOI: 10.2307/1971395

[4] E. BEDFORD & B.A. TAYLOR .– The Dirichlet problem for the complex Monge-Ampère equation ; Invent. Math. 37 (1976), pp. 1-44. MR 56 #3351 | Zbl 0315.31007 · Zbl 0315.31007 · DOI: 10.1007/BF01418826 · EUDML: 142425

[5] E. BEDFORD & B. A. TAYLOR .– A new capacity for plurisubharmonic functions ; Acta Math. 149 (1982), pp. 1-41. MR 84d:32024 | Zbl 0547.32012 · Zbl 0547.32012 · DOI: 10.1007/BF02392348

[6] U. CEGRELL .– On the discontinuity of the complex Monge-Ampère operator ; preprint, cf. note Comptes R. Acad. Sc. Paris (mai 1983). Zbl 0541.31008 · Zbl 0541.31008

[7] S. S. CHERN , H. I. LEVINE & L. NIRENBERG .– Intrinsic norms on a complex manifold ; Global Analysis (Papers in Honor of K. Kodaira) pp. 119-139, Univ. of Tokyo Press, Tokyo, 1969 . MR 40 #8084 | Zbl 0202.11603 · Zbl 0202.11603

[8] J. P. DEMAILLY .– Différents exemples de fibrés holomorphes non de Stein ; sém. P. Lelong-H. Skoda (Analyse) 1976 / 1977 , Lecture Notes in Math. n° 694, Springer-Verlag 1978 . Zbl 0418.32011 · Zbl 0418.32011

[9] J. P. DEMAILLY .– Un exemple de fibré holomorphe non de Stein à fibre $\mathbb{C}^2$ ayant pour base le disque ou le plan ; Inv. Math. 48 (1978), pp. 293-302. MR 81m:32036 | Zbl 0372.32012 · Zbl 0372.32012 · DOI: 10.1007/BF01390248 · EUDML: 142592

[10] J. P. DEMAILLY .– Un exemple de fibré holomorphe non de Stein à fibre $\mathbb{C}^2$ au-dessus du disque ou du plan ; à paraître au Sém. P. Lelong, P. Dolbeault, H. Skoda (Analyse) 1983 - 1984 . Zbl 0594.32030 · Zbl 0594.32030

[11] J. P. DEMAILLY .– Formules de Jensen en plusieurs variables et applications arithmétiques ; Bull. Soc. Math. France 110 (1982), pp. 75-102. Numdam | MR 84d:32014 | Zbl 0493.32003 · Zbl 0493.32003 · NUMDAM: BSMF_1982__110__75_0 · EUDML: 87429

[12] J. P. DEMAILLY .– Sur les nombres de Lelong associés à l'image directe d'un courant positif fermé ; Ann. Inst. Fourier 32, 2 (1982), pp. 37-66. Numdam | MR 84k:32011 | Zbl 0457.32005 · Zbl 0457.32005 · DOI: 10.5802/aif.872 · NUMDAM: AIF_1982__32_2_37_0 · EUDML: 74541

[13] J. P. DEMAILLY & H. SKODA .– Relations entre les notions de positivité de P. A. Griffiths et S. Nakano pour les fibrés vectoriels ; Sém. P. Lelong-H. Skoda (Analyse) 1978 / 1979 , Lect. Notes in Math. 822, Springer, 1980 . Zbl 0454.55011 · Zbl 0454.55011

[14] J. DIEUDONNE .– Cours de géométrie algébrique , tome 2 ; Coll. Sup., Presses Univ. de France, 1974 . Zbl 01958374 · Zbl 1092.14500

[15] H. EL MIR .– Sur le prolongement des courants positifs fermés ; Thèse de Doctorat Univ. de Paris VI (nov. 1982), publiée aux Acta Math., vol. 153 (1984), pp. 1-45 ; cf. aussi Comptes R. Acad. Sc. Paris, série I, t. 294 (1er février 1982) pp. 181-184 et t.295 (18 oct. 1982) pp. 419-422.

[16] J. E. FORNAESS & R. NARASIMHAN .– The Levi problem on complex spaces with singularities ; Math. Ann. 248 (1980), pp. 47-72. MR 81f:32020 | Zbl 0411.32011 · Zbl 0411.32011 · DOI: 10.1007/BF01349254 · EUDML: 163367

[17] J. E. GOODMAN .– Affine open subsets of algebraic varieties and ample divisors ; Ann. of Math. 89 (1969), pp. 160-183. MR 39 #4170 | Zbl 0159.50504 · Zbl 0159.50504 · DOI: 10.2307/1970814

[18] H. GRAUERT .– On Levi's problem and the imbedding of real analytic manifolds ; Ann. of Math. 68 (1958), pp. 460-472. MR 20 #5299 | Zbl 0108.07804 · Zbl 0108.07804 · DOI: 10.2307/1970257

[19] H. HIRONAKA .– Resolution of singularities of an algebraic variety , I-II, Ann. of Math. 79 (1964), pp. 109-326. MR 33 #7333 | Zbl 0122.38603 · Zbl 0122.38603 · DOI: 10.2307/1970486

[20] L. HÖRMANDER .– L^2 estimates and existence theorems for the $\overline{\partial}$-operator ; Acta Math. 113 (1965), pp. 89-152. Zbl 0158.11002 · Zbl 0158.11002 · DOI: 10.1007/BF02391775

[21] L. HÖRMANDER .– An introduction to complex analysis in several variables ; 2nd edition, North Holland, vol. 7, 1973 . Zbl 0271.32001 · Zbl 0271.32001

[22] C. O. KISELMAN .– Sur la définition de l'opérateur de Monge-Ampère complexe ; preprint.

[23] P. LELONG .– Fonctions plurisousharmoniques et formes différentielles positives ; Gordon and Breach, New-York, et Dunod, Paris, 1969 . Zbl 0195.11603 · Zbl 0195.11603

[24] P. LELONG .– Fonctionnelles analytiques et fonctions entières (n variables) ; Presses de l'Univ. de Montréal, 1968 , Sém. de Math. Supérieures, été 1967 , n° 28. Zbl 0194.38801 · Zbl 0194.38801

[25] P. LELONG .– Fonctions entières (n variables) et fonctions plurisousharmoniques d'ordre fini dans $\mathbb{C}^n$; J. Anal. de Jérusalem 62 (1964), pp. 365-407. MR 29 #3668 | Zbl 0126.29602 · Zbl 0126.29602 · DOI: 10.1007/BF02807441

[26] J. MILNOR .– Morse Theory ; Ann. of Math. Studies n° 51, Princeton Univ. Press, 1963 . MR 29 #634 | Zbl 0108.10401 · Zbl 0108.10401

[27] N. MOK .– Courbure bisectionnelle positive et variétés algébriques affines ; C. R. Acad. Sc. Paris, Série I, t. 296 (21 mars 1983), pp. 473-476. MR 85a:32015 | Zbl 0579.53043 · Zbl 0579.53043

[28] N. MOK .– An embedding theorem of complete Kähler manifolds of positive bisectional curvature onto affine algebraic varieties ; à paraître au Bull. Soc. Math. France, t. 112 (1984). Numdam | MR 87a:53103 | Zbl 0536.53062 · Zbl 0536.53062 · NUMDAM: BSMF_1984__112__197_0 · EUDML: 87459

[29] N. MOK .– Complete non compact Kähler manifolds of positive curvature (survey article) ; to appear in a special volume of the Madison conference on Several complex variables, 1982 . Zbl 0537.53054 · Zbl 0537.53054

[30] N. MOK , Y. T. SIU & S. T. YAU .– The Poincaré-Lelong equation on complete Kähler manifolds , Comp. Math., Vol. 44, fasc. 1-3 (1981), pp. 183-218. Numdam | MR 84g:32011 | Zbl 0531.32007 · Zbl 0531.32007 · NUMDAM: CM_1981__44_1-3_183_0 · EUDML: 89516

[31] S. NAKANO .– Vanishing theorems for weakly 1-complete manifolds II , Publ. R.I.M.S., Kyoto University, 1974 , pp. 101-110. Article | MR 52 #3617 | Zbl 0298.32019 · Zbl 0298.32019 · DOI: 10.2977/prims/1195192175 · http://minidml.mathdoc.fr/cgi-bin/location?id=00260309

[32] R. NARASIMHAN .– Introduction to the theory of analytic spaces ; Lecture Notes in Math. n° 25, 1966 , Springer-Verlag. MR 36 #428 | Zbl 0168.06003 · Zbl 0168.06003 · DOI: 10.1007/BFb0077071

[33] N. SIBONY .– Quelques problèmes de prolongement de courants en Analyse complexe ; prépublication Univ. de Paris-Sud, Orsay, n° 84 T 15.

[34] N. SIBONY & P. M. WONG .– Some remarks on the Casorati-Weierstrass thorem ; Ann. Polon. Math. 39 (1981), pp. 165-174. MR 82k:32015 | Zbl 0476.32005 · Zbl 0476.32005

[35] C. L. SIEGEL .– Meromorphe Funktionen auf compakten analytischen Mannigfaltigkeiten ; Cöttinger Nachr. (1955) pp. 71-77. MR 17,530c | Zbl 0064.08201 · Zbl 0064.08201

[36] C. L. SIEGEL .– On meromorphic functions of several variables ; Bull. Calcutta Math. Soc. 50 (1958) pp. 165-168. MR 21 #2748 | Zbl 0121.06802 · Zbl 0121.06802

[37] Y. T. SIU - S. T. YAU .– Complete Kähler manifolds with non positive curvature of faster than quadratic decay ; Ann. of Math. 105 (1977), pp. 225-264. MR 55 #10719 | Zbl 0358.32006 · Zbl 0358.32006 · DOI: 10.2307/1970998

[38] H. SKODA .– Estimations L^2 pour l'opérateur $\overline{\partial}$ et applications arithmétiques ; Sém. P. Lelong (Analyse) 1975 / 1976 , Lecture Notes in Math. n° 538, Springer-Verlag 1977 . Zbl 0363.32004 · Zbl 0363.32004

[39] H. SKODA .– Fibrés holomorphes à base et à fibre de Stein ; Inv. Math. 43 (1977), pp. 97-107. MR 58 #22657 | Zbl 0365.32018 · Zbl 0365.32018 · DOI: 10.1007/BF01390000 · EUDML: 142510

[40] H. SKODA .– Morphismes surjectifs et fibrés linéaires semi-positifs ; Sém. P. Lelong, H. Skoda (Analyse) 1976 / 1977 , Lecture Notes in Math. n° 694, Springer-Verlag 1978 . Zbl 0396.32009 · Zbl 0396.32009

[41] H. SKODA .– Morphismes surjectifs de fibrés vectoriels semi-positifs ; Ann. Scient. Ec. Norm. Sup., 4e série, t. 11 (1978), p. 577-611. Numdam | MR 80j:32047 | Zbl 0403.32019 · Zbl 0403.32019 · NUMDAM: ASENS_1978_4_11_4_577_0 · EUDML: 82026

[42] H. SKODA .– Prolongement des courants positifs fermés de masse finie ; Invent. Math. 66 (1982), pp. 361-376. MR 84k:32020 | Zbl 0488.58002 · Zbl 0488.58002 · DOI: 10.1007/BF01389217 · EUDML: 142887

[43] E. H. SPANIER .– Algebraic topology ; Mc Graw-Hill, 1966 . MR 35 #1007 | Zbl 0145.43303 · Zbl 0145.43303

[44] W. STOLL .– The growth of the area of a transcendental analytic set , I and II, Math. Ann. 156 (1964), pp. 47-78 and pp. 144-170. MR 29 #3670 | Zbl 0126.09502 · Zbl 0126.09502 · DOI: 10.1007/BF01359980 · EUDML: 161208

[45] W. STOLL .– The characterization of strictly parabolic manifolds ; Ann. Sc. Norm. Sup. Pisa, s. IV, vol. VII, n° 1 (1980), pp. 87-154. Numdam | MR 81h:32028 | Zbl 0438.32005 · Zbl 0438.32005 · NUMDAM: ASNSP_1980_4_7_1_87_0 · EUDML: 83834

[46] P. THIE .– The Lelong number of a point of a complex analytic set ; Math. Ann. 172 (1967), pp. 269-312. MR 35 #5661 | Zbl 0158.32804 · Zbl 0158.32804 · DOI: 10.1007/BF01351593 · EUDML: 161592

Demailly, Jean-Pierre

Propagation des singularités des courants positifs fermés. (French) Zbl 0566.32005

Ark. Mat. 23, 35-52 (1985).

Let T be a closed positive current in a bounded Runge open subset $\Omega \subset \mathbb{C}^n$. One studies sufficient conditions to be verified by the mass density of T in order that there exists a global extension of T to $\mathbb{C}^n$. Assume that T is of bidegree $(1,1)$ (resp. (q,q), $q>1$), that the cohomology class of T in Ω is 0 and that the trace measure $\sigma_T(z,r)$ of the ball $B(z,r)$ satisfies

$$(*) \qquad \sup_{z\in K}\int_0^\varepsilon \frac{\sigma_T(z,r)}{r^{2n-1}}dr<+\infty \quad \text{resp.} \quad \int_0^\varepsilon \left(\sup_{z\in K}\left(\sigma_T(z,r)\right)^{1/2}/r^n\right)dr<+\infty,$$

for every $K\subset\subset\Omega$ and $\varepsilon>0$ small enough. Then for every $\omega_1\subset\subset\omega_2\subset\subset\Omega$, one proves the existence of a closed positive current Θ in $\mathbb{C}^n$ which is equal to T in ω_1 and C^∞-smooth in $\mathbb{C}^n\smallsetminus\overline{\omega}_2$. Conversely, using the Skoda-El Mir structure theorems for closed ≥ 0 currents, one constructs various counterexamples to the extension problem. When T has bidegree $(1,1)$, one shows that $(*)$ is essentially the best possible condition allowing extension, whereas in bidegree (q,q) there exists a current T such that $\sup_{z\in K}\sigma_T(z,r)\leq C_K r^{2n-2q-1}$, whose singularities propagate up to $\partial\Omega$ along $(2n-2q-1)$-CR submanifolds. This last example rests upon the existence of totally real complete pluripolar $(n-1)$-submanifolds in $\mathbb{C}^n$, a result due to Diederich-Fornaess that we reprove in a new and simpler way.

Cited in 1 Review

Cited in 3 Documents

MSC:
32C30 Integration on analytic sets and spaces, currents
32D15 Continuation of analytic objects (several variables)

Keywords: global extension of closed positive current; propagation of singularities; density condition; bounded Runge open subset

References:

[1] Demailly, J. P., Courants positifs extrémaux et conjecture de Hodge;Inv. Math.,69, (1982), 347–374. · Zbl 0488.58001 · DOI: 10.1007/BF01389359

[2] Diederich, K. andFornaess, J. E., Smooth, but not complex analytic pluripolar sets; Manuscripta Math. 37, (1982), 121–125. · Zbl 0483.32012 · DOI: 10.1007/BF01239949

[3] El Mir, H Théorèmes de prolongement des courants positifs fermés; Thèse de Doctorat d'Etat soutenue à l'Université de Paris VI, novembre 1982; Acta Math. 153 (1984).

[4] Lelong, P.,Fonctions plurisousharmoniques et formes différentielles positives; Gordon and Breach, New York, et Dunod, Paris (1967).

[5] Siu, Y. T., Analyticity of sets associated to Lelong numbers and the extension of closed positive currents;Inv. Math. 27, pp. 53–156 (1974). · Zbl 0289.32003 · DOI: 10.1007/BF01389965

[6] Skoda, H., Prolongement des courants positifs fermés de masse finie;Inv. Math. 66, pp. 361–376 (1982). · Zbl 0488.58002 · DOI: 10.1007/BF01389217

Demailly, Jean-Pierre

Champs magnétiques et inégalités de Morse pour la d''-cohomologie. (Magnetic fields and Morse inequalities for d''-cohomology). (French) Zbl 0565.58017

Ann. Inst. Fourier 35, No.4, 189-229 (1985).

Nous démontrons des inégalités de Morse-Witten asymptotiques pour la dimension des groupes de cohomologie des puissances tensorielles d'un fibré holomorphe en droites hermitien au-dessus d'une variété ℂ-analytique compacte. La dimension du q-ième groupe de cohomologie se trouve ainsi majorée par une intégrale de courbure intrinsèque, étendue à l'ensemble des points d'indice q de la forme de courbure du fibré. La preuve repose sur un théorème spectral qui décrit la distribution asymptotique des valeurs propres de l'opérateur de Schrödinger associé à un champ magnétique assez grand. Comme application, nous obtenons une nouvelle démonstration de la conjecture de Grauert-Riemenschneider sur la caractérisation des espaces de Moišezon, résolue récemment par Siu, sous des hypothèses géométriques plus générales qui n'exigent pas nécessairement la semi-positivité ponctuelle du fibré.

MSC:
58E05 Abstract critical point theory

Keywords: cohomology; Morse inequalities; curvature; Schrödinger operator for magnetic field

Demailly, Jean-Pierre

Sur les transformées de Fourier de fonctions continues et le théorème de de Leeuw-Katznelson-Kahane. (On Fourier transforms of continuous functions and a theorem of de Leeuw-Katznelson-Kahane). (French) Zbl 0571.43003

C. R. Acad. Sci., Paris, Sér. I 299, 435-438 (1984).

Given any locally compact abelian group G and any function $\varphi \in L^2(\widehat{G})$, we prove the existence of a function $f \in L^2(G)$ continuous and vanishing at infinity such that $|\widehat{f}| \geq |\varphi|$ a.e. on $\widehat{G}$.

MSC:
43A25 Fourier and Fourier-Stieltjes transforms on locally compact and other abelian groups

Keywords: Fourier transforms; locally compact abelian group

Demailly, Jean-Pierre

Sur la propagation des singularités des courants positifs fermés. (English) Zbl 0551.32009

Analyse complexe, Proc. Journ. Fermat - Journ. SMF, Toulouse 1983, Lect. Notes Math. 1094, 53-64 (1984).

[For the entire collection see Zbl 0539.00009.]

Let T be a closed positive current in a bounded Runge open subset $\Omega \subset \mathbb{C}^n$. We study sufficient conditions to be verified by the mass densities of T in order that there exists a global extension of T to $\mathbb{C}^n$. Assume that the cohomology class of T in Ω is 0 and that the trace measure σ_T satisfies

$$(*) \qquad \int_0^{\delta/2} \Big(\sup_{z \in \Omega_\delta \setminus \Omega_\varepsilon} \big(\sigma_T(z,r)\big)^{\frac{1}{2}} / r^n \Big) dr < +\infty$$

for some $\varepsilon > \delta > 0$, where $\Omega\varepsilon = \{z \in \Omega\,;\ d(z, \mathbb{C}^n \setminus \Omega) > \varepsilon\}$. Then we prove the existence of a closed positive current Θ in $\mathbb{C}^n$ which is equal to T in Ω_ε and C^∞-smooth in $\mathbb{C}^n \setminus \overline{\Omega}_\delta$. Thus, there is no propagation of singularities in that case. Conversely, using the Skoda-El Mir structure theorems for closed ≥ 0 currents and a result of Diederich-Fornaess on the existence of totally real complete pluripolar $(n-1)$-submanifolds in $\mathbb{C}^n$, we construct specific counterexamples to the extension problem with density bounds. When T has bidegree $(1,1)$, we show that the best possible sufficient condition allowing extension is

$$(**) \qquad \sup_{z \in \Omega_\delta \setminus \Omega_\varepsilon} \int_0^{\delta/2} \Big(\sigma_T(z,r) / r^{2n-1} \Big) dr < +\infty,$$

whereas in bidegree (q,q) there exist currents T such that $\sup_{z \in \Omega_\delta} \sigma_T(z,r) \leq C r^{2n-q-1}$, whose singularities propagate up to $\partial\Omega$ along $(2n-q-1)$-CR submanifolds.

MSC:
32C30 Integration on analytic sets and spaces, currents
32Sxx Singularities (analytic spaces)

Keywords: extension problem; propagation of singularities; density condition; closed positive current; bounded Runge open subset

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Demailly, Jean-Pierre; Gaveau, Bernard

Majoration statistique de courbure d'une variété analytique. (French) Zbl 0533.53054

Sémin. d'Analyse P. Lelong-P. Dolbeault-H. Skoda, Années 1981/83, Lect. Notes Math. 1028, 96-124 (1983).

[For the entire collection see Zbl 0511.00025.]

Let $\Omega \subset \mathbb{C}^n$ be a bounded strictly pseudoconvex open subset. Given an analytic map $F : \Omega \to \mathbb{C}^p$, one studies the average growth of the Ricci curvature of the level varieties $X_\zeta = F^{-1}(\zeta)$. Especially, if $R = -\mathrm{Ricci}(X_\zeta) \geq 0$ denotes the positive Ricci $(1,1)$-form of X_ζ, δ the distance to $\partial\Omega$ and $\alpha = dd^c|z|^2$, it is shown that the estimate

$$\int_{\zeta\in\mathbb{C}^n} d\lambda(\zeta) \int_{X_\zeta} \delta^{p+q} \left[\log(1+1/\delta)\right]^{-q} R^q \wedge \alpha^{n-p-q} < +\infty$$

holds for every $q = 0, 1, ..., n-p$ when F is bounded. A more general estimate valid for any order of growth of F is also given. The proof consists essentially in integrations by parts of positive currents, using the explicit expression of R in terms of the derivatives of F. Denote now by $\Gamma = \det R$ the Gaussian curvature of a smooth complex hypersurface in $\mathbb{C}^n$. R is proved to verify the following Monge-Ampère equation:

$$i\partial\overline{\partial}\log\Gamma = -(n+1)R + 2\pi[Z]$$

where Z is the divisor of zeros of Γ.

MSC:
53C55 Hermitian and Kählerian manifolds (global differential geometry) 53C65 Integral geometry
32C30 Integration on analytic sets and spaces, currents

Keywords: growth estimate of Ricci curvature of level variety; positive current; Monge-Ampère equation; strictly pseudoconvex set; Gaussian curvature of a smooth complex hypersurface

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Demailly, Jean-Pierre

Sur la structure des courants positifs fermés. (French) Zbl 0529.32006

Inst. Elie Cartan, Univ. Nancy I 8, 52-62 (1983).

MSC:
32C30 Integration on analytic sets and spaces, currents 30C80 Maximum principle; Schwarz's lemma, Lindelöf principle, etc. (one complex variable)
14C30 Transcendental methods, Hodge theory, Hodge conjecture
32A30 Generalizations of function theory to several variables

Keywords: closed positive current; Lelong number; extremal current; Hodge conjecture; Lelong-Jensen formulas; plurisubharmonic weight; Schwarz lemma; integration currents

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Demailly, Jean-Pierre

Constructibilité des faisceaux de solutions des systèmes différentiels holonomes. (D'après Masaki Kashiwara). (French) Zbl 0529.32003

Sémin. d'Analyse P. Lelong - P. Dolbeault - H. Skoda, Années 1981/83, Lect. Notes Math. 1028, 83-95 (1983).

MSC:
32A45 Hyperfunctions (complex analysis) 58J15 Relations with hyperfunctions (PDE on manifolds)
14F10 Special sheaves; D-modules; Bernstein-Sato ideals and polynomials 58J10 Differential complexes; elliptic complexes
32L05 Holomorphic fiber bundles and generalizations

Keywords: microdifferential operator; coherent differential module; holonomic system; constructibility of extension sheaf; stratification; differential operators; Lagrangian characteristic variety; extension theorem; D-module

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Demailly, Jean-Pierre

Estimations L^2 pour l'opérateur $\overline{\partial}$ d'un fibré vectoriel holomorphe semi-positif au-dessus d'une variété Kählérienne complète. (French) Zbl 0507.32021

Ann. Sci. Éc. Norm. Supér. (4) 15, 457-511 (1982).

Cited in 5 Reviews

Cited in 38 Documents

MSC:
32L20 Vanishing theorems (analytic spaces)
32L05 Holomorphic fiber bundles and generalizations 53C55 Hermitian and Kählerian manifolds (global differential geometry)
32L10 Sections of holomorphic vector bundles
32U05 Plurisubharmonic functions and generalizations

Keywords: Kaehler manifold; vanishing theorems; plurisubharmonic weights; smoothing theorems; s-positive hermitian vector bundle; weakly pseudoconvex

References:

[1] C. A. BERENSTEIN and B. A. TAYLOR , Interpolation Problems in $\mathbb{C}^n$ with Applications to Harmonic Analysis (à paraître au Journal d'Analyse Math. de Jérusalem). · Zbl 0464.42003

[2] E. BOMBIERI , Algebraic Values of Meromorphic Maps (Invent. Math., vol. 10, p. 267-287, 1970 et 11, p. 163-166, 1970). MR 46 #5328 | Zbl 0214.33702 · Zbl 0214.33702 · DOI: 10.1007/BF01418775 · EUDML: 142035

[3] J. BRIANÇON , Sur la clôture intégrale d'un idéal de germes de fonctions holomorphes en un point de $\mathbb{C}^n$; preprint de l'Université de Nice, février 1974 (non publié). MR 49 #5394

[4] J. BRIANÇON et H. SKODA , Sur la clôture intégrale d'un idéal de germes de fonctions holomorphes en un point de $\mathbb{C}^n$ (C. R. Acad. Sc., t. 278, série A, 1974 , p. 949-951). MR 49 #5394 | Zbl 0307.32007 · Zbl 0307.32007

[5] J.-P. DEMAILLY , Scindage holomorphe d'un morphisme de fibrés vectoriels semi-positifs avec estimations L^2 (à paraître au Séminaire P. Lelong, H. Skoda, 1980 - 1981). Zbl 0481.32011 · Zbl 0481.32011

[6] J.-P. DEMAILLY , Relations entre les différentes notions de fibrés et de courants positifs (à paraître au Séminaire P. Lelong, H. Skoda, 1980 - 1981). Zbl 0481.32010 · Zbl 0481.32010

[7] J.-P. DEMAILLY et H. SKODA , Relations entre les notions de positivités de P. A. Griffiths et de S. Nakano pour les fibrés vectoriels [Séminaire P. Lelong, H. Skoda (Analyse), 19e année, 1978 - 1979 , Lecture Notes, n° 822, 1980 , Springer-Verlag, Berlin, Heidelberg, New York]. Zbl 0454.55011 · Zbl 0454.55011

[8] K. DIEDERICH und R. P. PFLUG , Über Gebiete mit vollständiger Kählermetrik (à paraître aux Math. Annalen). Zbl 0472.32011 · Zbl 0472.32011 · DOI: 10.1007/BF01458284 · EUDML: 163551

[9] A. DOUADY et J.-L. VERDIER , Séminaire de Géométrie analytique , E.N.S., 1972 - 1973 , Différents aspects de la positivité (Astérisque, 17, 1974 , Société Mathématique de France).

[10] H. GRAUERT , Charakterisierung der Holomorphie-gebiete durch die vollständige kählersche Metrik (Math. Annalen, t. 131, 1956 , p. 38-75). MR 17,1072a | Zbl 0073.30203 · Zbl 0073.30203 · DOI: 10.1007/BF01354665 · EUDML: 160478

[11] R. E. GREENE and H. WU , C^∞ Approximation of Convex, Subharmonic, and Plurisubharmonic Functions (Ann. scient. Éc. Norm. Sup., 4e série, t. 12, 1979 , p. 47 à 84). Numdam | MR 80m:53055 | Zbl 0415.31001 · Zbl 0415.31001 · NUMDAM: ASENS_1979_4_12_1_47_0 · EUDML: 82031

[12] P. A. GRIFFITHS , Hermitian Differential Geometry, Chern Classes and Positive Vector Bundles ; Global Analysis , Princeton University Press, 1969 , p. 185-251. MR 41 #2717 | Zbl 0201.24001 · Zbl 0201.24001

[13] L. HÖRMANDER , L^2 Estimates and Existence Theorem for the $\overline{\partial}$-Operator (Acta Math., 113, 1965 , p. 89-152). Zbl 0158.11002 · Zbl 0158.11002 · DOI: 10.1007/BF02391775

[14] L. HÖRMANDER , An Introduction to Complex Analysis, in Several Variables ; Princeton, van Nostrand Company, 1966 ; 2e édition, North-Holland/American Elsevier, 1973 . Zbl 0271.32001 · Zbl 0271.32001

[15] L. HÖRMANDER , Generators for Some Rings of Analytic Functions (Bull. Amer. Math. Soc., vol. 73, 1967 , p. 943-949). Article | MR 37 #1977 | Zbl 0172.41701 · Zbl 0172.41701 · DOI: 10.1090/S0002-9904-1967-11860-3 · http://minidml.mathdoc.fr/cgi-bin/location?id=00221347

[16] B. JENNANE , Extension d'une fonction définie sur une sous-variété avec contrôle de la croissance [Séminaire P. Lelong-H. Skoda (Analyse), 17e année, 1976 - 1977 , Lecture Notes in Math., n° 694, Springer-Verlag, Berlin, Heidelberg, New York, 1978]. Zbl 0403.32008 · Zbl 0403.32008

[17] J. J. KELLEHER and B. A. TAYLOR , Finitely Generated Ideals in Rings of Analytic Functions (Math. Ann., band 193, heft 3, 1971). MR 46 #2077 | Zbl 0207.12906 · Zbl 0207.12906 · DOI: 10.1007/BF02052394 · EUDML: 182723

[18] P. LELONG , Fonctionnelles analytiques et fonctions entières (n variables) ; Montréal, les Presses de l'Université de Montréal, 1968 (Séminaire de Mathématiques supérieures, été 1967 , n° 28). Zbl 0194.38801 · Zbl 0194.38801

[19] J. LE POTIER , Annulation de la cohomologie à valeurs dans un fibré vectoriel holomorphe positif de rang quelconque (Math. Ann., t. 218, 1975 , p. 35-53). MR 52 #6044 | Zbl 0313.32037 · Zbl 0313.32037 · DOI: 10.1007/BF01350066 · EUDML: 162783

[20] S. NAKANO , Vanishing Theorems for Weakly 1-Complete Manifolds II (Publ. R.I.M.S., Kyoto University, vol. 10, 1974 , p. 101). Article | MR 52 #3617 | Zbl 0298.32019 · Zbl 0298.32019 · DOI: 10.2977/prims/1195192175 · http://minidml.mathdoc.fr/cgi-bin/location?id=00260309

[21] R. RICHBERG , Stetige streng pseudokonvexe Funktionen (Math. Ann., t. 175, 1968 , p. 257-286). MR 36 #5386 | Zbl 0153.15401 · Zbl 0153.15401 · DOI: 10.1007/BF02063212 · EUDML: 161667

[22] H. SKODA , Application des techniques L^2 à la théorie des idéaux d'une algèbre de fonctions holomorphes avec poids (Ann. scient. Éc. Norm. Sup., t. 5, fasc. 4, 1972 , p. 545-579). Numdam | MR 48 #11571 | Zbl 0254.32017 · Zbl 0254.32017 · NUMDAM: ASENS_1972_4_5_4_545_0 · EUDML: 81906

[23] H. SKODA , Formulation hilbertienne du Nullstellensatz dans les algèbres de fonctions holomorphes ; paru dans l'Analyse harmonique dans le domaine complexe, Lecture Notes in Math., n° 336, Springer-Verlag, Berlin, Heidelberg, New York, 1973 . MR 52 #11114 | Zbl 0259.32004 · Zbl 0259.32004

[24] H. SKODA , Morphismes surjectifs et fibrés linéaires semi-positifs [Séminaire P. Lelong-H. Skoda (Analyse), 17e année, 196-77, Lecture Notes in Math., n° 694, Springer-Verlag, Berlin, Heidelberg, New York, 1978]. MR 80b:32027 | Zbl 0396.32009 · Zbl 0396.32009

[25] H. SKODA , Morphismes surjectifs de fibrés vectoriels semi-positifs (Annales scient. Éc. Norm. Sup., 4e série, t. 11, p. 577-611, 1978). Numdam | MR 80j:32047 | Zbl

0403.32019 · Zbl 0403.32019 · NUMDAM: ASENS_1978_4_11_4_577_0 · EUDML: 82026

[26] H. SKODA , Relèvement des sections globales dans les fibrés semi-positifs [Séminaire P. Lelong-H. Skoda (Analyse), 19e année, 1978 - 1979 , Lecture Notes in Math., n° 822, Springer-Verlag, Berlin, Heidelberg, New York, 1980]. Zbl 0443.32017 · Zbl 0443.32017

[27] H. SKODA , Estimations L^2 pour l'opérateur $\overline{\partial}$ et applications arithmétiques [Séminaire P. Lelong (Analyse), 16e année, 1975 - 1976 , p. 314-323, Lecture Notes in Math., n° 538, Springer-Verlag, Berlin, Heidelberg, New York, 1977]. Zbl 0363.32004 · Zbl 0363.32004

[28] H. SKODA , Sous-ensembles analytiques d'ordre fini ou infini dans $\mathbb{C}^n$ (Bull. Soc. Math. Fr., t. 100, 1972 , p. 353-408). Numdam | MR 50 #5004 | Zbl 0246.32009 · Zbl 0246.32009 · NUMDAM: BSMF_1972__100__353_0 · EUDML: 87191

[29] A. WEIL , Variétés kählériennes , Hermann, Paris, 1957 . Zbl 0137.41103 · Zbl 0137.41103

Demailly, Jean-Pierre

Formules de Jensen en plusieurs variables et applications arithmétiques. (French) Zbl 0493.32003

Bull. Soc. Math. Fr. 110, 75-102 (1982).

Cited in 1 Review

Cited in 12 Documents

MSC:
32A25 Integral representation; canonical kernels (several complex variables) 30C80 Maximum principle; Schwarz's lemma, Lindelöf principle, etc. (one complex variable)
32A15 Entire functions (several variables)
32C30 Integration on analytic sets and spaces, currents
32A30 Generalizations of function theory to several variables

Keywords: Lelong numbers; entire functions; Schwarz lemma; algebraic values of meromorphic maps; zero sets of polynomials

References:

[1] BOMBIERI (E.) .– Algebraic values of meromorphic maps , Inventiones Math., Vol. 10, 1970 , p. 267-287 et Vol. 11, 1970 , p. 163-166. Zbl 0214.33702 · Zbl 0214.33702 · DOI: 10.1007/BF01418775 · EUDML: 142035

[2] CHUDNOVSKY (G. V.) .– Singular points on complex hypersurfaces and multidimensional Schwarz lemma , Séminaire Delange-Pisot-Poitou, 21e année, 1979 - 1980 , Progress in Math., no 12, p. 29-69, Marie-José BERTIN, éd., Boston, Basel, Stuttgart, Birkhäuser, 1981 . Zbl 0455.32004 · Zbl 0455.32004

[3] DEMAILLY (J.-P.) .– Sur les nombres de Lelong associés à l'image directe d'un courant positif fermé , à paraître aux Ann. Inst. Fourier, t. 32, fasc. 2, 1982 . Numdam | MR 84k:32011 | Zbl 0457.32005 · Zbl 0457.32005 · DOI: 10.5802/aif.872 · NUMDAM: AIF_1982__32_2_37_0 · EUDML: 74541

[4] LELONG (P.) .– Fonctions plurisousharmoniques et formes différentielles positives , Gordon and Breach, New York, et Dunod, Paris, 1967 . Zbl 0195.11603 · Zbl 0195.11603

[5] LELONG (P.) .– Sur les cycles holomorphes à coefficients positifs dans C" et un complément au théorème de E. Bombieri , C. R. Math. Rep. Acad. Sc. Canada, vol. 1, no 4, 1979 , p. 211-213. MR 80j:32017 | Zbl 0421.32005 · Zbl 0421.32005

[6] MOREAU (J.-C.) .– Lemmes de Schwarz en plusieurs variables et applications arithmétiques , Séminaire Pierre Lelong-Henri Skoda, Analyse, année 1978 - 1979 , p. 174-190 ; Lectures Notes in Math., no 822, Springer Verlag, 1980 . Zbl 0452.10036 · Zbl 0452.10036

[7] NARASIMHAN (R.) .– Introduction to analytic spaces , lecture Notes in Math., no 25, Springer Verlag, 1966 . MR 36 #428 | Zbl 0168.06003 · Zbl 0168.06003 · DOI: 10.1007/BFb0077071

[8] SKODA (H.) .– Estimations L^2 pour l'opérateur $\overline{\partial}$ et applications arithmétiques , Séminaire Pierre Lelong, Analyse, année 1975 - 1976 , p. 314-323 ; Lecture Notes in Math., no 538, Springer Verlag, 1977 . Zbl 0363.32004 · Zbl 0363.32004

[9] WALDSCHMIDT (M.) .– Propriétés arithmétiques des fonctions de plusieurs variables (II) , Séminaire Pierre Lelong, Analyse, année 1975 - 1976 , p. 108-135 ; Lecture Notes in Math., no 538, Springer Verlag, 1977 . Zbl 0363.32003 · Zbl 0363.32003

[10] WALDSCHMIDT (M.) .– Nombres transcendants et groupes algébriques , Astérisque, no 69-70, 1979 . MR 82k:10041 | Zbl 0428.10017 · Zbl 0428.10017

Demailly, Jean-Pierre

Courants positifs extrémaux et conjecture de Hodge. (French) Zbl 0488.58001

Invent. Math. 69, 347-374 (1982).

MSC:
58A25 Currents (global analysis) 58A14 Hodge theory (global analysis)
14C30 Transcendental methods, Hodge theory, Hodge conjecture
14C20 Divisors, linear systems, invertible sheaves

Keywords: extremal elements on the cone of closed strongly positive currents which are not integration currents over analytic subsets; Hodge conjecture; approximation of currents of bidegree $(1,1)$ by irreducible divisors on projective varieties or Stein manifolds

References:

[1] Bourbaki, N.: Espaces vectoriels topologiques, chap. 1 et 2, Paris: Hermann, 1964 · Zbl 0205.34302

[2] Demailly, J.-P.: Construction d'hypersurfaces irréductibles avec lieu singulier donné dans $\mathbb{C}^n$. Ann. de l'Inst. Fourier 30, (fasc. 3) 219-236 (1980)

[3] Federer, H.: Geometric measure theory, Band 153. Berlin, Heidelberg, New York: Springer 1969 · Zbl 0176.00801

[4] Grauert, H.: On Levi's problem and the imbedding of real analytic manifolds. Ann. of Math.68, (no 2) 460-472 (1958) · Zbl 0108.07804 · DOI: 10.2307/1970257

[5] Harvey, R.: Holomorphic chains and their boundaries. Proceedings of Symposia in pure Mathematics of the Amer. Math. Soc., held at Williamstown, vol.30, Part 1, pp. 309-382 (1975)

[6] Harvey, R., Knapp, A.W.: Positive (p, p) forms, Wirtinger's inequality and currents. Value distribution theory. Part A: Proc. Tulane Univ. Program on Value Distribution Theory in Complex Analysis and Related Topics in differential Geometry, 1972-1973; pp. 43-62, New York Dekker 1974 · Zbl 0287.53046

[7] Lelong, P.: Intégration sur un ensemble analytique complexe. Bull. Soc. Math. France85, 239-262 (1957) · Zbl 0079.30901

[8] Lelong, P.: Fonctions plurisousharmoniques et formes différentielles positives. New York: Gordon and Breach, Paris: distribué par Dunod Editeur, 1968 · Zbl 0195.11603

[9] Lelong, P.: Eléments extrémaux sur le cône des courants positifs fermés. Séminaire P. Lelong (Analyse), 12e année, 1971-1972, Lecture Notes in Math., vol. 332. Berlin, Heidelberg, New York: Springer 1972

[10] Phelps, R.: Lectures on Choquet's theorem. Princeton, New Jersey: Van Nostrand, 1966 · Zbl 0135.36203

[11] Skoda, H.: Prolongement des courants positifs fermés de masse finie. Invent. Math.66, 361-376 (1982) · Zbl 0488.58002 · DOI: 10.1007/BF01389217

Demailly, Jean-Pierre

Scindage holomorphe d'un morphisme de fibrés vectoriels semi-positifs avec estimations L^2. (French) Zbl 0481.32011

Sémin. P. Lelong - H. Skoda, Analyse, Années 1980/81, et: Les fonctions plurisousharmoniques en dimension finie ou infinie, Colloq. Wimereux 1981, Lect. Notes Math. 919, 77-107 (1982).

Cited in 1 Review

Cited in 2 Documents

MSC:
32L05 Holomorphic fiber bundles and generalizations 53C55 Hermitian and Kählerian manifolds (global differential geometry)
32D15 Continuation of analytic objects (several variables)
32T99 Pseudoconvex domains

Keywords: holomorphic vector bundles; weakly pseudoconvex Kaehlerian manifold; holomorphic retraction; Hörmander-Bombieri-Skoda theorem; holomorphic splitting; extension problem

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Demailly, Jean-Pierre

Relations entre les différentes notions de fibrés et de courants positifs. (French) Zbl 0481.32010

Sémin. P. Lelong - H. Skoda, Analyse, Années 1980/81, et: Les fonctions plurisousharmoniques en dimension finie ou infinie, Colloq. Wimereux 1981, Lect. Notes Math. 919, 56-76 (1982).

Cited in 2 Documents

MSC:
32L05 Holomorphic fiber bundles and generalizations
32C30 Integration on analytic sets and spaces, currents 58A25 Currents (global analysis)

Keywords: semi-positive vector bundles; currents; hermitian holomorphic vector bundle; positive differential forms

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Demailly, Jean-Pierre

Extremal positive currents and Hodge conjecture. (English) Zbl 0476.58001

Invent. Math. 69 (1982) 347-374

Cited in 2 Reviews

MSC:
58A25 Currents (global analysis) 58A14 Hodge theory (global analysis)

14C30 Transcendental methods, Hodge theory, Hodge conjecture
14C20 Divisors, linear systems, invertible sheaves

Keywords: extremal elements on the cone of closed strongly positive currents which are not integration currents over analytic subsets; Hodge conjecture; approximation of currents of bidegree (1,1) by irreducible divisors on projective varieties or Stein manifolds

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Demailly, Jean-Pierre

Sur les nombres de Lelong associés à l'image directe d'un courant positif fermé. (French) Zbl 0457.32005

Ann. Inst. Fourier 32, No.2, 37-66 (1982).

Cited in 2 Reviews

Cited in 9 Documents

MSC:
32C30 Integration on analytic sets and spaces, currents 58A25 Currents (global analysis)

Keywords: Jensen formula; Lelong numbers of closed positive current; multiplicity

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Demailly, Jean-Pierre; Skoda, Henri

Relations entre les notions de positivités de P.A. Griffiths et de S. Nakano pour les fibres vectoriels. (French) Zbl 0454.55011

Sémin. P. Lelong - H. Skoda, Analyse, Années 1978/79, Lect. Notes Math. 822, 304-309 (1980).

Cited in 4 Documents

MSC:
55R25 Sphere bundles; vector space bundles
32L05 Holomorphic fiber bundles and generalizations
32L15 Bundle convexity

Keywords: positive holomorphic hermitian vector bundle in the sense of Griffiths; positive holomorphic hermitian vector bundle in the sense of Nakano

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Demailly, Jean-Pierre

Construction d'hypersurfaces irréductibles avec lieu singulier donné dans $\mathbb{C}^n$. (French) Zbl 0414.32004

Ann. Inst. Fourier 30, No.3, 219-236 (1980).

Cited in 4 Documents

MSC:
32Sxx Singularities (analytic spaces)
14J17 Singularities of surfaces
32A22 Nevanlinna theory (local); growth estimates; other inequalities (several complex variables)
32C25 Analytic subsets and submanifolds

Keywords: convolution ring; irreducible hypersurface; singular locus; transcendental Bezout problem; growth order

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Demailly, Jean-Pierre

Fonctions holomorphes à croissance polynomiale sur la surface d'équation $e^x + e^y = 1$. (French) Zbl 0412.32007

Bull. Sci. Math., II. Ser. 103, 179-191 (1979).

Cited in 1 Document

MSC:
32A22 Nevanlinna theory (local); growth estimates; other inequalities (several complex variables)
32U05 Plurisubharmonic functions and generalizations
32A10 Holomorphic functions (several variables)
32A07 Special domains in Cn (Reinhardt, Hartogs, circular, tube)
32D15 Continuation of analytic objects (several variables)
32E30 Holomorphic and polynomial approximation (several variables), Runge pairs, interpolation

Keywords: holomorphic functions with polynomial growth; extension theorem for holomorphic functions; special curve; plurisubharmonic function

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Demailly, Jean-Pierre

Fonctions holomorphes bornées ou à croissance polynomiale sur la courbe $e^x + e^y = 1$. (French) Zbl 0409.32009

C. R. Acad. Sci., Paris, Sér. A 288, 39-40 (1979).

MSC:
32D15 Continuation of analytic objects (several variables)
32A22 Nevanlinna theory (local); growth estimates; other inequalities (several complex variables)
32U05 Plurisubharmonic functions and generalizations

Keywords: Extension of Holomorphic Function; Plurisubharmonic Functions; Holomorphic Function of Polynomical Growth

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Demailly, Jean-Pierre

Différents exemples de fibrés holomorphes non de Stein. (French) Zbl 0418.32011

Sémin. Pierre Lelong - Henri Skoda (Anal.), Année 1976/77, Lect. Notes Math. 694, 15-41 (1978).

Cited in 4 Reviews

Cited in 2 Documents

MSC:
32E10 Stein spaces, Stein manifolds
32L05 Holomorphic fiber bundles and generalizations

Keywords: Steinness; Serre problem; holomorphic fiber bundle

Demailly, Jean-Pierre

Un exemple de fibré holomorphe non de Stein à fibre $\mathbb{C}^2$ ayant pour base le disque ou le plan. (French) Zbl 0372.32012

Invent. Math. 48, 293-302 (1978).

Cited in 2 Reviews

Cited in 2 Documents

MSC:
32L05 Holomorphic fiber bundles and generalizations
32U05 Plurisubharmonic functions and generalizations

References:

[1] Demailly, J.-P.: Différents exemples de fibrés holomorphes non de Stein, à paraître au Séminaire Lelong 1976/1977

[2] Hörmander, L.: An introduction to complex analysis in several variables. Second edition. North Holland Publishing Company, 1973 · Zbl 0271.32001

[3] Lelong, P.: Fonctionnelles analytiques et fonctions entières (n variables). Montréal, les Presses de l'Université de Montréal, 1968, séminaire de Mathématiques Supérieures, Eté 1967, no 28

[4] Serre, J.-P.: Quelques problèmes globaux relatifs aux variétés de Stein. Colloque sur les fonctions de plusieurs variables. Bruxelles, 1953

[5] Skoda, H.: Fibrés holomorphes à base et à fibre de Stein. C.R. Acad. Sc. de Paris, 16 mai 1977, A. 1159-1202 · Zbl 0353.32032

[6] Skoda, H.: Fibrés holomorphes à base et à fibre de Stein, Inventiones Mathematicae, p. 97-107, Vol. 43, Fasc. 2, 1977 · Zbl 0365.32018 · DOI: 10.1007/BF01390000

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132 publications authored by Jean-Pierre-Demailly since 1978, including 8 books