MR2296419 32Jxx

Debarre, Olivier (F-STRAS-I)

Classes de cohomologie positives dans les variétés kählériennes compactes (d'après Boucksom, Demailly, Nakayama, Păun, Peternell et al.). (French. French summary) [Positive cohomology classes in compact Kähler manifolds (after Boucksom, Demailly, Nakayama, Păun, Peternell et al.)]

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{A review for this item is in process.}

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MR2275023 (Review) 14B05 (13A35 14M05)
Takagi, Shunsuke [Takagi, Shunsuke[2]] (J-KYUSM)

Formulas for multiplier ideals on singular varieties. (English summary)

Amer. J. Math. **128** (2006), *no. 6,* 1345–1362.

The paper under review uses characteristic p methods including tight closure to generalize certain properties of multiplier ideals to the case of singular varieties.

Recall that for a $\mathbb{Q}$-Gorenstein normal variety X over a field of characteristic zero, an ideal sheaf $\mathfrak{a} \subset \mathcal{O}_X$, and a real number $t \geq 0$, one has the associated multiplier ideal sheaf $\mathcal{J}(\mathfrak{a}^t) \subseteq \mathcal{O}_X$. Details may be found in [R. K. Lazarsfeld, *Positivity in algebraic geometry. II*, Springer, Berlin, 2004; MR2095472 (2005k:14001b)]. One also defines "mixed" multiplier ideals $\mathcal{J}(\mathfrak{a}^t\mathfrak{b}^s)$. The multiplier ideals satisfy a number of interesting properties, including the following two:

Subadditivity: J.-P. Demailly, L. M. H. Ein and Lazarsfeld [Michigan Math. J. **48** (2000), 137–156; MR1786484 (2002a:14016)] showed the following subadditivity property of multiplier ideals on a smooth variety:

$$\mathcal{J}(\mathfrak{a}^t\mathfrak{b}^s) \subseteq \mathcal{J}(\mathfrak{a}^t)\mathcal{J}(\mathfrak{b}^s).$$

Summation: M. Mustaţă [Trans. Amer. Math. Soc. **354** (2002), no. 1, 205–217 (electronic); MR1859032 (2002k:14006)] showed the following summation formula for multiplier ideals on a smooth variety:

$$\mathcal{J}((\mathfrak{a}+\mathfrak{b})^t) = \sum_{\lambda+\mu=t} \mathcal{J}(\mathfrak{a}^\lambda)\mathcal{J}(\mathfrak{b}^\mu).$$

The paper under review generalizes these formulas to the case of $\mathbb{Q}$-Gorenstein normal X over a field K of characteristic zero. The generalizations involve the Jacobian ideal of X over K, $\mathfrak{J}(X/K)$:

$$\mathfrak{J}(X/K)\mathcal{J}(\mathfrak{a}^t\mathfrak{b}^s) \subseteq \mathcal{J}(\mathfrak{a}^t)\mathcal{J}(\mathfrak{b}^s),$$

$$\mathcal{J}((\mathfrak{a}+\mathfrak{b})^t) = \sum_{\lambda+\mu=t} \mathcal{J}(\mathfrak{a}^\lambda\mathfrak{b}^\mu).$$

The method relies on the theory of tight closure introduced by M. Hochster and C. L. Huneke [J. Amer. Math. Soc. **3** (1990), no. 1, 31–116; MR1017784 (91g:13010)]. A generalization, called $\mathfrak{a}^t$-tight closure, was introduced by N. Hara and K. Yoshida [Trans. Amer. Math. Soc. **355** (2003), no. 8, 3143–3174 (electronic); MR1974679 (2004i:13003)]. The test ideals $\tau(\mathfrak{a})$ introduced by Hochster and Huneke were generalized to $\mathfrak{a}^t$-test ideals $\tau(\mathfrak{a}^t)$ by Hara and Yoshida, who showed that the $\mathfrak{a}^t$-test ideals correspond to the multiplier ideal $\mathcal{J}(\mathfrak{a}^t)$ via reduction to characteristic $p \gg 0$.

In the paper under review, analogs of the subadditivity and summation formulas are proved for the test ideals $\tau(\mathfrak{a}^t)$, allowing the author to prove the promised generalizations for multiplier ideals on singular varieties. Similar formulas are proved for asymptotic multiplier ideals [L. M. H. Ein, R. K. Lazarsfeld and K. E. Smith, Invent. Math. **144** (2001), no. 2, 241–252; MR1826369 (2002b:13001)] using asymptotic versions of the test ideals.

Finally, the subadditivity formula for asymptotic multiplier ideals is applied to answer a question of Hochster and Huneke [Invent. Math. **147** (2002), no. 2, 349–369; MR1881923 (2002m:13002)] on the growth of symbolic powers of an ideal.

Reviewed by *Zach Teitler*

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MR2272098 (2007g:32022) 32U25 (32U05 32U35)
Rashkovskii, Alexander [Rashkovskiĭ, A. Yu.]
Relative types and extremal problems for plurisubharmonic functions.
Int. Math. Res. Not. **2006,** *Art. ID* 76283, 26 *pp.*

The paper under review is concerned with singularities of plurisubharmonic (psh) functions. A psh function u defined in a neighborhood of a point $\zeta \in \mathbf{C}^n$ is said to have a singularity at ζ if $u(\zeta) = -\infty$.

There are many ways of measuring the "strength" of the singularity. The most basic invariant is the Lelong number. It can be defined in two different ways: as a growth order of u at the origin, or as a Monge-Ampère mass (or intersection number). More precisely, if we set $\varphi(x) = \log|x - \zeta|$, then the first characterization of the Lelong number is as the $\liminf$ when $x \to \zeta$ of $u(x)/\varphi(x)$.

The second characterization is as the mass of the measure $dd^c u \wedge dd^c \varphi$ at ζ.

J.-P. Demailly [Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)] defined a generalized Lelong number $\nu(\varphi, u)$ with respect to an arbitrary psh weight φ (i.e. a psh function with an isolated singularity at ζ) using the second formula above. He observed that these generalized Lelong numbers share many of the same properties as the usual Lelong number, especially when the weight is locally maximal outside the origin.

Here the author instead generalizes the first characterization of the Lelong number, thus defining the relative type $\sigma(\varphi, u)$ which in general differs from the corresponding generalized Lelong number $\nu(\varphi, u)$. They do, however, agree when $\varphi = \max_{1\le k\le n} a_k^{-1} \log|x_k - \zeta_k|$ for some constants $a_k > 0$, and in this case one obtains the directional Lelong numbers defined by C. O. Kiselman [Ann. Polon. Math. **60** (1994), no. 2, 173–197; MR1301603 (95i:32024)].

One can define a tropical structure on the cone of psh functions, with the tropical addition given by $\max\{u, v\}$ and the tropical multiplication by $u + v$. Endow the set $[0, +\infty]$ with an analogous tropical structure, but where tropical addition is given by $\min\{s, t\}$. For a given psh weight φ, the generalized Lelong number $\nu(\varphi, \cdot)$ is then tropically multiplicative, but not tropically additive in general. On the other hand, the relative type $\sigma(\varphi, \cdot)$ is tropically additive, but not tropically multiplicative in general.

A main result in the paper (Theorem 4.3) characterizes relative weights as functionals on psh functions near ζ that are upper semicontinuous, positively homogeneous and tropically additive.

If φ is chosen such that the relative weight $\sigma(\varphi, \cdot) \equiv \nu(\varphi, \cdot)$, then the resulting functional is both tropically additive and multiplicative. It can then be checked that the assignment $f \mapsto \sigma(\varphi, \log|f|)$ defines a valuation on the ring of holomorphic germs at ζ [see C. Favre and M. Jonsson, Invent. Math. **162** (2005), no. 2, 271–311; MR2199007 (2006k:32064)].

As the author points out, "Tropical additivity of the relative types make them a perfect tool for dealing with upper envelopes of families of plurisubharmonic functions with prescribed singularities." The paper contains several applications of relative weights to pluricomplex Green functions and a Siu-type theorem on the analyticity of superlevel sets of relative weights.

Reviewed by *Mattias Jonsson*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2257847 (2007h:14021) 14F10 (32Q45)
Rousseau, Erwan (3-QU)
Équations différentielles sur les hypersurfaces de $\mathbb{P}^4$. (French. English, French summaries)
[Differential equations on hypersurfaces in $\mathbb{P}^4$]
J. Math. Pures Appl. (9) **86** (2006), *no. 4,* 322–341.

The paper under review deals with entire holomorphic curves into hypersurfaces in complex projective spaces.

After the studies of J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)] and Y. T. Siu [in *The legacy of Niels Henrik Abel*, 543–566, Springer, Berlin, 2004; MR2077584 (2005h:32061)], the author gives theorems on existence of global sections of holomorphic jet bundles. Let X be a smooth hypersurface of degree $d \geq 2$ in $\mathbf{P}^4$ and T_X its holomorphic cotangent bundle. Denote by $E_{k,m}T^*_X$ the vector bundle of jet differentials on X of order k and of degree m. Let A be an ample line bundle over X. Then he first shows that $H^0(X, E_{2,m}T^*_X) = 0$, and if $d \geq 97$, then $H^0(X, E_{3,m}T^*_X \otimes A^{-1}) \neq 0$ for sufficiently large m.

This yields that every entire holomorphic curve $f\colon \mathbf{C} \to X$ must satisfy an algebraic differential equation of third order, which is the main result in the present paper. The logarithmic version of this result is also obtained, that is, every entire curve in the complement of a smooth hypersurface in $\mathbf{P}^3$ of degree $d \geq 92$ must satisfy an algebraic differential equation of third order.

REVISED (June, 2007)

Current version of review. Go to earlier version.

Reviewed by *Yoshihiro Aihara*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2254806 (2007e:14066) 14J70 (14J25 32Q45)
Bogomolov, Fedor (1-NY-X); **De Oliveira, Bruno** (1-MIAM)
Hyperbolicity of nodal hypersurfaces. (English summary)
J. Reine Angew. Math. **596** (2006), 89–101.

S. Kobayashi [*Hyperbolic manifolds and holomorphic mappings*, Dekker, New York, 1970; MR0277770 (43 #3503)] conjectured that a generic surface X of $\mathbb{P}^3$ of degree $d \geq 5$ is Kobayashi hyperbolic, i.e., there is no nonconstant holomorphic map from $\mathbb{C}$ into X. J.-P. Demailly and J. El Goul [Amer. J. Math. **122** (2000), no. 3, 515–546; MR1759887 (2001f:32045)] proved the conjecture for very generic surfaces of degree $d \geq 21$. In the paper under review, the authors deal with algebraic quasi-hyperbolicity, i.e., the property of having only finitely many rational and elliptic curves. Their main result is that a nodal hypersurface X of $\mathbb{P}^3$ of degree d with a sufficiently large number l of nodes, $l > \frac{8}{3}(d^2 - \frac{5}{2}d)$, is algebraically quasi-hyperbolic. Such surfaces exist for degrees $d \geq 6$ [Y. Miyaoka, Math. Ann. **268** (1984), no. 2, 159–171; MR0744605 (85j:14060)].

The strategy of the proof is to study symmetric differentials on the minimal resolution Y of X, i.e., global sections of $S^m\Omega^1_Y$. In a previous work by one of the authors [F. A. Bogomolov, Dokl. Akad. Nauk SSSR **236** (1977), no. 5, 1041–1044; MR0457450 (56 #15655)], it was shown that the existence of sufficiently many symmetric differentials on a surface of general type implies its algebraic quasi-hyperbolicity. It is a well-known result of F. Sakai [in *Algebraic geometry (Proc. Summer Meeting, Univ. Copenhagen, Copenhagen, 1978)*, 545–563, Lecture Notes in Math., 732, Springer, Berlin, 1979; MR0555717 (82b:32043)] that smooth surfaces in $\mathbb{P}^3$ have no symmetric differentials. So here, the main observation of the authors is to show the contribution of the singularities to the existence of symmetric differentials. This is done using Riemann-Roch computations generalized to orbifolds.

Reviewed by *Erwan Rousseau*

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1287, English translation: Math. USSR Izv. **13** (1) (1979), 499–544. MR0522939 (80j:14014)
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MR2229475 (2007b:14034) 14E30 (14J40)
Peternell, Thomas (D-BAYR-IM)
Kodaira dimension of subvarieties. II. (English summary)
Internat. J. Math. **17** (2006), *no. 5,* 619–631.

In this paper the author proves that, given a subvariety A of a smooth projective variety X with various special properties, X is uniruled, continuing an investigation which was initiated in Part I [T. Peternell, M. H. Schneider and A. J. Sommese, Internat. J. Math. **10** (1999), no. 8, 1065–1079; MR1739364 (2001e:14016)].

To give the flavour of the type of results contained in this paper, suppose that the normal bundle of A is ample. Then it was proved in [op. cit.] that either A is of general type or the Kodaira dimension of X is $-\infty$. Using a beautiful result of S. Boucksom, J.-P. Demailly, M. Paun and Peternell ["The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension", preprint, arxiv.org/abs/math/0405285] which says that if K_X is not pseudo-effective, then X is uniruled, the author is able to conclude that if A is not of general type, then in fact X is uniruled, and he is able to weaken the hypothesis on the positivity of the normal bundle of A.

Reviewed by *James McKernan*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2228686 (2007e:14067) 14J70 (32Q45)
Debarre, Olivier (F-STRAS-I); **Pacienza, Gianluca** (F-STRAS-I); **Păun, Mihai** (F-NANC-IE)
Non-deformability of entire curves in projective hypersurfaces of high degree. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **56** (2006), *no. 1,* 247–253.

The Kobayashi conjecture asserts that the general hypersurface in $\mathbb{P}^n$ of degree $d \geq 2n-1$ is Kobayashi hyperbolic. Brody's criterion of hyperbolicity states that X is Kobayashi hyperbolic if and only if there are no entire curves on X, i.e. any holomorphic map $f\colon \mathbb{C} \to X$ is constant. For $n = 2$, J.-P. Demailly and J. El Goul showed that the very general hypersurface in $\mathbb{P}^3$ of degree $d \geq 21$ is hyperbolic [Amer. J. Math. **122** (2000), no. 3, 515–546; MR1759887 (2001f:32045)]. Previously M. McQuillan had done this for the case $d \geq 36$ [Geom. Funct. Anal. **9** (1999), no. 2, 370–392; MR1692470 (2000f:32035)]. More generally, Y. T. Siu showed that the general hypersurface of degree d in $\mathbb{P}^n$ for any n is hyperbolic if d is sufficiently large [in *The legacy of Niels Henrik Abel*, 543–566, Springer, Berlin, 2004; MR2077584 (2005h:32061)].

This article is short and well written. The authors prove that there is no entire curve on a smooth hypersurface X of degree $d \geq 2n$ that deforms with X along an open subset of the parameter space S of the universal family $\mathcal{X} \to S$ of hypersurfaces of $\mathbb{P}^n$ of degree d. This result says that the Kobayashi conjecture can only fail for $d \geq 2n$ if the general hypersurface X has an entire curve which is not preserved by any local universal deformation of X. As the authors mention, their result follows from the Kobayashi conjecture, hence it is only new for the cases not covered by the results stated in the previous paragraph. To prove their main theorem the authors, as Siu did in the paper previously mentioned, bring to the transcendental case the variational approach initiated by C. H. Clemens to prove algebraic hyperbolicity [Ann. Sci. École Norm. Sup. (4) **19** (1986), no. 4, 629–636; MR0875091 (88c:14037); see also L. M. H. Ein, Invent. Math. **94** (1988), no. 1, 163–169; MR0958594 (89i:14002); C. Voisin, J. Differential Geom. **44** (1996), no. 1, 200–213; MR1420353 (97j:14047)]. The authors use and prove Siu's result about global generation of $T_{\mathcal{X}} \otimes p^* \mathcal{O}_{\mathbb{P}^n}(1)$ to obtain a sequence of functions that proves their main theorem by contradiction using the standard negative curvature arguments.

Reviewed by *Bruno N. de Oliveira*

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J. Diff. Geom. (1998), no. 49, p. 601–611. MR1669712 (99m:14078)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2226021 (2007c:32031) 32Q45 (13A50)

Rousseau, Erwan (3-QU)

Étude des jets de Demailly-Semple en dimension 3. (French. English, French summaries)
[Study of Demailly-Semple jets in dimension 3]

Ann. Inst. Fourier (Grenoble) **56** (2006), *no. 2,* 397–421.

The paper under review deals with the characterization of Demailly-Semple jets, which is closely related to the hyperbolicity of algebraic varieties. After the study of J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)], the author gives an algebraic characterization of Demailly-Semple jets in dimension three by making use of the invariant theory of nonreductive groups. Let X be a complex manifold of dimension three and T^*_X its cotangent bundle. Denote by $E_{k,m}T^*_X$ the vector bundle of jet differentials on X of order k and of degree m. Put $A_k = \bigoplus_m (E_{k,m}T^*_X)_x$ for $x \in X$. The author describes the structure of A_3 and gives a characterization for $\mathrm{Gr}^\bullet E_{3,m}T^*_X$.

Namely, he proves that $\mathrm{Gr}^\bullet E_{3,m}T^*_X$ can be written as a direct sum of the spaces of Schur polynomials on $S^{a_j}T^*_X$ for some $a_j \in \mathbf{Z}^+$, which is the main result in this paper.

Some results of Riemann-Roch type are also obtained in the case in which X is a hypersurface in projective 4-space.

Reviewed by *Yoshihiro Aihara*

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MR2212883 (2007b:32039) 32Q20 (14J45 14J70)
Arezzo, Claudio (I-PARM); **Ghigi, Alessandro** (I-PAVI); **Pirola, Gian Pietro** (I-PAVI)
Symmetries, quotients and Kähler-Einstein metrics. (English summary)
J. Reine Angew. Math. **591** (2006), 177–200.

It is well known that it is extremely difficult to check whether a Fano manifold has a Kähler-Einstein metric or not. For instance, even the case of del Pezzo surfaces is far from being obvious, and in full generality it is not known for hypersurfaces of the projective space. G. Tian proved that Fermat hypersurfaces are Kähler-Einstein, showing the properness of a certain energy functional F_ω whose Euler-Lagrange equation is the Monge-Ampère equation. In this paper, the behaviour of F_ω under a Galois covering is studied, and some algebraic conditions on the covering maps are found to ensure the properness of this functional. This allows the authors to extend widely the work of Tian to other classes of Fano manifolds:

(1) hypersurfaces of the form
$$\{x_0^d+\cdots+x_{k-1}^d+f(x_k,\dots,x_{n+1})=0\}\subset\mathbb{P}^{n+1}$$
where f is a homogeneous polynomial of degree d, and $k>n+2-d$;
(2) n-dimensional intersections of hypersurfaces of the same form as above, all of the same degree d, and $k>n+2-d$;
(3) arbitrary intersections of two (hyper)quadrics;
(4) double covers of $\mathbb{P}^n$ ramified along a smooth hypersurface of degree $2d$ with $\frac{n+1}{2}<d\le n$;
(5) double covers of the n-dimensional quadric $Q_n\subset\mathbb{P}^{n+1}$ with smooth branching locus cut out by a hypersurface of degree $2d$ with $\frac{n}{2}<d<n$.

In order to get the result, the authors prove the following interesting fact. Let $\pi\colon M\to N$ be a ramified Galois covering of degree d with structure group G, ω_N a Kähler-Einstein metric on N and $\omega\in 2\pi c_1(M)$ a G-invariant Kähler metric on M. If we denote $R(\pi)$ the ramification divisor of π and assume that in homology $R(\pi)=-\beta K_M$ for a certain $\beta\in\mathbb{Q}_+$, then there is a constant C such that for all G-invariant smooth potential φ with $\omega+\sqrt{-1}\partial\bar\partial\varphi>0$,
$$F^0_\omega(\varphi)\ge\frac{1}{1+\beta}\log\left(\frac{1}{V}\int_M e^{-(1+\beta)\varphi}\pi^*\omega_N^n\right)-C.$$
Here M has complex dimension n and $V=\int_M\omega^n$. Note that for a potential φ such that $\frac{1}{V}\int_M e^{h(\omega)-\varphi}\omega^n=1$, where $\operatorname{Ric}(\omega)-\omega=\sqrt{-1}\partial\bar\partial h(\omega)$, one has $F^0_\omega(\varphi)=F_\omega(\varphi)$. We refer to the survey [G. Tian, *Canonical metrics in Kähler geometry*, Birkhäuser, Basel, 2000; MR1787650 (2001j:32024)] for the details about the functionals F_ω and F^0_ω and the equivalence between the existence of a Kähler-Einstein metric and the properness of F_ω.

With a view to applying their result to the examples above, the authors need to control the integral $\int_M e^{-(1+\beta)\varphi}\omega^n$ with the term $\int_M e^{-(1+\beta)\varphi}\pi^*\omega_N^n$ in order to get the required properness. Interestingly, this leads one to the consideration for $\eta=\frac{\pi^*\omega_N^n}{\omega^n}$ of the real number $c=\sup\{r\ge 0\colon \frac{1}{\eta^r}\in L^1(M,\omega^n)\}$, which is just the infimum over M of the complex singularity exponent of the ideal sheaf induced by the divisor $R(\pi)$ [J.-P. Demailly and J. Kollár, Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 4, 525–556; MR1852009 (2002e:32032)]. Although the singularities of the ramification divisor are quite mild, it is difficult to compute the complex singularity exponent in full generality. Nevertheless, if the reduced divisor associated to $R(\pi)$ is smooth at $p \in M$, then there is a holomorphic function f defined on a neighborhood of p such that $R(\pi) = \{f^m = 0\}$ with $m \leq \deg(\pi) - 1$ and $Df(p) \neq 0$. This gives the required lower bound for the complex singularity exponent at the point p under a natural assumption on the covering. Then, the manifolds quoted previously enter into this framework.

Reviewed by *Julien Keller*

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MR2211329 (2007f:32027) 32L10 (32L20 47B35)
Berman, Robert (S-CHAL)
Super Toeplitz operators on line bundles. (English summary)
J. Geom. Anal. **16** (2006), *no. 1,* 1–22.

Let L be a Hermitian line bundle over a compact complex manifold X. Denote by $X(q)$ the subset of X where the curvature form of L is nondegenerate and has exactly q negative eigenvalues, by $H^0(X,L)$ the space of all holomorphic sections of L, and by $\{\psi_j\}$ any orthonormal basis of $H^0(X,L)$. The Bergman kernel of $H^0(X,L)$ is then the holomorphic section of $L\otimes\overline{L}$ defined by $K(x,y)=\sum_j \psi_j(x)\otimes\overline{\psi_j(y)}$, and the author calls

$$B(x):=\|K(x,x)\|=\sup\{\|f(x)\|^2:\ f\in H^0(X,L),\ \|f\|\le 1\}$$

the Bergman function. Let $B_k(x)$ be similarly defined for L^k in the place of L. Using Demailly's holomorphic Morse inequalities, the author shows that if $X(1)=\varnothing$ then

$$k^{-n}B_k(x)\to\pi^{-n}\,\mathbf{1}_{X(0)}(x)\,\|\det\frac{i}{2}\partial\overline{\partial}\varphi(x)\|.$$

This is then elaborated on in two directions. First, applications are given to the asymptotic sets of sampling in $H^0(X,L^k)$ as $k\to\infty$, generalizing, in particular, the result of Boutet de Monvel and Guillemin on the counting function for the eigenvalues of a Toeplitz operator T_f with real-valued symbol f. Second, an analogous theory is developed if $H^0(X,L^k)$ is replaced by the spaces $\mathcal{H}^q(X,L^k)$ of harmonic $(0,q)$-forms on X with values in L^k. It turns out that this admits a very convenient formulation in the language of supermanifolds and superintegrals and, again, has implications concerning the asymptotic distribution of eigenvalues, as $k\to\infty$, of certain "super-Toeplitz" operators T_f whose symbols f are differential forms on X. The super formalism also allows a compact notation.

Reviewed by *Miroslav Engliš*

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MR2209220 (2006k:32046) 32Q15 (32Q05 53C25 53C55)
Fang, Fuquan (PRC-CAP)
Kähler manifolds with numerically effective Ricci class and maximal first Betti number are tori. (English, French summaries)
C. R. Math. Acad. Sci. Paris **342** (2006), *no. 6,* 411–416.

In [Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)], J.-P. Demailly, T. Peternell and M. H. Schneider generalized the notion of a numerically effective (nef) holomorphic line bundle over an algebraic variety to any compact complex manifold. A Kähler manifold with nef anticanonical bundle, or equivalently, numerically effective Ricci class, is called a nef Kähler manifold in the paper under review. For a nef Kähler manifold M, Demailly, Peternell and Schneider conjectured that the Albanese map $\alpha\colon M \to \mathrm{Alb}(M)$ is surjective. When M is a projective manifold of arbitrary dimension, Q. Zhang proved this conjecture in [J. Reine Angew. Math. **478** (1996), 57–60; MR1409052 (97m:14039)] by using relative deformation theory and mod p reductions (which were originally used by S. Mori to settle Hartshorne's conjecture). Using the analytic techniques of differential geometry, M. Paun proved the conjecture under the assumption that the Ricci class of M is integrable in [Comm. Anal. Geom. **9** (2001), no. 1, 35–60; MR1807951 (2001m:32050)]. F. Campana, Peternell and Zhang confirmed the conjecture when the dimension of M is not greater than four in [Proc. Amer. Math. Soc. **131** (2003), no. 2, 549–553 (electronic); MR1933346 (2004e:32020)].

Let M be a nef Kähler manifold of dimension n. In this well-written and very readable paper the author proves: (1) If the first Betti number $b_1(M) = 2n$, then M is biholomorphic to a complex torus of dimension n. In particular, if $b_1(M) = 2n$, then the Albanese map $\alpha\colon M \to \mathrm{Alb}(M)$ is surjective. (2) Let G be the fundamental group of M and $G' = [G, G]$ be the commutator subgroup of G. If the first Betti number $b_1(M) = 2n - 2$, and $G'/[G', G]$ has rank at least two, then the Albanese map $\alpha\colon M \to T^{n-1}_{\mathbb{C}}$ is surjective.

The author claims that the proof of the second result follows along the same lines as the first one. The following is the main idea of the proof of the first main result. By the Aubin-Calabi-Yau theorem, Demailly, Peternell and Schneider proved that a Kähler manifold is nef if and only if there exists a sequence of Kähler metrics $\{\omega_k\}$ such that for any $k > 0$, $\{\omega_k\}$ belongs to a fixed Kähler class $[\omega]$, and the Ricci curvature of ω_k is bounded from below by $-\frac{1}{k}$. Let $\widetilde{M}_k$ be the Riemannian covering space of M_k (the manifold M with metric ω_k). Let $\overline{M}_k = \widetilde{M}_k/G'$. Using the equivariant Gromov-Hausdorff convergence and Gromov compactness theorem, and a splitting theorem of Cheeger-Colding for limit spaces, the author proves that there is a finite index torsion-free subgroup Γ_k of $\Gamma = G/G'$ such that $(\overline{M}_k, \Gamma_k)$ converges to $(\mathbb{R}^{2n}, \mathbb{Z}^{2n})$. From this the author concludes that $\overline{M}_k/\Gamma_k$ is homeomorphic to a torus, and hence so is M. From the Poincaré-Lelong equation it follows that the Albanese map has no zeros and is actually a biholomorphism.

Reviewed by *Qi Lin Yang*

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MR2199007 (2006k:32064) 32U05 (13A18 32U25)
Favre, Charles (F-PARIS7-GDM); **Jonsson, Mattias** (1-MI)
Valuative analysis of planar plurisubharmonic functions. (English summary)
Invent. Math. **162** (2005), *no. 2,* 271–311.

This is the first of a series of papers in which the authors give applications of the theory of the valuative tree, a notion introduced in their book [*The valuative tree*, Lecture Notes in Math., 1853, Springer, Berlin, 2004; MR2097722 (2006a:13008)].

The valuations considered here are valuations acting on germs of holomorphic functions at $0 \in \mathbf{C}^2$, that is, functions $\nu\colon \mathcal{O}(\mathbf{C}^2, 0) \mapsto [0, \infty]$ such that:

(i) $\nu(\psi\psi') = \nu(\psi) + \nu(\psi')$;
(ii) $\nu(\psi + \psi') \geq \min(\nu(\psi), \nu(\psi'))$;
(iii) $\nu(0) = \infty, \nu(1) = 0, \min(\nu(x), \nu(y)) = 1$.

In their aforementioned work, the authors proved that the space $\mathcal{V}$ of such valuations has the structure of an $\mathbf{R}$-tree, hence is eligible to be treated by the methods of analysis and measure theory.

The most classical example of valuation is the multiplicity of a germ. The Lelong number can be seen as an extension of this multiplicity, acting on the space of germs of plurisubharmonic (psh) functions.

Using the formalism of the valuative tree, the authors first show that (quasimonomial) valuations can be evaluated on psh functions, giving rise to generalized Lelong numbers in the sense of J.-P. Demailly [Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)] (see also [C.-O. Kiselman, in *Séminaire d'Analyse Complexe et Géométrie 1985–1987*, 61–70, Fac. Sci. Tunis/Fac. Sci. Tech. Monastir; per bibl.]).

This allows them to describe singularities of psh functions quite accurately as follows. If u is a psh function with singularity at the origin, $\nu \mapsto \nu(u)$ is a function on the space of valuations, with special convexity properties that allow one to apply the methods of analysis on the valuative tree and obtain a "tree measure" associated to it.

The general idea of the paper is that this tree measure contains a lot of information about u. The authors give some applications of this principle.

The first application is a process of attenuation of singularities of positive closed currents. Given a germ of positive closed current at $0 \in \mathbf{C}^2$, there exists a composition of blowups π such that $\pi^* T$ decomposes as a current supported on the exceptional divisor plus a current with arbitrary small Lelong numbers. This extends results of Mimouni and Guedj.

The second application is an exact formula for the mass of $dd^c u \wedge dd^c v\{0\}$, where u and v are germs of singular psh functions, under some regularity assumptions on u and v. The value of the mass depends on the tree measures of u and v. As a consequence of this formula, it is proved that every generalized Lelong number is an average of valuations.

An important third application to the theory to multiplier ideal sheaves associated to psh functions appears in a separate paper [C. Favre and M. Jonsson, J. Amer. Math. Soc. **18** (2005), no. 3, 655–684 (electronic);; MR2138140 (2007b:14004)].

Reviewed by *Romain Dujardin*

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MR2191703 (2006k:31004) 31C10
Cegrell, Urban (S-UMEA-IM); **Wiklund, Jonas** (S-UMEA-IM)
A Monge-Ampère norm for delta-plurisubharmonic functions. (English summary)
Math. Scand. **97** (2005), *no. 2,* 201–216.

Let $\Omega \subset \mathbb{C}^n$ be a bounded hyperconvex domain and $\mathrm{PSH}(\Omega) \subset L^1_{\mathrm{loc}}(\Omega)$ be the cone of plurisubharmonic functions on Ω. It is well known from the pioneering work of Bedford and Taylor that the complex Monge-Ampère operator $(dd^c u)^n$ is well defined on the class of bounded plurisubharmonic functions u on Ω, but not for all plurisubharmonic functions on Ω. Later on, Cegrell introduced interesting classes of (singular) plurisubharmonic functions on Ω for which the complex Monge-Ampère operator is well defined as a Radon measure on Ω and which play an important role in the solution of the Dirichlet problem for the complex Monge-Ampère equation [see U. Cegrell, Acta Math. **180** (1998), no. 2, 187–217; MR1638768 (99h:32016)].

In the paper under review, the authors study one of these classes, namely, the class $\mathcal{F}(\Omega)$ of functions $\varphi \in \mathrm{PSH}(\Omega)$ for which there exists a decreasing sequence (φ_j) of plurisubharmonic functions on Ω with boundary values 0 which converges to φ on Ω and satisfies $\sup_j \int_\Omega (dd^c\varphi_j)^n < +\infty$. It follows from Cegrell's work that for $\varphi \in \mathcal{F}(\Omega)$ the Monge-Ampère measure $(dd^c\varphi)^n$ is well defined on Ω and is of finite mass on Ω.

The set $\mathcal{F}(\Omega)$ is a cone in the linear space $L^1_{\mathrm{loc}}(\Omega)$. It is then natural to consider the linear subspace generated by this cone. This is the set $\delta\mathcal{F}(\Omega)$ of functions $u \in L^1_{\mathrm{loc}}(\Omega)$ such that there exist $u_1, u_2 \in \mathcal{F}(\Omega)$ satisfying $u = u_1 - u_2$.

The authors then define a norm on $\delta\mathcal{F}(W)$ by

$$\|u\| := \inf\left\{\int_\Omega (dd^c(u_1+u_2))^n;\ u_1, u_2 \in \mathcal{F}(\Omega), u = u_1 - u_2\right\}.$$

The first main result of the paper states that $\|\cdot\|$ is a norm on the linear space $\delta\mathcal{F}(W)$ and this space is a Banach space. Moreover the authors characterize its dual as the linear space generated by the dual cone $\mathcal{F}'(\Omega)$.

Finally the authors prove that Demailly's generalized Lelong numbers functional is continuous on the Banach space $\delta\mathcal{F}(W)$.

Reviewed by *Ahmed Zeriahi*

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MR2192217 (2006k:32042) 32L20 (14F17)
Laytimi, F. (F-LILL); **Nahm, W.** (IRL-DIAS-P)
On a vanishing problem of Demailly.
Int. Math. Res. Not. **2005,** *no. 47,* 2877–2889.

Let X be a smooth projective complex algebraic variety of dimension n. In the first section of the paper the authors discuss several vanishing results for the cohomology of certain vector bundles on X, and explain that one of them (Theorem 1.4) implies all the other ones. (The proof of Theorem 1.4 is given in the second section.) For instance, two of the corollaries of the main result of the paper are the following generalizations of vanishing theorems of Demailly and Griffiths, respectively. In both of these results, we let E be a holomorphic vector bundle of rank e on X, we let L be a line bundle on X, we let p, q be integers with $0 \leq p, q \leq n$, and we put $r = \min(n-p, n-q)$.

(1) Let $k \geq 1$ be an integer, let $m = \min(e-1, k)$, and assume that $S^{k+(r+m)e} E \otimes L$ is ample. Then

$$H^{p,q}(X, E^{\otimes k} \otimes (\det E)^{m+r} \otimes L) = 0 \qquad \text{for } \; p+q-n > 0.$$

(2) Let $\alpha \geq 0$ be an integer, and assume that $S^{\alpha+r+re} E \otimes L$ is ample. Then

$$H^{p,q}(X, S^{\alpha} E \otimes (\det E)^{r+1} \otimes L) = 0 \qquad \text{for } \; p+q-n > 0.$$

However, the original motivation for the work was trying to answer the following question of Demailly (which is alluded to in the title of the paper). We keep the notation X, n, E, e, L, p, q and r introduced above. Let $a = (a_1, \ldots, a_l)$ be a nonincreasing sequence of positive integers of length $l \leq e$; its weight is defined to be $|a| = \sum_i a_i$, and a can then be thought of as a partition of the integer $|a|$. This partition determines a Schur functor on the category of vector bundles on X; the value of this functor on E will be denoted by $S_a E$. (Schur functors arise from the representation theory of the general linear group, and they generalize the symmetric power and exterior power functors.)

If E is ample and L is nef or vice versa, Demailly posed the problem of determining the smallest exponent $j_0 = j_0(n, p, q, a)$ such that $H^{p,q}(X, S_a E \otimes (\det E)^j \otimes L) = 0$ for $j \geq j_0$. Demailly also suggested that $j_0 = r + l$ is sufficient. The authors confirm Demailly's prediction with the following result (Theorem 1.2 in the paper) which turns out to be equivalent to a special case (Theorem 1.3) of the main theorem of the paper (Theorem 1.4). Namely, let $\widetilde{a} = (\widetilde{a}_1, \widetilde{a}_2, \dots)$ denote the transpose of the partition a, let $m \geq 0$ be an integer, and assume that $S^{|a|+(m+r)e}E \otimes L$ is ample. Then

$$H^{p,q}(X, S_a E \otimes (\det E)^{m+r} \otimes L) = 0$$

for $p + q - n > \sum_{\widetilde{a}_i > m}(e - \widetilde{a}_i)$. As the authors remark, the ampleness condition in this result is satisfied if E is ample and L is nef, or vice versa. If we take $m = l$ in the theorem, then the condition on the right-hand side of the last formula becomes $p + q - n > 0$, and we arrive at a solution of Demailly's problem, with the exponent being $j_0 = r + l$, as he suggested. In the third and final section of the paper the authors show that this result is optimal in a certain range of the parameters n, p, q and a, which, unfortunately, is not symmetric with respect to p and q.

Reviewed by *Dmitriy S. Boyarchenko*

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MR2190241 (2007c:53056) 53C25 (14J25)
Kollár, János (1-PRIN)
Einstein metrics on five-dimensional Seifert bundles. (English summary)
J. Geom. Anal. **15** (2005), *no. 3,* 445–476.

Summary: "The aim of this article is to study Seifert bundle structures on simply connected 5-manifolds. We classify all such 5-manifolds which admit a positive Seifert bundle structure, and in a few cases all Seifert bundle structures are classified. These results are then used to construct positive Ricci curvature Einstein metrics on these manifolds.

"The proof has 4 main steps: first, the study of the Leray spectral sequence of the Seifert bundle, based on work of P. Orlik and P. Wagreich [Invent. Math. **28** (1975), 137–159; MR0361150 (50 #13596)]; second, the study of log del Pezzo surfaces; third, the construction of Kähler-Einstein metrics on del Pezzo orbifolds using the algebraic existence criterion of J.-P. Demailly and J. Kollár [Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 4, 525–556; MR1852009 (2002e:32032)]; fourth, the lifting of the Kähler-Einstein metric on the base of a Seifert bundle to an Einstein metric on the total space using the Kobayashi-Boyer-Galicki method."

Reviewed by *Massimiliano Pontecorvo*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2188444 (2006j:14063) 14K12 (14F10 14M10)
Debarre, Olivier (F-STRAS-I)
Varieties with ample cotangent bundle. (English summary)
Compos. Math. **141** (2005), *no. 6,* 1445–1459.

The goal of the paper under review is to construct examples of projective algebraic varieties X with ample cotangent bundle. Such varieties are always of general type. Moreover, when defined over $\mathbb{C}$ they do not admit nontrivial holomorphic maps $\mathbb{C} \to X$ [J.-P. Demailly, in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)], and when defined over a number field K they are conjectured to have only finitely many K-rational points [A. Moriwaki, Math. Res. Lett. **2** (1995), no. 1, 113–118; MR1312981 (96b:14021)]. Although such varieties are expected to be abundant, there are few known concrete examples. This situation is now somewhat remedied.

The main part of the paper is Section 2, in which the author proves the following three theorems: (i) Let $L_1, \dots, L_c$ be very ample line bundles on a simple abelian variety of dimension n, where $c \geq n/2$. Let $H_1 \in |L_1^{e_1}|, \dots, H_C \in |L_C^{e_c}|$ be general divisors where $e_2, \dots, e_c$ are all $> n$; then the complete intersection $H_1 \cap \cdots \cap H_c$ has ample cotangent bundle. (ii) If we change the condition on the e_i's to be large enough and divisible enough, then the same property holds for any abelian variety. (iii) If the dimension n is 4 and $c = 2$, it suffices to take $e_1, e_2 \geq 5$ (this result has been subsequently improved by the author and E. Izadi ["Ampleness of intersections of translates of theta divisors in an abelian fourfold", preprint, arxiv.org/abs/math/0506374], who proved that for any 4-dimensional Jacobian the intersection of two general translates of the theta divisor has ample cotangent bundle). The author proves (i) by showing that the fibers of the natural map $\mathbb{P}(\Omega_X) \to \mathbb{P}(\Omega_{A,0}) \times X \to \mathbb{P}(\Omega_{A,0})$ are 0-dimensional. This is done by showing that if ∂ is a nontrivial constant vector field on A, then there is an inequality $\dim(H_1 \cap \partial H_1 \cap \cdots \cap H_c \cap \partial H_c) \leq \max(n - 2c, 0)$. This inequality is proved by induction on c and is the most technical part of the paper. The proofs of (ii) and (iii) are similar to the proof of (i).

Section 3 of the paper is mostly conjectural—the author conjectures that similar theorems to his theorems on abelian varieties would hold in $\mathbb{P}^n$. In Section 4 he reproduces an unpublished result of F. Bogomolov: Let $X_1, \dots, X_n$ be smooth projective varieties with big cotangent bundles, all of dimension $\geq d > 0$, and let V be a general linear section of $X_1 \times \cdots \times X_m$ such that $\dim(V) \leq$

$(d(m+1)+1)/2(d+1)$. Then the cotangent bundle of V is ample.

Reviewed by *David Lehavi*

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MR2178969 (2006j:53058) 53C25 (32Q20 57R60)
Boyer, Charles P. (1-NM); **Galicki, Krzysztof** (1-NM); **Kollár, János** (1-PRIN)
Einstein metrics on spheres.
Ann. of Math. (2) **162** (2005), *no. 1,* 557–580.

This paper provides existence theorems for large families of Einstein metrics on the sphere S^5, on all 28 oriented diffeomorphism classes on S^7 as well as on the standard and Kervaire spheres S^{4m+1}. This is a substantial advance on previous work and the techniques used here motivate the authors' conjecture that all odd-dimensional homotopy spheres which bound parallelizable manifolds admit Sasakian-Einstein metrics.

The study of special Riemannian metrics on exotic spheres has quite a long history. D. Gromoll and W. Meyer [Ann. of Math. (2) **100** (1974), 401–406; MR0375151 (51 #11347)] wrote explicitly a metric of non-negative sectional curvature on one of the Milnor 7-spheres. Much later, K. Grove and W. Ziller [Ann. of Math. (2) **152** (2000), no. 1, 331–367; MR1792298 (2001i:53047)] proved that all exotic 7-spheres which are S^3-bundles over S^4 admit such a metric. It is however not known whether exotic spheres admit metrics of positive sectional curvature. Concerning metrics with positive Ricci curvature, they are known to exist on all spheres which bound parallelizable manifolds by a result of D. Wraith [J. Differential Geom. **45** (1997), no. 3, 638–649; MR1472892 (98i:53058)]. In dimension 7, all homotopy spheres have this property, and part of the ideas in the paper under review come from a re-proving of Wraith's theorem by Boyer, Galicki and M. Nakamaye [Topology **42** (2003), no. 5, 981–1002; MR1978045 (2004c:53055)] in the framework of the classical Brieskorn presentation. Dimension 7 was also deeply analyzed by M. Kreck and S. Stolz [Ann. of Math. (2) **127** (1988), no. 2, 373–388; MR0932303 (89c:57042)], who proved the existence of 7-manifolds with the maximal number of 28 smooth structures each of which admits an Einstein metric with positive scalar curvature.

The present paper attacks the problem of existence of Einstein metrics on standard and exotic spheres by combining the original Brieskorn point of view through links L_f of isolated hypersurface singularities with their Sasakian geometry and with a continuity method to construct Kähler-Einstein metrics on their associated transversal structure.

We now sketch some of the ideas entering in the proof.

First, links L_f are defined by choosing a weight vector $\overrightarrow{w} = (w_0, \dots, w_n) \in \mathbb{Z}_+^{n+1}$ and a weighted homogeneous polynomial f of weighted degree $w(f)$: $f(\lambda^{w_0} z_0, \dots, \lambda^{w_n} z_n) = \lambda^{w(f)} f(z_0, \dots, z_n)$. The definition is $L_f = \{z \in \mathbb{C}^{n+1} :\ f(z) = 0\} \cap S^{2n+1}$, and when 0 is the only isolated singularity of f, the weighted S^1-fibration $S^{2n+1}_{\overrightarrow{w}} \to P^n_{\mathbb{C}}(\overrightarrow{w})$ gives rise to an induced fibration $L_f \to Z_f$, relating the Sasakian geometry of the smooth link L_f to its transversal Kähler geometry, encoded in the orbifold Z_f. As a first result, the authors prove that the orbifold Z_f is Fano if and only if $w(f) - \sum w_i < 0$.

A remarkable class of links L_f is given by Brieskorn-Pham links $L(\overrightarrow{a})$, $\overrightarrow{a} = (a_0, \ldots, a_n)$, corresponding to polynomials $f = \sum z_i^{a_i}$. Then $w(f) = \mathrm{lcm}\ (a_0, \ldots, a_n)$, the weights are $w_i = \frac{w(f)}{a_i}$, and the Fano condition reads $1 < \sum \frac{1}{a_i}$. It was proved in E. Brieskorn's famous 1966 paper [Invent. Math. **2** (1966), 1–14; MR0206972 (34 #6788)] that $L(\overrightarrow{a})$ is homeomorphic to a sphere provided some conditions hold on an associated graph $G(\overrightarrow{a})$. More generally, perturbed Brieskorn-Pham links $L(\overrightarrow{a}, p)$, corresponding to $f = \sum z_i^{a_i} + p(z_0, \ldots, z_n)$, satisfy the same Fano condition for the corresponding Kähler orbifolds $Z(\overrightarrow{a}, p)$. The strategy is now to apply a continuity method to $Z(\overrightarrow{a}, p)$ to ensure on them the existence of a Kähler-Einstein metric. A Sasakian-Einstein metric on $L(\overrightarrow{a}, p)$ will then be obtained from the classical Kobayashi circle bundle construction, suitably adapted to orbifolds.

A second key ingredient in the proof is the continuity method developed through the work of T. Aubin, Y. T. Siu, G. Tian and A. M. Nadel. The aim is to show the existence of a Kähler-Einstein metric of positive sign through the Monge-Ampére equation by starting from a Yau solution for the value $t = 0$ of the parameter and by getting conditions that ensure that the value $t = 1$ can be reached by continuity. The relevant theorem in the authors' proof is in the orbifold category and is due to J.-P. Demailly and Kollár [Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 4, 525–556; MR1852009 (2002e:32032)]. For Fano Kähler hypersurfaces $Z_f \subset \mathrm{P}^n_{\mathbb{C}}(\overrightarrow{w})$ associated to the links L_f, the Demailly and Kollár theorem is in fact used to show that a Kähler-Einstein metric of positive sign can be obtained on Z_f provided the following condition holds: there is a $\gamma > \frac{n}{n+1}$ such that for every weighted homogeneous polynomial g of weighted degree $s(\sum w_i - w(f))$, not identically zero on $Y_f = \{\{f = 0\} \subset \mathbb{C}^{n+1}\}$, the function $|g|^{-\frac{\gamma}{s}}$ is locally L^2 on $Y_f - \{0\}$. Then, by working out this condition for Brieskorn-Pham links $L(\overrightarrow{a}, p)$, the authors prove that a Kähler-Einstein metric on the orbifolds $Z(\overrightarrow{a}, p)$ exists if $1 < \sum \frac{1}{a_i} < 1 + \frac{n}{n+1} \min\{\frac{1}{a_i}, \frac{1}{b_i b_j}\}$, where $b_i = \gcd\ (C_i, a_i)$, $C_i = \mathrm{lcm}\ (a_0, \ldots, \widehat{a}_i, \ldots, a_n)$. Moreover, given two vectors $\overrightarrow{a}$, $\overrightarrow{a}'$ satisfying these conditions, the links $L(\overrightarrow{a})$ and $L(\overrightarrow{a}')$ are isometric if and only if $\overrightarrow{a}$ is a permutation of $\overrightarrow{a}'$.

An enumeration of all sequences $(a_0, \ldots, a_n)$ satisfying the above condition can thus be accomplished in some cases, also with the help of computer programs. Moreover, possible perturbing polynomials $p(z_0, \ldots, z_n)$ can be considered as well, and some significant cases are examined in the last section of the paper. This leads the authors to obtain 68 inequivalent families of Sasakian-Einstein metrics on S^5, from 231 to 452 families on each of the 28 oriented diffeomorphism classes on S^7, and a doubly exponential number of inequivalent families on standard and Kervaire S^{4m+1}. All these results of course suggest the examination also of homotopy spheres S^{4m+3} and the companion paper by Boyer et al. [Experiment. Math. **14** (2005), no. 1, 59–64; MR2146519 (2006a:53042)] gives a computer assisted proof for S^{11} and S^{15} of the authors' conjecture stated at the beginning of this review.

This extensive range of ideas and techniques has already originated some remarkable developments. Among them, the construction of Einstein metrics on 5-dimensional Seifert bundles, described by Kollár [J. Geom. Anal. **15** (2005), no. 3, 445–476;; MR2190241 (2007c:53056)], and of further new Einstein metrics on odd-dimensional spheres by A. Ghigi and Kollár ["Kaehler-Einstein metrics on orbifolds and Einstein metrics on spheres" preprint, arxiv.org/abs/math.DG/0507289], are certainly deserving of mention. Very likely, many other developments will follow from the present rich paper.

Reviewed by *Paolo Piccinni*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2172891 (2006d:32054) 32W20 (32U15)
Kołodziej, Sławomir
The complex Monge-Ampère equation and pluripotential theory. (English summary)
Mem. Amer. Math. Soc. **178** (2005), *no. 840,* *x*+64 *pp*.

This is a survey and update of results in pluripotential theory; mostly due to the author and primarily on existence theorems for the complex Monge-Ampère operator. Chapter 1 gives background material on positive currents and plurisubharmonic (psh) functions. The complex Monge-Ampère operator $(dd^c u)^n$ is defined for locally bounded psh functions in $\mathbf{C}^n$; convergence theorems and comparison theorems are proved for such functions. The relative extremal function and relative capacity cap are introduced and utilized to prove quasicontinuity of psh functions. Josefson's theorem, that locally pluripolar sets are globally pluripolar, and the Bedford-Taylor result, that negligible sets are pluripolar, are also proved. Much of the material is from the Bedford-Taylor papers [E. Bedford and B. A. Taylor, Invent. Math. **37** (1976), no. 1, 1–44; MR0445006 (56 #3351); Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)], but proofs of some of the convergence theorems and the negligible sets are pluripolar result have been slightly simplified. Chapter 2 introduces the Lelong classes $\mathcal{L}$ and $\mathcal{L}^+$ and the Siciak-Zaharjuta extremal function of a bounded set and proves the capacity comparison theorem of H. J. Alexander and Taylor [Math. Z. **186** (1984), no. 3, 407–417; MR0744831 (85k:32034)]. Chapter 3 solves the Dirichlet problem for the complex Monge-Ampère operator on a strictly pseudoconvex domain Ω: given $\varphi \in C(\partial\Omega)$ and $f \in C(\overline{\Omega})$ with $f \geq 0$, find $u \in \mathrm{PSH}(\Omega) \cap C(\overline{\Omega})$ with $(dd^c u)^n = f dV$ in Ω and $\lim_{z' \to z} u(z') = \varphi(z)$ for all $z \in \partial\Omega$. The procedure of Bedford and Taylor in [op. cit., 1976] is followed with some minor modifications of J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)].

The final three chapters cover more recent results—all within the past ten years—on the Dirichlet problem and the complex Monge-Ampère operator. In Chapter 4, the author generalizes the class of admissible data f for solvability of the Dirichlet problem. First, define a class of measures μ that satisfy a type of local domination by relative capacity: for an "admissible" function $h\colon \mathbf{R}_+ \to (1,\infty)$ and a positive constant A,

$$\mathcal{F}(A,h) := \{\mu\colon\ \mu(K) \leq F(\mathrm{cap}(K,\Omega)),\ F(x) = \frac{Ax}{h(x^{-1/n})},\ K \text{ compact}\}.$$

A priori estimates for the sup-norm of solutions of the Dirichlet problem with μ in place of $f dV$ are proved which imply the existence of continuous solutions of the Dirichlet problem for a wide class of absolutely continuous measures $f dV$ (Theorem 4.6). This class includes all $f \in L^p_{\mathrm{loc}}(\Omega)$ for $p > 1$. If one relaxes the assumption of continuity of the solution and asks for a bounded solution of the Dirichlet problem, Theorem 4.7 shows that if such a Dirichlet problem admits a subsolution then it admits a solution. These results are from [S. Kołodziej, Ann. Polon. Math. **65** (1996), no. 1, 11–21; MR1414748 (98a:32015); Indiana Univ. Math. J. **44** (1995), no. 3, 765–782; MR1375348 (96m:32013); Acta Math. **180** (1998), no. 1, 69–117; MR1618325 (99h:32017); Math. Z. **240** (2002), no. 4, 835–847; MR1922732 (2003f:32043)]. The complex Monge-Ampère operator can be defined in a reasonable way for certain unbounded psh functions; this is the content of Chapter 5. The so-called energy classes $\mathcal{E}_p$ and $\mathcal{F}_p$ of Cegrell are introduced and a characterization of the finite measures μ in a hyperconvex domain which are Monge-Ampère measures $(dd^c u)^n$ of a function u in $\mathcal{F}_p$ is given (Theorem 5.5). Much of this chapter follows [U. Cegrell, Acta Math. **180** (1998), no. 2, 187–217; MR1638768 (99h:32016)]. In addition, if $\mu \in \mathcal{F}(A, h)$, one obtains a continuous solution [S. Kołodziej, in *Complex geometric analysis in Pohang (1997)*, 241–243, Contemp. Math., 222, Amer. Math. Soc., Providence, RI, 1999; MR1653056 (99i:32019)]. Finally, Chapter 6 presents work of the author on the complex Monge-Ampère equation on a compact Kähler manifold M [S. Kołodziej, op. cit., 1998; Indiana Univ. Math. J. **52** (2003), no. 3, 667–686; MR1986892 (2004i:32062)]. Given a fundamental form ω normalized so that $\int_M \omega^n = 1$, a continuous function φ on M is called ω-plurisubharmonic ($\varphi \in \mathrm{PSH}(\omega)$) if $\omega_\varphi := \omega + dd^c\varphi \geq 0$. Given a nonnegative function f on M normalized so that $\int_M f\omega^n = 1$, the problem is to solve the Monge-Ampère equation $\omega_\varphi^n = f\omega^n$ for φ. Defining

$$\mathcal{F}(A, h) := \{f \in L^1(M)\colon f \geq 0,\ \int_E f\omega^n \leq F(\mathrm{cap}_\omega(E)),\ E \text{ Borel}\}$$

where cap_ω is the appropriate notion of capacity, the author shows (Theorem 6.7) that if h is admissible and $1 \in \mathcal{F}(A, h)$, then for any $f \in \mathcal{F}(A, h)$ there exists a continuous solution φ of $\omega_\varphi^n = f\omega^n$. If one normailizes φ so that $\max_M \varphi = 0$, the solution is unique; this follows from a stability estimate in the final section.

Reviewed by *Norman Levenberg*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2166340 (2006k:32070) 32U40 (32C30 32H50 37A25 37B40 37F10)
Guedj, Vincent (F-TOUL3-LM)
Courants extrémaux et dynamique complexe. (French. English, French summaries) [Extremal currents and complex dynamics]
Ann. Sci. École Norm. Sup. (4) **38** (2005), *no. 3,* 407–426.

The goal of the paper is to construct extremal positive closed currents of any degree in $\mathbb{P}^k$ (i.e., extremal elements in the closed convex cone of positive closed currents of bidimension $(p,p), 0 \leq p \leq k$) which are not currents of integration along irreducible analytic subsets. Such currents, of dynamical character, were constructed before by various authors [J.-P. Demailly, Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007); E. Bedford and J. Smillie, Math. Ann. **294** (1992), no. 3, 395–420; MR1188127 (93k:32062); J. E. Fornæss and N. Sibony, Duke Math. J. **65** (1992), no. 2, 345–380; MR1150591 (93d:32040)], but all examples were of bidimension $(k-1,k-1)$. The author considers a polynomial endomorphism $f\colon \mathbb{C}^k \to \mathbb{C}^k$ and its meromorphic extension to $\mathbb{P}^k$. Throughout the paper, he makes the assumption (H1): $X_f \cap I_f = \varnothing$, where $X_f := f((t=0)) \setminus I_f$, $(t=0)$ is the hyperplane at infinity and I_f is the indeterminacy set for f. Under this assumption, there exists a positive closed current T_+ in $\mathbb{P}^k$ such that

$$T_+ = \omega + dd^c g_+,\ \ f^*T_+ = dT_+,$$

where $d > 1$ is the first dynamical degree of f, ω is the Fubini-Study form in $\mathbb{P}^k$ and g_+ is continuous in $\mathbb{P}^k \setminus I_f$. Then $\dim X_f = r-1,\ \dim I_f = k-r-1$ for some integer $1 \leq r \leq k-1$ and it is possible to define $T_+^j,\ 1 \leq j \leq r+1$. T_+^r is one of the currents whose extremal properties are established in the paper. The key point in the author's proof is the existence of potentials that allow him to control sign. In Section 1, under the assumption (H2) that I_f is f^{-1}-attracting, the author observes that T_+^r admits almost positive potentials, i.e., $T_+^r = \omega^r + dd^c T_{\rm can}$, where $T_{\rm can}$ is a positive current in $\mathbb{P}^k \setminus W_{I_f}$, W_{I_f} is a suitable neighborhood of I_f and $\lambda^{-j}(f^j)^* T_{\rm can} \to 0$ in $\mathbb{C}^k$ with $\lambda = d^r$. Proposition 1.1 proves the existence of an almost everywhere negative potential (with control on mass) for any positive closed current S of bidegree (s,s) in $\mathbb{P}^k$ satisfying $0 \le S \le \sigma$ in $\mathbb{P}^k$, where σ is a given positive closed current of bidegree (s,s) in $\mathbb{P}^k$, $1 \le s \le k$.

Section 2 deals with extremality properties. Theorem 2.1 says the following: Let $f\colon \mathbb{C}^k \to \mathbb{C}^k$ be a polynomial automorphism satisfying conditions (H1) and (H2). Then T_+^r is extremal in the cone $\mathcal{T}^r(\mathbb{P}^k)$ of positive closed (r,r)-currents in $\mathbb{P}^k$. Also, it is extremal in $\mathcal{T}^r(\mathbb{C}^k)$. Theorem 2.3 is its counterpart for a polynomial endomorphism f of $\mathbb{C}^k$ satisfying (H1),(H2), with the conclusion that T_+^r is extremal in the sub-cone of $\mathcal{T}^r(\mathbb{P}^k)$ consisting of currents S satisfying $f^*S = \lambda S$. Both proofs are based on uniform convergence of sequences of pullbacks under iterates of f of some quite general currents to constant multiples of T_+^r. Later, the author makes an assumption (H3) about the degrees of f:

$$\lambda = d^r > \lambda_{r+1}(f) := \lim_{j\to\infty} (\delta_{r+1}(f^j))^{1/j},$$

where $\delta_{r+1}(f)$ is the $(r+1)$-st algebraic degree of f. For f satisfying (H1) and (H3) it is possible to construct a positive closed current $T_{k-r}^- = \theta + dd^c T_\infty^-$ such that $T_\infty^- \ge 0$, $f_* T_{k-r}^- = \lambda T_{k-r}^-$ and $\|T_{k-r}^-\| = 1$, where θ is a suitable positive closed smooth $(k-r,k-r)$-form. The existence of the positive potential T_∞^- implies that T_{k-r}^- is not a current of integration over an analytic set of dimension r. To deal with singularities of $f_*\theta$ when f is not invertible, the author assumes (H4): $\lim_{z\in C_f, |z|\to\infty} f(z) \in X_f$, where C_f is the critical set of f. By Theorem 2.6, for f satisfying conditions (H1), (H3) and (H4), the current T_{k-r}^- is extremal in the cone $\mathcal{T}_{f_*}^{k-r}(\mathbb{P}^k) = \{S \in \mathcal{T}^{k-r}(\mathbb{P}^k) \colon f_*S = \lambda S\}$. If f is an automorphism, T_{k-r}^- is also extremal in $\mathcal{T}^{k-r}(\mathbb{P}^k)$. Proposition 2.5 states that quasi-plurisubharmonic functions in $\mathbb{P}^k$ are integrable with respect to $T_{k-r}^- \wedge \omega^r$. In particular, T_{k-r}^- does not charge pluripolar sets. In Remark 2.7 it is observed that T_{k-r}^- can be extremal in $\mathcal{T}^{k-r}(\mathbb{P}^k)$ even though f is not invertible, as proved by the author in [Amer. J. Math. **124** (2002), no. 1, 75–106; MR1879000 (2003b:32021)] for $k=2, r=1$.

Section 3 is devoted to a canonical invariant measure for f. It is $\mu_f = T_+^r \wedge T_{k-r}^-$, which is a well-defined measure, since T_+^r and T_{k-r}^- have complementary bidegrees and T_+ has bounded potentials on the support of T_{k-r}^-. Theorem 3.1 states that μ_f is an f-invariant probability measure with compact support in $\mathbb{C}^k$ such that $\mathcal{L}(\mathbb{C}^k) \subset L^1(\mu_f)$, where $\mathcal{L}$ denotes the Lelong class of plurisubharmonic functions in $\mathbb{C}^k$ with logarithmic growth. In particular, μ_f does not charge pluripolar sets in $\mathbb{C}^k$. Theorem 3.2 says that μ_f is ergodic and of maximal entropy

$$h_{\mu_f}(f) = h_{\rm top}(f) = \log\lambda.$$

The inequality

$$h_{\mu_f}(f) \le h_{\rm top}(f)$$

between metric and topological entropy is the Misiurewicz's variational principle, and the inequality

$$h_{\text{top}}(f) \leq \max_{1\leq j\leq k} \log \lambda_j(f)$$

is due to M. L. Gromov [Enseign. Math. (2) **49** (2003), no. 3-4, 217–235; MR2026895 (2005h:37097)]. When f satisfies (H1) and (H3), λ is the largest dynamical degree. The equalities were conjectured by S. Friedland in [Ann. of Math. (2) **133** (1991), no. 2, 359–368; MR1097242 (92c:58115)].

Reviewed by *Małgorzata Stawiska*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2158978 (2006g:32007) 32A26 (32W05 53C60)
Qiu, Chunhui (PRC-XIAM-SM); **Zhong, Tongde** (PRC-XIAM-SM)
The Koppelman-Leray formula on complex Finsler manifolds. (English summary)
Sci. China Ser. A **48** (2005), *no. 6,* 847–863.

J.-P. Demailly and C. Laurent-Thiébaut constructed integral representations of Cauchy-Leray-Koppelman type for forms of arbitrary bidegree on complex manifolds [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)], and later B. Berndtsson obtained more precise results [Ann. Sci. École Norm. Sup. (4) **24** (1991), no. 3, 319–337; MR1100993 (92c:32012)]. The kernel of Demailly and Laurent-Thiébaut is essentially the leading term of the kernel of Berndtsson.

The authors adapted the method of Demailly and Laurent-Thiébaut to the setting of complex Finsler manifolds in a previous article [Sci. China Ser. A **47** (2004), no. 2, 284–296; MR2068946 (2005e:32005)]. The present article, which uses the kernel of Berndtsson in place of the kernel of Demailly and Laurent-Thiébaut, is completely parallel, even reproducing much of the earlier article verbatim. The authors have added two proofs that they omitted in the earlier article as well as three examples.

Reviewed by *Harold P. Boas*

MR2157164 (2006j:32016) 32L10 (32A25 32L20)

Berman, Robert (S-CHAL)

Holomorphic Morse inequalities on manifolds with boundary. (English, French summaries)

Ann. Inst. Fourier (Grenoble) **55** (2005), *no. 4,* 1055–1103.

The main result of this paper is a generalization of Demailly's weak holomorphic Morse inequalities to the case of compact n-dimensional complex manifolds with boundary.

To be more precise, let X be a compact complex manifold with boundary, let ρ be a defining function for the boundary and let $\mathcal{L} = i\partial\overline{\partial}\rho$ restricted to $T^{1,0}(\partial X)$ be the Levi form of the boundary. Further, assume that L is a holomorphic line bundle over X with fiber metric φ and let $\Theta = i\partial\overline{\partial}\varphi$ be its curvature form. Now let $X(q)$ be the subset of X where Θ has exactly q negative eigenvalues, i.e. the set where $\operatorname{index}(\Theta) = q$, and let

$$T(q)_{\rho,x} := \{t > 0\colon \operatorname{index}(\Theta + t\mathcal{L}) = q \text{ along } T^{1,0}_x(\partial X)\}.$$

The author shows that if the Levi form is nondegenerate on the boundary then, up to terms of order $o(k^n)$, the dimension of $H^{0,q}(X, L^k)$ can be estimated by

$$k^n(-1)^q\left(\frac{1}{2\pi}\right)^n\left(\int_{X(q)}\frac{\Theta^n}{n!} + \int_{\partial X}\int_{T(q)_{\rho,x}}\frac{(\Theta + t\mathcal{L})^{n-1}}{(n-1)!}\wedge\partial\rho\wedge dt\right).$$

The proof of this theorem uses in an essential way the estimates of some Bergman functions, which can be derived by explicitly computing some model cases. The author also gives some examples that illustrate the sharpness of the obtained result.

Some of the other results obtained in this paper are the strong holomorphic Morse inequalities, applications to the volume of semi-positive line bundles and some relations to hole filling and contact geometry.

Reviewed by *Bert Fischer*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2142242 (2006d:32033) 32Q15 (32G20 32J27)
Demailly, Jean-Pierre (F-GREN-F); **Eckl, Thomas** (D-KOLN);
Peternell, Thomas (D-BAYR-IM)
Line bundles on complex tori and a conjecture of Kodaira.
Comment. Math. Helv. **80** (2005), *no. 2,* 229–242.

A compact Kähler manifold is called almost algebraic if it can be approximated by smooth projective varieties. K. Kodaira proved in [Ann. of Math. (2) **78** (1963), 1–40; MR0184257 (32 #1730)] that every Kähler surface is almost algebraic. The statement that this should be true also in higher dimensions is known as the Kodaira conjecture. Recently, C. Voisin ["On the homotopy types of Kähler manifolds and the birational Kodaira problem", preprint, arxiv.org/abs/math/0410040] and K. Oguiso ["Automorphisms of hyperkähler manifolds in the view of topological entropy", preprint, arxiv.org/abs/math/0407476] constructed counterexamples by constructing rigid non-algebraic Kähler threefolds. The present paper, which was completed before the counterexamples appeared, gives some observations concerning the Kodaira conjecture. A certain blow-up of a $\mathbb{P}_1^3$-bundle over a 3-dimensional complex torus with Picard number ≥ 3 is shown to be rigid. It turns out, however, that these complex tori are algebraic. Some interesting generalizations are also considered.

Reviewed by *H. Lange*

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MR2140209 (2006a:30047) 30F60 (32G15)
Haïssinsky, Peter (F-PROV-LAT)

Déformation localisée de surfaces de Riemann. (French. English summary) [Localized deformations of Riemann surfaces]

Publ. Mat. **49** (2005), *no. 1,* 249–255.

Let Y be a Riemann surface with compact boundary embedded into a hyperbolic Riemann surface of finite type X. The following results are proved:

(1) The space of deformations $\mathcal{D}$ of the complex structure of Y in X is an open subset of the Teichmüller space $T(X)$ of X.

(2) The space $\mathcal{D}$ has compact closure in $T(X)$ if and only if Y is simply connected or isomorphic to a punctured disk.

(3) $\mathcal{D} = T(X)$ if and only if the components of $X \setminus Y$ are all disks or punctured disks.

Point (1) was already known by results of J.-P. Demailly and C. T. McMullen. Points (2) and (3) are original. The proof follows from the standard parametrization of Teichmüller space given by Fenchel-Nielsen coordinates and quasi-conformal Teichmüller theory.

Reviewed by *Marco Boggi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2128092 (2006i:32024) 32L15 (32J99 32L10)
Marinescu, George [Marinescu, Gheorghe] (D-HUMB-IM)
A criterion for Moishezon spaces with isolated singularities. (English summary)
Ann. Mat. Pura Appl. (4) **184** (2005), *no. 1,* 1–16.

Summary: "We give a criterion for a compact complex space with isolated singularities to be Moishezon in the spirit of Siu-Demailly's solution to the Grauert-Riemenschneider conjecture [see Y. T. Siu, J. Differential Geom. **19** (1984), no. 2, 431–452; MR0755233 (86c:32029); J.-P. Demailly, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)]. It refines a previous work by A. M. Nadel and H. Tsuji [J. Differential Geom. **28** (1988), no. 3, 503–512; MR0965227 (89m:32047)], and another one by S. Takayama [Tohoku Math. J. (2) **46** (1994), no. 2, 281–291; MR1272883 (95d:32032)], in a more specific situation."

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J. Geom. Anal. **3**, 37–61 (1993) MR1197016 (93m:32014)
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MR2113990 (2005m:32038) 32L05 (32J27)
Biswas, Indranil (6-TIFR-SM); **Subramanian, Swaminathan** (6-TIFR-SM)
Numerically flat principal bundles. (English summary)
Tohoku Math. J. (2) **57** (2005), *no. 1,* 53–63.

The notion of "numerical effectiveness" of vector bundles over smooth projective varieties is very well known. It was extended by J.-P. Demailly, T. Peternell and M. H. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)] to the case of vector bundles over compact Kähler manifolds. A vector bundle E is said to be numerically flat if it is numerically effective and its dual E^* is also numerically effective.

In the paper under review the authors extend the notion of numerical flatness to the case of principal bundles and give some characterizations of numerically flat principal bundles.

Let G be a semisimple algebraic group, M a compact Kähler manifold and E_G a holomorphic principal G-bundle over M. The principal G-bundle E_G is said to be numerically flat if for all pairs (P, χ), where P is a parabolic subgroup of G and χ is an anti-dominant character of P with respect to some Borel subgroup contained in P, the associated line bundle $E_\chi = E_G \times_\chi \mathbb{C}$ over E_G/P is a numerically effective line bundle. The main results of the paper under review are the following:

(1) If E_G is numerically flat and V is a finite dimensional complex G-module, then the associated vector bundle $E_G \times_G V$ is numerically flat. Conversely, let $\rho\colon G \to \mathrm{SL}(V)$ be a representation of G, where V is as above. If $\ker(\rho)$ is finite and if $E_G \times_G V$ is numerically flat then E_G is numerically flat. In particular a principal G-bundle E_G is numerically flat if and only if the adjoint vector bundle $\mathrm{ad}(E_G)$ is numerically flat.

(2) A numerically flat principal G-bundle E_G over a compact Kähler manifold M is semistable (with respect to the Kähler metric as a polarization) and all its (rational) characteristic classes of degree at least one vanish. The converse is true if we assume that M is projective. Hence a characterization of numerical flatness of holomorphic principal bundles in terms of semistability in the case M is projective.

In the last part of the paper, the authors give another characterization of numerical flatness of holomorphic principal bundles in terms of a reduction of the structure group and the existence of some "special connections". More precisely, they prove that:

(3) A principal G-bundle E_G over a compact Kähler manifold M is numerically flat if and only if there exist a parabolic subgroup P of G and a reduction E_P of the structure group of E_G to P, such that the principal P-bundle E_P admits a flat holomorphic connection ∇ with the property that the monodromy of the (flat) connection on $E_{L(P)}$ induced by ∇ is contained in a maximal compact subgroup of $L(P)$, where $L(P)$ is a Levi factor of the parabolic subgroup P and $E_{L(P)}$ is the principal $L(P)$-bundle obtained by extending the structure group of E_P to $L(P)$ via the morphism $P \longrightarrow L(P)$.

Reviewed by *Boudjemâa Anchouche*

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MR2099777 (2005i:14052) 14J45 (14C05 53D10)
Kebekus, Stefan (D-KOLN)
Lines on complex contact manifolds. II. (English summary)
Compos. Math. **141** (2005), *no. 1,* 227–252.

Let X be a complex projective manifold of dimension $2n+1$. The manifold X is called a contact manifold if there exist a line bundle L and a twisted holomorphic form $\theta \in H^0(X, \Omega_X \otimes L)$ such that $\theta \wedge (d\theta)^{\wedge n}$ is nowhere zero. Following [S. Kebekus et al., Invent. Math. **142** (2000), no. 1, 1–15; MR1784795 (2002a:14047)] and [J.-P. Demailly, in *Complex geometry (Göttingen, 2000)*, 93–98, Springer, Berlin, 2002; MR1922099 (2003f:32029)] it is known that either X is the projectivisation of the (co)tangent bundle of an $(n+1)$-fold (with the standard contact structure), or $b_2(X) = 1$ and X is Fano. In the latter case X is conjectured to be a rational homogeneous manifold and the closed orbit of the projectivisation of the adjoint representation of a simple algebraic group. It is expected that the conjecture should be proved by understanding rational curves on X. The paper under review concerns rational curves on X which are of degree 1 with respect to the line bundle L; they are called contact lines, or just lines. By [S. Kebekus, in *Complex geometry (Göttingen, 2000)*, 147–155, Springer, Berlin, 2002; MR1922103 (2003j:14065)] X is either the projective space $\mathbf{P}^{2n+1}$ or it is covered by lines. The main theorem of the paper under review is as follows: Let us choose an irreducible component H of the space parameterizing lines and for $x \in X$ let H_x denote the subvariety in H parameterizing lines passing through x. Then for a general x the variety H_x is irreducible and smooth, and the locus of curves in X parametrized by H_x forms a cone over H_x. In particular, all lines through x are smooth, they meet only in x and they do not share a common tangent direction at x. This implies, for example, that the variety of tangents to lines at x is a smooth Legendrian subvariety of the contact distribution $\mathbf{P}(\ker(\theta)_x) \subset \mathbf{P}(T_x X)$. In an appendix the author discusses some properties of jet bundles which are interesting for their own sake.

{For Part I see [S. Kebekus, J. Reine Angew. Math. **539** (2001), 167–177; MR1863858 (2002h:14069)].}

Reviewed by *Jarosław A. Wiśniewski*

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12. C. LeBrun, *Fano manifolds, contact structures and quaternionic geometry,* Internat. J. Math. **6** (1995), 419–437. MR1327157 (96c:53108)
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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2144160 32-06 (14-06)

Komplexe Analysis. [Complex analysis]

Abstracts from the workshop held August 22–28, 2004.
Organized by Jean-Pierre Demailly, Klaus Hulek and Thomas Peternell.
Oberwolfach Reports. Vol. 1, no. 3.

Oberwolfach Rep. **1** (2004), *no. 3,* 2171–2215.

{This item will not be reviewed individually. For details of the collection in which this item appears see MR2022954 (2006c:00027) .}

{For the entire collection see MR2022954 (2006c:00027)}

MR2132649 (2007h:14003) 14B05 (13H99)
Blickle, Manuel (D-DUES2); **Lazarsfeld, Robert** (1-MI)
An informal introduction to multiplier ideals. (English summary)
Trends in commutative algebra, 87–114, *Math. Sci. Res. Inst. Publ.*, 51, *Cambridge Univ. Press, Cambridge*, 2004.

From the introduction: "Given a smooth complex variety X and an ideal (or ideal sheaf) $\mathfrak{a}$ on X, one can attach to $\mathfrak{a}$ a collection of multiplier ideals $\mathcal{J}(\mathfrak{a}^c)$ depending on a rational weighting parameter $c > 0$. These ideals, and the vanishing theorems they satisfy, have found many applications in recent years. In the global setting they have been used to study pluricanonical and other linear series on a projective variety [J.-P. Demailly, J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007); U. Angehrn and Y. T. Siu, Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036); Y. T. Siu, Invent. Math. **134** (1998), no. 3, 661–673; MR1660941 (99i:32035); L. M. H. Ein and R. K. Lazarsfeld, J. Amer. Math. Soc. **10** (1997), no. 1, 243–258; MR1396893 (97d:14063); Invent. Math. **137** (1999), no. 2, 427–448; MR1705839 (2000j:14028); J.-P. Demailly, Astérisque No. 266 (2000), Exp. No. 852, 3, 59–90; MR1772670 (2001m:32042)]. More recently they have led to the discovery of some surprising uniform results in local algebra [L. M. H. Ein, R. K. Lazarsfeld and K. E. Smith, Invent. Math. **144** (2001), no. 2, 241–252; MR1826369 (2002b:13001); Amer. J. Math. **125** (2003), no. 2, 409–440; MR1963690 (2003m:13004); Duke Math. J. **123** (2004), no. 3, 469–506; MR2068967 (2005k:14004)]. The purpose of these lectures is to give an easy-going and gentle introduction to the algebraically-oriented local side of the theory.

"Multiplier ideals can be approached (and historically emerged) from three different viewpoints. In commutative algebra they were introduced and studied by J. Lipman [Bull. Soc. Math. Belg. Sér. A **45** (1993), no. 1-2, 223–244; MR1316244 (97a:13030)] (under the name 'adjoint ideals', which now means something else), in connection with the Briançon-Skoda theorem. On the analytic side of the field, A. M. Nadel [Ann. of Math. (2) **132** (1990), no. 3, 549–596; MR1078269 (92d:32038)] attached a multiplier ideal to any plurisubharmonic function, and proved a Kodaira-type vanishing theorem for them. (In fact, the 'multiplier' in the name refers to their analytic construction; see Section 2.4.) This machine was developed and applied with great success by Demailly, Siu and others. Algebro-geometrically, the foundations were laid in passing by Esnault and Viehweg in connection with their work involving the Kawamata-Viehweg vanishing theorem. More systematic developments of the geometric theory were subsequently undertaken by Ein, Kawamata and Lazarsfeld. We take the geometric approach here.

"The present notes follow closely a short course on multiplier ideals given by Lazarsfeld at the Introductory Workshop for the Commutative Algebra Program at the MSRI in September 2002. The three main lectures were supplemented with a presentation by Blicke on multiplier ideals associated to monomial ideals (which appears here in Section 3). We have tried to preserve in this write-up the informal tone of these talks: thus we emphasize simplicity over generality in statements of results, and we present very few proofs. Our primary hope is to give the reader a feeling for what multiplier ideals are and how they are used. For a detailed development of the theory from an algebro-geometric perspective we refer to the forthcoming Part III of [R. K. Lazarsfeld, *Positivity in algebraic geometry. I*, Springer, Berlin, 2004; MR2095471 (2005k:14001a); *II*, MR2095472

(2005k:14001b)]. The analytic picture is covered in J.-P. Demailly's lectures [in *School on Vanishing Theorems and Effective Results in Algebraic Geometry (Trieste, 2000)*, 1–148, Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2001; MR1919457 (2003f:32020)]."
{For the entire collection see MR2132646 (2005j:13002)}

MR2119243 (2006c:32045) 32U40 (32C30 32H04 32Q15 37B40)
Dinh, Tien-Cuong (F-PARIS11); **Sibony, Nessim** (F-PARIS11)
Regularization of currents and entropy. (English, French summaries)
Ann. Sci. École Norm. Sup. (4) **37** (2004), *no. 6,* 959–971.

In the paper, the authors prove several results about the regularization of (p,p)-currents on a compact Kähler manifold (X,ω) of dimension k. The main theorem is Theorem 1.1: Let (X,ω) be a compact Kähler manifold of dimension k. Then, for every positive closed (p,p)-current T on X there exist smooth positive closed (p,p)-forms T_n^+ and T_n^- such that $T_n^+ - T_n^-$ converges weakly to the current T. Moreover, $\|T^\pm\| \le c_X\|T\|$, where $c_X > 0$ is a constant independent of T. This is a generalization of a result by J.-P. Demailly [in *Several complex variables (Berkeley, CA, 1995–1996)*, 233–271, Cambridge Univ. Press, Cambridge, 1999; MR1748605 (2002e:32046)] on the regularization of positive closed $(1,1)$-currents on X. In Section 1, they derive other results as corollaries from Theorem 1.1. Corollary 1.2 extends their previous result on the regularization of positive closed (p,p)-currents on a projective manifold [T.-C. Dinh and N. Sibony, "Une borne supérieure pour l'entropie topologique d'une application rationnelle", Ann. of Math. (2), to appear]. Corollary 1.3 defines the pullback of the current T by a surjective holomorphic map $\Pi\colon X' \mapsto X$, where (X',ω') is another compact Kähler manifold of dimension $k' \ge k$. Theorem 1.4 says that if f is a dominating meromorphic self-map of X, then $h(f) \le \mathrm{lov}(f) = \max_{1\le p\le k} \log d_p$, where $h(f)$ is the topological entropy of f, d_p is the dynamical degree of order p of f and $\mathrm{lov}(f)$ measures the growth of the volume of the graphs of $(f,\dots,f^{n-1})$ (over the subset of X on which all iterates of f are defined). However, it should be noted that for a compact Kähler manifold X of dimension k and $f\colon X \mapsto X$ holomorphic, S. Friedland proved a stronger result, namely that $h(f) = \mathrm{lov}(f) = \max_{1\le p\le k} d_p$ (see [J. Fourier Anal. Appl. **1995**, Special Issue; MR1364875 (96f:00039)] for the whole collection). His proof does not rely on Lemma 3 in his previous publication [S. Friedland, Ann. of Math. (2) **133** (1991), no. 2, 359–368; MR1097242 (92c:58115)], which the authors of the paper under review point out to be wrong. The equality $h(f) = \mathrm{lov}(f)$ for f meromorphic still remains a conjecture. In Section 2, the authors prove an auxiliary Lemma 2.1, which gives properties of a linear operator P defined on the set $\mathcal{M}$ of Radon measures on $\mathbb{R}^m$ as integration against a kernel K with compact support on $B\times B$, smooth in $B\times B\setminus\Delta$ and bounded pointwise by a constant multiple of the fundamental kernel of classical potential theory. Section 3 contains the proof of Theorem 1.1. First they give a weak regularization of the current of integration $[\Delta]$, i.e., they construct positive closed (k,k)-forms $K_n^{\pm}$ with coefficients in L_1 and smooth out Δ such that $K_n^+ - K_n^- \to [\Delta]$ weakly and $\|K_n^{\pm}\| \leq c_1$, where $c_1 > 0$. Then they define $T_n^{\pm}$ by wedging $K_n^{\pm}$ with T, show that $T_n^+ - T_n^- \to T$ and $\|T_n^{\pm}\| \leq c\|T\|$, and obtain the smoothness of $T_n^{\pm}$ by repeated use of Lemma 2.1. In Section 4, they prove Theorem 4.1, which is an analog of Theorem 1.1 for positive dd^c-closed (p,p)-currents on X, with $T_n^{\pm}$ smooth, positive and dd^c-closed. Later, they introduce a special class $\mathrm{DSH}^p(X)$ of (p,p)-currents equipped with a suitable norm. Theorem 4.4 is a regularization result for currents in this class. Proposition 4.6 is a variant of Theorem 1.1 for a continuous form T. Section 5 is devoted to the study of the intersection $S \wedge T$ of a positive closed $(1,1)$-current S with a positive pluriharmonic (p,p)-current T, $1 \leq p \leq k-1$. S can be written as $S = \alpha + dd^c u$, where u, called the potential of S, is a quasi-psh function. Theorem 5.1 says that if u is continuous, then $S \wedge T$ is well defined and is a positive dd^c-closed current, which depends continuously on S and T in a suitable sense. Theorem 5.3 is a similar result for T in the class $\mathrm{DSH}^p(X)$; $S \wedge T$ is then in $\mathrm{DSH}^{p+1}(X)$. Proposition 5.5 is an analog of Theorem 5.1 for T of bidegree $(1,1)$ and S with bounded potential.

Reviewed by *Małgorzata Stawiska*

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preprint, 2003, math.DS/0306095. cf. MR 2004e:37063
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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2112584 (2006b:14011) 14C25 (32J27)
Demailly, Jean-Pierre (F-GREN-IF)
On the geometry of positive cones of projective and Kähler varieties. (English summary)
The Fano Conference, 395–422, *Univ. Torino*, *Turin*, 2004.

Some of the more fundamental problems and results in complex geometry revolve around such questions as when a complex manifold would be projective or Kähler, or how much of its geometry could be determined by divisors and curves. All projective manifolds are Kähler, and a famous theorem of Kodaira proves that a Kähler manifold (X,ω) is projective precisely when the class

$[\omega] \in H^{1,1}(X) \subset H^2(X,\mathbb{R})$ moreover represents a class in $H^2(X,\mathbb{Z})$. Kodaira had also conjectured that a compact complex surface admits a Kähler metric if and only if the first Betti number is even. A closely related question concerns the ampleness of holomorphic line bundles L on X. When X is projective, the Nakai-Moishezon criterion establishes that ampleness of L is equivalent to having a strictly positive integral for the p-th exterior power of the Chern class of L over any algebraic subset of dimension p for $1 \leq p \leq n = \dim(X)$. Mori's theory of complex three-manifolds brought new techniques to bear on the projective context via the geometry of cones of divisors and curves lying within their respective cohomology groups. For example, a conjecture of Fano asserts that a projective X is "uniruled" by rational curves precisely when the Chern class of the canonical line bundle lies outside the closure of the cone of effective divisors. The article under review is a survey of relatively recent achievements of the author and his collaborators, S. Boucksom, M. Paun and T. Peternell, in further unifying and extending the theory surrounding these questions. Central to their programme are the powerful techniques associated with positive currents of type $(1,1)$ on compact Kähler manifolds, and the interplay between the open convex cone of Kähler forms and the enveloping closed convex cone of positive $(1,1)$-currents (the "pseudo-effective" cone). While some basic familiarity with Kähler geometry and the theory of currents is assumed, the author's exposition is designed to be informative to the non-specialist. Among the results surveyed, some highlights are a generalization of the Nakai-Moishezon criterion and its application to the characterisation of Kähler currents on compact complex manifolds, as well as a theory of Poincaré duality between cones of positive currents of type $(1,1)$ and $(n-1,n-1)$, which leads in particular to a proof of Fano's conjecture.

{For the entire collection see MR2112562 (2005g:14003)}

Reviewed by *Adam Gregory Harris*

MR2112562 (2005g:14003) 14-06

★**The Fano Conference.**

Proceedings of the conference to commemorate the 50th anniversary of the death of Gino Fano (1871–1952) held in Torino, September 29–October 5, 2002.
Edited by Alberto Collino, Alberto Conte and Marina Marchisio.
Università di Torino, *Dipartimento di Matematica*, *Turin*, 2004. *xiv*+804 *pp*. €99.90.
ISBN 88-900876-1-7

Contents: Robert Fano, In loving memory of my father Gino Fano (1–4) MR2112563; Herbert C. Clemens [C. Herbert Clemens], A version of Abel's theorem for surfaces (5–23); MR2112564 (2006d:14010); Vasilij A. Iskovskikh [Vasiliĭ Alekseevich Iskovskikh], On the Noether-Fano inequalities (25–35); MR2112565 (2005m:14017); Shigefumi Mori and Shigeru Mukai, Extremal rays and Fano 3-folds (37–50); MR2112566 (2005k:14085); Jacob P. Murre, Fano varieties and algebraic cycles (51–68); MR2112567 (2005i:14053); Valery A. Alexeev and Michel Brion, Boundedness of spherical Fano varieties (69–80); MR2112568 (2005k:14081); Vincenzo Ancona and Marco Maggesi, On the quantum cohomology of Fano bundles over projective spaces (81–98); MR2112569 (2005m:14103); Marco Andreatta, The projective space and other simple varieties (99–108); MR2112570 (2005j:14019); Victor V. Batyrev, Toric degenerations of Fano varieties and constructing mirror manifolds (109–122); MR2112571 (2006b:14070); Ingrid C. Bauer and Fabrizio Catanese, Some new surfaces with $p_g = q = 0$ (123–142); MR2112572 (2005k:14079).

Arend Bayer and Yuri I. Manin [Yuriĭ Ivanovich Manin], (Semi)simple exercises in quantum cohomology (143–173); MR2112573 (2006a:14090); Arnaud Beauville, Fano threefolds and $K3$ surfaces (175–184); MR2112574 (2006b:14071); Mauro C. Beltrametti, Maria L. Fania and Andrew J. Sommese, Mukai varieties as hyperplane sections (185–208); MR2112575 (2005i:14008); Tzachi Ben-Itzhak and Mina Teicher, Topological algorithm for conjugation of half-twists in the braid group with application to Hurwitz equivalence (209–221); MR2112576 (2006e:20064); Fedor Bogomolov and Yuri Tschinkel [Yu. Tschinkel], Special elliptic fibrations (223–234); MR2112577 (2005k:14017); Gavin Brown [Gavin Brown^2], Alessio Corti and Francesco Zucconi, Birational geometry of 3-fold Mori fibre spaces (235–275); MR2112578 (2005k:14031); Alberto Calabri, Ciro Ciliberto, Flaminio Flamini and Rick Miranda, On the geometric genus of reducible surfaces and degenerations of surfaces to unions of planes (277–312); MR2112579 (2005j:14010); Frédéric Campana, L'anneau ordonné de la géométrie algébrique [Ordered rings in algebraic geometry] (313–338); MR2112580 (2006k:14017); Frédéric Campana, Orbifoldes à première classe de Chern nulle [Orbifolds of zero first Chern class] (339–351); MR2112581 (2005i:32019); Lucia Caporaso, Moduli theory and arithmetic of algebraic varieties (353–378); MR2112582 (2005m:14015).

Olivier Debarre, Seshadri constants of abelian varieties (379–394); MR2112583 (2005k:14093); Jean-Pierre Demailly, On the geometry of positive cones of projective and Kähler varieties (395–422); MR2112584 (2006b:14011); Igor V. Dolgachev, Abstract configurations in algebraic geometry (423–462); MR2112585 (2005k:14091); Antonella Grassi, Open-closed string dualities in geometry and modified open Gromov-Witten invariants (463–478); MR2112586 (2005k:14118); Joe Harris [Joseph Daniel Harris] and Klaus Hulek, A remark on the Schottky locus in genus 4 (479–483); MR2112587 (2005h:14077); Yasuyuki Kachi and Shashikant B. Mulay, Local-to-global correspondence in algebraic geometry (485–514); MR2112588 (2006a:14018); János Kollár and Frank-Olaf Schreyer, Real Fano 3-folds of type V_{22} (515–531); MR2112589 (2005k:14084); Viktor S. Kulikov, Generic coverings of the plane and braid monodromy invariants (533–558); MR2112590 (2005j:14050); Yoichi Miyaoka, Numerical characterisations of hyperquadrics (559–562); MR2112591 (2005h:14103); Shigeru Mukai, Plane quartics and Fano threefolds of genus twelve (563–572); MR2112592 (2006d:14043).

Viacheslav V. Nikulin [Vyacheslav V. Nikulin], On algebraic varieties with finite polyhedral Mori cone (573–589); MR2112593 (2006c:14019); Christian Okonek and Andrei Teleman, Gauge theoretical Gromov-Witten invariants and virtual fundamental classes (591–623); MR2112594 (2005i:14072); Thomas Peternell and Andrew J. Sommese, Ample vector bundles and branched coverings. II (625–645); MR2112595 (2005k:14088); Yuri G. Prokhorov, A remark on Fano threefolds with canonical Gorenstein singularities (647–657); MR2112596 (2005k:14086); Aleksandr V. Pukhlikov, Birationally rigid Fano varieties (659–681); MR2112597 (2005j:14017); Kristian Ranestad, Non-abelian Brill-Noether loci and the Lagrangian Grassmannian LG$(3,6)$ (683–692); MR2112598 (2005j:14031); Nicholas I. Shepherd-Barron, Stably rational irrational varieties (693–700); MR2112599 (2006b:14021); Andrei N. Tyurin, Fano versus Calabi-Yau (701–734); MR2112600 (2005h:14098); Alessandro Verra, The Prym map has degree two on plane sextics (735–759); MR2112601 (2005k:14057); Claire Voisin, Intrinsic pseudo-volume forms and K-correspondences (761–792); MR2112602 (2006b:14020); Pelham M. H. Wilson, Metric limits of Calabi-Yau manifolds (793–804); MR2112603 (2005h:32057).

{Most of the papers are being reviewed individually.}

MR2113021 (2005i:32020) 32J27 (32Q15)

Demailly, Jean-Pierre (F-GREN); **Paun, Mihai** (F-STRAS)

Numerical characterization of the Kähler cone of a compact Kähler manifold. (English summary)

Ann. of Math. (2) **159** (2004), *no. 3,* 1247–1274.

This article gives a beautiful solution of a long-standing basic problem in Kähler geometry and, as such, can be viewed as a classic. It is likely to have a lasting impact on the field.

The problem was to generalize the classical Nakai-Moishezon criterion of ampleness to a numerical characterization of the Kähler cone of a compact Kähler manifold.

Let us first recall the statement of the Nakai-Moishezon theorem. Let k be a field and X be a projective scheme over k. Let L be a Cartier divisor on X. Then L is ample iff for every positive dimensional reduced closed subscheme $Z\subset X$, $L^{\dim Z}\cdot Z>0$.

If $k=\mathbf{C}$ and X is smooth, we can reformulate this using Kodaira's theorem that ample divisors L on X are characterized by the existence of a smooth Hermitian metric of positive curvature or, in an equivalent fashion, by the fact that the first Chern class $c_1(L)$, as an element of the vector space $H^{1,1}(X)$ of degree 2 de Rham real cohomology classes represented by closed $(1,1)$-forms, has a Kähler representative. The open convex cone in $H^{1,1}(X)$ consisting of classes with a Kähler representative is called the Kähler cone and will be denoted by $\mathcal{K}(X)$.

Thus, we get the following statement: Let X be a complex projective manifold. Let $\mathrm{NS}(X)\subset H^{1,1}(X)$ be the subset of $H^{1,1}(X)$ consisting of classes with integral periods. A class $\omega\in\mathrm{NS}(X)$ lies in $\mathcal{K}(X)$ iff $\int_Z\omega^{\dim Z}>0$ for every positive-dimensional closed analytic subset Z of X.

We will denote by $\mathcal{P}(X)$ the set of classes ω cut out by the conditions that $\int_Z\omega^{\dim Z}>0$ for every positive-dimensional closed analytic subset Z of X.

It was widely believed that a similar result holds for general real $(1,1)$ classes, namely that

$\mathcal{K}(X) = \mathcal{P}(X)$.

The article under review confirms this conjecture in the more general case of compact Kähler manifolds. Here the statement should be modified to the effect that $\mathcal{K}(X)$ is a connected component of $\mathcal{P}(X)$.

The proof consists in a reduction to the nef case and a subtle application of Yau's fundamental work on the solution of the inhomogeneous complex Monge-Ampère equation in which the volume form acquires a singularity.

Reviewed by *Philippe P. Eyssidieux*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2111016 (2006d:32027) 32J25 (32C30)
Eckl, Thomas (D-BAYR-IM)
Numerically trivial foliations. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **54** (2004), *no. 4,* 887–938.

In the present paper, the author continues his investigation of numerically trivial fibrations on projective complex manifolds from [T. Eckl, J. Algebraic Geom. **13** (2004), no. 4, 617–639; MR2072764 (2005f:14014)] by studying the localized notion of foliations with numerically trivial leaves.

Tsuji's criterion for numerical triviality of a pair (L, h) involving the curvature current of the singular metric h on the pseudo-effective line bundle L is taken as the definition for the numerical triviality of an arbitrary closed positive $(1, 1)$-current. The main result is the existence of a maximal foliation with numerically trivial leaves with respect to such a current. Y. T. Siu's decomposition theorem [Invent. Math. **27** (1974), 53–156; MR0352516 (50 #5003)] plays an important role in the proof. On a projective complex manifold, Tsuji's numerically trivial fibration with respect to a singular metric having the given current as curvature current turns out to be maximal among those fibrations whose fibers are contained in the leaves. Using an appropriate metric, the Iitaka fibration can also be characterized in this way (i.e., it coincides with Tsuji's fibration).

Furthermore, based on ideas of Demailly and Boucksom regarding moving intersection numbers, the author describes the nef fibration of a nef line bundle as the maximal fibration contained in the appropriately defined numerically trivial maximal foliation with respect to the first Chern class (called the nef foliation). The Iitaka fibration in turn is shown to contain the nef foliation. As a consequence, when the nef foliation is not a fibration, the Iitaka dimension is strictly less than the numerical dimension. It is not known whether the converse to this statement holds true.

In fact, to define the numerically trivial foliation in the preceding paragraph, only a pseudo-effective class α is needed; under the assumption that the singularities of the numerically trivial foliation with respect to α are isolated points, it is shown that the codimension of the leaves is an upper bound for the numerical dimension of α. It is unknown to what extent the assumption of isolated singularities can be removed.

Finally, some explicit surface examples are discussed, and an appendix gives the basics of singular foliations.

Reviewed by *Gordon Heier*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2099189 (2005f:14103) 14N05 (32Q45)

Duval, Julien (F-TOUL3-LM)

Une sextique hyperbolique dans $\mathrm{P}^3(\mathbf{C})$. (French. English, French summaries) [A hyperbolic sextic in $\mathrm{P}^3(\mathbf{C})$]

Math. Ann. **330** (2004), *no. 3,* 473–476.

From the text (translated from the French): "A subset of $\mathrm{P}^3(\mathbf{C})$ is said to be hyperbolic if it does not contain an entire curve, i.e. a non-constant holomorphic image of $\mathbf{C}$. The Kobayashi conjecture in projective space stipulates that a generic surface of degree ≥ 5 in $\mathrm{P}^3(\mathbf{C})$ is hyperbolic. It was proved for degrees ≥ 36 by M. McQuillan [Geom. Funct. Anal. **9** (1999), no. 2, 370–392; MR1692470 (2000f:32035)] and then for degrees ≥ 21 by J.-P. Demailly and J. El Goul [Amer. J. Math. **122** (2000), no. 3, 515–546; MR1759887 (2001f:32045)]. In a parallel and more modest effort, a number of authors (see the references in [M. Zaidenberg, "Hyperbolic surfaces in $\mathbf{P}^3$: examples", preprint, arxiv.org/abs/math/0311394]) have sought to construct examples of hyperbolic surfaces of the lowest possible degree. Until now, the best bound was degree 8. Our goal in this note is to show the existence of hyperbolic surfaces of degree 6."

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MR2099122 (2006c:32041) 32S65 (32Q25 37D20 37F10)

Cantat, Serge (F-RENNB-IM)

Difféomorphismes holomorphes Anosov. (French. English summary) [Anosov holomorphic diffeomorphisms]

Comment. Math. Helv. **79** (2004), *no. 4,* 779–797.

The theme of this article is the classification of compact complex manifolds X which admit a holomorphic Anosov diffeomorphism $f\colon X \to X$. When X is a complex surface, it has been proved by É. Ghys (see Theorem A in [Invent. Math. **119** (1995), no. 3, 585–614; MR1317651(95k:58116)]) that X is a complex torus and f is a linear automorphism.

The situation is not as simple in higher dimensions, as shown by Example 1.5 of the article under review. The transverse properties of the stable/unstable foliations $\mathcal{F}^{s/u}$ play an important role here. In the case where the stable (or unstable) leaves have dimension 1, Ghys has proved that these foliations are holomorphic foliations. Moreover, if f has a dense orbit, then (f, X) is topologically conjugate to a linear automorphism of a complex torus (see Theorem B in [op. cit.]).

The purpose of this article is to establish similar results when the stable/unstable leaves have complex dimension ≥ 2. It is not clear in this case that $\mathcal{F}^{s/u}$ are holomorphic foliations (the transverse structure is a priori merely continuous). The main result of the paper (Theorem 1.4.a) asserts that if $\mathcal{F}^{s/u}$ are holomorphic foliations and X is projective, then (f, X) is—up to an étale cover—a linear automorphism of a complex torus.

The proofs use a lot of complex analytic and algebraic geometry, hence they are quite different in nature from those given by Ghys. They rely notably on a recent alternative of S. Boucksom, J.-P. Demailly, M. Paun and T. Peternell which says that either X is uniruled or K_X is pseudoeffective.

Reviewed by *Vincent Guedj*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2095471 (2005k:14001a) 14-02 (14C20)
Lazarsfeld, Robert (1-MI)

★**Positivity in algebraic geometry. I.**
Classical setting: line bundles and linear series.
Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 48.
Springer-Verlag, *Berlin*, 2004. *xviii*+387 *pp*. $129.00. *ISBN* 3-540-22533-1

MR2095472 (2005k:14001b) 14-02 (14C20 14F05 14F17)
Lazarsfeld, Robert (1-MI)

★**Positivity in algebraic geometry. II.**
Positivity for vector bundles, and multiplier ideals.
Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 49.
Springer-Verlag, *Berlin*, 2004. *xviii*+385 *pp*. $129.00. *ISBN* 3-540-22534-X

Positivity has long been a major theme in various branches of algebraic geometry, both as an object of study and as a technical tool. A first book attempting to treat positivity, under the heading of ampleness, appeared already three and a half decades ago. That is R. Hartshorne's book [*Ample subvarieties of algebraic varieties*, Springer, Berlin, 1970; MR0282977 (44 #211)], where the main goal was to extend the notion of ample divisor to arbitrary subvarieties of a given variety. Since then, the subject has seen major developments in a large number of different directions: intersection theory, singularities, topology of algebraic varieties, vanishing theorems and their applications to linear series, and higher dimensional geometry, to give a certainly incomplete list. However, these developments have mostly remained scattered in the literature and some, maybe precisely for this reason, have not been worked out in a systematic fashion. In the book under review the author succeeds wonderfully in putting together under the same heading most of the areas of classical and modern complex algebraic geometry dedicated to, or influenced by, the study of positivity.

The book is divided into three parts, each with a separate introduction. In addition, each chapter contains introductory remarks and concluding notes which emphasize the history of the topic, sources of inspiration, and further references. I will present in what follows the rough contents of each part, chapter by chapter.

The first part is equivalent to the content of Volume I, and is devoted to a fundamental topic, namely that of line bundles and linear series. The main notion of positivity here is that of ampleness. Its importance has become widely recognized after the foundational papers of J.-P. Serre [Ann. of Math. (2) **61** (1955), 197–278; MR0068874 (16,953c)] on the algebraic viewpoint, and K. Kodaira [Proc. Nat. Acad. Sci. U. S. A. **39** (1953), 1268–1273; MR0066693 (16,618b)] on the analytic one. The author presents in Chapter 1 the basic theory of ampleness, with an accent on modern concepts like $\mathbf{Q}$- and $\mathbf{R}$-divisors, nefness, and cones of divisors in the Néron-Severi space of numerical equivalence classes. He continues in Chapter 2 by presenting the theory of linear series which may not be ample. The following is in my view the most remarkable and novel feature of this part of the book: a systematic study of the asymptotic theory of non-ample linear series, with a special emphasis on the role of big divisors (the birational analogues of ample divisors). Chapter 3 takes up a more geometric and topological approach to positivity. Its main focus is on the Lefschetz and Bertini theorems, together with subsequent generalizations by Barth, Fulton and Hansen and others, and we get a glimpse of the interesting geometry associated with subvarieties of small codimension in projective spaces. Chapter 4 contains a treatment of vanishing theorems, including the classical Kodaira and Nakano theorems for ample line bundles, and the very useful generalization by Kawamata and Viehweg to the case of big and nef divisors (but not the $\mathbf{Q}$-divisors case, which is treated separately later, in §9.1). With respect to this part, the author mentions that the exposition is somewhat more elementary than the standard presentations. The main generic vanishing theorem of M. L. Green and Lazarsfeld [Invent. Math. **90** (1987), no. 2, 389–407; MR0910207 (89b:32025)] is also presented. Finally, Chapter 5 deals with the topic of local positivity. This is a more recent development, based on Demailly's notion of Seshadri constant, which measures how much of the positivity of a given line bundle is concentrated at a given point of the variety. The most important result presented here is a lower bound for the Seshadri constant given by L. M. H. Ein, O. Küchle and Lazarsfeld [J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)]. It should be said, however, that the picture is not definitive, and a substantially better bound is proposed, partly in view of Fujita's famous conjecture on the freeness of adjoint bundles.

A note about a couple of things I particularly enjoyed reading in the first part: first, it is nice to see the notion of Castelnuovo-Mumford regularity take a quite prominent role in a text on positivity. The idea roughly speaking is to quantify directly how much one has to twist a sheaf by a positive line bundle in order to achieve vanishing. This makes later arguments particularly concise, for example in the context of multiplier ideals. Second, the machinery of Cutkosky of producing examples of line bundles on higher dimensional varieties with interesting base-locus or volume behavior (e.g. disproving the existence of Zariski decomposition on threefolds or higher) is systematized and made widely available.

The second part of the book (first part of Volume II) focuses on positivity for vector bundles of higher rank. This is an area of study started in the 1960's by people like Grauert, Griffiths and Hartshorne, with the aim of generalizing to higher rank as much of the geometry of ample divisors as possible. Important and beautiful applications of these generalizations have emerged during the subsequent decades, and they naturally belong to the general context of positivity. Chapter 6 is devoted to introducing the notions of ampleness and nefness for vector bundles, mainly through many examples associated to interesting geometric contexts. The adopted definition is that of Hartshorne [Inst. Hautes Études Sci. Publ. Math. No. 29 (1966), 63–94; MR0193092 (33 #1313)], namely a vector bundle E on a projective variety X is ample (or nef) if the Serre line bundle $\mathcal{O}_{\mathbf{P}E}(1)$ is so on the projectivized bundle $\mathbf{P}E \to X$. Some of the most basic examples to be mentioned are the following: the normal bundles of smooth subvarieties in projective space and in simple abelian varieties are ample, smooth projective varieties with ample cotangent bundle are Kobayashi hyperbolic, the duals of Picard bundles parametrizing sufficiently positive linear series of fixed numerical class are ample, push-forwards of relative canonical bundles are nef (under mild hypotheses). Chapter 7 focuses on the geometric properties of ample vector bundles, particularly on the topology associated with zero loci of their sections, or more generally to degeneracy loci. After presenting the Lefschetz type theorems of A. J. Sommese [Math. Ann. **233** (1978), no. 3, 229–256; MR0466647 (57 #6524)] and S. Bloch and D. Gieseker [Invent. Math. **12** (1971), 112–117; MR0297773 (45 #6825)], the author discusses his joint theorem with W. Fulton on the connectedness of degeneracy loci [Acta Math. **146** (1981), no. 3-4, 271–283; MR0611386 (82k:14016)], together with its beautiful applications to Brill-Noether theory and to other contexts. Vanishing theorems for vector bundles, for example the celebrated result of J. Le Potier [Math. Ann. **218** (1975), no. 1, 35–53; MR0385179 (52 #6044)], are also presented. Chapter 8 deals with numerical consequences of ampleness for vector bundles, and it is here that the idea of positivity comes to the forefront. The central result is the theorem of Fulton and Lazarsfeld [Ann. of Math. (2) **118** (1983), no. 1, 35–60; MR0707160 (85e:14021)], stating that the cone of numerically positive polynomials in the Chern classes of ample vector bundles is spanned by Schur polynomials.

An interesting original thing in this part of the book is the formalism developed by the author regarding twisting vector bundles by $\mathbf{Q}$-divisors. This allows one to keep track formally of the finer positivity properties of vector bundles, particularly when studying nefness.

The third part (second part of Volume II) makes the transition from classical to the most modern developments in the field. It takes up ideas and techniques from higher dimensional geometry, under the form of multiplier ideals. This is the star attraction of the book, since the theory of multiplier ideals has only recently taken a well-defined shape, due in part to efforts of people like Siu, Demailly, Ein and the author. To quote Lazarsfeld: “it seems safe to predict that multiplier ideals and their variants are destined to become fundamental tools in algebraic geometry”. Multiplier ideals appeared first in the complex analytic setting, where they are defined naturally in terms of integrability conditions. It was noted later that they can be defined in purely algebro-geometric terms, using log-resolutions. In fact this definition had already been worked-out in passing by H. Esnault and E. Viehweg in [*Lectures on vanishing theorems*, Birkhäuser, Basel, 1992; MR1193913 (94a:14017)]. For those less familiar with the topic, let me comment that multiplier ideals arise in this context from the wish to avoid passing to a birational modification in order to satisfy the normal crossing hypothesis required for the Kawamata-Viehweg vanishing theorem for **Q**-divisors (which by the Kawamata-Reid-Shokurov technique had become an essential tool in the study of linear series). Another, in fact very much related, approach to this circle of ideas is the notion of singularities of pairs. The author explains the connection between the two, but does not go deeper into the context of singularities of pairs. For this there are already excellent references, such as J. Kollár’s survey [in *Algebraic geometry—Santa Cruz 1995*, 221–287, Proc. Sympos. Pure Math., 62, Part 1, Amer. Math. Soc., Providence, RI, 1997; MR1492525 (99m:14033)] and Kollár and S. Mori’s book [*Birational geometry of algebraic varieties*, Translated from the 1998 Japanese original, Cambridge Univ. Press, Cambridge, 1998; MR1658959 (2000b:14018)].

The author begins Chapter 9 with a proof of the Kawamata-Viehweg vanishing theorem for **Q**-divisors, and continues with the basic definitions, properties and examples of multiplier ideals. Very interesting results presented at the end of the chapter are the Skoda and Briançon-Skoda theorems, which are local statements concerning multiplier ideals (and the integral closure) of powers of ideals, together with global version proved by Ein and the author [Invent. Math. **137** (1999), no. 2, 427–448; MR1705839 (2000j:14028)]. Chapter 10 is devoted to various applications of multiplier ideals to the general theory of divisors and linear series. Among the most interesting ones, where multiplier ideals seem to have provided a real breakthrough, are the Ein-Lazarsfeld theorems on the singularities of theta divisors [J. Amer. Math. Soc. **10** (1997), no. 1, 243–258; MR1396893 (97d:14063)], the approach of Y. T. Siu [Houston J. Math. **28** (2002), no. 2, 389–409; MR1898197 (2003i:32038)] and J.-P. Demailly [Invent. Math. **124** (1996), no. 1-3, 243–261; MR1369417 (97a:32035)] to Matsusaka’s big theorem, and the Angehrn-Siu Fujita-type theorem on the global generation of adjoint bundles [U. Angehrn and Y. T. Siu, Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036)]. Finally, Chapter 11 introduces probably the most modern concept in the book, asymptotic multiplier ideals, stemming from work of Siu on the deformation invariance of plurigenera. The author develops (for the first time) a coherent theory of asymptotic multiplier ideals, following the basic properties presented earlier in the ordinary case. This is then applied in a few directions. First, an application to uniform results in commutative algebra is described, following work of Ein, K. E. Smith and the author [Invent. Math. **144** (2001), no. 2, 241–252; MR1826369 (2002b:13001)]. We are then guided through a nice presentation of the context of Fujita’s approximation theorem and its applications to the study of volumes of line bundles. Here the author includes a very nice recent result of S. Boucksom, Demailly, M. Paun and T. Peternell [“The pseudo-effective cone of a compact Kähler manifold and varieties of negative

Kodaira dimension", preprint, arxiv.org/abs/math/0405285], which describes the pseudoeffective cone of an irreducible variety as that dual to the cone of mobile curves (roughly speaking curves which move in families covering an open dense subset of the variety). The book concludes with what is widely regarded as the most spectacular application of multiplier ideals to date—the proof of the invariance of plurigenera for varieties of general type—due to Siu [Invent. Math. **134** (1998), no. 3, 661–673; MR1660941 (99i:32035)]. Although the original proof was analytic, the author presents here a relatively quick proof based on the algebro-geometric ideas developed in the text. (It should be said, however, that in the meantime, Siu [in *Complex geometry (Göttingen, 2000)*, 223–277, Springer, Berlin, 2002; MR1922108 (2003j:32027a)] has proved the deformation invariance of plurigenera for arbitrary varieties still based on the analytic theory of multiplier ideals, and in this case, no purely algebraic proof is currently known.)

Very much in this last part of the book is original, but as a single example expressing simply a preference of the reviewer, it is again remarkable, how Castelnuovo-Mumford regularity is used, together with vanishing theorems, in order to give purely conceptual proofs to key results on non-vanishing or global generation for twists of multiplier ideal sheaves.

Switching to overall comments, one interesting feature of the text is the fact that at times it provides new simplified proofs of, or new approaches to, well-established results. One such example is the proof of the Campana-Peternell theorem (Theorem 2.3.18), which is an analogue of the Nakai-Moishezon ampleness criterion in the setting of $\mathbf{R}$-divisors. Another is a generalization of the Nadel product lemma (Theorem 8.4.10), using the positivity of cone classes. More importantly, the book contains genuinely new results. A favorite of the reviewer is Theorem 2.2.44 on the continuity of the volume function. This has inspired a great deal of subsequent work on asymptotic invariants of line bundles.

There are numerous examples scattered throughout the text (together with various applications, they form about a third of the book). Some are very explicit, making essentially every concept introduced in the book quite easy to grasp. Others serve as a guide to further literature and encourage independent study. One recognizes here one of the features that has also made Fulton's *Intersection theory* [Second edition, Springer, Berlin, 1998; MR1644323 (99d:14003)] so successful.

For understanding the material presented in this text, the reader is assumed to have some familiarity only with standard introductory texts like Hartshorne's *Algebraic geometry* [Springer, New York, 1977; MR0463157 (57 #3116)] and P. Griffiths and J. Harris' *Principles of algebraic geometry* [Wiley-Intersci., New York, 1978; MR0507725 (80b:14001)], with only very occasional need to go beyond these sources. The author emphasizes that there is relatively little in the book about the Hodge-theoretic and complex analytic side of the story. For this he suggests the texts of C. Voisin [*Hodge theory and complex algebraic geometry. I*, Translated from the French original by Leila Schneps, Cambridge Univ. Press, Cambridge, 2002; MR1967689 (2004d:32020); *Hodge theory and complex algebraic geometry. II*, Translated from the French by Leila Schneps, Cambridge Univ. Press, Cambridge, 2003; MR1997577 (2005c:32024b)] and Demailly's *Complex analytic and algebraic geometry*, fragments of which can already be found in various places in the literature.

The book under review is exceptionally well written. It treats a large number of concepts and topics, but always in a gentle and explicit manner. It can be used both as a textbook and as a source for current research problems. As such, it will be of great value to both students and experts in the field. It is also excellent as a guide to further literature. In my opinion, Lazarsfeld's book will become one of the fundamental references in the field of complex algebraic geometry.

Reviewed by *Mihnea Popa*

MR2088931 (2005g:32024) 32L10 (32A25 32L20)
Berman, Robert (S-CHAL)
Bergman kernels and local holomorphic Morse inequalities. (English summary)
Math. Z. **248** (2004), *no. 2,* 325–344.

The author proves a local version of J.-P. Demailly's holomorphic Morse inequalities [Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)] by a clever comparison of the Bergman kernel of a complex Hermitian manifold together with a Hermitian holomorphic line bundle and the Bergman kernel of the model $\mathbb{C}^n$ with flat metric and trivial line bundle with constant metric. After scaling, the comparison relies on elliptic theory.

Reviewed by *Christophe Mourougane*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2087040 (2005f:32041) 32Q45 (32L05)
El Goul, Jawher (F-TOUL3-LM)
Demailly's 2-jet negativity of certain hyperbolic fibrations. (English summary)
Complex analysis in several variables—Memorial Conference of Kiyoshi Oka's Centennial Birthday, 85–93, *Adv. Stud. Pure Math.*, 42, *Math. Soc. Japan*, *Tokyo*, 2004.

The paper under review concerns a conjecture of Demailly on k-jet negativity. J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)] conjectured that the existence of a metric with k-negativity on a k-jet bundle should characterize Kobayashi hyperbolicity for compact complex manifolds. In this paper the author deals with the special case of this conjecture. He considers the case of 2-jet bundles of a hyperbolic (singular) fibration on hyperbolic curves with certain conditions on the singularities of special fibers and proves a weak negativity property on this bundle. It is noticed that his method only works up to the 2-jet stage.
{For the entire collection see MR2087033 (2005c:32002)}

Reviewed by *Yoshihiro Aihara*

MR2087033 (2005c:32002) 32-06 (00B30)
★**Complex analysis in several variables—Memorial Conference of Kiyoshi Oka's Centennial Birthday.**
Papers from the conference held in Kyoto, October 30–November 5, 2001 and Nara, November 6–8, 2001.
Edited by Kimio Miyajima, Mikio Furushima, Hideaki Kazama, Akio Kodama, Junjiro Noguchi, Takeo Ohsawa, Hajime Tsuji and Tetsuo Ueda.
Advanced Studies in Pure Mathematics, 42.
Mathematical Society of Japan, *Tokyo*, 2004. *x*+345 *pp*. *ISBN* 4-931469-27-2

Contents: Oka, Kiyoshi (3–5) MR2087034; Toshio Nishino, Mathematics of Professor Oka—a landscape in his mind (17–30) MR2087035; Yoshihiro Aihara, Uniqueness problem for meromorphic mappings under conditions on the preimages of divisors (31–35); MR2087036 (2005f:32029); Takao Akahori, On the middle dimension cohomology of A_l singularity (37–44); MR2087037 (2005g:32042); Takashi Aoki, Takahiro Kawai and Yoshitsugu Takei, The exact steepest descent method—a new steepest descent method based on the exact WKB analysis (45–61); MR2087038 (2005h:34237); Eric Bedford, Excursions of a complex analyst into the realm of dynamical systems (63–84); MR2087039 (2005e:37095); Jawher El Goul, Demailly's 2-jet negativity of certain hyperbolic fibrations (85–93); MR2087040 (2005f:32041); John Erik Fornæss, Short $\mathbb{C}^k$

(95–108); MR2087041 (2005h:32027); Hirotaka Fujimoto, Some constructions of hyperbolic hypersurfaces in $P^n(\mathbf{C})$ (109–114); MR2087042 (2005h:32059); Kengo Hirachi, A link between the asymptotic expansions of the Bergman kernel and the Szegö kernel (115–121); MR2087043 (2005e:32004); Andrei Iordan, On the non-existence of smooth Levi-flat hypersurfaces in $\mathbb{CP}_n$ (123–126); MR2087044 (2005f:32061); Su-Jen Kan, Recent development on Grauert domains (127–133); MR2087045 (2005m:32021).

Kang-Tae Kim, Analytic polyhedra with non-compact automorphism group (135–140); MR2087046 (2005e:32039); Shoshichi Kobayashi, Problems related to hyperbolicity of almost complex structures (141–146); MR2087047 (2005g:32035); Joseph J. Kohn, Ideals of multipliers (147–157); MR2087048 (2005e:32056); Gen Komatsu, The Bergman kernel of Hartogs domains and transformation laws for Sobolev-Bergman kernels (159–164); MR2087049 (2005f:32005); Masatake Kuranishi, An approach to the Cartan geometry. II. CR manifolds (165–187); MR2087050 (2006e:32046); László Lempert, The $\overline{\partial}$ equation in N variables, as N varies (189–202); MR2087051 (2005g:32053); Kazuko Matsumoto, Levi form of logarithmic distance to complex submanifolds and its application to developability (203–207); MR2087052 (2005h:32016); Yoichi Miyaoka, Numerical characterisations of hyperquadrics (209–235); MR2087053 (2005h:14123); Seiki Mori, Meromorphic mappings and deficiencies (237–242); MR2087054 (2005h:32040); Junjiro Noguchi, Intersection multiplicities of holomorphic and algebraic curves with divisors (243–248); MR2087055 (2005f:32030); Takeo Ohsawa, Generalization of a precise L^2 division theorem (249–261); MR2087056 (2005h:32012).

Mikael Passare, Amoebas, convexity and the volume of integer polytopes (263–268); MR2087057 (2005e:32011); R. Michael Range, On the decomposition of holomorphic functions by integrals and the local CR extension theorem (269–273); MR2087058 (2005e:32055); Oswald Riemenschneider, The monodromy covering of the versal deformation of cyclic quotient surface singularities (275–282); MR2087059 (2006b:32041); Georg Schumacher, Moduli as algebraic spaces (283–288); MR2087060 (2005g:14027); Satoru Shimizu [Satoru Shimizu[1]], Prolongation of holomorphic vector fields on a tube domain and its applications (289–299); MR2087061 (2005h:32051); Manabu Shirosaki, Hypersurfaces and uniqueness of holomorphic mappings (301–305); MR2087062 (2005f:32031); Shigeharu Takayama, Seshadri constants and a criterion for bigness of pseudo-effective line bundles (307–312); MR2087063 (2005h:14018); Hajime Tsuji, Subadjunction theorem (313–318); MR2087064 (2005j:32010); Tetsuo Ueda, Fixed points of polynomial automorphisms of C^n (319–324); MR2087065 (2005e:32033); Katsutoshi Yamanoi, On Nevanlinna theory for holomorphic curves in abelian varieties (325–331); MR2087066 (2005d:32026); Stephen S.-T. Yau, Numerical characterization for affine varieties be a cone over nonsingular projective varieties (333–338); MR2087067 (2005f:32047); Ken-Ichi Yoshikawa, Nikulin's $K3$ surfaces, adiabatic limit of equivariant analytic torsion, and the Borcherds Φ-function (339–345); MR2087068 (2005h:14093).

{Most of the papers are being reviewed individually.}

MR2086081 (2005i:14009) 14C20 (14K05)
Tutaj-Gasińska, Halszka (PL-JAGL)
Seshadri constants in half-periods of an abelian surface. (English summary)
J. Pure Appl. Algebra **194** (2004), *no. 1-2,* 183–191.

In recent years there has been a major interest in studying the local positivity of ample line bundles on algebraic varieties. Seshadri constants, introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)], partially in connection with the study of T. Fujita's conjecture [in *Algebraic geometry, Sendai, 1985*, 167–178, Adv. Stud. Pure Math., 10, North-Holland, Amsterdam, 1987; MR0946238 (89d:14006)], incorporate in a natural way information about the local positivity of a line bundle.

Let X be a smooth projective variety of dimension n, $x_1,\dots,x_r \in X$ distinct points and L an ample line bundle on X. Let π be the blow-up of X in the considered points with $E_1,\dots,E_r$ the exceptional divisors. Then the real number

$$\varepsilon(L,x_1,\dots,x_r)=\sup\{\varepsilon\in\mathbf{R}|\ \pi^*L-\varepsilon\sum_{i=1}^r E_i \text{ is nef}\}$$

is the Seshadri constant of L at $x_1,\dots,x_r$ (called the multiple point Seshadri constant if $r\geq 2$). Equivalently

$$\varepsilon(L,x_1,\dots,x_r):=\inf_{C\ni x_1,\dots,x_r}\frac{L\cdot C}{\sum_{i=1}^r \operatorname{mult}_{x_i}C},$$

(where the infimum is taken over all irreducible curves passing through $x_1,\dots,x_r$).

Despite their apparently "easy" definition, Seshadri constants are very difficult to compute. In fact their exact value is known only for a few cases (even for $r=1$) and even on surfaces it is difficult to control them, as was already pointed out by Demailly. To put things into perspective, the computation of multiple point Seshadri constants for $\mathbf{P}^2$ is equivalent to the unsolved Nagata conjecture [M. Nagata, Chinese J. Math. **11** (1983), no. 1, 1–4; MR0692988 (84f:14008)]. Therefore, any contribution towards bounding or calculating them is of interest.

One has the bounds $0<\varepsilon(L,x_1,\dots,x_r)\leq\sqrt[n]{\frac{L^n}{r}}$ from [L. M. H. Ein, O. Küchle and R. K. Lazarsfeld, J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)]. It is still not known whether a Seshadri constant can be non-rational. However, some useful information is that if $\varepsilon(L,x_1,\dots,x_r)<\sqrt[n]{\frac{L^n}{r}}$, then $\varepsilon(L,x_1,\dots,x_r)$ is rational, by A. Steffens [Math. Z. **227** (1998), no. 3, 505–510; MR1612681 (99c:14009)].

Seshadri constants on abelian surfaces have been studied in [T. Bauer, Math. Ann. **312** (1998), no. 4, 607–623; MR1660259 (2000a:14054)] and in [T. Bauer, Math. Ann. **313** (1999), no. 3, 547–583; MR1678549 (2000d:14006)]. In the appendix of the latter paper it is proved that a one-point Seshadri constant on an abelian surface is always rational and an upper bound is given, which is shown to be attained in the case of Picard number one in the first paper.

In the very nice paper under review the author obtains the following result for multiple point Seshadri constants of an ample line bundle L of type $(1, d)$ at any r of the 16 half-periods $e_1, \dots, e_{16}$ of an abelian surface $S \simeq \mathbf{C}^2/\Lambda$, i.e., $e_1, \dots, e_{16}$ are the elements of $\frac{1}{2}\Lambda$:

(a) If $\sqrt{\frac{2d}{r}} \in \mathbf{Q}$, then $\varepsilon(L, e_1, \dots, e_r) = \sqrt{\frac{2d}{r}}$.

(b) If $\sqrt{\frac{2d}{r}} \notin \mathbf{Q}$, then $\varepsilon(L, e_1, \dots, e_r) \le 2dk_0/l_0$, where (k_0, l_0) is the primitive solution of Pell's equation $2rdk^2 + 1 = l^2$.

In particular $\varepsilon(L, e_1, \dots, e_r)$ is rational.

The method of proof consists of using results from the last two papers mentioned and from [H. Lange and C. Birkenhake, *Complex abelian varieties*, Springer, Berlin, 1992; MR1217487 (94j:14001)] to explicitly construct curves in $|mL|$, for suitable $m > 0$, passing through $e_1, \dots, e_r$ with the desired multiplicities.

Reviewed by *Andreas Leopold Knutsen*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2078708 (2005c:14018) 14E30
Solá Conde, Luis Eduardo; **Wiśniewski, Jarosław A.** (PL-WASW-IM)
On manifolds whose tangent bundle is big and 1-ample.
Proc. London Math. Soc. (3) **89** (2004), *no. 2,* 273–290.

Much of the recent work in Fano geometry and in the classification of complex manifolds is related to the problem of characterizing complex-projective manifolds X whose tangent bundles T_X have certain positivity properties.

After S. Mori's spectacular solution to the Hartshorne conjecture, asserting that the projective space $\mathbb{P}_n$ is the only manifold whose tangent bundle is ample, the next important case is that of manifolds whose tangent bundle is nef—recall that T_X is called ample or nef if the line bundle $\mathcal{O}_{\mathbb{P}(T_X)}(1)$ on the projectivization $\mathbb{P}(T_X)$ is ample or nef. It has been conjectured by Campana and Peternell that T_X nef implies that X is homogeneous.

While the Campana-Peternell conjecture is still open, the paper under review studies a more special situation where the tangent bundle T_X is assumed to be almost, but not quite, ample. More precisely, the authors classify manifolds whose tangent bundle is big and 1-ample. The assumptions imply that global sections in the bundle $\mathcal{O}_{\mathbb{P}(T_X)}(m)$, for $m \gg 0$, give rise to a morphism $\mathbb{P}(T_X) \to Y$ whose fibers are at most 1-dimensional. A part of the argumentation is then based on the observation that the complement of the zero section of the cotangent bundle $T_X^\vee$ is a complex-symplectic manifold and makes use of earlier works where morphisms from symplectic manifolds were studied.

The paper contains an appendix where the authors fill two gaps in earlier papers of J. Wierzba [J. Algebraic Geom. **12** (2003), no. 3, 507–534; MR1966025 (2003m:14023)] and J.-P. Demailly, T. Peternell and M. H. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)].

Reviewed by *Stefan Kebekus*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2069123 (2005b:32058) 32Q28 (32A07 32E10 32L05)
Pflug, Peter (D-OLD-M); **Zwonek, Wlodzimierz** (PL-JAGL)
The Serre problem with Reinhardt fibers. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **54** (2004), *no. 1,* 129–146.

The Serre problem asks whether a holomorphic fiber bundle with a Stein fiber F and a Stein basis is itself Stein. Although many positive answers were given in special cases, the first counterexamples were given by H. Skoda [Invent. Math. **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] and a few afterwards by Demailly. In 1985 G. Coeuré and the reviewer [Ann. of Math. (2) **122** (1985), no. 2, 329–334; MR0808221 (87c:32033)] gave a counterexample with F a bounded Reinhardt domain in $\mathbb{C}^2$. In this paper, the authors characterize the hyperbolic Reinhardt domains F in $\mathbb{C}^2$ which can be produced as fibers for a counterexample to the Serre problem. Such an F a priori can be of three types: Type 0: F is complete; Type 1: the intersection of F with exactly one axis is nonempty; Type 2: the intersection with the two axes is empty.

The main result here is that for Types 0 and 1, F cannot be a counterexample, but there are counterexamples for Type 2, and in this paper the counterexamples are described using a matrix of $\mathrm{GL}(2,\mathbb{Z})$, generalizing the description given by Coeuré and the reviewer.

The Type 0 case was proved by K. Königsberger [Math. Ann. **189** (1970), 178–184; MR0268410 (42 #3308)] and can also be seen as a special case of general results about the Serre problem.

The Type 1 case uses explicit results of Shimizu and Kruzhilin on the automorphism group of Reinhardt domains. A theorem of Stehle and an ad hoc extending lemma are also used.

The Type 2 case also uses results of Shimizu about the algebraicity of the automorphism group. In order to construct counterexamples, the method of the paper of Coeuré and the reviewer is applied.

It should also be noted that recently K. Oeljeklaus and D. Zaffran have obtained new results in this direction.

Reviewed by *Jean-Jacques Loeb*

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MR2068946 (2005e:32005) 32A25 (32W05 53C56 53C60)
Qiu, Chunhui (PRC-XIAM); **Zhong, Tongde** (PRC-XIAM)
Integral formulas for differential forms of type (p, q) on complex Finsler manifolds. (English summary)
Sci. China Ser. A **47** (2004), *no. 2,* 284–296.

In this paper, using the invariant integral kernel introduced by J.-P. Demailly and C. Laurent-Thiébaut [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)], the authors obtain Koppelman and Koppelman-Leray formulas for relatively compact domains with C^1 boundary in strongly pseudoconvex complex Finsler manifolds, and then apply them to solve the $\overline{\partial}$-equation in such domains.

Reviewed by *Marco Abate*

MR2066940 (2005g:32020) 32H50 (32J27 32Q15 37B40 37C85 37F10)
Dinh, Tien-Cuong (F-PARIS11); **Sibony, Nessim** (F-PARIS11)

Groupes commutatifs d'automorphismes d'une variété kählérienne compacte. (French. English, French summaries) [Commutative groups of automorphisms of a compact Kähler manifold]

Duke Math. J. **123** (2004), *no. 2,* 311–328.

It is quite difficult to construct automorphisms of a compact, complex manifold V and it is even harder to find automorphisms that commute. One would therefore expect abelian subgroups of $\mathrm{Aut}(V)$ to be quite special. The main result in the paper under review is a kind of structure theorem for abelian subgroups of $\mathrm{Aut}(V)$ in the case in which V is a compact Kähler manifold.

Recall that the topological entropy of an endomorphism f of V is a number which measures the complexity of f. The dynamically most interesting endomorphisms are those with positive entropy.

Main Theorem: let $\mathcal{G}'$ be an abelian subgroup of $\mathrm{Aut}(V)$ and let U be the set of elements of $\mathcal{G}'$ of zero entropy. Then U is a group and $\mathcal{G}'$ is isomorphic to the direct product $U \times \mathcal{G}$, where $\mathcal{G}$ is a subgroup of $\mathcal{G}'$ such that all elements of $\mathcal{G} \setminus \{\mathrm{id}\}$ have positive entropy. Moreover, $\mathcal{G}$ is a free abelian subgroup of index at most $\dim V - 1$. This estimate is sharp and, in the case of equality, U is finite.

The proof proceeds by studying the action of $\mathcal{G}'$ on the Dolbeault cohomology group $H^{1,1}(V, \mathbf{R})$. To this end, the authors exploit the structure of the Kähler cone $\mathcal{H} \subset H^{1,1}(V, \mathbf{R})$, in particular a version of the Hodge-Riemann theorem due to M. L. Gromov [in *Advances in differential geometry and topology*, 1–38, World Sci. Publishing, Teaneck, NJ, 1990; MR1095529 (92d:52018)] and a recent result by J.-P. Demailly and M. Paun [Ann. of Math. (2) **159** (2004), no. 3, 1247–1274; MR2113021]. They also use the characterization by Gromov [Astérisque No. 145-146 (1987), 5, 225–240; MR0880035 (89f:58082); Enseign. Math. (2) **49** (2003), no. 3-4, 217–235; MR2026895] and Y. Yomdin [Israel J. Math. **57** (1987), no. 3, 285–300; MR0889979 (90g:58008)] that $f \in \mathrm{Aut}(V)$ has positive entropy if and only if the spectral radius of the induced action of f on $H^{1,1}(V, \mathbf{R})$ is strictly larger than one.

By the Perron-Frobenius theorem, each $f \in \mathrm{Aut}(V)$ of positive entropy admits a class in $\overline{\mathcal{H}}$ which is an eigenvector for f^*. Roughly speaking, then, if there were too many (commuting, without relations) such f, we would find a large invariant subspace of $H^{1,1}$ generated by elements of $\overline{\mathcal{H}}$. Using the Hodge-Riemann theorem, the authors show that this is impossible.

Overall, the paper is well-written and contains a nice mix of ideas from dynamics and analytic geometry.

Reviewed by *Mattias Jonsson*

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MR2058513 (2005d:13016) 13B22 (14J17)

Takagi, Shunsuke [Takagi, Shunsuke[2]] (J-TOKYOGM);
Watanabe, Kei-ichi [Watanabe, Keiichi[1]] (J-NIHOH)

When does the subadditivity theorem for multiplier ideals hold? (English summary)

Trans. Amer. Math. Soc. **356** (2004), *no. 10,* 3951–3961 (*electronic*).

An important property of multiplier ideals is sub-additivity, which in its simplest incarnation predicts that the multiplier ideal of a product of two ideal sheaves $\mathfrak{a}$ and $\mathfrak{b}$ on a smooth complex variety is contained in the product of the multiplier ideal of $\mathfrak{a}$ and the multiplier ideal of $\mathfrak{b}$. (Subadditivity was proved by J.-P. Demailly, L. M. H. Ein and R. K. Lazarsfeld in [Michigan Math. J. **48** (2000), 137–156; MR1786484 (2002a:14016)].) The subadditivity theorem is at the heart of many of the applications of mutliplier ideals to commutative algebra and algebraic geometry. It is natural to hope that some form of subadditivity might hold in settings more general than smooth varieties.

The paper under review shows that the subadditivity theorem, as stated in the simple form above, holds when the ambient variety is a surface with log terminal singularities. It also proves that if the general form of subadditivity (in which coefficients are allowed) holds for some surface, then that surface must be smooth. That is, if X is a two dimensional normal $\mathbf{Q}$-Gorenstein surface, and $\mathcal{J}(\mathbf{a}^c\mathbf{b}^d) \subset \mathcal{J}(\mathbf{a}^c)\mathcal{J}(\mathbf{b}^d)$ for all ideals $\mathbf{a}$, $\mathbf{b}$ and all positive rational numbers c, d, then the surface X is in fact smooth. The paper also provides simple examples of higher dimensional varieties with very nice singularities (including the toric case) in which even the simplest form of subadditivity fails.

Reviewed by *Karen E. Smith*

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MR2051683 (2005d:32056) 32U25 (32H50 32U40 37F10)
Coman, Dan (1-SRCS); **Guedj, Vincent** (F-TOUL3-LM)
Invariant currents and dynamical Lelong numbers. (English summary)
J. Geom. Anal. **14** (2004), *no. 2,* 199–213.

Let $f=(P_1,\dots,P_k)\colon \mathbf{C}^k\to\mathbf{C}^k$ be a polynomial automorphism with $\lambda:=\max\deg P_j\geq 2$. The meromorphic extension, which we still denote by f, to $\mathbf{P}^k=\mathbf{C}^k\cup(t=0)$ is not well-defined on the indeterminacy locus I^+. In this article, the authors consider those f for which $X^+:=f((t=0)\setminus I^+)$ reduces to a single point lying outside of I^+. Such an f is algebraically stable and it follows that the sequence of currents $\lambda^{-n}(f^n)^*\omega$, where ω is the Fubini-Study form on $\mathbf{P}^k$, converges to a positive closed current T_+ of bidegree $(1,1)$ with $f^*T_+=\lambda T_+$ [cf. N. Sibony, in *Dynamique et géométrie complexes (Lyon, 1997)*, ix–x, xi–xii, 97–185, Soc. Math. France, Paris, 1999; MR1760844 (2001e:32026); V. Guedj and N. Sibony, Ark. Mat. **40** (2002), no. 2, 207–243; MR1948064 (2004b:32029)]. Define $\lambda_2(f):=\lim_{n\to\infty}[\delta_2(f^n)]^{1/n}$ where $\delta_2(f^n)$ is the degree of $f^{-n}(L)$ for L a generic linear subspace of codimension 2. Under the additional assumption that $\lambda>\lambda_2(f)$—this includes, e.g., complex Hénon maps in $\mathbf{C}^2$—a positive closed current σ_- of bidegree $(k-1,k-1)$ of unit mass with $(f^{-1})^*\sigma_-=\lambda\sigma_-$ can be constructed [V. Guedj and N. Sibony, op. cit. (Theorem 3.1)]. This construction is carried out again in Section 2.1 where it is shown that σ_- puts no mass on the hyperplane at infinity (Theorem 2.2). In the case where I^+ is f^{-1}-attracting, this was shown in [V. Guedj and N. Sibony, op. cit.]. It follows that for a positive closed current S of bidegree $(1,1)$ and unit mass on $\mathbf{P}^k$, $S\wedge\sigma_-$ is well-defined as a probability measure. We define the generalized Lelong number of S with respect to σ_- as $\nu(S,\sigma_-):=S\wedge\sigma_-(\{X^+\})$ (Definition 2.3). In particular, let f be a regular automorphism, i.e., $I^+\cap I^-=\varnothing$. In this case f^{-1} is algebraically stable and the invariant Green current T_- for f^{-1} is well-defined. Here the number $\nu(S,\sigma_-)$ reduces to the Demailly number of S with respect to the weight T_- [J.-P. Demailly, in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)]. Moreover, in this setting, $\nu(S,\sigma_-)$ is positive if and only if the standard Lelong number $\nu(S,X^+)$ at the point X^+ is positive.

The main result of this article, Theorem 1.1, concerns polynomial automorphisms f with $X^+\cap I^+=\varnothing$, $\lambda>\lambda_2(f)$, and such that I^+ is an attracting set for f^{-1}. It states that if S is a positive closed current of bidegree $(1,1)$ and unit mass on $\mathbf{P}^k$, then

$$\frac{1}{\lambda^n}(f^n)^*S\to c_S[t=0]+(1-c_s)T_+$$

as currents on $\mathbf{P}^k$, where $c_S = \nu(S, \sigma_-)$. Moreover, as in the regular automorphism setting, $\nu(S, \sigma_-) > 0$ if and only if $\nu(S, X^+) > 0$. In Section 2.3, an interesting invariant probability measure $\mu_f = T^+ \wedge \sigma_-$ is introduced; in particular, plurisubharmonic functions of logarithmic growth are shown to be integrable with respect to μ_f (Remark 2.13). The proof of Theorem 1.1 is given in Section 3. In the fourth and final section, the authors utilize the classification of J. E. Fornæss and H. Wu [Publ. Mat. **42** (1998), no. 1, 195–210; MR1628170 (99e:14015)] to check their hypotheses on families of quadratic polynomial automorphisms of $\mathbf{C}^3$.

Reviewed by *Norman Levenberg*

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MR2050205 (2005i:32018) 32J18 (32C30)
Boucksom, Sébastien (F-GREN-F)
Divisorial Zariski decompositions on compact complex manifolds. (English, French summaries)
Ann. Sci. École Norm. Sup. (4) **37** (2004), *no. 1,* 45–76.

It was originally shown by Zariski that any effective $\mathbb{Q}$-divisor D on a projective surface X can be decomposed uniquely into a sum $D = P + N$, where P is a numerically effective $\mathbb{Q}$-divisor and $N = \sum a_j D_j$ is an effective $\mathbb{Q}$-divisor such that the Gram matrix $(D_i \cdot D_j)$ is negative definite. In terms of the intersection form, P is moreover orthogonal to N. The spaces of global holomorphic sections corresponding to $H^0(kP)$ and $H^0(kD)$ are then isomorphic, so that the ring $R(X, D) = \bigoplus_{k\geq 0} H^0(X, \mathcal{O}(kD))$ is equivalent to $R(X, P)$ via an isomorphism that respects the natural decomposition. In order to generalise this result to an arbitrary compact complex manifold, the author of the present paper employs Demailly's theory of regularization of almost positive $(1,1)$-currents to obtain a Zariski-type decomposition of a pseudo-effective class $\alpha \in H^{1,1}_{\partial\overline{\partial}}(X, \mathbb{R})$. Recall that in the world of complex manifolds beyond Kähler geometry it is convenient to work with the cohomology of d-closed smooth $(1,1)$-forms modulo $\partial\overline{\partial}$-exact ones, such that the classes are easily seen to represent a more general affine space of closed $(1,1)$-currents. Moreover, when X is compact the $\partial\overline{\partial}$-operator has closed range, so that $H^{1,1}_{\partial\overline{\partial}}(X, \mathbb{C})$ is finite-dimensional. The author's generalisation of the Zariski decomposition is therefore in the spirit of the Siu decomposition for closed positive currents. A real $(1,1)$-current T is said to be "almost positive" if there exists a smooth real $(1,1)$-form γ such that $T \geq \gamma$. Given the definition of the Lelong number of T at each $x \in X$, one defines $\nu(T, D)$ to be the infimum of the Lelong numbers over all points of a given prime divisor D, and arrives at the Siu decomposition of T accordingly. A cohomology class α, as above, is said to be "pseudoeffective" iff it contains a positive current. It is "nef" iff for each $\varepsilon > 0$, α contains a smooth form $\vartheta_\varepsilon \geq -\varepsilon\omega$, where ω denotes the Hermitian form associated with a Gauduchon metric on X. It is "big" iff it contains a Kähler current, i.e., a closed $(1,1)$-current T such that $T \geq -\varepsilon\omega$ for sufficiently small ε. Let $\alpha[\gamma]$ denote the set of closed almost positive currents such that $T \geq \gamma$ for a smooth real $(1,1)$-form γ. It follows that any family of elements of $\alpha[\gamma]$ has an infimum with respect to a given preorder relation. In particular, $T_{\min,\gamma}$ will denote the infimum within $\alpha[\gamma]$ itself. The author introduces the notion of "minimal multiplicity" for a pseudoeffective class α as

$$\nu(\alpha, x) = \sup_{\varepsilon>0} \nu(T_{\min,-\varepsilon\omega}, x),$$

and hence $\nu(\alpha, D)$ as the infimum of this multiplicity along a given prime divisor D. α is then nef iff $\nu(\alpha, x)$ vanishes everywhere, while the "non-nef locus" corresponds to the set of points of X where the multiplicity is positive. The main result of the paper is to derive a decomposition of the form $\alpha = N + Z$, where $N = \sum \nu(\alpha, D)D$ is the summation over all prime divisors of X, and Z is a real pseudoeffective $(1,1)$-class. The author proves that there are only finitely many prime divisors D for which $\nu(\alpha, D) > 0$ (the non-nef locus of α), while the non-nef locus of Z, though not empty in general, contains no prime divisors (hence it is said to belong to the "modified nef-cone" of the Néron-Severi space). This decomposition is naturally induced by the Siu decomposition of a closed positive current with the minimality condition above inside α when it is big. When X is a surface, the distinction between modified-nef and nef classes disappears, and when X is moreover projective one recovers the original Zariski decomposition if α is the class of an effective $\mathbb{Q}$-divisor D. Extensions of the surface theory to compact hyper-Kähler manifolds are also explored via the Beauville-Bogomolov form on $H^{1,1}(X, \mathbb{R})$.

Reviewed by *Adam Gregory Harris*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2045101 (2004m:32059) 32S65 (32Q30 32U40 37F75)
Brunella, Marco (F-DJON-IM)
On the regularity of the leafwise Poincaré metric. (English summary)
Bull. Sci. Math. **128** (2004), *no. 3,* 189–195.

This article is the continuation of the author's previous work [Invent. Math. **152** (2003), no. 1, 119–148; MR1965362 (2003m:32028)]. Let X, $\mathcal{F}$ and Ω be as in the review of the above-mentioned article. Moreover, assume that the singularities of $\mathcal{F}$ are reduced in Seidenberg's sense (one can obtain this after a finite number of blow-ups in X). On one hand, according to [Y. T. Siu, Invent. Math. **27** (1974), 53–156; MR0352516 (50 #5003)] we have $\Omega = \Omega_{\rm alg} + \Omega_{\rm res}$, where $\Omega_{\rm alg}$ is a finite sum of integration currents over algebraic cycles and $\Omega_{\rm res}$ is a closed positive current with vanishing Lelong number outside a finite set of X. On the other hand, we have Lebesgue's decomposition $\Omega = \Omega_{\rm sing} + \Omega_{\rm ac}$ of Ω into singular and absolutely continuous parts [see, for instance, J.-P. Demailly, J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)]. In general for a closed positive current we have $\Omega_{\rm alg} \leq \Omega_{\rm sing}$ and the main result of this article is the equality $\Omega_{\rm alg} = \Omega_{\rm sing}$. According to [M. McQuillan, in *European Congress of Mathematics, Vol. II (Barcelona, 2000)*, 47–53, Progr. Math., 202, Birkhäuser, Basel, 2001; MR1905350 (2003j:14048)], for a foliation $\mathcal{F}$ of general type, i.e. $\mathrm{kod}(\mathcal{F}) = 2$, we have $\Omega_{\rm alg} = 0$ and so in this case Ω is absolutely continuous. The local analysis of Ω is used in the proof of the main result.

Reviewed by *Hossein Movasati*

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MR2038086 (2004k:32039) 32Q15 (32C30)

Popovici, Dan [Popovici, Dan[2]] (F-GREN-IF)

Estimation effective de la perte de positivité dans la régularisation des courants. (French. English, French summaries) [Effective estimate of positivity loss in current regularizations]

C. R. Math. Acad. Sci. Paris **338** (2004), *no. 1,* 59–64.

Summary: "Let (X, ω) be a compact complex Hermitian manifold, and let $T \geq \gamma$ be a d-closed $(1,1)$ almost positive current on X. A variant of Demailly's regularization-of-currents theorem states that T is the weak limit of a sequence of $(1,1)$-currents T_m with analytic singularities of coefficient $1/m$, lying in the same cohomology class as T, whose Lelong numbers converge to those of T, and with a loss of positivity decaying to zero. We prove that if the $(1,1)$-form γ is assumed to be closed and C^∞, the regularizing currents T_m can be chosen such that $T_m \geq \gamma - \frac{C}{m}$ for a constant $C > 0$ independent of m."

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3. D. Popovici, Quelques applications des méthodes effectives en géométrie analytique, Thèse Univ. de Grenoble, en préparation.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2036329 (2004m:32074) 32W20
Błocki, Zbigniew (PL-JAGL)
On the definition of the Monge-Ampère operator in $\mathbb{C}^2$. (English summary)
Math. Ann. **328** (2004), *no. 3,* 415–423.

Let u be plurisubharmonic (psh) on an open subset Ω in $\mathbf{C}^N$; we write $u \in \mathrm{PSH}(\Omega)$. If $u \in C^2(\Omega)$, the complex Monge-Ampère operator $(dd^c(\cdot))^N$ applied to u,

$$(dd^c u)^N = dd^c u \wedge \cdots \wedge dd^c u = 4^N N! \det [\frac{\partial^2 u}{\partial z_j \partial \bar{z}_k}] d\lambda,$$

is a nonnegative function on Ω times the volume form $d\lambda$ on $\mathbf{C}^N$. If u is locally bounded, or, more generally, if the set of points where u is not locally bounded is relatively compact in Ω, then $(dd^c u)^N$ is well-defined [E. Bedford and B. A. Taylor, Invent. Math. **37** (1976), no. 1, 1–44; MR0445006 (56 #3351); Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024); J.-P. Demailly, in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)] as a positive Radon measure and has the desirable property that it is continuous under decreasing sequences; i.e., if $u_j \downarrow u$, then $(dd^c u_j)^N \to (dd^c u)^N$ in the weak-* topology. Define $\mathcal{D}(\Omega)$ to be the class of psh functions u on Ω such that there exists a positive Radon measure μ on Ω with the property that if $\Omega' \subset \Omega$ is open and $\{u_j\}$ is any sequence of smooth, psh functions on Ω' which decrease to u in Ω', then $(dd^c u_j)^N \to \mu|_{\Omega'}$ weak-*. The main result of this paper, Theorem 1.1, is that for $N = 2$,

$$\mathcal{D}(\Omega) = \mathrm{PSH}(\Omega) \cap W^{1,2}_{\mathrm{loc}}(\Omega).$$

As observed by Bedford and Taylor in [op. cit., 1976], an integration by parts shows that $(dd^c u)^2$ is well-defined for $u \in \mathrm{PSH}(\Omega) \cap W^{1,2}_{\mathrm{loc}}(\Omega)$, $\Omega \subset \mathbf{C}^2$. The author verifies that this class coincides with $\mathcal{D}(\Omega)$, and in the final section of the paper he shows that if Ω is a bounded hyperconvex domain in $\mathbf{C}^2$ (there exists a negative psh function φ in Ω with $\lim_{z\to\partial\Omega}\varphi(z)=0$), then the negative psh functions in $\mathcal{D}(\Omega)=\mathrm{PSH}(\Omega)\cap W^{1,2}_{\mathrm{loc}}(\Omega)$ coincide with the class $\mathcal{E}(\Omega)$ defined by U. Cegrell [in *Actes des Rencontres d'Analyse Complexe (Poitiers-Futuroscope, 1999)*, 39–42, Atlantique, Poitiers, 2002; MR1944194 (2003j:32047)].

In a forthcoming paper, the author gives a characterization of the class $\mathcal{D}(\Omega)$ when Ω is an open subset of $\mathbf{C}^N$ for any $N\geq 2$, and he shows that for a negative psh function u in a hyperconvex domain Ω, $u\in\mathcal{E}(\Omega)$ if and only if $u\in\mathcal{D}(\Omega)$.

Reviewed by *Norman Levenberg*

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★**Complex analysis and geometry.**

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Contents: Henri Skoda, Présence de l'œuvre de Pierre Lelong dans les grands thèmes de recherches d'aujourd'hui [The presence of Pierre Lelong in modern research] (1–30); Bibliographie de Pierre Lelong [Bibliography of Pierre Lelong] (31–37); Józef Siciak, Wiener's type sufficient conditions for regularity in $\mathbf{C}^N$ (39–46); Jean-Pierre Demailly, On the Ohsawa-Takegoshi-Manivel L^2 extension theorem (47–82); Giovanni Bassanelli, A geometrical application of the product of two positive currents (83–90); Michael Christ, Hypoellipticity: geometrization and speculation (91–109); Klas Diederich and Takeo Ohsawa, Moment problems for weighted Bergman kernels (111–122); Gregor Herbort, On the Bergman metric near a plurisubharmonic barrier point (123–132); Ulrich Hiller, On the vanishing order of a holomorphic germ along a complex analytic germ in $\mathbf{C}^n$ (133–142); S. M. Webster, Stationary curves and complete integrability in the complex domain (143–150); C. Laurent-Thiébaut and J. Leiterer, The Malgrange vanishing theorem with support conditions (151–162); Tien-Cuong Dinh, Mesures orthogonales à support compact de longueur finie et applications [Orthogonal measures with compact support of finite length and applications] (163–172); P. Dingoyan, Convexity and Hartogs's theorem in some open subset of a projective manifold (173–181); Klas Diederich and Jeffery D. McNeal, Pointwise nonisotropic support functions on convex domains (183–192); Emmanuel Mazzilli, Un exemple d'obstruction géométrique à l'extension des fonctions holomorphes bornées [Example of geometric obstruction to the extension of bounded holomorphic functions] (193–201); Eric Bedford and Mattias Jonsson, Potential theory in complex dynamics: regular polynomial mappings of $\mathbf{C}^k$ (203–211); Giuseppe Tomassini, Boundaries of Levi-flat hypersurfaces of $\mathbf{C}^2$ (213–219); François Berteloot and Julien Duval, Une démonstration directe de la densité des cycles répulsifs dans l'ensemble de Julia [A direct proof of the density of repulsive cycles in a Julia set] (221–222); Charles L. Epstein and Gennadi M. Henkin, Embeddings for 3-dimensional CR-manifolds (223–236); List of problems (237–241).

{Most of the papers are being reviewed individually.}

MR1782659 (2001m:32041) 32L10 (32D15 32J25 32U05)
Demailly, Jean-Pierre (F-GREN-F)

On the Ohsawa-Takegoshi-Manivel L^2 extension theorem. (English, French summaries)

Complex analysis and geometry (*Paris*, 1997), 47–82, *Progr. Math.*, 188, *Birkhäuser*, *Basel*, 2000.

This paper provides a rather deep insight into the current status of L^2 extension techniques for sections [resp. $(0, q)$-forms] of vector bundles over complex analytic submanifolds. The fundamental extension theorem of T. Ohsawa and K. Takegoshi [Math. Z. **195** (1987), no. 2, 197–204; MR0892051 (88g:32029)], refined in many ways by Ohsawa, and in a more geometric setting by L. Manivel [Math. Z. **212** (1993), no. 1, 107–122; MR1200166 (94e:32050)], is proven here in its most general form. Unfortunately, a gap in the proof of Manivel is pointed out, regarding the regularity of the extension in the case of $(0, q)$-forms when $q > 0$. It thus reappears here as a conjecture, which is discussed in detail, but without being settled.

This theorem can yield powerful constructions that have been used in transcendental algebraic geometry. First, any psh function on a pseudoconvex open set in $\mathbb{C}^n$ can be approximated accurately with functions of the form $c \log |f|$ where f is a holomorphic function; this can be applied for instance to approximate the curvature current of a singular metric by divisors with multiplicities controlled by the Lelong numbers of the current. Other implications are detailed, among them a Briançon-Skoda theorem for multiplier ideal sheaves, and an analytical proof of Fujita's approximate Zariski decomposition for big line bundles.

{For the entire collection see MR1782699 (2001c:32002)}

Reviewed by *Thierry Bouche*

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MR1782656 (2001h:32001) 32-03 (01A70 32C30 32J25 32U25)
Skoda, Henri (F-PARIS6-MI)

Présence de l'œuvre de Pierre Lelong dans les grands thèmes de recherches d'aujourd'hui. (French. English, French summaries) [The presence of the work of Pierre Lelong in the big picture of modern research]

Complex analysis and geometry (*Paris*, 1997), 1–30, *Progr. Math.*, 188, *Birkhäuser*, *Basel*, 2000.

The goal of this article is to discuss the major lines of the research of Pierre Lelong.

The author begins by outlining the first topics explored by Lelong: plurisubharmonic functions and their approximation by logarithmic functions; closed positive currents and the relation between these currents and plurisubharmonic functions; the potential associated with a closed positive current; the Lelong number of a closed positive current T and of a plurisubharmonic function. He shows that the Lelong number of a closed positive current coincides with that of the potential associated with that current. Hörmander's L^2-estimates allowed Lelong to prove Siu's theorem, namely that for all $c > 0$, the set $E_c = \{z \in \Omega\colon\ \nu_T(z) \geq c\}$ is an analytic subset for every closed positive current on a domain $\Omega \subset \mathbb{C}^n$, where $\nu_T(z)$ denotes the Lelong number of T at the point z.

In the third section Skoda shows how Lelong's methods intersect with those of the geometry of manifolds and algebraic geometry. These methods have been implemented in a clear way by Demailly, for example in his treatment of the Fujita conjecture [J.-P. Demailly, J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)]. However, one can in fact encounter Lelong's ideas in many other fields, for example transcendental number theory, the theory of potentials of several variables, etc.

In the last section the author gives a sketch of the Hodge conjecture, which has not yet been proved, and shows that the Poincaré-Lelong equation has provided partial results. (For a study of the connections between the Hodge conjecture and the theory of closed positive currents see a paper by Demailly [Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007)].)

{For the entire collection see MR1782699 (2001c:32002)}

Reviewed by *Mongi Blel*

MR1772670 (2001m:32042) 32L10 (14C20 32J25)

Demailly, Jean-Pierre (F-GREN-F)

Méthodes L^2 et résultats effectifs en géométrie algébrique. (French. French summary) [L^2-methods and effective results in algebraic geometry]

Séminaire Bourbaki, Vol. 1998/99.
Astérisque No. 266 (2000), *Exp. No.* 852, 3, 59–90.

This paper surveys the recent work that has been done by Demailly, Siu and Nadel among others about effective results in algebraic geometry obtained through Hörmander's L^2-methods for the $\overline{\partial}$ equation with singular metrics. A first section provides good insight into the basic tools, which are defined, and the results, whose proofs are outlined: singular metrics of holomorphic line bundles over complex analytic manifolds, the Bochner-Kodaira-Nakano identity for the antiholomorphic Laplace-Beltrami operator (in the case where the metric is smooth), L^2 estimates with singular metrics, Nadel's multiplier ideal sheaves and the corresponding vanishing theorem. Two important applications of these techniques are then described in detail: the Fujita conjecture [see Y. T. Siu, in *Modern methods in complex analysis (Princeton, NJ, 1992)*, 291–318, Ann. of Math. Stud., 137, Princeton Univ. Press, Princeton, NJ, 1995; MR1369144 (98f:32032)] and Siu's theorem about the invariance of plurigenera under deformation [see Y. T. Siu, Invent. Math. **134** (1998), no. 3, 661–673; MR1660941 (99i:32035)].

{For the entire collection see MR1772667 (2001b:00028)}

Reviewed by *Thierry Bouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1772667 (2001b:00028) 00B25

★Séminaire Bourbaki. Vol. 1998/99. (French) [Bourbaki Seminar. Vol. 1998/99]

Exposés 850–864.
Astérisque No. 266 (2000).
Société Mathématique de France, *Paris*, 2000. *pp. i–iv and* 1–483.

{Vol. 1997/98 has been reviewed [MR1685659 (99m:00026)].}

Contents: Jean-Yves Chemin, Explosion géométrique pour certaines équations d'ondes non linéaires (d'après Serge Alinhac) [Geometric blowup of some nonlinear wave equations (following Serge Alinhac)] (Exp. No. 850, 3, 7–20); Pierre Colmez, Fonctions L p-adiques [p-adic L-functions] (Exp. No. 851, 3, 21–58); Jean-Pierre Demailly, Méthodes L^2 et résultats effectifs en géométrie algébrique [L^2-methods and effective results in algebraic geometry] (Exp. No. 852, 3, 59–90); Tomasz Komorowski, Brownian motion in a Poisson obstacle field (Exp. No. 853, 3, 91–111); Stefano Marmi, Chaotic behaviour in the solar system (following J. Laskar) (Exp. No. 854, 3, 113–136); Michel Boileau, Uniformisation en dimension trois [Uniformization in dimension three] (Exp. No. 855, 4, 137–174); Nicolas Burq, Formules de trace, résonances et quasi-modes (d'après Sjöstrand-Zworski, Stefanov-Vodev et al.) [Trace formulas, resonances and quasimodes (following Sjöstrand-Zworski, Stefanov-Vodev et al.)] (Exp. No. 856, 4, 175–190); Henri Carayol, Preuve de la conjecture de Langlands locale pour GL_n: travaux de Harris-Taylor et Henniart [Proof of the local Langlands conjecture for GL_n: works of Harris and Taylor and of Henniart] (Exp. No. 857, 4, 191–243); Olivier Mathieu, Classification des algèbres de Lie simples [Classification of simple Lie algebras] (Exp. No. 858, 4, 245–286); Michel Talagrand, Verres de spin et optimisation combinatoire [Spin glasses and combinatorial optimization] (Exp. No. 859, 4, 287–317); Christophe Breuil, Intégration sur les variétés p-adiques (d'après Coleman, Colmez) [Integration on p-adic manifolds (following Coleman and Colmez)] (Exp. No. 860, 5, 319–350);

Robert Bryant, Recent advances in the theory of holonomy (Exp. No. 861, 5, 351–374); Yuri I. Manin [Yuriĭ Ivanovich Manin], Classical computing, quantum computing, and Shor's factoring algorithm (Exp. No. 862, 5, 375–404); Joseph Oesterlé, Densité maximale des empilements de sphères en dimension 3 (d'après Thomas C. Hales et Samuel P. Ferguson) [Maximal density of three-dimensional sphere packings (following Thomas C. Hales and Samuel P. Ferguson)] (Exp. No. 863, 5, 405–413); Jean-Pierre Serre, Sous-groupes finis des groupes de Lie [Finite subgroups of Lie groups] (Exp. No. 864, 5, 415–430).

{The papers are being reviewed individually.}

MR1775582 (2001j:14037) 14G40 (11G50 14M25)

Maillot, Vincent (F-ENS-MI)

Géométrie d'Arakelov des variétés toriques et fibrés en droites intégrables. (French. English, French summaries) [Arakelov geometry of toric varieties and integrable line bundles]

Mém. Soc. Math. Fr. (N.S.) No. 80 (2000), *vi*+129 *pp*.

In the present monograph, the author develops the arithmetic intersection theory which involves characteristic classes of line bundles with singular Hermitian metrics and applies it to smooth projective toric varieties.

An arithmetic variety X is a regular scheme which is flat and projective over $\mathbf{Z}$. In a remarkable paper H. Gillet and C. Soulé [Inst. Hautes Études Sci. Publ. Math. No. 72 (1990), 93–174 (1991); MR1087394 (92d:14016)] constructed the arithmetic Chow groups $\widehat{\mathrm{CH}}^*(X)$ of X and their intersection product. Let X_∞ be the complex manifold associated to the generic fiber $X_{\mathbf{Q}}$ and $F_\infty\colon X_\infty \to X_\infty$ the complex conjugate of X_∞. For a vector bundle E on X with an F_∞-invariant smooth Hermitian metric h on the generic fiber, the Chern class of (E,h) is defined in $\widehat{\mathrm{CH}}^*(X)$ [H. A. Gillet and C. Soulé, Ann. of Math. (2) **131** (1990), no. 1, 163–203; MR1038362 (91m:14032a)].

In Arakelov geometry, we often need to deal with line bundles with non-smooth Hermitian metrics. But the Chern classes of these bundles cannot be defined in $\widehat{\mathrm{CH}}^*(X)$. One of the purposes of this monograph is to extend the arithmetic Chow groups so that their Chern classes can be defined.

A Hermitian line bundle (L,h) on X is called admissible if L is generated by global sections and if h is continuous, positive and uniformly approximated by positive smooth metrics. (L,h) is called integrable if it is written as a difference of two admissible line bundles. Let $\widehat{\mathrm{CH}}^p(X)$ denote a group of pairs (Z,g) consisting of a closed subscheme $Z \subset X$ of codimension p and a real F_∞-invariant $(p-1,p-1)$-current g, subject to rational equivalence. We note that $\widehat{\mathrm{CH}}^p(X)$ is a subgroup of $\widetilde{\mathrm{CH}}^p(X)$. For a rational section s of an integrable line bundle (L,h), the pair $(\mathrm{div}(s), -\log h(s,s))$ determines an element in $\widetilde{\mathrm{CH}}^1(X)$, which is denoted by $\widehat{c}_1(L,h)$.

When the metric h is not smooth, $-\log h(s,s)$ is no longer a Green current of logarithmic type. But for finitely many global sections s_i of admissible line bundles (L_i,h_i) and a Green current g_Z for $Z\subset X$ such that $\mathrm{div}(s_1),\cdots,\mathrm{div}(s_q)$ and Z intersect properly, the $*$-product of g_Z, $-\log h_1(s_1,s_1),\cdots,-\log h_q(s_q,s_q)$, is defined by using the theory of products of positive currents due to E. Bedford and B. A. Taylor [Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)] and J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)]. This generalized $*$-product yields the product $\alpha\widehat{c}_1(L_1,h_1)\cdots\widehat{c}_1(L_q,h_q)$ in $\widetilde{\mathrm{CH}}^p(X)$, where $\alpha\in\widehat{\mathrm{CH}}^{p-q}(X)$ and (L_i,h_i) are integrable line bundles on X. The subgroup of $\widetilde{\mathrm{CH}}^p(X)$ generated by such products as mentioned above is denoted by $\widehat{\mathrm{CH}}^p_{\mathrm{int}}(X)$ and called the generalized arithmetic Chow group of codimension p. It is obvious that $\widehat{\mathrm{CH}}^*(X)\subset\widehat{\mathrm{CH}}^*_{\mathrm{int}}(X)$. The most important property of $\widehat{\mathrm{CH}}^*_{\mathrm{int}}(X)$ is that it possesses a multiplicative structure.

The author applies this generalized arithmetic intersection theory to smooth projective toric schemes over $\mathbf{Z}$. Let Δ be a complete regular fan such that the associated toric scheme $\mathbf{P}(\Delta)$ is smooth and projective over a ground ring. $\mathbf{P}(\Delta)$ has a natural action of a torus T. When the ground ring is a field, the Chow ring of $\mathbf{P}(\Delta)$ is completely known: Let $\Delta(1)$ be the set of all 1-dimensional cones. We can associate to $\sigma\in\Delta(1)$ a T-invariant line bundle L_σ on $\mathbf{P}(\Delta)$. Then the Chow ring of $\mathbf{P}(\Delta)$ over a field is generated by the Chern classes $c_1(L_\sigma)$ for all $\sigma\in\Delta(1)$ and their relations are given in terms of the fan.

In order to seek an analogy of the above result on the arithmetic Chow ring, we have to choose a canonical Hermitian metric on any T-invariant line bundle on $\mathbf{P}(\Delta)$ and to define its arithmetic Chern class. In this paper three equivalent ways to construct the metric are introduced. Although this metric is not smooth, the T-invariant line bundle with this metric, which is denoted by $\overline{L}_\sigma$, becomes integrable. Hence its arithmetic Chern class is defined in $\widehat{\mathrm{CH}}^1_{\mathrm{int}}(\mathbf{P}(\Delta))$.

The author computes the intersection product, $\widehat{c}_1(\overline{L}_{\sigma_1})\cdots\widehat{c}_1(\overline{L}_{\sigma_q})$, of these Chern classes for $\sigma_1,\cdots,\sigma_q\in\Delta(1)$. The main result is the following: If the usual intersection $c_1(L_{\sigma_1})\cdots c_1(L_{\sigma_q})$ is zero in the Chow ring, then the arithmetic intersection $\widehat{c}_1(\overline{L}_{\sigma_1})\cdots\widehat{c}_1(\overline{L}_{\sigma_q})$ is also zero. Furthermore the author shows that the canonical heights of hypersurfaces in $\mathbf{P}(\Delta)$ over $\mathbf{Z}$ are expressed by their Mahler measures. At the end, he proves an arithmetic analogue of the Bernstein-Kushnirenko theorem.

Reviewed by *Yuichiro Takeda*

MR1768171 (2001e:32051) 32U35
Herbort, Gregor (D-WUPP)
The pluricomplex Green function on pseudoconvex domains with a smooth boundary. (English summary)
Internat. J. Math. **11** (2000), *no. 4,* 509–522.

Let D be a bounded hyperconvex domain in $\mathbf{C}^n$ and let $g(z,w)$ denote the pluricomplex Green function on D with a (single) pole at w. In the paper under review, the author studies the behaviour of g as the pole tends to a boundary point. It is well known that for a fixed pole w, $g(z,w)$ tends to zero as z tends to the boundary, but in general g is not symmetric, and in fact it is unknown whether $\lim_{w\to w_0} g(z,w)=0$ for any w_0 in the boundary of an arbitrary bounded hyperconvex domain. This question is interesting not only in itself, but also has consequences for the study of the boundary behaviour of the Bergman metric.

The main result of the paper is that if D admits a Hölder continuous bounded plurisubharmonic exhaustion function, then $\lim_{w\to w_0} \inf_{z\in K} g(z,w)=0$ for every $w_0\in\partial D$ and every compact set $K\subset D$. In particular, this statement holds if D is a bounded pseudoconvex domain with C^2 boundary.

The proof of the main result is rather technical and depends on careful upper and lower bounds of the integral $\mathcal{I}_j=\int |g(\cdot,w_j)|(dd^c\max\{g(\cdot,z_j),-\eta_j\})^n$ (where (z_j) is a sequence in some fixed compact set K and η_j is a suitable sequence of positive numbers tending to ∞). The difficult part is to get a lower bound for $\mathcal{I}_j$ in terms of g itself and a Hölder continuous plurisubharmonic exhaustion function, and this is done by adapting a construction by Demailly.

Reviewed by *Frank Wikström*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1762959 (2001f:32039) 32Q05 (14C30 32M15)

Hwang, Jun-Muk; **To, Wing-Keung** (SGP-SING)

On Seshadri constants of canonical bundles of compact quotients of bounded symmetric domains. (English summary)

J. Reine Angew. Math. **523** (2000), 173–197.

Let L be an ample line bundle over a projective manifold X. To measure the "local positivity" of L at a given point x in X, Demailly introduced the Seshadri number $\varepsilon(L,x)$. Lower bounds of these numbers yield precise results about the generation of s-jets by global sections of K_X+L at x, while upper bounds also carry some local geometric information on the polarized manifold

(X, L).

The paper under review gives lower and upper bounds for $\varepsilon(L, x)$ in terms of metric invariants when L is the canonical line bundle over a smooth compact quotient of a bounded symmetric domain of $\mathbf{C}^n$, endowed with its natural Poincaré-like metric of Ricci curvature -1. The method of proof involves the construction of some singular Hermitian metric on K_X with prescribed pole order at x.

Reviewed by *Thierry Bouche*

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MR1760443 (2001i:32040) 32Q45 (32H02 32J15)
Cantat, Serge (F-ENSLY)
Deux exemples concernant une conjecture de Serge Lang. (French. English, French summaries) [Two examples related to a conjecture of Serge Lang]
C. R. Acad. Sci. Paris Sér. I Math. **330** (2000), *no. 7,* 581–586.

A compact complex analytic space X is hyperbolic if every holomorphic map from the complex plane $\mathbf{C}$ to X must be constant. M. Green and P. Griffiths [in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557 (82h:32026)] conjectured that if X is a pseudo-canonical (Lang's "functorial" terminology for "general type") projective variety, then the image of any holomorphic map from $\mathbf{C}$ to X will be contained in a proper Zariski subvariety of X. Thus, S. Lang [Bull. Amer. Math. Soc. (N.S.) **14** (1986), no. 2, 159–205; MR0828820 (87h:32051)] conjectured that if X is a projective variety, then every subvariety of X, including X itself, is pseudo-canonical if and only if X is hyperbolic. Lang also conjectured that if X is a projective variety which is not pseudo-canonical, then there will be an abelian variety A and a non-constant rational map from A to X. Thus, Lang conjectured that a projective variety X is hyperbolic if and only if every rational map from every abelian variety to X must be constant. Note that the existence of subvarieties, and hence the projectivity assumption on X, is essential to Lang's reasoning sketched here. However, some authors, for example J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)(Conjecture 2.6)] and J. Winkelmann [Mém. Soc. Math. Fr. (N.S.) No. 72-73 (1998), x+219 pp.; MR1654465 (99g:32058)(Question 4.14.4)], left out the projectivity assumption and asked simply: If X is a compact complex analytic space such that every holomorphic map from a complex torus to X must be constant, then must X be hyperbolic?

The paper under review gives examples of non-projective compact complex manifolds that show that this generalization of Lang's conjecture to non-projective manifolds is false, even if one restricts oneself to compact Kähler manifolds. The author's first examples are non-projective $K3$ surfaces, which are Kähler. He shows that if X is a $K3$ surface without any projective curves, then there are no non-constant holomorphic maps from complex tori into X. Indeed, if there were a non-constant holomorphic map from a complex torus, it would necessarily be surjective by the proper mapping theorem and the absence of one-dimensional subvarieties. This would then contradict the fact that X is simply connected. The fact that hyperbolic $K3$ surfaces form an open subset of the moduli space of all $K3$ surfaces, and the fact that Kummer surfaces are dense in that moduli space, imply that no $K3$ surface is hyperbolic. The author's second examples of non-hyperbolic surfaces with no nontrivial images of complex tori are certain non-Kähler quotients of $\mathbf{D} \times \mathbf{C}$, known as Inoue surfaces [M. Inoue, Invent. Math. **24** (1974), 269–310; MR0342734 (49 #7479)] The paper concludes with a discussion of why one cannot use a construction like Inoue's to produce higher-dimensional projective counterexamples to Lang's original conjecture.

Reviewed by *William A. Cherry*

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MR1758586 (2001j:32037) 32U40 (32C30 32U25 32W20)
Ben Messaoud, Hedi (TN-SFAXS); **ElMir, Hassine** (TN-TUNISM)
Opérateur de Monge-Ampère et tranchage des courants positifs fermés. (French)
[Monge-Ampère operator and slicing of closed positive currents]
J. Geom. Anal. **10** (2000), *no. 1,* 139–168.

For $k \le p \le n$, R a current of bidimension (p,p) in the unit polydisk Δ^n in $\mathbf{C}^n$ ($R \in \mathcal{D}'_{(p,p)}(\Delta^n)$), and $\alpha \ge 0$ a bounded, measurable function with compact support in $\mathbf{C}^k$ such that $\int_{\mathbf{C}^k} \alpha\, d\lambda_k = 1$, the slice of R at $a \in \Delta_k$, denoted by $\langle R, \pi, a\rangle_\alpha$, is the weak limit in $\mathcal{D}'_{(p-k,p-k)}(\Delta^n)$ of

$$R \wedge \pi^*\Big(\frac{1}{\varepsilon^{2k}}\alpha\Big(\frac{z'-a}{\varepsilon}\Big)\cdot\frac{1}{4^k k!}(dd^c|z'|^2)^k\Big)$$

as $\varepsilon \to 0$, provided this limit exists. Here, $z = (z', z'') \in \Delta^k \times \Delta^{n-k}$ and $\pi(z) = z'$ (warning: in the second paragraph of the introduction, where this definition is given, "ε" is mistakenly written "e"). If α is the (normalized) characteristic function of the unit ball in $\mathbf{C}^k$, this agrees with the definition of Federer; if α is a smooth, compactly supported function, this agrees with the notion of R. Harvey and B. Shiffman [Ann. of Math. (2) **99** (1974), 553–587; MR0355095 (50 #7572)] (we remark that Federer, as well as Harvey-Shiffman, allows the projection π to be replaced by an arbitrary C^∞-map of an open set in $\mathbf{C}^n = \mathbf{R}^{2n}$ to $\mathbf{C}^k = \mathbf{R}^{2k}$).

When $R = F + dG$ where F and G have locally integrable coefficients (i.e., R is locally flat), it is well-known that the slice of R exists over each point $a \in \Delta_k$ outside of a set of $2k$-dimensional Lebesgue measure zero. Let T be a positive, closed current of bidimension (p,p) in a neighborhood of the closed unit polydisk $\overline{\Delta^n}$ in $\mathbf{C}^n$. The main result of the paper is Theorem 1.2: there exists a pluripolar set $E \subset \Delta^k$, independent of α, such that for all $a \in \Delta^k \setminus E$, the slice $\langle T, \pi, a\rangle_\alpha$ exists and is independent of α; moreover, for a smooth, compactly supported $(p-k, p-k)$-form φ on Δ^n and a locally bounded plurisubharmonic function v in Δ^k,

$$\int_{\Delta^n} T \wedge (dd^c(v\circ\pi))^k \wedge \varphi = \int_{a\in\Delta^k} \langle T, \pi, a\rangle(\varphi)(dd^c v)^k. \qquad (1)$$

The proof uses a special regularization procedure: one considers the regularized currents $T * \alpha_j$, where $\alpha_j(|z|) := j^{2n}\alpha(j|z|)$, and then studies the weak convergence of the sequence $\{(T*\alpha_j)\wedge(dd^c v_j)^k\}_j$ where $\{v_j\}_j$ are plurisubharmonic (psh) functions decreasing to a psh function v whose unbounded locus $L(v)$ avoids $\operatorname{supp} T$ in a local sense (Theorem 1.3). A key tool in proving Theorem 1.3, developed in Section 2, is the potential $U = U(\eta, T) = U(\Omega, T)$ associated to a positive, closed current T of bidimension (p,p) in an open set $\Omega_1 \subset \mathbf{C}^n$. For $\Omega \subset\subset \Omega_1$ and $\eta \in \mathcal{D}(\Omega_1)$, $0 \le \eta \le 1$ with $\eta \equiv 1$ on a neighborhood of $\overline{\Omega}$, U is the negative current of bidimension $(p+1,p+1)$ in $\mathbf{C}^n$ defined by

$$U(z) := \frac{-1}{(n-1)(4\pi)^n} \int_{x\in\mathbf{C}^n} \eta(x)T(x) \wedge \frac{(dd^c(|z-x|^2))^{n-1}}{|z-x|^{2n-2}}.$$

Utilizing techniques developed by J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)], the authors obtain weak convergence of the sequence $\{(T*\alpha_j)\wedge(dd^c v_j)^k\}_j$.

Section 3 begins the discussion on slicing. After the definition and basic properties are given, results on slicing the potential U are proved; it is shown that slices of U exist outside a pluripolar set and (1) holds for U; from this, one deduces the analogous results on the positive, closed current T. Section 4 includes applications of the slicing results; for example, modifications, using (1), are indicated which give a simplification of the proof from [H. Ben Messaoud and H. El Mir, C. R. Acad. Sci. Paris Sér. I Math. **316** (1993), no. 11, 1173–1176; MR1221644 (94e:32020)] of the main result in that paper: if A is a closed, complete pluripolar set in the unit polydisk Δ^n (there exists u psh in Δ^n with $A = \{z\in\Delta^n\colon\ u(z) = -\infty\}$), and if T is a positive, closed current in $\Delta^n \setminus A$ of bidimension (p,p) such that T has finite mass in $\{(z',z'')\in\Delta^n\colon\ r<|z''|\}$, some $r<1$, and $\langle T,\pi,z'\rangle$ exists and has finite mass for every z' in a nonpluripolar subset F of Δ^k, then the trivial extension of T by zero on A is a closed, positive current. At the end of this section, a pair of interesting results are proved. First, a nice sufficient condition for the existence of the slice $\langle T,\pi,a\rangle_\alpha$ at $a\in\Delta_k$ is provided in Theorem 4.4: the function $x\to h_k(a-x')$, where $h_k(x)$ is the standard Newtonian kernel in $\mathbf{C}^k=\mathbf{R}^{2k}$, should be in $L^1_{\rm loc}(\Delta^n,\sigma'_T)$, where

$$\sigma'_T = T\wedge(dd^c|z'|^2)^{k-1}T\wedge(dd^c|z''|^2)^{p-k+1}$$

(the trace measure of $T\wedge(dd^c|z'|^2)^{k-1}$). Then it is shown that for any pluripolar set $E\subset\Delta^k$, there exists a positive, closed current T of bidimension (p,p) in Δ^n such that the slice of T at $a\in E$ does not exist. Finally, in Section 5, it is shown that the Lelong number is preserved under slicing, outside of an exceptional pluripolar set.

Reviewed by *Norman Levenberg*

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MR1759887 (2001f:32045) 32Q45 (14J29 14J70)
Demailly, Jean-Pierre (F-GREN-F); **El Goul, Jawher** (F-TOUL3)

Hyperbolicity of generic surfaces of high degree in projective 3-space. (English summary)

Amer. J. Math. **122** (2000), *no. 3,* 515–546.

S. Kobayashi [*Hyperbolic manifolds and holomorphic mappings*, Dekker, New York, 1970; MR0277770 (43 #3503)] conjectured that a generic hypersurface of dimension n in the projective space $\mathbf{P}^{n+1}$ is hyperbolic, i.e., every holomorphic map from the affine complex line $\mathbf{C}$ into such a hypersurface is constant. In the paper under review the authors verify the above conjecture for a very generic surface in $\mathbf{P}^3$ of degree $d \geq 21$ (i.e., away from a possible countable union of subvarieties in the moduli space of surfaces of degree d). More precisely, the surfaces for which the claim holds are of general type, have Picard number 1 and their Chern classes satisfy certain inequalities. The methods and techniques developed and used in the paper might be of independent interest but they are far too elaborate to be discussed here.. For the purpose of this review we outline briefly the key ideas of the proof which goes as follows. Using the Riemann-Roch theorem one produces a branched covering Z of X living in the projectivized tangent bundle of X. If $f\colon \mathbf{C} \longrightarrow X$ is a non-constant holomorphic map, then its first differential extends to a holomorphic map whose image is contained in a leaf of an algebraic foliation on Z. By a recent argument of M. McQuillan [Inst. Hautes Études Sci. Publ. Math. No. 87 (1998), 121–174; MR1659270 (99m:32028)], the resulting curve must be algebraically degenerate, i.e., contained in a proper algebraic subvariety of Z. In order to apply this result one is in fact forced to consider 2-jets. Then the closure of the image of f is either a rational or an elliptic curve. On the other hand, by a result of H. Clemens [Ann. Sci. École Norm. Sup. (4) **19** (1986), no. 4, 629–636; MR0875091 (88c:14037)], a generic surface of degree at least 7 in the projective space contains no rational or elliptic curves which implies that f is in fact constant.

Similar results in a more general context (implying, in particular, the Kobayashi conjecture for generic surfaces of degree at least 36 in $\mathbf{P}^3$) were obtained recently in [M. McQuillan, Geom. Funct. Anal. **9** (1999), no. 2, 370–392; MR1692470 (2000f:32035)].

Reviewed by *Tomasz Szemberg*

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MR1453597 (98f:32024)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1750448 (2001b:32006) 32A26 (32E10)
Zhan, Hui Rong (PRC-XIAM); **Yao, Zong Yuan** (PRC-XIAM)

A generalization of the Koppelman formula for differential forms of type (p, q) on Stein manifolds. (Chinese. English, Chinese summaries)

Xiamen Daxue Xuebao Ziran Kexue Ban **39** (2000), *no. 2,* 147–151.

Summary: “(p, q)-type differential forms on Stein manifolds cannot adopt the Euclidean metric as they can in $\mathbf{C}^n$ because the Euclidean metric on a Stein manifold is not invariant under holomorphic transformations. This article adopts Demailly and Laurent-Thiebaut’s methods to solve the problem of the invariant metric by using a Hermitian metric and the Chern connection. A generalization of the Koppelman formula for differential forms of (p, q)-type on Stein manifolds is obtained by introducing a chosen parameter m, a natural number which is greater than or equal

to 2. When m is 2, the formula is just the original Koppelman formula for differential forms of (p,q)-type on Stein manifolds. When m is equal to $3, 4, \cdots, N$ $(N < +\infty)$, respectively, a series of Koppelman formulas with different forms can be given."

MR1741779 (2001h:32034) 32M12 (53C56)
Huckleberry, Alan T. (D-BCHMM); **Kebekus, Stefan** (D-BAYR-M8);
Peternell, Thomas (D-BAYR-M1)
Group actions on S^6 and complex structures on $\mathbf{P}_3$.
Duke Math. J. **102** (2000), *no. 1,* 101–124.

It has been known for a long time that the 6-sphere S^6 admits almost complex structures, while the other spheres S^{2n} for $n > 1$ have no almost complex structures. But it is not known whether any of these almost complex structures on S^6 are integrable.

In this paper the authors assume that X is S^6 with a complex structure. Under this assumption, it is then proved that X is not an almost homogeneous manifold, i.e., that the group of holomorphic automorphisms (which is a complex Lie group) does not have an open orbit on X.

The proof roughly goes as follows. By a result of F. Campana, J.-P. Demailly and T. Peternell [Compositio Math. **112** (1998), no. 1, 77–91; MR1622747 (99e:32047)] the manifold X does not have any nonconstant meromorphic functions. As a consequence, if X contained an open orbit of its automorphism group it would necessarily be of the form G/Γ, where G is a 3-dimensional complex Lie group and Γ is a discrete subgroup of G. Hence G would be either semisimple or solvable. The first case is easily eliminated. The second is more complicated and proceeds by elimination of the various 3-dimensional solvable complex Lie groups.

This paper also contains some observations about complex structures on $\mathbf{P}_3$ obtained by assuming that S^6 has a complex structure and blowing up a point.

Reviewed by *B. Gilligan*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1748605 (2002e:32046) 32U40 (32C30 32C37 32F10 53C60)

Demailly, Jean-Pierre (F-GREN-F)

Pseudoconvex-concave duality and regularization of currents. (English summary)

Several complex variables (*Berkeley*, *CA*, 1995–1996), 233–271, *Math. Sci. Res. Inst. Publ.*, 37, *Cambridge Univ. Press*, *Cambridge*, 1999.

The goal of this paper is to investigate some duality properties connecting pseudoconvexity and pseudoconcavity in a certain perspective to obtain a geometric version of the Serre duality theorem. These duality properties are related to several geometric problems, such as the conjecture of Hartshorne asserting that the complement of a q-codimensional algebraic subvariety Y with ample normal bundle N_Y in a projective algebraic variety X is q-convex in the sense of Andreotti-Grauert. M. Schneider proved the conjecture in the case that the normal bundle is positive in the sense of Griffiths. Using Sommese's result, the author proves the conjecture in the case that N_Y^* has a strictly convex plurisubharmonic Finsler metric.

Let X be a complex manifold of dimension n and E a holomorphic vector bundle of rank r. Demailly treats the problem of approximation of closed positive $(1,1)$-currents and the attenuation of their singularities. In general a closed positive current T cannot be approximated in the weak topology by smooth closed positive currents. J.-P. Demailly [Ann. Sci. École Norm. Sup. (4) **15** (1982), no. 3, 457–511; MR0690650 (85d:32057); J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015); in *Contributions to complex analysis and analytic geometry*, 105–126, Vieweg, Braunschweig, 1994; MR1319346 (96k:32012)] proved that this approximation is possible if we allow the regularization T_ε to have a small negative part. The main point is to control the negative part in terms of the global geometry of the ambient geometry X. It turns out that more or less optimal bounds can be described in terms of the convexity of a Finsler metric on the tangent bundle T_X. The author gives an easy proof based on the use of symmetric products of Finsler metrics.

{For the entire collection see MR1748597 (2000k:32002)}

Reviewed by *Mongi Blel*

AMERICAN MATHEMATICAL SOCIETY
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Article

MR1748597 (2000k:32002) 32-06

★**Several complex variables.**

Papers from the MSRI Program held in Berkeley, CA, 1995–1996.
Edited by Michael Schneider and Yum-Tong Siu.
Mathematical Sciences Research Institute Publications, 37.
Cambridge University Press, *Cambridge*, 1999. *xii*+564 *pp*. \$59.95. *ISBN* 0-521-77086-6

Contents: M. Salah Baouendi and Linda Preiss Rothschild, Local holomorphic equivalence of real analytic submanifolds in $\mathbf{C}^N$ (1–24); Daniel Barlet, How to use the cycle space in complex geometry (25–42); Edward Bierstone and Pierre D. Milman, Resolution of singularities (43–78); Harold P. Boas and Emil J. Straube, Global regularity of the $\overline{\partial}$-Neumann problem: a survey of the L^2-Sobolev theory (79–111); Frédéric Campana and Thomas Peternell, Recent developments in the classification theory of compact Kähler manifolds (113–159); Michael Christ, Remarks on global irregularity in the $\overline{\partial}$-Neumann problem (161–198); John P. D'Angelo and Joseph J. Kohn, Subelliptic estimates and finite type (199–232); Jean-Pierre Demailly, Pseudoconvex-concave duality and regularization of currents (233–271); John Erik Fornæss and Nessim Sibony, Complex dynamics in higher dimension (273–296); John Erik Fornæss and Brendan Weickert, Attractors in $\mathbf{P}^2$ (297–307); Peter Heinzner and Alan Huckleberry, Analytic Hilbert quotients (309–349); Jun-Muk Hwang and Ngaiming Mok, Varieties of minimal rational tangents on uniruled projective manifolds (351–389); Christian Okonek and Andrei Teleman, Recent developments in Seiberg-Witten theory and complex geometry (391–428); Yum-Tong Siu, Recent techniques in hyperbolicity problems (429–508); Domingo Toledo, Rigidity theorems in Kähler geometry and fundamental groups of varieties (509–533); Paul Vojta, Nevanlinna theory and Diophantine approximation (535–564).

{The papers are being reviewed individually.}

MR1724404 (2000i:32057) 32U25 (32C30)

Favre, Charles (S-RIT)

Note on pull-back and Lelong number of currents. (English, French summaries)

Bull. Soc. Math. France **127** (1999), *no. 3,* 445–458.

The main result of this paper is the following: If $f\colon (\mathbf{C}^m,0)\to(\mathbf{C}^n,0)$ is a holomorphic map of maximal rank equal to n, and T a positive closed current of bidegree (1,1), then the pull-back f^*T is well defined and there exists a constant $C>0$ (depending only on f) such that one has the inequalities $\nu(T,0)\le\nu(f^*T,0)\le C\cdot\nu(T,0)$. The author proves a semilocal version of the main result and gives some relations of his result with the results of J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)], M. Meo

[C. R. Acad. Sci. Paris Sér. I Math. **322** (1996), no. 12, 1141–1144; MR1396655 (97d:32013)] and C. O. Kiselman ["Le nombre de Lelong des images inverses des fonctions plurisousharmoniques" Bull. Sci. Math., to appear].

Reviewed by *Mongi Blel*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1722815 (2000i:32066) 32W20

Xing, Yang (S-UMEA)

Complex Monge-Ampère equations with a countable number of singular points. (English summary)

Indiana Univ. Math. J. **48** (1999), *no. 2,* 749–765.

As proven by J.-P. Demailly [Math. Z. **194** (1987), no. 4, 519–564; MR0881709 (88g:32034)], one can define $(dd^c u)^n$ if u is a plurisubharmonic function such that the set where u is not locally bounded is relatively compact. In the paper under review the author proves that under certain technical assumptions on a nonnegative Borel measure μ (including the assumption that μ has at most countably many singular points) there exists u with $(dd^c u)^n = \mu$.

Reviewed by *Zbigniew Błocki*

MR1714825 (2000i:14008) 14C20 (14J10)
Küchle, Oliver (D-BAYR-IM); **Steffens, Andreas** (D-BAYR-IM)
Bounds for Seshadri constants.
New trends in algebraic geometry (*Warwick*, 1996), 235–254, *London Math. Soc. Lecture Note Ser.*, 264, *Cambridge Univ. Press*, *Cambridge*, 1999.

In recent years there has been considerable interest in understanding the local positivity of ample line bundles on algebraic varieties. Seshadri constants, introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)], emerged as a natural measure of the local positivity of a line bundle. These numbers are very hard to control and their exact value is known only in very few cases. Therefore, given a polarized variety (X, L) of dimension n, it is interesting to ask for bounds for the Seshadri constant $\varepsilon(L, x)$ at a point $x \in X$. Whereas there is a universal upper bound $\varepsilon(L, x) \leq (L^n)^{1/n}$, the non-existence of a universal lower bound follows from examples given by Miranda. On the other hand, L. M. H. Ein and R. K. Lazarsfeld [Astérisque No. 218 (1993), 177–186; MR1265313 (95f:14031)] showed that if X is a surface then $\varepsilon(L, x) \geq 1$ for all but countably many points $x \in X$ (in fact, all but finitely many provided $L^2 > 1$). It is conjectured that the bound $\varepsilon(L, x) \geq 1$ is valid in any dimension, at least for $x \in X$ very general i.e. away from a countable union of proper subvarieties. A weaker result $\varepsilon(L, x) \geq 1/n$ for x very general was shown by Ein, Küchle and Lazarsfeld [J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)].

In the paper under review the authors refine the study of lower bounds for Seshadri constants. The strategy consists of finding via the Riemann-Roch theorem an effective divisor in $|kL|$, a high multiple of L, having large multiplicity at the given point x. The procedure splits then according to whether the singularity at x is isolated or not, the second case imposing existence of a subvariety on X with unusual low degree with respect to L. The bounds obtained look technically involved but they can be flexibly adjusted to a concrete situation at hand.

The methods presented in the paper combined with the effective very ampleness results due to U. Angehrn and Y. T. Siu [Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036)] allow the authors to give bounds valid at arbitrary points of X. These bounds depend of course on the geometry of X.

{For the entire collection see MR1714817 (2000e:14001)}

Reviewed by *Tomasz Szemberg*

MR1713311 (2000i:32034) 32J25 (14C20 32M15 32Q45)
Hwang, Jun-Muk (KR-SNU); **To, Wing-Keung** (SGP-SING)

On Seshadri constants of canonical bundles of compact complex hyperbolic spaces. (English summary)

Compositio Math. **118** (1999), *no. 2,* 203–215.

Seshadri constants were introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)] as a way to measure the local positivity of an ample line bundle. They emerged first in connection with problems revolving around Fujita's conjecture concerning global generation and very ampleness of adjoint line bundles. In the course of time they have constantly gained more and more interest in their own right [L. M. H. Ein, O. Küchle and R. K. Lazarsfeld, J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007); T. Bauer, Math. Ann. **313** (1999), no. 3, 547–583; MR1678549 (2000d:14006)]. Since these numbers are very hard to control and their exact values are known only in very few cases, it is interesting, given a polarized variety (X, L) of dimension n, to ask for bounds for the Seshadri constant $\varepsilon(L, x)$ at a point $x \in X$. Kleiman's nefness criterion provides a universal upper bound $\varepsilon(L, x) \leq (L^n)^{1/n}$. If the actual value of $\varepsilon(L, x)$ is strictly lower than the upper bound, it has strong geometric consequences for X. On the other hand, examples due to Miranda show there is no universal lower bound. However Ein, Küchle and Lazarsfeld [op. cit.] showed that $1/n$ is such a bound if x is sufficiently general. It is conjectured that an ample line bundle behaves as a very ample line bundle at a very general point (i.e. away from a countable union of proper subvarieties); in particular, the actual lower bound at such points is expected to be 1. This conjecture was proved for surfaces by Ein and Lazarsfeld [Astérisque No. 218 (1993), 177–186; MR1265313 (95f:14031)].

For all the above reasons it is interesting and important to study bounds for Seshadri constants for specific classes of polarized varieties. In the article under review the authors provide lower and upper bounds for Seshadri constants of the canonical bundle on compact quotients of the unit ball in $\mathbf{C}^n$. These bounds are expressed in terms of the Poincaré metric invariants. The proof for the lower bound builds upon ideas of Lazarsfeld [Math. Res. Lett. **3** (1996), no. 4, 439–447; MR1406008 (98e:14044)] applied originally to abelian varieties. The proof for the upper bound is more involved. It relies on properties of plurisubharmonic functions which are shown in the second half of the paper.

Reviewed by *Tomasz Szemberg*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1704301 (2000i:32021) 32F45 (32F17 32U10)

Chen, Bo-Yong (PRC-FUDAN-IM)

Completeness of the Bergman metric on non-smooth pseudoconvex domains. (English summary)

Ann. Polon. Math. **71** (1999), *no. 3,* 241–251.

The author proves that if Ω is either a bounded pseudoconvex domain with Lipschitz boundary in $\mathbf{C}^n$ or a bounded regular domain in $\mathbf{C}$ then it is complete with respect to the Bergman metric. He also gives an example of a bounded domain in $\mathbf{C}$ which is Bergman complete but not regular. As proven by J.-P. Demailly [Math. Z. **194** (1987), no. 4, 519–564; MR0881709 (88g:32034)], pseudoconvex domains with Lipschitz boundary in $\mathbf{C}^n$ are hyperconvex (that is, they admit a bounded plurisubharmonic exhaustion function), whereas regularity of domains in $\mathbf{C}$ is equivalent to hyperconvexity. It has been recently shown by P. Pflug and the reviewer [Nagoya Math. J. **151** (1998), 221–225; MR1650305 (2000b:32065)] and, independently, by G. Herbort [Math. Z. **232** (1999), no. 1, 183–196; MR1714284 (2000i:32020); see the preceding review], that all hyperconvex domains are Bergman complete. In the first of these works the article under review was used, whereas Herbort did not know about it.

Reviewed by *Zbigniew Błocki*

MR1696762 (2000m:14007) 14C20 (14E25 14M25)
Di Rocco, Sandra (S-RIT)
Generation of k-jets on toric varieties. (English summary)
Math. Z. **231** (1999), *no. 1,* 169–188.

In recent years, there has been interest in understanding higher order embeddings of algebraic varieties. Several notions of higher order embeddings were introduced [M. C. Beltrametti and A. J. Sommese, in *Problems in the theory of surfaces and their classification (Cortona, 1988)*, 33–48, Sympos. Math., XXXII, Academic Press, London, 1991; MR1273371 (95d:14005); in *Complex analysis and geometry*, 355–376, Plenum, New York, 1993; MR1211891 (94g:14006)], two of which—k-very ampleness and k-jet ampleness—attracted quite a lot of attention in the past decade. Whereas by now well understood in the case of surfaces, these notions remain mostly unexplored for polarized varieties of arbitrary dimension. The paper under review contributes towards understanding higher order embeddings of the broad class of varieties: toric varieties. The author shows that for toric varieties both notions mentioned above are equivalent and that they are equivalent to higher convexity for a Δ-support function as defined by the author. This is a nice generalization of strict convexity introduced by Demazure and Oda, which in turn is known to be equivalent to very ampleness of the line bundle in question.

In the last part of the paper the author shows that local and global positivity of a line bundle on a toric variety are closely related. In particular, she shows (Corollary 6.5) that if the Seshadri constant [see J.-P. Demailly, in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)] of a line bundle at a point is at least k then the line bundle is k-jet ample, the converse being true for arbitrary varieties.

Reviewed by *Tomasz Szemberg*

MR1688140 (2000d:32034) 32J27 (32C30 32J15 32Q15)
Lamari, Ahcène
Courants kählériens et surfaces compactes. (French. English, French summaries) [Kähler currents and compact surfaces]
Ann. Inst. Fourier (Grenoble) **49** (1999), *no. 1, vii*, *x*, 263–285.

It has been known for a while that every compact complex surface with even first Betti number is Kähler. The classical proof (completed in 1983 by Y. T. Siu's paper [Invent. Math. **73** (1983), no. 1, 139–150; MR0707352 (84j:32036)]) relies on the Kodaira classification, and a case by case examination. The paper under review provides a relatively short and self-contained unified proof of this fact. The strategy, inspired by Harvey-Lawson's work on intrinsic characterization of Kähler manifolds through currents, is to show the existence of a "Kähler current" (closed positive $(1,1)$-current bounded below by a Hermitian metric), which in turn provides a smooth Kähler metric in codimension 2. These constructions are not limited to the surface case; they are interesting in themselves. The main tool here is the regularization theorem of J.-P. Demailly [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)].

Reviewed by *Thierry Bouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1678549 (2000d:14006) 14C20 (14C21 14E25 14K05)
Bauer, Thomas (D-ERL-MI)
Seshadri constants on algebraic surfaces.
Math. Ann. **313** (1999), *no. 3,* 547–583.

Seshadri constants are invariants introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)]. They encode information on the local positivity of an ample line bundle. The name originates in the Seshadri criterion for ampleness [R. Hartshorne, *Ample subvarieties of algebraic varieties*, Springer, Berlin, 1970; MR0282977 (44 #211)].

Given a smooth variety X of dimension n, a line bundle L on X and a point $x \in X$ we denote by $\varepsilon(L, x)$ the Seshadri constant of L at x. This is the biggest number ε such that the $\mathbf{R}$-line bundle $f^*L - \varepsilon E$ is nef, where f is the blowup of X at x with exceptional divisor E. It is natural to ask what restrictions, if any, can be imposed on that quantity. An upper bound $\varepsilon(L, x) \le (L^n)^{1/n}$ follows easily from S. L. Kleiman's criterion [Ann. of Math. (2) **84** (1966), 293–344; MR0206009 (34 #5834)]. On the other hand, Miranda gave examples which show that there is no general lower bound in any dimension. Somehow surprisingly, in this context L. M. H. Ein, O. Küchle and R. K. Lazarsfeld [J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)] showed that $\varepsilon(L, x) \ge 1/n$ at a very general point $x \in X$, i.e. outside a countable union of divisors on X. It is conjectured that the actual bound can be improved to $\varepsilon(L, x) \ge 1$, some evidence being provided by earlier work of Ein and Lazarsfeld on surfaces [Astérisque No. 218 (1993), 177–186; MR1265313 (95f:14031)].

The bound $\varepsilon(L, x) \ge 1$ is obvious if X is an abelian variety (and it does not depend on x, as X is homogeneous). M. Nakamaye [Amer. J. Math. **118** (1996), no. 3, 621–635; MR1393263 (97k:14005)] observed that the equality has strong geometric implications, namely X splits as a product of an elliptic curve and an abelian variety of dimension $n - 1$. Proceeding along these lines, Lazarsfeld [Math. Res. Lett. **3** (1996), no. 4, 439–447; MR1406008 (98e:14044)] and Bauer [Math. Ann. **312** (1998), no. 4, 607–623; MR1660259 (2000a:14054)] showed that Jacobians, respectively Prym varieties, have small Seshadri constants among principally polarized abelian varieties. This is equivalent to saying that they have a period of unusually short length, as explained by Lazarsfeld, building upon results of P. Buser and P. C. Sarnak [Invent. Math. **117** (1994), no. 1, 27–56; MR1269424 (95i:22018)].

Seshadri constants are very hard to compute in general. They are not known even in the seemingly easy case of surfaces $X \subset \mathbf{P}^3$ of degree $d \ge 5$ (see Bauer [Math. Ann. **309** (1997), no. 3, 475–481; MR1474202 (98i:14009)] for $d \le 4$). In the paper under review the author restricts his attention to algebraic surfaces and proves a long list of interesting results in this setup.

The paper consists of eight sections. The first one has an introductory character. Every other section could be viewed as a paper on its own.

In the second section a list of possible values for the Seshadri constant of a smooth surface in $\mathbf{P}^3$ is given. The author shows that there are only a few choices for small Seshadri numbers and that they are related to the global geometry of the surface.

In the next section the author gives a lower bound for the Seshadri constant of an ample line bundle L in terms of the canonical slope of L, which is defined as the minimal real number

$\sigma = \sigma(L)$ such that $\sigma L - K_X$ is nef. Furthermore, Miranda's examples, originally constructed on rational surfaces, are generalized to arbitrary surfaces.

The next two sections contain a bound on the degree of curves which cause $\varepsilon(L, x)$ to be small at a very general point $x \in X$. This bound is used to give a quick proof of Nakamaye's result mentioned above in the case of abelian surfaces. The author also discusses the question of how many curves can cause $\varepsilon(L, x)$ to be sub-maximal, i.e. $\varepsilon(L, x) < \sqrt{L^2}$.

The last three sections deal with different aspects of Seshadri constants on abelian surfaces. If (X, L) is a polarized abelian surface with Picard number $\rho(X) = 1$ then the author in fact computes the Seshadri constant of L. Abelian surfaces thus constitute the first nontrivial class of algebraic varieties for which Seshadri constants have been computed. The numbers in effect were conjectured in the previous joint work of the author and the reviewer [T. Bauer, op. cit., 1998 (Appendix)]. The next result presented here is a detailed description of the nef cone of an abelian surface with arbitrary Picard number (it is well known that $1 \leq \rho(X) \leq 4$). Finally, multiple point Seshadri constants are introduced. These invariants are even harder to control in general; it suffices to say that their computation for $\mathbf{P}^2$ is equivalent to the unsolved Nagata conjecture [M. Nagata, Chinese J. Math. **11** (1983), no. 1, 1–4; MR0692988 (84f:14008)]. In the case of abelian surfaces the author gives interesting lower bounds for multiple point Seshadri constants valid in any points. Previous results along these lines proved by Küchle [Ann. Inst. Fourier (Grenoble) **46** (1996), no. 1, 63–71; MR1385510 (97d:14010)] are valid only for very general points.

One of the upshots of the paper under review is that Seshadri constants of abelian surfaces are rational numbers. Although there is not much other evidence, it is tempting to finish with a conjecture, not addressed directly in the paper, that Seshadri constants are always rational.

Reviewed by *Tomasz Szemberg*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1678537 (2000b:32017) 32C30 (32E20 32U25)
Guedj, Vincent (F-PARIS11)
Approximation of currents on complex manifolds.
Math. Ann. **313** (1999), *no. 3,* 437–474.

The purpose of this work is the approximation of positive closed currents of bidegree $(1,1)$ on complex projective algebraic manifolds by rational divisors. For X a pseudoconvex domain of $\mathbf{C}^n$ and $H^2(X,\mathbf{R})=0$, in 1972 P. Lelong proved this kind of approximation. In 1982 J.-P. Demailly [Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007)] generalized this result to the case where X is a Stein or projective algebraic manifold modulo some cohomological assumption, and gave a control of the Lelong numbers of the approximation. In 1995 J. Duval and N. Sibony [Duke Math. J. **79** (1995), no. 2, 487–513; MR1344768 (96f:32016)] showed that one can approximate a $(1,1)$ positive current T by rational divisors whose support converges to the support of T in the Hausdorff metric.

The paper under review can be seen as a combination of the result of Demailly, and that of Duval and Sibony. The first main result of this work is the following: Every positive closed current T of bidegree $(1,1)$ on the projective space $\mathbf{C}\mathrm{P}^n$ [resp. the Grassmann manifold $G_{k,m}(\mathbf{C})$ of k-planes of $\mathbf{C}^m$, resp. the hyperquadric $Q_m(\mathbf{C})$ for $m\geq 4$] can be weakly approximated by rational divisors whose support converges to $\operatorname{Supp} T$.

In the second section the author generalizes the notion of rational convexity and gives a generalization of the result of Duval and Sibony. In the last section the author gives the second main result on the approximation of closed positive currents. He proves the following: Let T be a positive closed current of bidegree $(1,1)$ on a projective algebraic manifold X. Let $\lambda > 0$ be such that $[\lambda T] = c_1(L)$ for some holomorphic line bundle L which we assume is positive. Assume $T = [H] + R$, where $H = \sum_{j=1}^{p} \lambda_j [Z_j]$, where Z_j is an irreducible algebraic hypersurface of X and R is a positive closed current of bidegree $(1,1)$ on X such that the level sets of the Lelong numbers of R are of codimension greater than 2. If T satisfies some condition of convexity ($\forall K \subset\subset X \setminus \operatorname{Supp} T$, $\widehat{K}^T \subset\subset X \setminus \operatorname{Supp} T$) then we can approximate T by rational divisors with control of the Lelong numbers of the approximation.

Reviewed by *Mongi Blel*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1676295 (2000e:14057) 14J25 (14H45 32Q45)
Chiantini, Luca (I-SIN); **Lopez, Angelo Felice** (I-ROME3)
Focal loci of families and the genus of curves on surfaces. (English summary)
Proc. Amer. Math. Soc. **127** (1999), *no. 12,* 3451–3459.

In the context of the problem of characterizing which projective algebraic varieties over the complex field are hyperbolic and in view of the Kobayashi-Lang conjecture, the authors pay attention to the intermediate concept of algebraically hyperbolic varieties and their properties [J.-P. Demailly, in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)] where the problem is connected with the study of the geometric genus of curves in the varieties.

They obtain several results concerning the genus of curves in general surfaces of $\mathbf{P}^3$ that let them conclude the algebraic hyperbolicity of these surfaces when certain conditions are satisfied. The key point is that they apply the classical theory of focal loci, recently rephrased in modern terms by C. Ciliberto and E. Sernesi [J. Algebraic Geom. **1** (1992), no. 2, 231–250; MR1144438 (92j:14034)] for this purpose.

In this way, they are able to give a short proof of one of the main theorems of G. Xu [J. Differential Geom. **39** (1994), no. 1, 139–172; MR1258918 (95d:14043)] by translating Xu's local analysis with a global property of the focal locus of a family of curves.

With the same method, they also study surfaces in $\mathbf{P}^3$ that are general in a given component of the Noether-Lefschetz locus and finally they obtain the algebraic hyperbolicity of some general projectively Cohen-Macaulay surfaces in $\mathbf{P}^4$.

Reviewed by *Raquel Mallavibarrena*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR1658236 (99k:14030) 14E20 (14C30 14F17 32L20)
Takayama, Shigeharu (J-OSAKEGS)
Nonvanishing theorems on an algebraic variety with large fundamental group.
J. Algebraic Geom. **8** (1999), *no. 1,* 181–195.

Let X be a smooth projective manifold over $\mathbf{C}$ with infinite fundamental group. J. Kollár's Shafarevich map [Invent. Math. **113** (1993), no. 1, 177–215; MR1223229 (94m:14018)] is an important tool for analyzing the influence of $\pi_1(X)$ on the algebro-geometric properties of X. F. Campana [Bull. Soc. Math. France **122** (1994), no. 2, 255–284; MR1273904 (95f:32036)] gave an independent construction also valid in the Kähler case. Kollár [*Shafarevich maps and automorphic forms*, Princeton Univ. Press, Princeton, NJ, 1995; MR1341589 (96i:14016)(1.8)] refined this construction and defined a quasi-fibration $S: X \to Y$ whose general fiber F is a subvariety of X with finite fundamental group which is maximal among such subvarieties.

This article gives a partial solution to Conjecture (18.9.1) in Kollár's book [op. cit., 1995]. Its main theorem states that, given a Cartier divisor L on X satisfying $h^0(F, K_F + L) \neq 0$ and $L \equiv M + \Delta$, where M is a nef and big $\mathbf{Q}$-divisor and (X, Δ) is Kawamata log terminal, then $h^0(X, K_X + L) \neq 0$.

Here is a sketch of the proof. Let $\pi: \tilde{X} \to X$ be the universal covering space of X. Atiyah's L_2-index theorem and the Demailly-Nadel version of the Kawamata-Viehweg vanishing theorem reduces the problem to proving that $\pi^*(K_X + L)$ has a nonzero square-integrable holomorphic section. Let Φ be a generic fiber of the Shafarevich map. Assume, inductively, that $h^0(\Phi, K_\Phi + L) \neq 0$. Then, for all $s > 0$, $\pi^* L$ has a singular Hermitian metric with an isolated pole along Φ of order $\geq s$. Existence of the sought for nonzero L_2 holomorphic section then follows from the Demailly-Nadel theorem.

A slight variant of this theorem (with F replaced by Φ) has been obtained independently by the reviewer using similar arguments [Ann. Inst. Fourier (Grenoble) **49** (1999), no. 1, vi, ix–x, 141–176; MR1688152 (2000d:32036)].

Reviewed by *Philippe P. Eyssidieux*

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17. S. Takayama, *The Levi Problem and the Structure Theorem for Non-Negatively Curved Complete Kähler Manifolds*, preprint (1997). cf. MR 99j:32034

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1707735 (2000k:32020) 32L10 (32C30 32L20)

Bonavero, Laurent (F-GREN-FM)

Inégalités de morse holomorphes singulières. (French. English summary) [Singular holomorphic Morse inequalities]

J. Geom. Anal. **8** (1998), *no. 3,* 409–425.

Let E be a holomorphic line bundle on a compact complex manifold X, and suppose that E is endowed with a smooth Hermitian metric. Demailly's holomorphic Morse inequalities [J.-P. Demailly, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)] give asymptotic estimates for the dimensions of the Dolbeault cohomology groups of $E^{\otimes k}$, in terms of certain curvature integrals depending on the metric.

In the paper under review, these inequalities are extended to the case of singular metrics (with restrictions on the type of the singularities). The estimates are then the same as the original ones of Demailly, provided $E^{\otimes k}$ is twisted by a suitable multiplier ideal sheaf.

One of the main applications of Demailly's inequalities was a solution of the Grauert-Riemenschneider conjecture (another proof was given independently by Y. T. Siu [in *Workshop Bonn 1984 (Bonn, 1984)*, 169–192, Lecture Notes in Math., 1111, Springer, Berlin, 1985; MR0797421 (87b:32055)]) which, in the spirit of the famous projectivity criterion of Kodaira, gave a sufficient condition for a compact complex variety to be Moishezon. As the author points out, this sufficient condition is not necessary: an explicit counterexample is provided. Then he deduces from his singular holomorphic Morse inequalities a complete characterization of Moishezon varieties in terms of the existence, not of a Hermitian line bundle, but of a $(1,1)$-current with certain positivity properties.

Reviewed by *Laurent Manivel* (Saint-Martin-d'Hères)

MR1703223 (2000i:32032) 32J17 (32Q15)

Alessandrini, L. (I-MILANP)

Curves which are obstructions to the existence of Kähler metrics on threefolds. (English, Italian summaries)

Rend. Mat. Appl. (7) **18** (1998), *no. 4,* 683–706 (1999).

In this paper, the author studies complex compact threefolds M containing a smooth curve

C of strictly positive genus such that $M \setminus C$ is Kähler. The main result is the following: put $M_0 = M$ and let by induction M_{n+1} be the manifold obtained by blowing up M_n along the curve C_n of minimal self-intersection in the exceptional divisor E_n of $M_n \to M_{n-1}$ (of course $C_0 = C$). Let $e_n = -C_n \cdot C_n$. Then, under the assumption that $e_n \geq 0$ and $E_n \cdot C_n \geq 0$ for every $n \geq 1$, M is Kähler if and only if C is not homologous to zero in the Aeppli group $V^{2,2}_{\mathbf{R}}(M)$. The proof uses Demailly's regularization theorem of closed positive currents, the Harvey-Lawson criterion and the machinery of ruled surfaces.

Reviewed by *Laurent Bonavero*

MR1689425 (2000d:32041) 32Q10 (32E40 32L20 32Q15 32Q28)
Takayama, Shigeharu (J-NARU)

The Levi problem and the structure theorem for non-negatively curved complete Kähler manifolds.

Analysis and geometry appearing in multivariable function theory (Japanese) (Kyoto, 1997).
Sūrikaisekikenkyūsho Kōkyūroku No. 1058 (1998), 105–113.

The main result is as follows: Let X be a complex manifold with negative canonical bundle K_X. Then X is holomorphically convex if and only if X is pseudoconvex. It generalizes the following result of Ohsawa [T. Ohsawa, Publ. Res. Inst. Math. Sci. **17** (1981), no. 1, 153–164; MR0613939 (82j:32031); supplement; MR0650217 (83h:32021)]: Let X be a 2-dimensional complex manifold with negative canonical bundle K_X. Then X is holomorphically convex if and only if it is weakly 1-complete. The author also proves a structure theorem: Every complete Kähler manifold with non-negative sectional curvature and positive Ricci curvature has a structure of holomorphic fiber bundle over a Stein manifold whose typical fiber is biholomorphic to some compact Hermitian symmetric manifold. The author uses some new techniques from recent developments in complex geometry and analysis on adjoint bundles on projective manifolds by J.-P. Demailly [in *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zürich, 1994)*, 817–827, Birkhäuser, Basel, 1995; MR1403982 (98e:32055)] and Siu [U. Angehrn and Y. T. Siu, Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036)].

{For the entire collection see MR1689414 (2000a:00014)}

Reviewed by *Shanyu Ji*

MR1690918 (2000e:32043) 32Uxx (32-06 32C30)

Lelong, Pierre

★Positivity in complex spaces and plurisubharmonic functions/Positivité dans les espaces complexes et fonctions plurisousharmoniques. (French summary)

Edited and with a note by Paulo Ribenboim.
Queen's Papers in Pure and Applied Mathematics, 112.
Queen's University, Kingston, ON, 1998. *x*+243 *pp. ISBN* 0-88911-828-0

This book is a collection of previously published works by the author on the theory of closed positive currents and plurisubharmonic functions.

The first article originally appeared as a book [*Fonctions plurisousharmoniques et formes différentielles positives*, Gordon & Breach, Paris, 1968; MR0243112 (39 #4436)] that served as a reference text in the theory of closed positive currents and plurisubharmonic functions. In particular, the author outlines the connections among holomorphic functions, plurisubharmonic functions and closed positive currents. Finally he introduces the integration of a differential form on an analytic subset.

In the second article [Bull. Soc. Math. France **85** (1957), 239–262; MR0095967 (20 #2465)] Lelong shows that if X is an analytic subset of pure dimension p in a complex-analytic manifold, then the integration current $[X]$ on X defines a closed positive (p,p) current. This current is defined as the continuation of the integration current on the regular points of X. In particular, Lelong proves that the mass of this current near the singular points is finite.

In the third article [in *Les probabilités sur les structures algébriques (Actes Colloq. Internat. CNRS, No. 186, Clermont-Ferrand, 1969)*, 251–263, Éditions Centre Nat. Recherche Sci., Paris, 1970; MR0409897 (53 #13649)] the author studies the frequency of obtaining certain functions in the algebra $\mathcal{A}(\Omega)$ of holomorphic functions on Ω, a domain of holomorphy in $\mathbf{C}^n$, with $n \geq 2$. He shows that if η is a subset of functions of $\mathcal{A}(\Omega)$ that can be continued outside Ω, then η is a thin set. The notions of negligible and polar sets are introduced.

The fourth article [in *Séminaire Pierre Lelong (Analyse), Année 1971-1972*, 112–131. Lecture Notes in Math., 332, Springer, Berlin, 1973; MR0412474 (54 #600)] is a study of the extremal elements of the cone of closed positive currents on a complex-analytic manifold Ω that is countable at infinity. In particular, Lelong proves that if X is an irreducible analytic subset of pure dimension p, then the integration current $[X]$ on X is an extremal current in the cone of closed positive (p,p) currents on Ω. This kind of problem is associated with the Hodge conjecture, as has been studied by J.-P. Demailly [Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007)]. Lelong shows that in a pseudoconvex domain Ω in $\mathbf{C}^n$ for which $H^2(\Omega,\mathbf{C})=0$, the cone of integration currents on analytic subsets of pure dimension $n-1$ is dense in the cone of closed positive $(n-1,n-1)$ currents.

In the fifth article [in *Séminaire Pierre Lelong (Analyse) (année 1972–1973)*, 97–106. Lecture Notes in Math., 410, Springer, Berlin, 1974; MR0372904 (51 #9108)] the author proves a support theorem for currents T in a manifold M such that T and ∂T are of order zero and $\operatorname{Supp} T$ is in a C^∞ submanifold that is smoothly embedded in M.

In the sixth article [in *Séminaire Pierre Lelong (Analyse) (année 1975/76)*, 136–156. Lecture Notes in Math., 578, Springer, Berlin, 1977; MR0486608 (58 #6328)] Lelong associates with every closed positive (p,p) current T on a pseudoconvex domain a closed positive $(1,1)$ current that has the same Lelong number at every point of the domain. This current is obtained from a potential associated with T which is an almost plurisubharmonic function. This result allows the author to give a simpler proof of Siu's theorem on the analyticity of the density set of a closed positive current.

The seventh article [Exposition. Math. **3** (1985), no. 2, 149–164; MR0816400 (87f:32001)] is a discussion of three important directions of research: the representation of analytic sets as density sets, the continuation of closed positive currents and the notion of capacity in complex analysis (see the article for details).

In the eighth article [Exposition. Math. **3** (1985), no. 2, 187–191; MR0816405 (87m:32002)] the author notes the priority of S. Lang and E. Bombieri's 1970 article [Invent. Math. **11** (1970), 1–14; MR0296028 (45 #5089)] over a paper by Bombieri of the same year [Invent. Math. **10** (1970), 267–287; MR0306201 (46 #5328)].

The ninth article [in *Geometrical and algebraical aspects in several complex variables (Cetraro, 1989)*, 211–229, EditEl, Rende, 1991; MR1222216 (94g:32016)] concerns the existence of a principal part $\tilde{f}_\xi(x)$ at every point ξ in the class of plurisubharmonic functions of minimal (logarithmic) growth for every plurisubharmonic function f in a locally convex complex vector space (see the paper for details).

In the last article [Math. Ann. **299** (1994), no. 4, 673–695; MR1286891 (95g:32025)] Lelong studies plurisubharmonic functions of logarithmic type on $\mathbf{C}^n$. He considers the existence of constants independent of R for the inequalities (A) $0 \le m(f,0,R) - \lambda(f,0,R) \le \sigma C_n$, (B) $0 \le M(f,0,R) - \lambda(f,0,R) \le \sigma\gamma_n$. Moreover, assuming that f_k is of logarithmic type σ_k for $k = 1,\cdots,m$, and that therefore $f = \sum f_k$ is of type $\sigma = \sum \sigma_k$, he studies the existence of constants δ_n independent of m and R such that

$$\sum_1^m M(f_k,0,R) \le M\Big(\sum_1^m f_k,0,R\Big) + \sigma\delta_n$$

(see the paper for further details).

Reviewed by *Mongi Blel*

MR1677100 (2000b:14020) 14E30 (14J30)
Peternell, Thomas (D-BAYR-IM); **Serrano, Fernando**
Threefolds with nef anticanonical bundles. (English summary)
Dedicated to the memory of Fernando Serrano.
Collect. Math. **49** (1998), *no. 2-3,* 465–517.

Mori theory, the cornerstone of birational classification theory, aims at finding a minimal model of a non-singular variety, i.e., a birational model with only "mild" singularities and with a nef canonical bundle. At this time the theory is complete in dimension 3 only, but the framework is laid out in all dimensions, except for one (albeit very hard) step.

The subject of the paper under review is a class of varieties at the other end of the spectrum: varieties with a nef anticanonical bundle. The main goal of the paper is to confirm the conjecture that for smooth varieties with a nef anticanonical bundle the Albanese map is a surjective submersion. The authors succeed in proving this in dimension 3 with the help of Mori theory.

Considerable work had been done on questions related to this one before this article. The first author along with J.-P. Demailly and M. Schneider [Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)] had proved the same statement for Kähler manifolds with a semi-positive Ricci curvature. They also proved surjectivity for this larger class of varieties in dimension 3, while Q. Zhang [J. Reine Angew. Math. **478** (1996), 57–60; MR1409052 (97m:14039)] proved surjectivity in all dimensions. Therefore the interesting part of the statement remaining was the smoothness of the Albanese map, which is proved in the present article in dimension 3 and is still open in higher dimensions.

The main idea of the proof is the following: First one can assume that the canonical bundle is not nef, otherwise the statement would follow by the Beauville-Bogomolov-Kobayashi decomposition theorem [A. Beauville, J. Differential Geom. **18** (1983), no. 4, 755–782 (1984); MR0730926 (86c:32030)]. Now if the canonical bundle is not nef, then Mori theory produces an extremal ray that can be contracted and either one gets a fibration over a smaller-dimensional variety or the second Betti number drops.

Certainly not everything is so easy. First of all, in order to run Mori theory one has to allow singularities, but this is not a major concern, as the singularities appearing are indeed very mild and have been extensively studied. The big drawback is that working with them makes the arguments longer and more technical (and of course it is somewhat harder to work with them than with smooth varieties).

The fibration case is relatively well understood, but the other one poses some problems. It is not at all clear that the resulting variety will still have a nef anticanonical bundle. The authors' way of dealing with this problem is to further enlarge the category, namely, they study varieties with an almost nef canonical bundle, i.e., they allow the canonical bundle to be negative on finitely many curves. This actually does the trick, and the induction works.

The authors prove a theorem that has evaded researchers for a long time and they find solutions to many problems along the way; yet one hopes that there is a simpler proof. Indeed the proof given here is dependent on the dimension restriction in more than one way. The dependency does not stop at the one coming from Mori theory. The way the fibration case is handled seems at times ad hoc and uses the benefits of low dimension every now and then. One hopes that perhaps these parts could be replaced with arguments working in all dimensions and one could confine the dimension restriction to the one coming from Mori theory.

{For the entire collection see MR1673613 (99i:00032)}

Reviewed by *Sándor J. Kovács*

MR1660941 (99i:32035) 32L10 (32G13 32J18)

Siu, Yum-Tong (1-HRV)

Invariance of plurigenera.

Invent. Math. **134** (1998), *no. 3,* 661–673.

FEATURED REVIEW.

Starting with the works of Euler and Abel on integrals on algebraic curves it has become clear that all the analytic information about a compact Riemann surface is contained in global holomorphic 1-forms. These are objects that locally can be written as $f(z)dz$, where z is a local coordinate and f is a holomorphic function. Riemann established that the dimension of the vector space of global holomorphic 1-forms is the same as the topological genus of the underlying surface. This is one of the earliest results establishing the topological nature of certain analytically defined numbers.

In the past 100 years much effort was directed towards finding higher-dimensional versions of the result of Riemann. One direction is to replace the dimension of the vector space of global holomorphic 1-forms with the Euler characteristic of the line bundle of holomorphic n-forms where n is the dimension. The Hirzebruch-Riemann-Roch theorem expresses this number in terms of the Chern classes of the complex manifold. The situation becomes murkier if we would like to study the vector space of global holomorphic n-forms. Hodge theory shows that this is naturally a subspace of one of the topological cohomology groups, but it is still unknown how to convert this information into an explicit formula. The theory does, however, imply the following weaker statement: If X_t is a family of smooth, complex, projective varieties depending continuously on a parameter t then the dimension of the vector space of global holomorphic n-forms on X_t is a locally constant function of t.

It has also been realized that in higher dimensions this single number does not carry enough information about a variety. Instead, one should look at global sections of tensor powers of the line bundle of n-forms. This line bundle is frequently denoted by $K_X^{\otimes m}$ and its local sections can be written as

$$f(z_1,\cdots,z_n)(dz_1\wedge\cdots\wedge dz_n)^{\otimes m}.$$

The dimension of the vector space of global sections of $K_X^{\otimes m}$ is called the mth plurigenus of X and it is denoted by $P_m(X)$. A very natural question is: are the numbers $P_m(X)$ topological in nature? Very little is known about this problem. The following weaker conjecture received much attention because it is especially important in moduli problems: Let X_t be a family of smooth, complex, projective varieties depending continuously on a parameter t. Are the $P_m(X_t)$ locally constant functions of t?

The traditional approach is to convert $P_m(X_t)$ into an Euler characteristic and then use the Hirzebruch-Riemann-Roch theorem. Unfortunately, the higher cohomology groups of $K_X^{\otimes m}$ are usually nonzero and they vary wildly within the birational equivalence class of X. This, however, led to the first approaches to the conjecture. In many cases one can find a suitable birational model X^* such that $P_m(X) = P_m(X^*) = \chi(K_{X^*}^{\otimes m})$. If such a model can be found for every X_t in a family in a continuous manner then we arrive at a solution of the conjecture. This approach has been successfully carried out for surfaces [S. Iitaka, J. Math. Soc. Japan **22** (1970), 247–261; MR0261639 (41 #6252)] and for threefolds [J. Kollár and S. Mori, J. Amer. Math. Soc. **5** (1992), no. 3, 533–703; MR1149195 (93i:14015)]. Some cases were settled in all dimensions by M. N. Levine [Invent. Math. **74** (1983), no. 2, 293–303; MR0723219 (85d:32054)].

The paper under review approaches the question differently. Instead of changing the variety X, we would like to change the line bundle $K_X^{\otimes m}$ in such a way that we do not change the space of global sections but we do eliminate higher cohomology groups. (In general this is only possible if $K_X^{\otimes m}$ is replaced by a sheaf which is not locally free.) The key step is to introduce a metric on the line bundle $K_X^{\otimes m}$ which blows up along a subvariety in a carefully controlled manner. These ideas were introduced in algebraic geometry by Iitaka and his school [cf. Y. Kawamata, K. Matsuda and K. Matsuki, in *Algebraic geometry, Sendai, 1985*, 283–360, Adv. Stud. Pure Math., 10, North-Holland, Amsterdam, 1987; MR0946243 (89e:14015)] and in complex manifold theory by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)].

In the paper under review the author proves the deformation invariance of plurigenera for varieties where sections of $K_X^{\otimes m}$ separate points over a dense open set. These are usually called varieties of general type. For such varieties there is a natural choice of the singular metric. Let g_i be a basis of the global sections of $K_X^{\otimes m}$; then we can declare that $(\sum |g_i|^2)^{1/2m}$ is a section of K_X which has norm 1 everywhere. We get a singular metric which depends on the choice of the basis and m. The proof now depends on two key observations. First, all global sections of $K_X^{\otimes m}$ are L^2 in these metrics and this remains so after small perturbations of the metric. In particular, the dependence on m is not a serious problem. Second, the extension theorems of T. Ohsawa and K. Takegoshi [Math. Z. **195** (1987), no. 2, 197–204; MR0892051 (88g:32029)] and L. Manivel [Math. Z. **212** (1993), no. 1, 107–122; MR1200166 (94e:32050)] can be globalized to extend L^2 sections from X_0 to nearby X_t with a suitable perturbation of the metric. The perturbations required for the second part are just small enough that they do not matter if all choices are made carefully.

Besides solving a long-standing problem, the author's method is applicable to several other problems concerning deformations of varieties. Two such results are in papers by Kawamata [J. Amer. Math. Soc. **12** (1999), no. 1, 85–92; MR1631527 (99g:14003)] and N. Nakayama ["Invariance of the plurigenera of algebraic varieties", Preprint, Res. Inst. Math. Sci., Kyoto Univ., Kyoto, 1998; per revr.].

Reviewed by *János Kollár*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1647555 (2000c:32067) 32L20 (32L10)

de Cataldo, Mark Andrea A. (D-MPI)

Singular Hermitian metrics on vector bundles. (English summary)

J. Reine Angew. Math. **502** (1998), 93–122.

The purpose of the paper under review is to extend the techniques related to singular Hermitian metrics on line bundles to the higher-rank case. This generalization is carefully carried out so that it yields natural extensions to works of Nadel, Demailly and Siu, among which the following are well known: multiplier ideal sheaves, Nadel's version of the Kawamata-Viehweg vanishing theorem, and effective very ampleness.

(See [Y. T. Siu, Ann. Inst. Fourier (Grenoble) **43** (1993), no. 5, 1387–1405; MR1275204 (95f:32035); J.-P. Demailly, Invent. Math. **124** (1996), no. 1-3, 243–261; MR1369417 (97a:32035); G. Fernández del Busto, J. Algebraic Geom. **5** (1996), no. 3, 513–520; MR1382734 (98d:14007)] for related work.)

Reviewed by *Thierry Bouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1643933 (99k:14062) 14J30 (14J40 32J18 32J25)
Peternell, Thomas (D-BAYR-IM)
Moishezon manifolds and rigidity theorems. (English summary)
Bayreuth. Math. Schr. No. 54 (1998), 1–108.

Let X be a complex compact manifold of dimension n; X is said to be a Moishezon manifold if the transcendence degree of the field of meromorphic functions over $\mathbf{C}$ is equal to n. B. Moishezon studied these manifolds and also proved that they become projective after a finite number of monoidal transformations (i.e. blow-ups) with nonsingular center. In the present paper the author discusses some central questions relative to these manifolds with special emphasis on dimension 3.

He starts by recalling some projectivity criteria: a Moishezon manifold is projective if and only if it is a Kähler manifold (this is due to Moishezon) or if and only if it has a line bundle whose curvature is semipositive and positive in at least one point (this is due to Siu and Demailly).

Then he proves a new projectivity criterion for Moishezon 3-folds X which says that X is projective if and only if there is no irreducible curve $C \subset X$ homologous to zero and $\mathrm{NE}(X) \cap -\overline{\mathrm{NE}(X)} = 0$, where $\mathrm{NE}(X)$ is the cone of effective curves in the vector space of 1-cycles modulo numerical equivalence.

The proof uses the so-called Mori theory; it is not known whether the closure can be omitted, and also, at the moment, there is not a clear generalization to higher dimension.

Among others things, in the rest of the paper the author proves that a Moishezon 3-fold X homeomorphic to $\mathbf{P}^3$ is actually (isomorphic to) $\mathbf{P}^3$ and that every nonprojective Moishezon 3-fold contains a rational curve. The latter is a conjecture in higher dimension.

The interested reader can find similar results and arguments in [J. Kollár, in *Surveys in differential geometry (Cambridge, MA, 1990)*, 113–199, Lehigh Univ., Bethlehem, PA, 1991; MR1144527 (93b:14059)].

Reviewed by *Marco Andreatta*

MR1645092 (99h:32035) 32L20
Koziarz, Vincent (F-NANCS-IE)
Annulation de la cohomologie pour les fibrés semi-positifs. (French. English, French summaries) [Vanishing theorems for semipositive bundles]
C. R. Acad. Sci. Paris Sér. I Math. **327** (1998), *no. 2,* 143–148.

This paper presents a general technique for proving vanishing theorems for holomorphic vector bundles over some special Hermitian complex-analytic manifolds. The main idea, which goes back to J.-P. Demailly [in *Séminaire d'analyse P. Lelong-P. Dolbeault-H. Skoda, années 1983/1984*, 88–97, Lecture Notes in Math., 1198, Springer, Berlin, 1986; MR0874763 (88f:32069)], is to describe these groups as an inductive limit of L^2 weighted cohomology groups, and reduce to L^2-cohomology vanishing theorems. This only requires a sufficient collection of weights so that any C^∞ section of E eventually becomes L^2. This is formalized here under the term "having enough metrics", further arguments boiling down to Demailly's. The rest of the paper presents various consequences of this technique, among them a relative Nakano vanishing theorem over irreducible Kähler analytic spaces. Unfortunately, the required collection of weights is not made explicit here.

Reviewed by *Thierry Bouche*

MR1642740 (99f:14017) 14E05 (14J99)
Tsai, I-Hsun (RC-NTAI)
Chow varieties and finiteness theorems for dominant maps.
J. Algebraic Geom. **7** (1998), *no. 4,* 611–625.

The Iitaka-Severi conjecture states that the number of smooth complex projective varieties of general type that arise as a dominant rational image of X is finite up to birational equivalence.

This article is one of many of the author's that are related to this topic. The central goal of the present article is giving effective estimates in the case when the target is assumed to be a surface.

The estimates heavily depend on effective very ampleness of adjoint bundles (as in Fujita's conjecture) of J.-P. Demailly, J. Kollár, L. Ein-R. Lazarsfeld, and Y. T. Siu. Therefore, although these estimates are probably far from being sharp, as soon as there are better estimates for very ampleness they should yield better ones in this case.

Besides using effective very ampleness results, the paper studies Chow varieties and uses results of F. M. E. Catanese [J. Algebraic Geom. **1** (1992), no. 4, 561–595; MR1174902 (93j:14005)].

Reviewed by *Sándor J. Kovács*

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MR1639560 (99m:58204) 58G26 (32L07 32S20)
Yoshikawa, Ken-ichi (J-NAGO-GM)
Smoothing of isolated hypersurface singularities and Quillen metrics. (English summary)
Asian J. Math. **2** (1998), *no. 2,* 325–344.

The author considers the following. Let $\pi\colon X\to S=\{t\in\mathbf{C};\ |t|<1\}$ be a proper surjective holomorphic map of complex manifolds. Suppose that π is of maximal rank outside of a finite number of points in $X_0=\pi^{-1}(0)$; this family (π,X,S) is then said to be a smoothing of isolated hypersurface singularities (IHS). Let g_X be a Kähler metric of X and $g_{X/S}$ be the induced metric on TX/S. Let (E,h) be a holomorphic Hermitian vector bundle on X. Write $\lambda(E)$ for the determinant line bundle associated with cohomologies, and $\|\cdot\|_Q$ for its Quillen metric relative to $g_{X/S}$ and h. Assume that (π,X,S) is projective over S. Then the main theorem of this paper computes the curvature current of $\|\cdot\|_Q$:

$$c_1(\lambda(E),\|\cdot\|_Q)=\frac{(-1)^{n+1}}{(n+2)!}r(E)\mu(X_0)\delta_0+\pi_*(\mathrm{Td}(TX/S,g_{X/S})\mathrm{ch}(E,h))^{(1,1)}.$$

Here $\dim X=n+1$, $r(E)$ is the rank of E, δ_0 the Dirac measure at 0 and $\mu(X_0)$ is the Milnor number of the singular fiber. Moreover the second term on the right-hand side lies in $L^p_{\mathrm{loc}}(S)$ for some $p>1$ depending only on $\mathrm{Sing}\,X_0$. (See [J.-M. Bismut and J.-B. Bost, Acta Math. **165** (1990), no. 1-2, 1–103; MR1064578 (91h:58122); J.-M. Bismut, J. Algebraic Geom. **6** (1997), no. 1, 19–149; MR1486991 (2000a:58084)] for the case of ordinary singularities.) The main theorem is applied in this paper to study the asymptotic behavior of analytic torsion in the case of smoothing of IHS, whose principal term turns out to be determined by the total Milnor number of the singular fiber and determinant of period integrals. (See [M. S. Farber, J. Differential Geom. **41** (1995), no. 3, 528–572; MR1338482 (96e:58165)] for a related result.) Concerning the proof, the author first proves the main theorem in the case where (X,E) is globalizable, i.e. X can be embedded in a projective algebraic manifold of the same dimension, and E extends as a coherent sheaf. For this part a main tool is a theorem of Bismut and G. Lebeau [Inst. Hautes Études Sci. Publ. Math. No. 74 (1991), ii+298 pp. (1992); MR1188532 (94a:58205)] and the method of proof is similar to that of Bismut [op. cit.]. After this is done, using an approximation result by J.-P. Demailly, L. Lempert and B. Shiffman [Duke Math. J. **76** (1994), no. 2, 333–363; MR1302317 (95i:32022)] the author proceeds with the approximation by algebraic families and completes the proof of the main theorem. All of the proofs in this paper are essentially analytical in nature; concerning an algebraic approach to similar problems, see [Y. L. L. Tong and I. H. Tsai, Comm. Math. Phys. **171** (1995), no. 3, 589–606; MR1346173 (96j:58177)].

Reviewed by *I-Hsun Tsai*

MR1637927 (99k:32049) 32L10 (32C17)
Takayama, Shigeharu (J-OSAKEGS)
Adjoint linear series on weakly 1-complete Kähler manifolds. I. Global projective embedding.
Math. Ann. **311** (1998), *no. 3,* 501–531.

The paper under review successfully applies the recent developments of effective adjunction theory for ample line bundles over weakly 1-complete Kähler manifolds. It has been known for a while (T. Ohsawa [Proc. Japan Acad. Ser. A Math. Sci. **55** (1979), no. 5, 193–194; MR0533546 (80e:32017)], answering a question of Nakano) that a line bundle L may be positive over such a manifold, but not ample, in the sense that global holomorphic sections of a high tensor power generate 1-jets at every point (and hence define a global one-to-one immersion into some projective space). The original idea, based on recent work related to the Fujiki conjecture, is to show that some adjoint bundles $K_X \otimes L^{\otimes m}$ will be ample for large m. This idea is shown to work extremely well on the mentioned problem. The main theorem, whose proof is rather involved but very cleanly exposed, yields an explicit bound for the separation of points by sections of $K_X \otimes L^{\otimes m}$ over "pseudo-balls" (or level sets of an exhaustive psh function). Its proof follows the scheme developed by Angehrn and Siu or Tsuji in the compact case: the Riemann-Roch formula is replaced by Demailly's holomorphic Morse inequalities (applied on relatively compact level sets) in order to construct singular metrics with controlled singularity at some points, thanks to Shokurov's concentration method; the Nadel vanishing theorem is then applied to that situation. Among the fine results derived from these main technical results, we can single out: "Let X be an n-dimensional weakly 1-complete manifold with a positive line bundle L. Then the line bundle $(K_X \otimes L^{\otimes m})^{\otimes n+2}$ is very ample for $m > n(n+1)/2$." Also, "If the anticanonical bundle $K_X^{\otimes -1}$ is positive, X is Stein iff X has no compact complex subspaces of positive dimension."

Reviewed by *Thierry Bouche*

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MR1632996 (99e:32013) 32C30

Meo, Michel (F-ANGR)

Inégalités d'auto-intersection pour les courants positifs fermés définis dans les variétés projectives. (French) [Self-intersection inequalities for closed positive currents defined in projective varieties]

Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **26** (1998), *no. 1,* 161–184.

Let T be a closed positive current of bidimension (p,p) defined on a complex variety X of dimension n. For all $c>0$, the sets $E_c=\{x\in X:\ \nu_T(x)\geq c\}$ are analytic subsets of dimension at most p. For X compact and equipped with a Kähler metric ω, J.-P. Demailly [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)] gave an upper bound on the degree with respect to ω of q-dimensional irreducible components in E_c in terms of the cohomology class of T when T is of bidimension $(n-1,n-1)$; namely, he proved that

$$\sum_{k\geq 1}(\nu_q,k-b_{n-1})\cdots(\nu_q,k-b_q)\{Z_q,k\}\{\omega\}^q \leq$$

$$(\{T\}+b_{n-1}\{u\})\cdots(\{T\}+b_q\{u\})\{\omega\}^q,$$

with u a semipositive cohomology class in X such that $c_1(\mathcal{O}_{TX}(1))+\pi_X^*\{u\}$ is semipositive, $b_q=\inf\{c>0;\ \dim E_c\leq q\}$, $b_{-1}=\max_{x\in X}\nu(T,x)$ and $(Z_{q,k})_{k\geq 1}$ is the at most countable family of q-dimensional irreducible components in E_c for $c\in(b_q,b_{q-1}]$ and $\nu_{q,k}=\min_{x\in Z_{q,k}}\nu(T,x)$.

In the case $X=\mathbf{P}^n$, one can take $u=0$, and the above inequality becomes

$$\sum_{k\geq 1}(\nu_{q,k}-b_{n-1})\cdots(\nu_{q,k}-b_q)\delta(Z_{q,k})\leq\delta(T)^{n-q}.$$

The author's goal in this paper is to prove an analogous inequality for a closed positive current T in $\mathbf{P}^n$:

$$\sum_{k\geq 1}(\nu_{q,k}-b_{n-1})\cdots(\nu_{q,k}-b_q)\delta(Z_{q,k})\leq\delta(T)^{p+1-q}.$$

The idea is to construct a closed positive current of bidegree $(1,1)$ in $\mathbf{P}^n$ which has the same degree as T and the same Lelong number at every point. This construction is similar to that of Lelong and Skoda, which consists of constructing a potential associated with the current T.

When T is defined on a projective variety X and ω is a Kähler metric on X defining an entire cohomology class, the author proves the inequality

$$\sum_{k\geq 1}(\nu_{q,k}-b_{n-1})\cdots(\nu_{q,k}-b_q)\delta(Z_{q,k})\leq C\delta(T)^{p+1-q}.$$

He uses Matsusaka's embedding theorem to prove the existence of the constant $C>0$.

Reviewed by *Mongi Blel*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1622747 (99e:32047) 32J17
Campana, Frédéric (F-NANC); **Demailly, Jean-Pierre** (F-GREN-F);
Peternell, Thomas (D-BAYR-IM)
The algebraic dimension of compact complex threefolds with vanishing second Betti number. (English summary)
Compositio Math. **112** (1998), *no. 1,* 77–91.

A compact complex threefold with vanishing second Betti number cannot be algebraic or Kähler. Then the natural question is: What possibilities are there for the algebraic dimension of such manifolds? (Algebraic dimension is the transcendence degree of the field of meromorphic functions over **C**.)

The main result of this article is that if the algebraic dimension is positive, then the topological Euler characteristic is 0 and then either $b_1 = 0$ and $b_3 = 2$ or $b_1 = 1$ and $b_3 = 0$. An interesting corollary is that S^6 does not admit a complex structure with a non-constant meromorphic function. The authors deduce the main result as a straightforward consequence of a vanishing theorem for vector bundles twisted by generic elements of Pic^0. Examples of threefolds with positive algebraic dimension and vanishing second Betti number and topological Euler characteristics are also given showing that the result is optimal.

The authors also investigate more deeply threefolds with vanishing second Betti number and whose algebraic dimension is 1.

This is another interesting article from these distinguished authors presenting ideas of great interest to algebraic and complex analytic geometers alike.

Reviewed by *Sándor J. Kovács*

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MR1613763 (99d:32037) 32L07 (53C55)

Mihai, Paun [Păun, Mihai] (F-GREN-FM)

Sur les variétés kählériennes compactes à classe de Ricci numériquement effective. (French) [On compact Kähler manifolds with numerically effective Ricci class]

Bull. Sci. Math. **122** (1998), *no. 2,* 83–92.

In this paper compact Kähler manifolds with numerically effective (nef) anticanonical bundle are investigated; in particular the structures of the Albanese map and of the fundamental group are studied. The main result is the following: Let X be a compact Kähler manifold of complex dimension n with nef anticanonical bundle. Then: (i) $h^1(X, \mathcal{O}_X) \leq n$; (ii) if Γ is a finitely generated subgroup of the fundamental group of X, then there exists a normal subgroup Γ_1 of Γ of finite index generated by at most $4^{2n}+1$ elements. The proof makes use of the Aubin-Calabi-Yau theorem and of some results of Gromov and Demailly-Peterell-Schumacher. This paper continues the results obtained in papers by F. Campana and T. Peternell [Math. Ann. **289** (1991), no. 1, 169–187; MR1087244 (91m:14061)] and J.-P. Demailly, Peternell and M. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037); Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)].

Reviewed by *Antonella Nannicini*

MR1632491 (99f:32021) 32F05 (32F07)

Lelong, Pierre

Remarks on pointwise multiplicities in complex spaces.

Dedicated to Professor Vyacheslav Pavlovich Zahariuta.

Linear Topol. Spaces Complex Anal. **3** (1997), 112–119.

Let F be a holomorphic function in a neighborhood of the closed unit polydisk $D = \{x \in \mathbf{C}^n\colon \sup_k |x_k| \leq 1\}$ with $F(0) = 0$. The theme of the paper is a discussion of generalizations of the classical notion of the multiplicity m of the zero of F at the origin; i.e., the order of the zero of the restriction of F to a generic complex line; the non-generic lines form a small exceptional set. For any n-tuple $a=(a_1,\cdots,a_n)$ with $a_k>0$, the index $I(F,0,a)$ of F at the origin 0 is defined as follows: for $0\le w\le 1$, replace x_k by $w^{a_k}x_k$ in the power series for $F(x)$ at 0 to get $F(w,x):=\sum_{J\ge I}w^J P_J(x)$, where $P_J(x)$ are polynomials. Then

$$I=I(F,0,a)=\lim_{w\to 0}\frac{\log|F(w,x)|}{\log w}$$

is the degree in w of $w\to F(w,x)$ for generic x, the exceptional set being the algebraic set $P_I(x)=0$. This recovers the classical notion of multiplicity of F at 0 if $a_1=\cdots=a_n=1$. This notion is extended to nonpositive plurisubharmonic (psh) functions f which are $-\infty$ at 0; the index, denoted $s(f,0,a)$, is defined by considering

$$h(w,x):=\frac{f(w^{a_1}x_1,\cdots,w^{a_n}x_n)}{\log 1/w};$$

then $K(x):=\limsup_{w\to 0}h(w,x)$ has the property that its upper semicontinuous regularization $K^*(x)$ is constant; this constant value is defined to be $-s(f,0,a)$. Here, the exceptional set $A:=\{x\in\mathbf{C}^n\colon\ K(x)<-s(f,0,a)\}$ is pluripolar. The index $s(f,0,a)$ reduces to the Lelong number ν of the current $(1/2\pi)dd^cf$ at the origin when $a_1=\cdots=a_n=1$. In particular, if

$$f(x):=\frac{1}{2}\log\sum_j|F_j(x)|^2$$

where F_j are holomorphic in a neighborhood of D, we have $s(f,0,a)=\inf_j I(F_j,0,a)$. J.-P. Demailly's notion of a generalized Lelong number $\nu(T,0,\varphi)$ of a closed positive current for a psh weight φ (with $\exp\varphi$ continuous) is recalled [Mém. Soc. Math. France (N.S.) No. 19 (1985), 124 pp.; MR0813252 (87g:32030); Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)]; for certain special weights φ, one gets nice formulas for $\nu(dd^cf,0,\varphi)$ for f psh. Finally, the author studies polynomial mappings $P=(P_1,\cdots,P_n)\colon\mathbf{C}^n\to\mathbf{C}^n$, $\deg P_k=d_k$, near points x of their zero sets W_P which have regular multiplicity $\nu(x)$, i.e.,

$$\nu(x):=\lim_{y\to x}\frac{\log|P(x)|}{\log|x-y|}\ge 1.$$

Let $\mu_P:=(2\pi)^{-n}(dd^c\log|P|)^n$ be the Monge-Ampère measure of $\log|P|$. Let W_P be compact and of dimension zero. It is shown that if $\{\zeta_s\}$ are the isolated zeros of P with regular multiplicity, then

$$\mu_P=\sum_s[\nu(\zeta_s)]^n\le d_1\cdots d_n.$$

{For the entire collection see MR1632477 (99a:00052)}

Reviewed by *Norman Levenberg*

MR1622653 (99c:32006) 32C30 (32F07)
Coman, Dan (R-CLUJMI)

Integration by parts for currents and applications to the relative capacity and Lelong numbers.

Mathematica **39(62)** (1997), *no. 1,* 45–57.

Let Ω be a bounded domain in $\mathbf{C}^n$, let u and v be locally bounded plurisubharmonic functions in Ω, and let T be a closed positive current of bidimension (1,1). When is the integration by parts formula

(1) $$\int_\Omega u dd^c v \wedge T = \int_\Omega v dd^c u \wedge T$$

or the inequality

(2) $$\int_\Omega u dd^c v \wedge T \leq \int_\Omega v dd^c u \wedge T$$

valid? The author first shows that if u and v are negative and agree outside a compact subset of Ω, then equality (1) holds; he then relaxes the hypotheses a bit to give weaker conditions sufficient to guarantee (1) or (2); e.g., (1) holds if Ω is hyperconvex, u and v are negative exhaustion functions for Ω, and both $\int_\Omega dd^c u \wedge T$ and $\int_\Omega dd^c v \wedge T$ are finite. These results are used to obtain upper and lower bounds for the relative capacity of a regular compact sublevel set $K = \{z \in \Omega\colon\ \psi(z) \leq -\gamma < 0\}$ of a hyperconvex domain $\Omega = \{z \in \Omega'\colon\ \psi(z) < 0\} \subset\subset \Omega'$ where $\psi \in C^2(\Omega)$ and $\psi = 0$ on $\partial\Omega$ and also to generalize slightly a formula of J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)] regarding Lelong numbers with weights.

Reviewed by *Norman Levenberg*

MR1614576 (2000a:32042) 32L05 (14F05 32J27)
Mourougane, Christophe (F-GREN-F)
Images directes de fibrés en droites adjoints. **(French. English summary)** **[Direct images of adjoint line bundles]**
Publ. Res. Inst. Math. Sci. **33** (1997), *no. 6,* 893–916.

Recall that some results for the direct image of the canonical fiber bundle over a projective manifold had been obtained by Y. Kawamata [Compositio Math. **43** (1981), no. 2, 253–276; MR0622451 (83j:14029)] and J. Kollár [Ann. of Math. (2) **123** (1986), no. 1, 11–42; MR0825838 (87c:14038)]. A study of the numerically effective properties of fiber bundles was given by J.-P. Demailly, T. Peternell and M. H. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)]. In the paper under review the more general situation of holomorphic line bundles L over compact complex manifolds M is considered and especially its direct images $\varphi_* L$ under a smooth holomorphic mapping $\varphi\colon M \to N$. The author's attention is focussed on the properties of ampleness and positivity and on the question of their implications for the direct image $\varphi_* L$ or at least for that of the adjoint line bundle. The methods used include a variety of techniques, algebraic and analytic in spirit, which help the author to separate some original algebraic version based on some algebraic treatment of positivity, without curvature tensors, etc. Applications are given. Sometimes the reviewer found the author's exposition hard to understand.

Reviewed by *S. Dimiev*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1492596 (99a:32033) 32H20 (32J10 32L05)

Demailly, Jean-Pierre (F-GREN-F)

Variétés projectives hyperboliques et équations différentielles algébriques. (French) [Hyperbolic projective varieties and algebraic differential equations]

Journée en l'Honneur de Henri Cartan, 3–17, *SMF Journ. Annu.*, 1997, *Soc. Math. France*, *Paris*, 1997.

This is a very well-written survey of some of the more recent developments in the theory of holomorphic curves in algebraic varieties, a holomorphic curve in an algebraic variety being a holomorphic mapping from the complex plane to the variety. The author's survey concentrates in particular on the recent work of Y. T. Siu and S.-K. Yeung [Amer. J. Math. **119** (1997), no. 5, 1139–1172; MR1473072 (98h:32044)]. An extensive bibliography is also provided. Although probably best suited for those readers already familiar with the language of complex differential geometry, and in particular the language used when working with Hermitian vector bundles and meromorphic connections, the survey is for the most part a very accessible introduction to some of the latest developments in the field and assumes little prior knowledge of Nevanlinna theory, algebraic geometry, or the other techniques commonplace in the study of holomorphic curves.

The reviewer's translation of the author's first paragraph reads as follows: "The goal of this text is to offer an introduction, which is as elementary as possible, to an important result concerning the geometry of the images of holomorphic curves in complex algebraic varieties. This result finds its origin in the fundamental work of A. Bloch [J. Math. Pures Appl. (9) **5** (1926), 19–66; JFM 52.0373.05] and in the thesis of H. Cartan [Ann. Sci. École Norm. Sup. **45** (1928), 255–346; JFM 54.0357.06]. The proof that we give here is a very recent contribution by Siu and Yeung [op. cit.]. It proceeds in a relatively simple manner with help from classical estimates in Nevanlinna theory, like the lemma on the logarithmic derivative, and by making use of differential operators such as Wronskians, all ideas whose germs were already sown in Henri Cartan's thesis [op. cit.]."

More specifically, the author explains techniques for showing that a holomorphic curve in an algebraic variety is algebraically degenerate, meaning that its image is contained in a proper algebraic subvariety. A fundamental conjecture along these lines is the conjecture of Green and Griffiths stating that a holomorphic curve in a variety of general type must be algebraically degenerate. The survey is centered around the following fundamental vanishing theorem. If $f\colon \mathbf{C} \to X$ is a holomorphic curve in a projective variety X, if L is a positive line bundle on X, and if P is an algebraic differential operator on X with values in L^{-1}, then P applied to f is zero. For hypersurfaces in projective space, this theorem can be applied to Wronskian-like differential operators coming from explicitly constructed meromorphic connections, as in the work of A. M. Nadel [Duke Math. J. **58** (1989), no. 3, 749–771; MR1016444 (91a:32036)]. This results in specific examples of general type projective varieties in which every holomorphic curve is algebraically degenerate. This method also proves that in some of these varieties, the image of every holomorphic curve must be constant; such varieties are called hyperbolic.

{For the entire collection see MR1492594 (98h:00041)}

Reviewed by *William A. Cherry*

MR1492594 (98h:00041) 00B30
Hirzebruch, Friedrich; **Demailly, Jean-Pierre** (F-GREN-F); **Lannes, Jean**
★**Journée en l'Honneur de Henri Cartan. (French) [Conference in Honor of Henri Cartan]**
SMF Journée Annuelle [SMF Annual Conference], 1997.
Société Mathématique de France, *Paris*, 1997. iv+27 *pp*.

Contents: F. Hirzebruch, Learning complex analysis in Muenster-Paris, Zuerich and Princeton from 1945 to 1953 (1–2); Jean-Pierre Demailly, Variétés projectives hyperboliques et équations différentielles algébriques [Hyperbolic projective varieties and algebraic differential equations] (3–17); Jean Lannes, Divers aspects des opérations de Steenrod [Various aspects of the Steenrod operations] (18–27).
{Most of the papers are being reviewed individually.}

MR1492539 (99b:32037) 32H20 (14J40 32L10)
Demailly, Jean-Pierre (F-GREN-F)
Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. (English summary)
Algebraic geometry—Santa Cruz 1995, 285–360, *Proc. Sympos. Pure Math.*, 62, *Part* 2, *Amer. Math. Soc.*, *Providence*, *RI*, 1997.

The article under review is an expanded version of five lectures delivered at the Santa Cruz AMS Summer School on Algebraic Geometry. It proposes an important framework for solving several geometry questions related to hyperbolicity in the sense of Kobayashi. This framework was initiated by M. Green and P. Griffiths [in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557 (82h:32026)]. Aiming, among other things, to fix a gap in Green-Griffiths' proof of the pointwise version of the Ahlfors-Schwarz lemma for jet differentials, Demailly introduces the concept of "directed manifold" and an associated tower of projective bundles over X (called Semple jet bundles). The Ahlfors-Schwarz lemma is then established in this setting, and the proof of Bloch's theorem is recovered following the approach of Green and Griffiths. Several important new results are also obtained in this paper. Moreover, the author believes that the Semple bundle construction should be an efficient tool to calculate the case locus; therefore several important open problems in the theory of complex hyperbolicity hopefully could be settled under this framework. It should be noted that, since the appearance of this article, Demailly has proved jointly with J. El Goul that every generic surface in $\mathbf{P}^3$ of degree greater than or equal to 42 is Kobayashi hyperbolic. It has been conjectured by Kobayashi that every generic surface in $\mathbf{P}^3$ of degree greater than or equal to 5 is Kobayashi hyperbolic.

This paper is a quite important contribution to the theory of complex hyperbolicity. The paper is self-contained and the exposition is excellent. It is highly recommended to the experts in this field, as well as to anyone who desires a general overview of this subject.

We will now try to outline this article. A complex directed manifold is a pair (X,V) where X is a complex manifold and V is a holomorphic subbundle of T_X; here T_X is the tangent bundle of X. To study the complex hyperbolicity of (X,V), a well-known major technique is the so-called "negative curvature method". The method is based on the following observation: by the Ahlfors-Schwarz lemma, the existence of a Hermitian metric on the line bundle $\mathcal{O}_{\mathbf{P}(V)}(-1)$ over $\mathbf{P}(V)$ (i.e. a Finsler metric on V) with negative curvature implies that (X,V) is hyperbolic. Let us recall here how to construct such a metric. Assume that V^* is "very big" in the following sense: there exist an ample line bundle L and a sufficiently large integer m such that the global sections $H^0(X, S^mV^*\otimes L^{-1})$ generate all fibers over $X\smallsetminus Y$, for some analytic subset $Y\subset X$. Let $\sigma_1,\cdots,\sigma_N$ be such global sections, and define

$$N(\xi)=\Big(\sum_{1\le j\le N}|\sigma_j(x)\cdot\xi^m|^2\Big)^{1/2m},\quad \xi\in V_x^*;$$

N then gives rise to such a metric. Therefore we have the following result: Let (X,V) be a directed complex manifold. Assume that V^* is "very big". Then every entire curve $f\colon \mathbf{C}\to X$ tangent to V satisfies $f(\mathbf{C})\subset Y$, where Y is the subset of X defined above. In particular, if V^* is ample, then (X,V) is hyperbolic.

The heart of the article consists of Chapters 4 to 7. They are devoted to extending the above result to k-jet differentials. The idea is based on the important fact, first observed by Green and Griffiths, that the Ahlfors-Schwarz lemma still works for k-jet differentials, and thus k-jet negativity also implies hyperbolicity. Unfortunately, there is a slight technical gap in Green and Griffiths' approach in the step proving the pointwise Ahlfors-Schwarz lemma for jet differentials. In his paper, Demailly fills the gap in the case of invariant jet differentials, and also extends the result to the more general situation of directed manifolds. (Note: Another solution has been provided later by Y. T. Siu and S.-K. Yeung by means of Nevanlinna's second main theorem [see Amer. J. Math. **119** (1997), no. 5, 1139–1172; MR1473072 (98h:32044)].)

To do this, Demailly introduces a canonical tower of projective bundles (also called Semple jet bundles). Given a complex directed manifold (X,V), a new complex directed manifold $(\widetilde{X},\widetilde{V})$ is produced as follows. Let $\widetilde{X}=\mathbf{P}(V)$ be the projectivized bundle of lines of V, and let $\widetilde{V}\subset T_{\widetilde{X}}$ be the subbundle of $T_{\widetilde{X}}$ defined as follows: for every point $(x,[v])\in\widetilde{X}$ associated with a vector $v\in V_x\smallsetminus\{0\}$,

$$\widetilde{V}_{(x,[v])}=\{\xi\in T_{\widetilde{X},(x,[v])}\colon \pi_*\xi\in\mathbf{C}\},\quad V_x\subset T_{X,x},$$

where $\pi\colon\widetilde{X}=\mathbf{P}(V)\to X$ is the natural projection. The projectivized k-jet bundle $\mathbf{P}_kV=X_k$ (or Semple k-jet bundle) and the associated subbundle $V_k\subset T_{X_k}$ are defined inductively by $(X_0,V_0)=$

(X,V), $(X_k,V_k)=(\widetilde{X}_{k-1},\widetilde{V}_{k-1})$. Every non-constant tangent trajectory $f\colon \Delta_R\to X$ of (X,V) lifts to a well-defined and unique tangent trajectory $f_{[k]}\colon \Delta_R\to X_k$ of (X_k,V_k).

The author shows that the Ahlfors-Schwarz lemma works at each level of the tower of projective bundles. That is: If (X,V) has a k-jet metric h_k on the line bundle $\mathcal{O}_{\mathbf{P}_kV}(-1)$ (i.e. a Finsler metric on the vector bundle V_{k-1} over $\mathbf{P}_{k-1}V$), with negative jet curvature, then every entire curve $f\colon \mathbf{C}\to X$ tangent to V satisfies $f_{[k]}(\mathbf{C})\subset\Sigma_{h_k}$, where Σ_{h_k} is the singularity set of the metric h_k.

To produce such metrics h_k, one uses global sections of $H^0(\mathbf{P}_kV,\mathcal{O}_{\mathbf{P}_kV}(m)\otimes\pi_{0,k}^*L^{-1})$, where L is an ample line bundle on X. The author also shows that the direct images $(\pi_{0,k})_*\mathcal{O}_{\mathbf{P}_kV}(m)$ can be viewed as bundles of algebraic differential operators of order k and degree m, acting on germs of curves and invariant under reparametrization. This bundle is denoted by $E_{k,m}(V^*)$. Therefore $H^0(\mathbf{P}_kV,\mathcal{O}_{\mathbf{P}_kV}(m)\otimes\pi_{0,k}L^{-1})\simeq H^0(X,E_{k,m}(V^*)\otimes L^{-1})$.

The above discussion leads to the following result: Assume that there exist integers $k,m>0$ and an ample line bundle L on X such that $H^0(X,E_{k,m}(V^*)\otimes L^{-1})$ has nonzero sections $\sigma_0,\cdots,\sigma_N$. Let $Z\subset\mathbf{P}_kV$ be the base locus of these sections. Then every entire curve $f\colon\mathbf{C}\to X$ tangent to V satisfies $f_{[k]}(\mathbf{C})\subset Z$. In other words, for every global parametrization invariant polynomial differential operator P with values in L^{-1}, every entire curve f as above must satisfy the algebraic differential equation $P(f)=0$.

The dimension

$$h^0(X,E_{k,m}(V^*)\otimes L^{-1})=\dim H^0(X,E_{k,m}(V^*)\otimes L^{-1})$$

can be computed by using the Riemann-Roch theorem and a vanishing theorem due to Bogomolov. In particular, in the surface case, the Riemann-Roch theorem yields the following (see Chapter 13, Corollary 13.9): If X is an algebraic surface of general type and L an ample line bundle over X, then

$$h^0(X,E_{2,m}T^*X\otimes\mathcal{O}(-L))\geq\frac{m^4}{648}(13c_1^2-9c_2)-O(m^3).$$

In particular, every smooth surface $X\subset\mathbf{P}^3$ of degree $d\geq 15$ admits a nontrivial section, and every entire function $f\colon\mathbf{C}\to X$ must satisfy the corresponding algebraic differential equations.

However, it seems very difficult to conclude that f satisfies an algebraic equation. The author suggests in Chapter 13 that the Riemann-Roch calculations might be helpful to locate the base locus, thus to conclude the algebraic degeneracy.

Another important part of this article is Chapter 2 and Chapter 9, where Demailly shows that Kobayashi hyperbolicity is related to other properties of a more algebraic nature. A projective directed manifold (X,V) is called algebraically hyperbolic if there exists $\varepsilon>0$ such that every algebraic curve $C\subset X$ tangent to V satisfies $2\,g(\overline{C})-2\geq\varepsilon\deg_\omega(C)$ ($\overline{C}$ is the normalization of C). The main result of Chapter 2 is that if (X,V) is hyperbolic, then (X,V) is algebraically hyperbolic. Chapter 9 extends this result to k-jet metrics and shows that the negativity of k-jet curvature implies strong restrictions of an algebraic nature on curve genera and their singularity indices.

Chapter 11 recalls the "meromorphic connection" method introduced by Siu [Y. T. Siu, Duke Math. J. **55** (1987), no. 1, 213–251; MR0883671 (89a:32030); A. M. Nadel, Duke Math. J. **58** (1989), no. 3, 749–771; MR1016444 (91a:32036)]. Using this method, the author reports on a joint work with J. El Goul, where examples of hyperbolic surfaces in $\mathbf{P}^3$ are produced for any degree ≥ 11.

{For the entire collection see MR1492532 (98h:14003)}

Reviewed by *Min Ru*

MR1492532 (98h:14003) 14-06

★**Algebraic geometry—Santa Cruz 1995.**

Proceedings of the AMS Summer Research Institute held at the University of California, Santa Cruz, CA, July 9–29, 1995.

Edited by János Kollár, Robert Lazarsfeld and David R. Morrison.

Proceedings of Symposia in Pure Mathematics, 62, Part 2.

American Mathematical Society, *Providence*, *RI*, 1997. xviii+449 *pp*. $89.00; $159.00 the two-volume set. *ISBN* 0-8218-0895-8; 0-8218-0493-6

{For Part 1 see the preceding review [MR1492516 (98h:14002)].}

Contents: Ron Y. Donagi, Seiberg-Witten integrable systems (3–43); W. Fulton and R. Pandharipande, Notes on stable maps and quantum cohomology (45–96); Richard Hain and Eduard Looijenga, Mapping class groups and moduli spaces of curves (97–142); Jun Li [Jun Li[1]] and Gang Tian, Algebraic and symplectic geometry of Gromov-Witten invariants (143–170); L. Katzarkov, On the Shafarevich maps (173–216); Carlos Simpson, The Hodge filtration on nonabelian cohomology (217–281); Jean-Pierre Demailly, Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials (285–360); Claude LeBrun, Twistors for tourists: a pocket guide for algebraic geometers (361–385); David A. Cox, Recent developments in toric geometry (389–436); Bernd Sturmfels, Equations defining toric varieties (437–449).

{The papers are being reviewed individually.}

MR1481120 (98i:32022) 32F07 (32F05 35J65)

Zeriahi, Ahmed (F-TOUL3-LM)

Pluricomplex Green functions and the Dirichlet problem for the complex Monge-Ampère operator.

Michigan Math. J. **44** (1997), *no. 3,* 579–596.

In this paper the notion of the pluricomplex Green function (first studied by Lempert, Klimek and Demailly) is generalized in the following way. Fix a function φ which is plurisubharmonic (psh) in a hyperconvex domain D such that e^{φ} is continuous and the singular set $S_{\varphi} := \{\varphi = -\infty\}$ is compact and contains a dense subset of points A_{φ} where the Lelong number of φ is positive. To such φ one can associate a generalized Green function $G_D(\cdot, \varphi)$ which is the supremum over the family of negative psh functions u in D which satisfy the inequality for Lelong numbers $\nu(u, a) \geq \nu(\varphi, a)$.

The author proves the basic properties of $G = G_D(\cdot, \varphi)$. It is continuous on $D \setminus S_{\varphi}$ and satisfies the complex Monge-Ampère equation $(dd^cG)^n = 0$ in this set. Moreover, $G = -\infty$ on A_{φ}. The function is also the unique solution of the Dirichlet problem for the Monge-Ampère equation where one preassigns boundary values, Lelong numbers $\nu(\varphi, a)$ on A_{φ}, and the measure $(2\pi)^n \sum \nu(\varphi, a)^n \delta_a$.

It is not clear if $G = -\infty$ on the whole set S_{φ}. The author poses a question concerning that problem. In particular, it would be interesting to know if given S_{φ} one can find $u \in \mathrm{PSH}(D)$ with $S_u = S_{\varphi}$ and $(dd^cu)^n = 0$ on $D \setminus S_{\varphi}$.

Reviewed by *Sławomir Kołodziej*

MR1474805 (98i:32051) 32L30 (14J99 32J15)

Brunella, Marco (F-DJON-T)

Feuilletages holomorphes sur les surfaces complexes compactes. (French. English, French summaries) [Holomorphic foliations on compact complex surfaces]

Ann. Sci. École Norm. Sup. (4) **30** (1997), *no. 5,* 569–594.

FEATURED REVIEW.

In this remarkable, carefully written and deep work, the classification of (non-singular) holomorphic foliations on compact complex surfaces is achieved. The first two sections of the paper recall and exploit the basic and essential material needed, especially P. Baum and R. Bott's formulae in the two-dimensional case [see J. Differential Geometry **7** (1972), 279–342; MR0377923 (51 #14092)], as well as behaviour under blowing-up. For instance, a nice and simple application of the contents of Section 1 is the following: Suppose X is a compact complex surface and $\mathcal{F}$

and $\mathcal{G}$ are non-singular holomorphic foliations on X which are transverse. Let $S \subset X$ be a curve which is neither $\mathcal{F}$-invariant nor $\mathcal{G}$-invariant. It follows that $S \cdot S \geq \chi(S)$ and hence, that X does not contain holomorphic spheres with negative self-intersection; in particular, X is minimal.

The remainder of the work can be divided into two parts, one dealing with surfaces of non-general type, the other with surfaces of general type.

In the first part, Sections 3, 4 and 5, foliations on surfaces of non-general type are studied. The Enriques-Kodaira classification is used throughout [see W. P. Barth, C. A. M. Peters and A. J. H. M. Van de Ven, *Compact complex surfaces*, Springer, Berlin, 1984; MR0749574 (86c:32026)]. General arguments are developed in Sections 3 and 4, especially exploiting the existence of fibrations, over "most" algebraic surfaces X, with generic fibre a rational or an elliptic curve. These fibrations are cleverly used as "reference fibrations", with which a foliation $\mathcal{F}$ in X is compared, that is, either $\mathcal{F}$ coincides with the fibration, is transverse to it, or else is a "turbulent" foliation. The concept of turbulent foliation is due to Reeb and was generalized, in the complex context, by É. Ghys [Ann. Fac. Sci. Toulouse Math. (6) **5** (1996), no. 3, 493–519; MR1440947 (98d:32037)]. Inspired by the definition of Ghys, the author defines a turbulent foliation on a compact complex surface X as a foliation $\mathcal{F}$ with the following property: there is a regular elliptic fibration of X such that: (i) a finite number of fibres are $\mathcal{F}$-invariant and (ii) all other fibres are transverse to $\mathcal{F}$. The results in the case of algebraic surfaces can be summarized as: If X is a compact complex algebraic surface and $\mathcal{F}$ is a non-singular holomorphic foliation on X, then we have one of the following (non-exclusive) possibilities: (1) $\mathcal{F}$ is a fibration; (2) $\mathcal{F}$ is transverse to a fibration; therefore, it is a suspension of an automorphism group of an algebraic curve; (3) $\mathcal{F}$ is a linear foliation on a complex torus; (4) $\mathcal{F}$ is a transversely hyperbolic foliation with dense leaves whose universal cover is a fibration over the disc, with the disc as typical fibre; (5) $\mathcal{F}$ is a turbulent foliation over an elliptic surface.

Section 5 of the paper displays the structure of foliations over surfaces of non-general type, and the results are: Let $\mathcal{F}$ be a non-singular holomorphic foliation on a compact complex surface X of Kodaira dimension < 2. Then $\mathcal{F}$ is one of the following objects (observe that such surfaces satisfy $c_1^2(X) = 2c_2(X)$): (1) an elliptic or rational fibration; (2) a foliation transverse to an elliptic or rational fibration; (3) a linear foliation on a complex torus; (4) certain (precisely described) foliations on Hopf and Inoue surfaces; (5) a turbulent foliation.

In the second part, Sections 6 and 7, the case of foliations on surfaces of general type is treated by considering dynamical properties of the foliation; essentially, a transverse invariant metric is constructed, and the arguments here are quite involved and make use of a result of J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)]. The main result of Sections 6 and 7 is: If $\mathcal{F}$ is a non-singular foliation on a surface of general type, then $\mathcal{F}$ is either a fibration or a transversely hyperbolic foliation with dense leaves whose universal cover is a fibration over the disc, with the disc as typical fibre.

Let us point out some amazing consequences of the results above: (i) The only non-minimal surface which admits a non-singular foliation is $\mathbf{P}^2_{\mathbf{C}}$ blown up at a point, and this foliation is the blow-up of the radial foliation centered at this point. (ii) If a surface X admits a regular foliation, then its signature is non-negative, that is, $\frac{1}{3}(c_1^2(X) - 2c_2(X)) \geq 0$. In particular, the only complete intersection which admits a non-singular foliation is $\overline{\mathbf{C}} \times \overline{\mathbf{C}}$.

In conclusion, Brunella has synthesized and clearly presented a key and very important result. This is a very substantial contribution to the area.

Reviewed by *M. G. Soares*

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16. S. Kobayashi, *Differential geometry of complex vector bundles, Princeton Univ. Press*, 1987. MR0909698 (89e:53100)
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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1326617 (96k:32001) 32-02 (32-06 32Cxx 32Dxx 32Fxx 32Jxx 32Lxx)

★**Several complex variables. VII.**

Sheaf-theoretical methods in complex analysis.

A reprint of *Current problems in mathematics. Fundamental directions. Vol. 74* (Russian), Vseross. Inst. Nauchn. i Tekhn. Inform. (VINITI), Moscow.
Edited by H. Grauert, Th. Peternell and R. Remmert.
Encyclopaedia of Mathematical Sciences, 74.
Springer-Verlag, *Berlin*, 1994. vi+369 *pp*. $99.00. *ISBN* 3-540-56259-1

This volume contains nine chapters arranged so as to provide a systematic introduction to and survey of the theory of complex spaces. Below I discuss each chapter separately, and provide some general comments at the end.

I. R. Remmert, "Local theory of complex spaces", 7–96: Starting from the Weierstrass division and preparation theorems, this chapter develops much of the algebraic and sheaf-theoretic background required to study the theory of complex spaces. The four main results that are considered here are the coherence of structure sheaves, the finite mapping theorem and the Rückert Nullstellensatz, the coherence of ideal sheaves, and the coherence of normalization sheaves. Most results in this chapter have sketches of proofs, except for the results that are purely commutative algebra.

II. Th. Peternell and R. Remmert, "Differential calculus, holomorphic maps and linear structures on complex spaces", 97–144: From the basic structure theorems, it is natural to progress to the behavior of cotangent sheaves as the next level of subtlety. The authors first discuss sheaves of germs of differential forms and criteria for smoothness and for submersions. Then they turn to questions of flatness (although the semicontinuity theorem is postponed until after cohomology has been discussed in Chapter III) and the correspondence $\{\text{vector bundles}\} \leftrightarrow \{\text{locally free sheaves}\}$ and its generalization to arbitrary coherent sheaves, leading eventually to the concept of analytic spectra. The remaining topics here are formal completions, Cohen-Macaulay spaces and dualizing sheaves. The discussions in this chapter are somewhat restricted because sheaf cohomology has not yet been discussed.

III. Th. Peternell, "Cohomology", 145–182: This chapter introduces the main ideas of cohomology theory for complex spaces, using the notions of flabby cohomology (for complexes, and de Rham's theorem) and Čech cohomology (and Dolbeault's theorem). Then the author turns to Stein spaces, Theorems A and B and the solution of the Cousin problems, and then to compact complex spaces, the direct image theorem, the comparison, base change, continuity and Riemann-Roch theorems, and Serre duality. Finally he gives a brief discussion of spectral sequences. Needless to say, this chapter does not contain proofs, although there are many examples, as elsewhere in the book.

IV. G. Dethloff and H. Grauert, "Seminormal complex spaces", 183–220: The first main result in this chapter is that analytically branched coverings are normal complex spaces. The proof of this uses L^2 methods, which are developed in a brief form and applied to other results. Probably the most important result on seminormal complex spaces is the criterion for a quotient space of a seminormal complex space to be seminormal. Most of the rest of this chapter is spent on developing the necessary abstract information about equivalence relations on complex spaces, so as to enable the reader to understand the sketch of the proof of this main result. Going outside of the realm of holomorphic equivalence relations, the authors turn to meromorphic equivalence relations and their applications to the meromorphic dependence of maps, the reduction of a complex space to a Moishezon space, and other types of non-regular behavior.

V. Th. Peternell, “Pseudoconvexity, the Levi problem and vanishing theorems”, 221–257: This chapter contains discussions of plurisubharmonic functions, pseudoconvex domains, 1-convex spaces, the classical Levi problem, and the characterization of exceptional analytic sets. The notion of pseudoconvexity is extended to bundles and the author discusses positive bundles and various vanishing theorems, such as those of Kodaira, Demailly and Grauert-Riemenschneider, and Hodge theory. The author makes a convincing case for treating the case of 1-convex spaces differently from that of q-convex spaces, $q \geq 2$.

VI. H. Grauert, “Theory of q-convexity and q-concavity”, 259–284: This chapter contains basic material such as the extension of the notions of q-convexity and q-concavity from domains in $\mathbf{C}^n$ to spaces, and the finite-dimensionality of the cohomology vector space in some cases. These results are applied to such topics as filling in holes in a complex space and the existence of hulls for cohomology classes. Finally, the author proves Serre’s duality theorem for q-convex spaces and the q-concavity of the fundamental domain for Siegel modular groups of degree $n > 1$.

VII. Th. Peternell, “Modifications”, 285–317: Beginning with the definition of a modification, the author gives a survey of results on the bimeromorphic geometry of complex spaces. He discusses blow-ups and blow-downs, the use of formal objects, the extension of analytic objects, Moishezon spaces, and desingularization.

VIII. F. Campana and Th. Peternell, “Cycle spaces”, 319–349: This chapter provides an introduction to one of the most interesting (to the reviewer) parts of complex analysis, the construction of the Douady space $\mathcal{D}(X)$ and Barlet space $\mathcal{C}(X)$ of a complex space X. (These are the analytic analogues of the Hilbert and Chow schemes from the algebraic case.) The authors prove the result of Lieberman and Fujiki that if X is a compact Kähler manifold, then the connected components of the Douady and Barlet spaces are compact. The Barlet space is applied to various structure results, in particular to manifolds of class $\mathcal{C}$, i.e., manifolds bimeromorphic to Kähler manifolds. Finally, convexity results for $\mathcal{C}(X)$ are discussed.

IX. H. Grauert and R. Remmert, “Extension of analytic objects”, 351–360: This brief chapter contains an extremely summary treatment of some of the basic results on extension, such as the Remmert-Stein theorems, the Stoll-Bishop theorem and Kneser’s Kontinuitätssatz. The authors express the hope that someone will come forth to write a detailed commentary on the results obtained in this field.

Some general remarks: As can be seen from the above summary, this volume contains an enormous amount of material, with at least hints at how the proofs go. There are very many examples and a lot of important remarks which will assist the reader in understanding some of the subtleties. If complete proofs had been included, the volume would probably have been four times its current size. The volume has a subject index, but unfortunately no notation index.

Unfortunately, the exposition is not always clear: important definitions and remarks are buried in the middle of paragraphs devoted to some other topic (and not always indicated in the index, either), bibliographic references are sometimes to items not included in the bibliographies (at the end of each chapter), and the level of English could have benefited from some serious editing by a native English speaker; there are many places which read as if they have been translated directly from German. (Here is an example from p. 14: "Analytic algebras exist two a penny. The more it will surprise that many of them are hidden in the algebra. . . .") In addition, the senior authors have kindly included extended passages in German and French, untranslated.

Despite these defects, the book should be very useful for students and for reference.

{Volume VI has been reviewed [MR1095088 (91i:32001)].}

Reviewed by *J. S. Joel*

MR1322991 (96f:32001) 32-02 (14J45 14J60 32J18 32J27 32L07)

Peternell, Thomas (D-BAYRMP);
Schneider, Michael [Schneider, Michael Hellmut] (D-BAYRMP)

Neuere Entwicklungen in der komplexen Geometrie. (German. German summary) [Recent developments in complex geometry]

Duration and change, 275–306, *Springer*, *Berlin*, 1994.

This is a survey written by two specialists for beginners or non-experts. They manage to give a short introduction to modern notions, recent results and open problems of an interesting field of mathematics. Although they do not give all possible references, at many places the reader is referred to other more detailed survey articles. This makes this paper comparatively short and easy to read.

The authors report on developments in complex geometry of the previous 20 years. By complex geometry is meant the study of complex manifolds with the methods of algebraic geometry, complex differential geometry and complex analysis. Of course, the authors cannot consider in this paper all aspects of this large and dynamically developing area of modern mathematics. Instead they focus on those parts which are closely related to their own contributions in the field of complex geometry [see, e.g., J.-P. Demailly, T. Peternell and M. H. Schneider, J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037); Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)].

The main subject of this paper is classification theory, which includes classification up to biholomorphisms or bimeromorphic transformations, up to deformations or up to homeomorphisms. One section explains some basic notions, results and open questions of Mori theory, which deals with the birational classification of projective manifolds. The results and conjectures of this theory suggest investigating Fano manifolds more closely. A manifold is called Fano if it has ample anticanonical bundle. The section on these manifolds gives a first impression of the theory of Fano manifolds. A central result of importance for the development of higher-dimensional classification theory is S. Mori's solution of the Hartshorne-Fraenkel conjecture [Ann. of Math. (2) **110** (1979), no. 3, 593–606; MR0554387 (81j:14010)] stating that any compact manifold with ample tangent bundle is isomorphic to projective space. This leads to the interesting question of classifying manifolds with "semipositive" tangent bundle. Ideas centered around this problem are discussed in the fourth section. The final part is devoted to the theory of holomorphic vector bundles. After explaining the basic notion of stability and some of its consequences, the usage of moduli spaces of vector bundles over algebraic surfaces to study the differential topology of these manifolds is explained (Donaldson polynomials). Finally, the Kobayashi-Hitchin correspondence is discussed. It forms a bridge between algebraic geometry and complex differential geometry by relating the notion of stability of vector bundles to the existence of a Hermite-Einstein metric.

{For the entire collection see MR1322982 (95i:00037)}

Reviewed by *Bernd Kreussler*

MR1319346 (96k:32012) 32C30 (32J25)

Demailly, Jean-Pierre (F-GREN-F)

Regularization of closed positive currents of type $(1,1)$ by the flow of a Chern connection.

Contributions to complex analysis and analytic geometry, 105–126, *Aspects Math.*, *E*26, *Vieweg*, *Braunschweig*, 1994.

Let X be a compact n-dimensional complex manifold and let T be a closed positive current of bidegree $(1,1)$ on X. In general, T cannot be approximated by closed positive currents of class C^∞: a necessary condition for this is that the cohomology class $\{T\}$ be numerically effective in the sense that $\int_Y \{T\}^p \geq 0$ for every p-dimensional subvariety $Y \subset X$. The author proves that it is always possible to approximate a closed positive current T of type $(1,1)$ by closed real currents admitting a small negative part, and that this negative part can be estimated in terms of the Lelong numbers of T and the geometry of X. Let α be a smooth closed $(1,1)$-form representing the same $\partial\overline{\partial}$-cohomology class as T and let ψ be a quasi-psh function on X such that $T = \alpha + (i/\pi)\partial\overline{\partial}\psi$ (a function is said to be quasi-psh if it is locally the sum of a psh function and a smooth function). Such a decomposition exists even when X is non-Kähler. If ψ_ε is an approximation of ψ, then $T_\varepsilon = \alpha + (i/\pi)\partial\overline{\partial}\psi_\varepsilon$ is an approximation of T. The author proves the following: Theorem. Let T be a closed almost positive $(1,1)$-current and let α be a smooth real $(1,1)$-form in the same $\partial\overline{\partial}$-cohomology class as T, i.e. $T = \alpha + (i/\pi)\partial\overline{\partial}\psi$, where ψ is an almost psh function. Let γ be a continuous real $(1,1)$-form such that $T \geq \gamma$. Suppose that the tangent bundle T_X is equipped with a smooth Hermitian metric ω such that the Chern curvature form $\Theta(T_X)$ satisfies $(\Theta(T_X) + u \otimes$

$\mathrm{Id}_{T_X})(\theta\otimes\xi,\theta\otimes\xi)\geq 0$ for all $\theta,\xi\in T_X$ with $\langle\theta,\xi\rangle=0$, for some continuous nonnegative $(1,1)$-form u on X. Then there is a family of closed almost positive $(1,1)$-currents $T_\varepsilon=\alpha+(i/\pi)\partial\overline{\partial}\psi_\varepsilon$, $\varepsilon\in(0,\varepsilon_0)$, such that ψ_ε is smooth over X, increases with ε, and converges to ψ as ε tends to 0 (in particular, T_ε is smooth and converges weakly to T on X), and such that (i) $T_\varepsilon\geq\gamma-\lambda_\varepsilon u-\delta_\varepsilon\omega$, where (ii) $\lambda_\varepsilon(x)$ is an increasing family of continuous functions on X such that $\lim_{\varepsilon\to 0}\lambda_\varepsilon(x)=\nu(T,x)$ (Lelong number of T at x) at every point, and (iii) δ_ε is an increasing family of positive constants such that $\lim_{\varepsilon\to 0}\delta_\varepsilon=0$.

For the proof, the author uses a smoothing procedure involving a convolution kernel constructed by means of the exponential map associated to the Chern connection on T_X. From a calculation of the Taylor expansion of the exponential map at order 3, a precise estimate of the complex Hessian of the regularized function is derived. Kiselman's singularity attenuation technique is then applied in combination with the above theorem to obtain a family of approximating currents $T_{c,\varepsilon}$ which are smooth in the complement $X\setminus E_c(T)$ of the Lelong sublevel set

$$E_c(T)=\{x\in X\colon\ \nu(T,x)\geq c\}$$

and have Lelong numbers $\nu(T_{c,\varepsilon},x)=\nu(T,x)-c$ along $E_c(T)$. It should be observed that similar results have been proved by the author in a related paper [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)], under slightly different curvature assumptions. Some geometric applications of the smoothing theorem to the study of compact complex manifolds with partially semipositive curvature are given.

{For the entire collection see MR1319341 (95j:32001)}

Reviewed by *Mongi Blel*

MR1319341 (95j:32001) 32-06 (00B30)

★**Contributions to complex analysis and analytic geometry.**

Dedicated to Pierre Dolbeault.

Edited by Henri Skoda and Jean-Marie Trépreau.

Aspects of Mathematics, E26.

Friedr. Vieweg & Sohn, *Braunschweig*, 1994. xiv+250 *pp*. $70.00. *ISBN* 3-528-06633-4

Contents: H. Skoda and J.-M. Trépreau, Foreword (Dedication to Pierre Dolbeault, on the occasion of his retirement) (French) (vi–xi); Vincenzo Ancona and Bernard Gaveau, The de Rham complex of a reduced analytic space (1–26); Bo Berndtsson, Some recent results on estimates for the $\overline{\partial}$-equation (27–42); Evgeni M. Chirka [E. M. Chirka] and Edgar Lee Stout, Removable singularities in the boundary (43–104); Jean-Pierre Demailly, Regularization of closed positive currents of type $(1,1)$ by the flow of a Chern connection (105–126); Klas Diederich and Gregor

Herbort, Pseudoconvex domains of semiregular type (127–161); Pierre Dolbeault and Gennadi Henkin [G. M. Khenkin], Surfaces de Riemann de bord donné dans $\mathbf{C}P^n$ [Riemann surfaces with given boundary in $\mathbf{C}P^n$] (163–187); Alan Huckleberry, Subvarieties of homogeneous and almost homogeneous manifolds (189–232); Mikael Passare, August Tsikh [A. K. Tsikh] and Oleg Zhdanov, A multidimensional Jordan residue lemma with an application to Mellin-Barnes integrals (233–241); Bernard Shiffman, Separately meromorphic mappings into compact Kähler manifolds (243–250).

{Most of the papers are being reviewed individually.}

MR1318282 (96a:32034) 32F20 (32E10)

Wu, Xiao Qin (PRC-JMU)

The solution of $\overline{\partial}_b$-equation of (p,q)-forms and its L^p & Hölder estimates on a Stein manifold. (English summary)

The collection of theses of Symposium on Real Analysis (Xiamen, 1993).
J. Math. Study **27** (1994), *no. 1,* 174–180.

Let M be a Stein manifold of dimension n and let $\Omega \subset\subset M$ be a strongly pseudoconvex domain with smooth boundary. In this paper, the author uses a method developed by Demailly and Laurent-Thiébaut to construct an integral kernel to solve the tangential Cauchy-Riemann equation $\overline{\partial}_b$ of (p,q)-forms. More precisely, let $T(M)$ be the holomorphic tangent bundle of M and let $\tilde{T}(M\times M)$ be the pullback of $T(M)$. Furthermore, we assume the curvature C of $\tilde{T}(M\times M)$ is 0. Then, for $f\in L^S_{p,q}(\partial\Omega)$ satisfying the compatibility condition, $f=(-1)^p\overline{\partial}_b(T(f)-S(f))$. Using this integral representation for the solution of $\overline{\partial}_b$, the author shows that $\|T(f)-S(f)\|_{\Lambda^r_{1/2}} \lesssim \|f\|_{L^r(\partial\Omega)}$. Here Λ^r_α is the Besov space equipped with the norm

$$\|f\|_{\Lambda^r_\alpha} = \|f\|_{L^r(\partial\Omega)} + \sup_{z(t)\in C,\ 0\le t\le 1} \frac{\|f(\gamma(t))-f(\gamma(0))\|_{L^r}}{|t|^\alpha},$$

where $0<\alpha<1$, $1\le r\le\infty$ and $\gamma\colon(0,1]\to\partial\Omega$ is a C^1 curve with $\|\gamma'(t)\|_{C^1}\le 1$.

{For the entire collection see MR1318248 (95i:00035)}

Reviewed by *Der-Chen E. Chang*

MR1302317 (95i:32022) 32E30 (32H02 32H15 32H20)
Demailly, Jean-Pierre (F-GREN-F); **Lempert, László** (1-PURD);
Shiffman, Bernard (1-JHOP)
Algebraic approximations of holomorphic maps from Stein domains to projective manifolds.
Duke Math. J. **76** (1994), *no. 2,* 333–363.

This paper studies the problem of approximation of holomorphic maps by algebraic maps. The authors show that algebraic approximation is always possible in the case of holomorphic maps to quasiprojective manifolds and of locally free sheaves. In particular, they obtain that any holomorphic map from a Runge domain Ω in an affine algebraic variety S into a quasiprojective algebraic manifold X can be approximated by Nash algebraic maps uniformly on every relatively compact domain $\Omega_0 \subset\subset \Omega$. As an application, they describe how both the Kobayashi-Royden pseudometric and the Kobayashi pseudodistance on projective algebraic manifolds can be given in terms of algebraic curves.

Reviewed by *Min Ru*

MR1299059 (95k:14065) 14K05 (14C20)
Debarre, O. (1-IA); **Hulek, K.** (D-HANN); **Spandaw, J.** (D-BAYRMP)
Very ample linear systems on abelian varieties.
Math. Ann. **300** (1994), *no. 2,* 181–202.

Let X be a "generic" abelian manifold of dimension g and L an ample line bundle. The authors restrict their study to the case in which L is of type $(1,\cdots,1,d)$, which means that L is the pullback of the principal polarization under a cyclic isogeny of degree d. Then they establish that the linear system $|L|$ is base-point-free if and only if $d \geq g+1$, the morphism defined by this linear system being birational if and only if $d \geq g+2$.

These results are part of a wider conjectural picture according to which, for $d > g$, Φ_L (the morphism of X in $\mathbf{P}^{d-1}$ defined by L) should be an embedding outside a set of dimension $2g+1-d$ (in particular, L should be very ample if and only if $d \geq 2g+2$). The authors then show that, for (X,L) generic of type $(1,\cdots,1,d)$, L is very ample for $d > 2^g$.

Their results are connected in the case $g=3$ to a conjecture of Griffiths and Harris. In the same case, Ein and Lazarsfeld have also given some very explicit sufficient conditions for a linear system $|L|$ on X to be base-point-free. Moreover, in the case of arbitrary g, Demailly has given effective bounds on the degree of L, with (X, L) always assumed to be generic, such that $|L|$ is base-point-free or very ample. The authors compare their results with the Ein-Lazarsfeld and Demailly results, respectively.

Reviewed by *Jean-Claude Douai*

MR1299050 (95k:32026) 32J27 (14J40 32J18)
Zhang, Qi [Zhang, Qi[1]] (1-MO)
A note on compact Kähler manifolds with nef tangent bundles. (English summary)
Manuscripta Math. **85** (1994), *no. 1,* 89–96.

Let us assume that X is a compact Kähler manifold of dimension n. A famous conjecture by Frankel and Hartshorne, proved by S. Mori [Ann. of Math. (2) **110** (1979), no. 3, 593–606; MR0554387 (81j:14010)], asserts that, if the sectional curvature of X is positive, or, equivalently, if the tangent bundle T_X is ample, then X is biholomorphic to the projective space $\mathbf{P}^n$. In view of the result of Mori it is natural to ask about a classification of manifolds with tangent bundle satisfying a weaker numerical condition. Namely, one can consider a manifold X with numerically effective (or nef) tangent bundle, which means that the tautological bundle $\mathcal{O}(1)$ on $\mathrm{Proj}(T_X)$ has non-negative degree on any curve $C \subset \mathrm{Proj}(T_X)$. Manifolds of this type have been studied by several authors [see, e.g., J.-P. Demailly, T. Peternell and M. H. Schneider, J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)]. The main theorem of the paper under review is as follows: Suppose that T_X is nef and that some positive multiple $-mK_X$ of the anticanonical line bundle $-K_X = \det(T_X)$ is spanned by global sections. Then the image of the associated map $X \to \mathrm{Proj}(H^0(X, -mK_X))$ is of dimension 1 if and only if there exists an étale covering $\mathbf{P}^1 \times A \to X$, where A is a complex torus. The proof of the theorem depends on results of Demailly, Peternell and Schneider.

Reviewed by *Jarosław A. Wiśniewski*

MR1298871 (95h:32032) 32L20 (32L05)
Marinescu, George [Marinescu, Gheorghe] (R-AOS)
The curvature of numerically effective line bundle and vanishing theorems. (English summary)
Rev. Roumaine Math. Pures Appl. **39** (1994), *no. 1,* 37–42.

In this paper nef line bundles are investigated; in particular, the following theorem is proved: Theorem 1.1. Let E be a big nef line bundle over a compact projective manifold. Then for any Hermitian metric on E the associated curvature form is positive on a nonempty set. The proof makes use of Demailly's asymptotic Morse inequalities. Moreover, vanishing theorems are obtained: Theorem 1.2. Let E be a nef line bundle over a compact Kähler manifold of dimension n. Assume that $\int_{X(\leq 1)}(ic(E))^n$ is positive. Then $H^q(X,E^{-1})=0$ for $q<n$. $ic(E)$ is the curvature form of a Hermitian metric on E and $X(\leq 1)$ is the set where $ic(E)$ is nondegenerate and has exactly 0 or 1 negative eigenvalues (§2). Theorem 1.3. Let X be a quasiprojective, weakly 1-complete manifold such that its projective closure has only isolated singularities. Let E be a holomorphic line bundle over X which is positive outside an algebraic subvariety of dimension m. Then $H^q(E,\Omega^p(E))$ vanishes for $p+q>m+n$, where $n=\dim_{\mathbf{C}} X$.

Reviewed by *Antonella Nannicini*

MR1298282 (95f:32040) 32L15 (32F15)
Asserda, Saïd
Convexité holomorphe des domaines pseudoconvexes par rapport à un fibré holomorphe de rang un positif. (French. English, French summaries) [Holomorphic convexity of pseudoconvex domains with respect to a positive holomorphic line bundle]
C. R. Acad. Sci. Paris Sér. I Math. **319** (1994), *no. 6,* 559–562.

Let D be a relatively compact pseudoconvex domain in an n-dimensional complex analytic manifold M and let L be a rank-1 holomorphic line bundle on M. The open subset D is said to be L-convex if for every infinite discrete sequence $(x_m)_m$ of D there exists a holomorphic section $g\in H^0(D,L^q)$ such that the sequence $(g(x_m))_m$ is unbounded. The author proves that if D is a relatively compact pseudoconvex domain in M, there exists an integer p such that, for all $q\geq p$, for all $\varepsilon\in(0,1]$ and for every infinite discrete sequence $(x_m)_m$ in D, there exists a holomorphic section $g\in H^0(D,L^q)$ such that the sequence $(g(x_m))_m$ is unbounded and satisfies $(*)$ $\int_D \|g\|^2\delta^{2\varepsilon+3}dV_M<+\infty$, where $\delta(z)=d(z,M\smallsetminus D)$ for all $z\in D$. This theorem generalizes and sharpens a result of K. R. Pinney [Math. Z. **206** (1991), no. 4, 605–615; MR1100844 (92f:32054)] concerning the particular case in which M is compact and ∂D is of class C^2. Asserda's proof rests essentially on a result of Elencwajg and on L^2-estimates of Hörmander, Skoda and Demailly. Using the Richberg and Greene-Wu approximation theorem as well as the above theorem, the author shows that every relatively compact pseudoconvex domain in a Kähler manifold has a complete Kähler metric. (This result had been obtained by Pinney under the restrictive conditions mentioned above.) From this the author derives the corollary that every locally trivial holomorphic fibration with the unit disk as fiber and a projective manifold as base is meromorphically convex.

In the last part the author considers the submanifold of $\mathbf{P}^n \times \mathbf{P}^{n-1}$ defined by

$$M = \{([z],[t]) \in \mathbf{P}^n \times \mathbf{P}^{n-1}\colon\ z_j t_k - t_j z_k = 0,\ 1 \le j,k \le n\},$$

which is exactly the blowup of $\mathbf{P}^n$ at the point $a = [1,0,\cdots,0]$, with $n \ge 2$, and considers the domain $D = M \setminus S$, where S is the exceptional divisor of the blowup. He constructs a holomorphic section h over D satisfying the inequality $(*)$, with ε as exponent instead of $2\varepsilon + 3$.

Reviewed by *Mongi Blel*

MR1278916 (95d:32001) 32-02 (32H20 32H25 32L05)

Siu, Yum Tong (1-HRV)

Problems and recent results in several complex variables.

Complex analysis and its applications (*Hong Kong*, 1993), 38–49, *Pitman Res. Notes Math. Ser.*, 305, *Longman Sci. Tech.*, *Harlow*, 1994.

This paper is a survey on problems and recent results concerning the construction of holomorphic sections of holomorphic line bundles. Precisely two problems are illustrated: Matsusaka's big theorem and the hyperbolicity of the complement in the complex projective plane of a generic smooth complex curve of sufficiently high degree. Regarding the first problem the main result described here is the following theorem recently obtained by the author. Theorem 2: Let X be a compact complex manifold of complex dimension n and let L be an ample line bundle over X and B be a numerically effective holomorphic line bundle over X. Let C be the Chern number $((n+2)L + B + K_X)L^{n-1}$. Then the line bundle $mL - B$ is very ample over X for $m \ge (24n^n C(1+C)^n)^{n(6n^3)^n}$. The proof of Theorem 2 is obtained by using Demailly's asymptotic strong Morse inequality and Nadel's vanishing theorem.

Regarding the second problem the following conjecture is investigated: Conjecture 2: There exists some effective positive number e with the following property. For $d \ge e$ there exists a Zariski open subset $\mathcal{H}$ in the space of all complex curves of degree d such that any holomorphic map from $\mathbf{C}$ to $\mathbf{P}_2 - C$ is constant for $C \in \mathcal{H}$.

{For the entire collection see MR1278912 (94m:30001)}

Reviewed by *Antonella Nannicini*

MR1257325 (95f:32037) 32J27 (14J45 32L07)
Demailly, Jean-Pierre (F-GREN-F); **Peternell, Thomas** (D-BAYR);
Schneider, Michael [Schneider, Michael Hellmut] (D-BAYR)
Compact complex manifolds with numerically effective tangent bundles.
J. Algebraic Geom. **3** (1994), *no. 2,* 295–345.

A vector bundle E on a projective variety is said to be numerically effective (nef) if the tautological line bundle $\mathcal{O}_E(1)$ on the associated projective bundle $\mathbf{P}(E)$ is numerically effective. This notion is extended to vector bundles over compact complex manifolds as follows. Let X be a compact complex manifold and let E be a vector bundle on X. Fix a Hermitian metric ω on $\mathbf{P}(E)$. Then E is nef if, for every positive number ε, we can find a Hermitian metric h_ε on $\mathcal{O}_E(1)$ (or a Hermitian Finsler metric on E) such that its curvature Θ_{h_ε} satisfies $\Theta_{h_\varepsilon} \geq -\varepsilon\omega$. This definition does not depend on the choice of the Hermitian metric ω.

The main result of the paper is a structure theorem for Kähler manifolds with nef tangent bundles. Main Theorem: Let X be a compact Kähler manifold with nef tangent bundle. Then there exists an étale finite cover $\tilde{X}$ such that the Albanese mapping $\alpha\colon \tilde{X} \to \mathrm{Alb}(\tilde{X})$ is a surjective, smooth morphism, every fibre of which is a Fano manifold with nef tangent bundle.

The result states that X is essentially constructed by a torus and Fano manifolds. The torus part defines a flat quotient E of T_X, the tangent bundle of X (or, more precisely, of $T_{\tilde{X}}$). Since a Fano manifold is always simply connected, the main theorem implies that the fundamental group of X, a compact Kähler manifold with nef tangent bundle, is an extension of a finite group by a free abelian group of even rank $\mathbf{Z}^{2q}$.

One of the essential steps toward the main theorem is to show the following. Proposition: Let X be an n-dimensional compact Kähler manifold with T_X nef. If $c_1(X)^n = 0$, then (1) $\chi(\mathcal{O}_X) = 0$, (2) there is a nowhere vanishing p-form u for suitable odd p, and, (3) by lifting to a suitable étale cover $\tilde{X}$, the irregularity $q(\tilde{X})$ becomes positive.

The proof of this part depends on a result of J. Tits on subgroups of linear groups [J. Algebra **20** (1972), 250–270; MR0286898 (44 #4105)] and on the authors' result to the effect that $\pi_1(X)$ has subexponential growth [Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)].

Choosing $\tilde{X}$ such that its irregularity attains the maximum, we get a subsheaf $E^* \subset \Omega^1_{\tilde{X}} = (T_X)^*$ generated by global sections. It is easy to show that E^* is a subbundle with trivial Chern classes, and results of Uhlenbeck-Yau and S. Kobayashi tell us that it has a filtration with Hermitian-flat graded pieces. Taking the duals, we get a quotient E of T_X which defines the torus part of $\tilde{X}$, or the image of the Albanese map. The smoothness of the fibres of the Albanese fibration is proved via the theory of Mori contractions.

As by-products of the proof, the authors obtain (a) the projectivity of Moishezon manifolds with nef tangent bundles and (b) the classification of nonalgebraic compact 3-folds with nef tangent bundles (up to finite étale coverings) into nonalgebraic tori and some $\mathbf{P}^1$-bundles over nonalgebraic two-dimensional tori.

Reviewed by *Yoichi Miyaoka*

MR1275204 (95f:32035) 32J25 (14C20 14C30 14J60)
Siu, Yum Tong (1-HRV)
An effective Matsusaka big theorem. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **43** (1993), *no. 5,* 1387–1405.

Let us recall the Matsusaka Big Theorem [T. Matsusaka, Amer. J. Math. **94** (1972), 1027–1077; MR0337960 (49 #2729)]: let $P(k)$ be a polynomial in one variable of degree n with rational coefficients whose values are integers at integral values of k. Then there is a positive integer k_0 depending on $P(k)$ such that, for every compact projective algebraic manifold X of complex dimension n and every ample line bundle L over X with $\sum_{v=0}^{n}(-1)^v \dim H^v(X,kL)=P(k)$ for every k, the line bundle kL is very ample for $k\geq k_0$. It is interesting to find an effective bound for the positive integer k_0. By a result of J. Kollár and Matsusaka [Amer. J. Math. **105** (1983), no. 1, 229–252; MR0692112 (85c:14007)], k_0 can be made to depend only on the coefficients of k^{n-1} and k^n in the polynomial $P(k)$.

In this paper, the author proves an effective version of Matsusaka's Big Theorem (Theorem (0.1)): Let L be an ample holomorphic line bundle over a compact complex manifold X of complex dimension n. Then mL is very ample for any $m\geq$ an explicit positive constant which depends only on $n, L^n, K_X\cdot L^{n-1}$. Moreover, if in addition B is any numerically effective line bundle over X, then (see Theorem (0.2)) $mL-B$ is very ample for any $m\geq$ an explicit positive constant which depends only on H^n, $H^{n-1}\cdot B$, $L^{n-1}\cdot B$, and $L^{n-2}\cdot B\cdot K_X$, where $H:=2(K_X+3(3n-2)^nL)$.

J.-P. Demailly's recent result [J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)] that $12n^nL+2K_X$ is very ample for any ample line bundle L over X can also be regarded as an effective version of Matsusaka's Big Theorem. But in the paper under review, the ampleness of $mL-B$ does not involve $2K_X$ and $-B$ can be used for any ample line bundle B.

The main idea of the proof of the effective Matsusaka Big Theorem is to show that for the given L and B, there is an effective lower bound m such that $mL-B$ is numerically effective, because of the very ampleness criterion of Demailly and Kollár. This is done in three steps. The first step is a lemma on the existence of nontrivial holomorphic sections of a multiple of the difference of two ample line bundles whose Chern classes satisfy a certain inequality. Here the strong Morse inequality of Demailly is used. The second step is to produce, for any d-dimensional irreducible subvariety Y of X and any very ample line bundle H of X, a nontrivial holomorphic section over Y of some line bundle related to H. The third step is to use induction to get the numerical effectiveness of $mL - B$.

Reviewed by *Shanyu Ji*

MR1261816 (95e:32033) 32L20 (32C17 32F30)

Hamada, Hidetaka

Cohomology vanishing theorems on a domain exhausted by complete Kähler domains.

Math. Rep. Kyushu Univ. **19** (1993), 17–25.

Let E be a holomorphic vector bundle on a weakly pseudoconvex Kähler manifold X. If E satisfies some positivity condition such as Nakano's positivity, Griffiths' positivity or Demailly's s-positivity [J.-P. Demailly, Ann. Sci. École Norm. Sup. (4) **15** (1982), no. 3, 457–511; MR0690650 (85d:32057)] that generalized the former, we have corresponding cohomology vanishing theorems for the sheaves of holomorphic forms with values in E. The author generalizes these vanishing theorems to the case when X is a complex manifold that is exhausted by a sequence of complete Kähler domains and when the restriction of E on each Kähler manifold satisfies the above-mentioned positivity conditions, but for the cohomologies of order higher than 2. The vanishing of the first cohomology is most important but, as is indicated in the paper, we cannot expect it.

Reviewed by *Tosiaki Kori*

MR1257232 (94i:32047) 32L10 (32J20 58E05)

Bonavero, Laurent (F-GREN-FM)

Inégalités de Morse holomorphes singulières. (French. English, French summaries)
[Singular holomorphic Morse inequalities]

C. R. Acad. Sci. Paris Sér. I Math. **317** (1993), *no. 12,* 1163–1166.

Summary: "We generalize J.-P. Demailly's holomorphic Morse inequalities for a line bundle E equipped with a singular metric on a complex compact manifold X. Our inequalities give an estimate of the cohomology groups with values in the tensor powers $E^{\otimes k}$, twisted by the corresponding sequence of Nadel's multiplier ideal sheaves. The singularities allowed are of the following type: the metric is locally given by a weight $\exp(-\varphi)$, where $\varphi(x) \sim c \cdot \log(\sum |f_j|^2)$ with holomorphic f_j. As a consequence, we obtain a necessary and sufficient condition, invariant under bimeromorphism, for a manifold X to be Moishezon."

MR1256437 (95e:32032) 32L10 (14J15 14J60 32L20)

Takayama, Shigeharu (J-TOKYM)

Ample vector bundles on open algebraic varieties.

Publ. Res. Inst. Math. Sci. **29** (1993), *no. 6,* 885–910.

The main result in this paper is the following: Theorem 1. There exists a function $C(n)$ in $n \in \mathbf{N}$ with the following property: let X be a projective manifold, let D be a reduced effective divisor on X with only simple normal crossings such that $K_X + D$ is nef, let L be a nef line bundle on X which is ample modulo D; then $2(K_X + D) + mL$ is very ample modulo D for any $m \geq C(\dim X)$. In the case $D = 0$ Theorem 1 was proved by J.-P. Demailly [J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)]. The methods used for the proof of Theorem 1 are similar to those used by Demailly. Theorem 1 has interesting consequences, for example, it can be used in the construction of the moduli space of open algebraic varieties of general type.

Reviewed by *Antonella Nannicini*

MR1255695 (95b:32044) 32J27 (14J40 32C17 53C55)
Demailly, Jean-Pierre (F-GREN-F); **Peternell, Thomas** (D-BAYR);
Schneider, Michael [Schneider, Michael Hellmut] (D-BAYR)
Kähler manifolds with numerically effective Ricci class.
Compositio Math. **89** (1993), *no. 2,* 217–240.

The purpose of this paper is to contribute to the solution of the following conjectures: Let X be a compact Kähler manifold with numerically effective (nef) anticanonical bundle $-K_X$; then: Conjecture 1: The fundamental group $\pi_1(X)$ of X has polynomial growth. Conjecture 2: The Albanese map $\alpha\colon X \to \mathrm{Alb}(X)$ is surjective. Section 1 is devoted to proving the following theorem, which is the main contribution to Conjecture 1. Theorem 1: Let X be a compact Kähler manifold with nef anticanonical bundle; then $\pi_1(X)$ has subexponential growth. The main tools used in order to prove Theorem 1 are the solution of the Calabi conjecture and volume bounds for geodesic balls due to Bishop and Gage. It should be mentioned that from the proof of Theorem 1 it follows that Conjecture 1 holds in the case $-K_X$ is Hermitian semipositive (Theorem 2).

In Section 2 the following theorem concerning Conjecture 2 is proved. Theorem 3: Let X be an n-dimensional compact Kähler manifold such that $-K_X$ is nef. Then the Albanese map α is surjective as soon as $\dim \alpha(X)$ is $0, 1$ or n, and, if X is projective, also for $n-1$; moreover, if X is projective and if the generic fibre F of α has $-K_F$ big, then α is surjective. Finally, Section 3 is devoted to the study of the structure of projective 3-folds with nef anticanonical bundles; in particular Conjecture 2 is proved in dimension 3 with purely algebraic methods, except in one very special case.

Reviewed by *Antonella Nannicini*

MR1249407 (94m:32015) 32C30 (32F05)
Blel, M. (TN-TUNISM)
Sur le cône tangent à un courant positif fermé. (French. English, French summaries) [On the tangent cone of a closed positive current]
J. Math. Pures Appl. (9) **72** (1993), *no. 6,* 517–536.

Suppose $1 \le p \le n-1$, and let T be a closed positive current of bidegree (p,p) on an open subset of $\mathbf{C}^n$ containing the origin. The tangent cone to T, if it exists, is the weak limit of dilates of T. Even when the tangent cone fails to exist, every weak limit of dilates of T is a closed, positive, conical current on $\mathbf{C}^n$ with the same Lelong number at the origin as T, as was shown by M. Blel, J.-P. Demailly and M. Mouzali [Ark. Mat. **28** (1990), no. 2, 231–248; MR1084013 (92f:32017)]. The author's main theorem is a characterization of the sets of currents that can arise in this way.

Namely, let K_p denote the set of closed, positive, conical currents on $\mathbf{C}^n$ of bidegree (p,p) with Lelong number equal to one at the origin. If M is a connected, closed subset of K_p, then there exists a closed positive current T of bidegree (p,p) on the unit ball of $\mathbf{C}^n$ for which the set of weak limits of dilates is M. The case $p=1$ is due to C. O. Kiselman [in *International Symposium in Memory of Hua Loo Keng, Vol. II (Beijing, 1988)*, 157–167, Springer, Berlin, 1991; MR1135833 (92i:32011)]. The author also gives a sufficient condition, in terms of limits of the trace measure, for the existence of the tangent cone when $p=1$ or $p=n-1$.

Reviewed by *Harold P. Boas*

MR1247749 (94k:32008) 32C30
Blel, Mongi (TN-TUNISM)
Sur le cône tangent associé à un courant positif fermé. (French. French summary) [On the tangent cone associated with a closed positive current]
Colloque d'Analyse Complexe et Géométrie (Marseille, 1992).
Astérisque No. 217 (1993), 5, 29–38.

Let T be any positive closed current of bidegree (p,p) on an open subset $\Omega\subset\mathbf{C}^n$ containing 0, $1\le p\le n-1$. In 1977 R. Harvey [in *Several complex variables (Williamstown, MA, 1975)*, 309–382, Proc. Sympos. Pure Math., 30, Part 1, Amer. Math. Soc., Providence, RI, 1977; MR0447619 (56 #5929)] raised the question whether there exists an associated "tangent cone" to T, i.e., whether there exists the weak limit of the family $\{(h_r)^*T\}$ as $r\searrow 0$, where $h_r\colon B(0,1)\to B(0,r)$, $z\mapsto rz$. The question has a negative answer by the work of C. O. Kiselman [in *International Symposium in Memory of Hua Loo Keng, Vol. II (Beijing, 1988)*, 157–167, Springer, Berlin, 1991; MR1135833 (92i:32011)] and the author, J.-P. Demailly and M. Mouzali [Ark. Mat. **28** (1990), no. 2, 231–248; MR1084013 (92f:32017)]. In fact, they showed that for any T as above, the set of value limits of the family $\{(h_r)^*T\}$ is a compact connected subset of the set $m\cdot K_p$, where m is the Lelong number of T at 0, and K_p is the set of positive closed conical currents of bidegree (p,p) on $\mathbf{C}^n$ with the Lelong number at 0 equal to 1; conversely, given a connected closed subset M of K_1, there exists a positive closed current T of bidegree $(1,1)$ such that M is equal to the set of limit values of the family $\{(h_r)^*T\}$. In the current paper, the author makes the above result more complete: Given any connected closed subset M of K_p, where $p\ge 2$, there exists a positive closed current of bidegree (p,p) on the unit ball of $\mathbf{C}^n$ such that M is equal to the set of limit values of the family $\{(h_r)^*T\}$. The author also discusses some necessary and sufficient conditions for a current T of bidegree $(1,1)$ or $(n-1,n-1)$ to admit a tangent cone.

{For the entire collection see MR1247746 (94f:32002)}

Reviewed by *Shanyu Ji*

Citations
From References: 0
From Reviews: 0

MR1237810 (94h:32005) 32A25 (32E10)
Zhong, Tong De (PRC-XIAM)
A jump formula for the Bochner-Martinelli-Koppelman transform of differential forms on Stein manifolds. (Chinese. English, Chinese summaries)
Xiamen Daxue Xuebao Ziran Kexue Ban **32** (1993), *no. 5,* 525–527.

Let D be a relatively compact domain with C^2-smooth boundary in a Stein manifold M. The author gives a jump formula for C^1-smooth (p,q)-forms on ∂D. This jump formula was asserted by J.-P. Demailly and C. Laurent-Thiébaut [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)].

Reviewed by *Lan Ma*

Citations
From References: 0
From Reviews: 0

MR1215287 (95d:14039) 14J45 (32J18)
Nadel, Alan M. (1-IASP)
Relative bounds for Fano varieties of the second kind.
Einstein metrics and Yang-Mills connections (*Sanda*, 1990), 181–191, *Lecture Notes in Pure and Appl. Math.*, 145, *Dekker*, *New York*, 1993.

From the introduction: "This article is concerned with bounding Fano varieties of the second kind. Recall that a Fano variety is a smooth projective variety with ample anticanonical class. A Fano variety is said to be of the first kind if its Picard number is one, and of the second kind if its Picard number is at least two. Fano varieties of the second kind have nontrivial extremal rays, which can be contracted in accordance with Mori's program, and are thus sometimes regarded as being less primitive than Fano varieties of the first kind.

"There is a well-known conjecture asserting that there are only finitely many deformation types of Fano varieties in each dimension. There is another well-known conjecture asserting that the anticanonical degree of a Fano variety is bounded from above by a universal constant depending only on the dimension. These two boundedness conjectures are equivalent, by work of Kollár and Matsusaka or Demailly.

"Boundedness of Fano varieties of dimension three or less follows from the classification. Indeed, the only Fano 1-fold is $\mathbf{P}^1$; the Fano 2-folds are the del Pezzo surfaces; and the classification of Fano 3-folds by Fano, Iskovskikh, Mori, Mukai, and Shokurov implies that there are precisely 104 deformation types of Fano 3-folds. Boundedness of toric Fano varieties was established by Batyrev. Boundedness of Fano varieties of the first kind was established in an earlier paper of ours ["A finiteness theorem for Fano 4-folds", Preprint; per bibl.] in dimension four, and in papers by J. Kollár, Y. Miyaoka and S. Mori [in *Classification of irregular varieties (Trento, 1990)*, 100–105, Lecture Notes in Math., 1515, Springer, Berlin, 1992; MR1180339 (94f:14039)] and us [J. Amer. Math. Soc. **4** (1991), no. 4, 681–692; MR1115788 (93g:14048)] in arbitrary dimensions.

"Here we are interested in the following setup. Let $\pi: M \to X$ be a connected (i.e., having connected fibers) surjective morphism from a (smooth) Fano variety M onto a (possibly singular) projective variety X. Let L be an ample (actually, nef and big is enough) line bundle on X. Let F be a general fiber of our morphism; it too is a Fano variety. Let k be the least integer such that K_F^{-k} is very ample on F; according to J.-P. Demailly [J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)], $k \leq 12f^f$, where $m = \dim M$, $f = \dim F$, and $x = \dim X$. Our main result is as follows. Theorem A. In the above situation, we have the estimate $c_1(M)^m \leq 3(m\tau)^m |(X,L)|$, where $|(X,L)| = \min_{0\leq\eta\leq x} h^0(X, L^\eta)$, and where $\tau = k^{f-1}c_1(F)^f + 2$ if $f > 0$ and $\tau = 2$ if $f = 0$.

"This article is organized as follows. The first section contains material on coherent sheaves of ideals associated to almost-plurisubharmonic functions, and culminates in a cohomology vanishing theorem. The second section contains various bits and pieces, mainly in the forms of lemmas. The third section contains the actual proof of Theorem A."

{For the entire collection see MR1215274 (93j:53002)}

MR1211891 (94g:14006) 14E25 (14C20)
Beltrametti, Mauro C. (I-GENO); **Sommese, Andrew J.** (1-NDM)
On k-jet ampleness.
Complex analysis and geometry, 355–376, *Univ. Ser. Math.*, *Plenum*, *New York*, 1993.

The authors previously introduced and studied different notions of higher order embeddings for projective complex manifolds [cf. in Algebraic geometry (L'Aquila, 1988), 24–51, Lecture Notes in Math., 1417, Springer, Berlin, 1990; MR1040549 (91g:14029); in 1988 Cortona Proceedings: projective surfaces and their classification, 33–48, Symposia Math., 32, Academic Press, New York, 1991; per bibl.; M. C. Beltrametti, P. Francia and A. J. Sommese, Duke Math. J. 58 (1989), no. 2, 425–439; MR1016428 (90h:14021)]. In the paper under review they study the stronger of these notions, k-jet ampleness. A line bundle L on a projective n-fold X is k-jet ample for a nonnegative integer k if, given any r integers $k_1 + \cdots + k_r = k$ and any r distinct points x_i on X with maximal ideals m_i, the restriction map on the global section $\Gamma(L) \to \bigoplus_{i=1}^{r} \Gamma(L/m_i^{k_i})$ is onto.

Relationships with the previously introduced k-very ampleness and k-spannedness are investigated, together with the behaviour of k-jet ampleness under tensor products. A lower bound for the degree of a k-jet ample line bundle is obtained.

A large part of the paper is devoted to obtaining results of k-jet ampleness for adjoint bundles $K + tL$, where K is the canonical bundle of the n-fold. Sharp results relating the k-jet ampleness of L with the τ-jet ampleness of $K + tL$ are obtained by showing that the latter follows from the vanishing of a suitable first cohomology group, obtained via results of Kawamata and Viehweg. On surfaces, conditions on L are relaxed to nef, and under additional assumptions on the dimension of the linear system associated to L and on its self-intersection, assumptions which allow the authors to use Ramanujam vanishing, k-jet ampleness of the adjoint is proved. Further results are given for special surfaces. Some of the results on surfaces were independently obtained by M. Andrea de Cataldo. The authors call attention to a related paper by J.-P. Demailly [J. Differential Geom. 37 (1993), no. 2, 323–374; MR1205448 (94d:14007)].

{For the entire collection see MR1211876 (93j:32001)}.

{For the entire collection see MR1211876 (93j:32001)}

Reviewed by *Gian Mario Besana*

MR1211880 (94k:32009) 32C30 (32F07)

Demailly, Jean-Pierre (F-GREN-F)

Monge-Ampère operators, Lelong numbers and intersection theory.

Complex analysis and geometry, 115–193, *Univ. Ser. Math.*, *Plenum*, *New York*, 1993.

This article surveys the theory of Lelong numbers and their applications for studying intersection theory of analytic cycles. For this point of view, we define a plurisubharmonic (psh) function on a complex manifold X to be an upper semicontinuous function u for which $dd^c u$ is a positive closed current of bidegree $(1,1)$. If u is a locally bounded psh function on X and T is a positive, closed current of bidimension (p,p), the wedge product $dd^c u \wedge T := dd^c(uT)$ defines a closed positive current; by induction, if $1 \le q \le p$ and $u_1, \cdots, u_q$ are locally bounded psh functions on X then

$$dd^c u_1 \wedge \cdots \wedge dd^c u_q \wedge T := dd^c(u_1 dd^c u_2 \wedge \cdots \wedge dd^c u_q \wedge T)$$

is a closed positive current. If $u_1^k, \cdots, u_q^k$ are decreasing sequences of psh functions converging pointwise to $u_1, \cdots, u_q$ then, following E. Bedford and B. A. Taylor [Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)], it is shown in Section 2 that

$$(1) \qquad u_1^k dd^c u_2^k \wedge \cdots \wedge dd^c u_q^k \wedge T \to u_1 dd^c u_2 \wedge \cdots \wedge dd^c u_q \wedge T$$

and

$$(2) \quad dd^c u_1^k \wedge \cdots \wedge dd^c u_q^k \wedge T \to dd^c u_1 \wedge \cdots \wedge dd^c u_q \wedge T$$

weakly.

Section 3 discusses the definition of $dd^c u_1 \wedge \cdots \wedge dd^c u_q \wedge T$ for certain psh u_i and closed positive currents T even if some of the u_i are not necessarily bounded below. Define the unbounded locus $L(u)$ of u to be the set of points $x \in X$ such that u is unbounded in every neighborhood of x. Modifying the arguments of the previous section, the following result is proved: Theorem 1. Let $u_1, \cdots, u_q$ be psh functions on X. The currents $u_1 dd^c u_2 \wedge \cdots \wedge dd^c u_q \wedge T$ and $dd^c u_1 \wedge \cdots \wedge dd^c u_q \wedge T$ are well-defined and have locally finite mass in X provided $q \le p =$ dimension of T and

$$\mathcal{H}_{2p-2m+1}(L(u_{j_1}) \cap \cdots \cap L(u_{j_m}) \cap \operatorname{Supp} T) = 0$$

for all choices of indices $j_1 < \cdots < j_m$ in $\{1, \cdots, q\}$. Here, $\mathcal{H}_s(E)$ denotes the s-dimensional Hausdorff measure of E. In addition, it is shown that the analogues of the monotone convergence theorems in Section 2 remain valid.

In Section 4, the definition of generalized Lelong numbers is given. For a Stein manifold X, let T be a positive, closed current of bidimension (p,p) and let $\varphi\colon X \to [-\infty, +\infty)$ be a semiexhaustive weight function on $\operatorname{Supp} T$, i.e., φ is continuous and psh on X and there exists R such that $B(R) \cap \operatorname{Supp} T \subset\subset X$, where $B(R) = \{x \in X\colon\ \varphi(x) < R\}$. It follows that $\{\varphi = -\infty\} \cap \operatorname{Supp} T$ is compact and $T \wedge (dd^c\varphi)^p$ is well-defined. For each $r \in (-\infty, R)$, set $\nu(T, \varphi, r) = \int_{B(r)} T \wedge (dd^c\varphi)^p$ and define the generalized Lelong number of T with respect to φ as

$$(3) \qquad \nu(T, \varphi) = \int_{\{\varphi=-\infty\}} T \wedge (dd^c\varphi)^p = \lim_{r \to -\infty} \nu(T, \varphi, r).$$

For $X = \mathbf{C}^n$ and $\varphi(z) = \log |z|$, this agrees with the ordinary Lelong number of T at 0,

$$\nu(T, 0) = \lim_{r \to 0} \frac{\sigma_T(B(r))}{\pi^p r^{2p}/p!},$$

where $\sigma_T = T \wedge (dd^c|z|^2)^p$ is the trace measure of T and $B(r)$ is the (Euclidean) ball of radius r centered at 0.

In Section 5, a Lelong-Jensen type formula is proved. Suppose $B(R) \subset\subset X$. Let $\mu_r = (dd^c[\max\{\varphi, r\}])^n - \mathbf{1}_{X-B(r)}(dd^c\varphi)^n$, $r < R$. In the case $X = \mathbf{C}^n$ and $\varphi(z) = \log |z|$, this is just a normalized surface area measure on the sphere of radius e^r. For any psh function V on X, V is μ_r-integrable for each $r < R$ and

$$\mu_r(V) - \int_{B(r)} V (dd^c\varphi)^n = \int_{-\infty}^r \nu(dd^c V, \varphi, t)\, dt.$$

This reduces to the classical Jensen formula if $n = 1$ ($X = \mathbf{C}$) and $V = \log |f|$. If $(dd^c\varphi)^n = 0$ on $X - \{\varphi = -\infty\}$, one obtains the formula $\nu(dd^c V, \varphi) = \lim_{r \to -\infty} \mu_r(V)/r$. An interesting case occurs in $\mathbf{C}^n$ if one takes $\varphi(z) = \log \max |z_j|^{\lambda_j}$, where $\lambda_j > 0$. It can be shown that $(dd^c\varphi)^n =$

$\lambda_1 \cdots \lambda_n \delta_0$ and $\mu_r = \lambda_1 \cdots \lambda_n (2\pi)^{-n} d\theta_1 \cdots d\theta_n$ on the distinguished boundary $\{z\colon\ |z_j| = e^{r/|\lambda_j|}\}$ of the polydisk $B(r)$. The Lelong number $\nu(dd^c V, \varphi)$ is then

$$\lim_{r\to-\infty} \frac{\lambda_1 \cdots \lambda_n}{r} \int_{\theta_j \in [0,2\pi]} V(e^{r/\lambda_1 + i\theta_1}, \cdots, e^{r/\lambda_n + i\theta_n}) \frac{d\theta_1 \cdots d\theta_n}{(2\pi)^n},$$

which is the directional Lelong number of $dd^c V$ at 0 with coefficients $\lambda := (\lambda_1, \cdots, \lambda_n)$ introduced by Kiselman. In general, for any current T, define $\nu(T, x, \lambda) = \nu(T, \log \max |z_j - x_j|^{\lambda_j})$; in $\mathbf{C}^n$, taking $\varphi(z) = \log |z - x|$, it follows that the usual Lelong numbers $\nu(T, x)$ agree with the Kiselman numbers $\nu(T, x, (1, \cdots, 1))$.

In Section 6, a new proof is given of Thie's theorem: If A is an analytic set of pure dimension p and $[A]$ is the current of integration over A, then for each $x \in A$, $\nu([A], x)$ is the multiplicity of A at x. Also, a result of Siu's on stability of Lelong numbers for closed, positive currents under slicing along linear subspaces is presented. In Section 7, a generalization of Siu's upper semicontinuity theorem is proved [J.-P. Demailly, Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)]. Section 8 describes the behavior of Lelong numbers under proper morphisms. As a concrete application, it is shown that for T a closed, positive current of bidimension (p, p) and for $0 < \lambda_1 \le \cdots \le \lambda_n$, the directional Lelong numbers of Kiselman satisfy $\lambda_1 \cdots \lambda_p \nu(T, x) \le \nu(T, x, \lambda) \le \lambda_{n-p+1} \cdots \lambda_n \nu(T, x)$. A type of Schwarz lemma relating growth of zeros of entire functions f in $\mathbf{C}^n$ which involves Lelong numbers of the current of integration $[Z_f]$ of the zero variety of f is proved in Section 9. This yields a theorem of Bombieri on algebraic values of meromorphic maps satisfying algebraic differential equations. Finally, in Section 10, a self-intersection inequality for closed positive currents of bidegree (1,1) is given. The motivation behind this inequality is the following. The wedge product of smooth differential forms defines a ring structure on de Rham cohomology, and, for two currents Θ_1, Θ_2 on X, there is a well-defined intersection class $\{\Theta_1\} \cdot \{\Theta_2\}$ in the cohomology ring, even if $\Theta_1 \wedge \Theta_2$ is not defined pointwise as a current. But the wedge product of closed, positive currents cannot always be defined in a reasonable way, and, moreover, such currents cannot necessarily be approximated in the weak topology by smooth closed, positive currents. Indeed, for T a closed, positive current, a necessary condition for such an approximation to be possible is that $\{T\}^p \cdot \{Y\} \ge 0$ for every p-dimensional subvariety $Y \subset X$. The author showed [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)] that T can be approximated by closed, real currents with small negative part governed by the curvature of X. Then, by regularizing and taking weak limits, one can compute self-intersections.

{For the entire collection see MR1211876 (93j:32001)}

Reviewed by *Norman Levenberg*

MR1211876 (93j:32001) 32-06

★**Complex analysis and geometry.**

Edited by Vincenzo Ancona and Alessandro Silva.

The University Series in Mathematics.

Plenum Press, *New York*, 1993. xvi+412 *pp*. $85.00. *ISBN* 0-306-44179-9

Contents: Daniel Barlet, Theory of (a, b)-modules. I (1–43); Jürgen Bingener and Hubert Flenner, On the fibers of analytic mappings (45–101); Paolo De Bartolomeis, Twistor constructions for vector bundles (103–114); Jean-Pierre Demailly, Monge-Ampère operators, Lelong numbers and intersection theory (115–193); Pierre Dolbeault, CR analytic varieties with given boundary (195–207); John Erik Fornæss and Nessim Sibony, Smooth pseudoconvex domains in $\mathbf{C}^2$ for which the corona theorem and L^p estimates for $\overline{\partial}$ fail (209–222); Alan T. Huckleberry and G. Fels, A characterization of K-invariant Stein domains in symmetric embeddings (223–234); László Lempert, Complex structures on the tangent bundle of Riemannian manifolds (235–251); Ngaiming Mok and Sai-Kee Yeung, Geometric realizations of uniformization of conjugates of Hermitian locally symmetric manifolds (253–270); Mauro Nacinovich, Approximation and extension of Whitney CR forms (271–283); Takeo Ohsawa, The existence of right inverses of residue homomorphisms (285–291); Thomas Peternell, Tangent bundles, rational curves, and the geometry of manifolds of negative Kodaira dimension (293–310); Robert Braun, Giorgio Ottaviani, Michael Schneider [Michael Hellmut Schneider] and Frank-Olaf Schreyer, Boundedness for nongeneral-type 3-folds in $\mathbf{P}_5$ (311–338); Georg Schumacher, The curvature of the Petersson-Weil metric on the moduli space of Kähler-Einstein manifolds (339–354); Mauro C. Beltrametti and Andrew J. Sommese, On k-jet ampleness (355–376); Giuliana Gigante and Giuseppe Tomassini, Deformations of complex structures on a real Lie algebra (377–385); Edoardo Ballico, A problem list on vector bundles (387–402).

{The papers are being reviewed individually.}

MR1207864 (94f:32060) 32L10 (32F10 32L15)

Demailly, Jean-Pierre (F-GREN)

Holomorphic Morse inequalities on q-convex manifolds.

Several complex variables (*Stockholm*, 1987/1988), 245–257, *Math. Notes*, 38, *Princeton Univ. Press*, *Princeton*, *NJ*, 1993.

This note is a report on a paper by T. Bouche [Ann. Sci. École Norm. Sup. (4) **22** (1989), no. 4, 501–513; MR1026747 (91a:32041)] in which holomorphic Morse inequalities for strongly q-convex manifolds were obtained, extending the results of the author [in *Séminaire d'analyse P. Lelong-P.*

Dolbeault-H. Skoda, années 1983/1984, 88–97, Lecture Notes in Math., 1198, Springer, Berlin, 1986; MR0874763 (88f:32069)]. The author carefully describes the main ideas and techniques used by Bouche and illustrates some interesting applications.

{For the entire collection see MR1207850 (93j:32002)}

Reviewed by *Antonella Nannicini*

MR1207850 (93j:32002) 32-06

★**Several complex variables.**

Proceedings of the Special Year held at the Mittag-Leffler Institute, Stockholm, 1987/1988.
Edited by John Erik Fornæss.
Mathematical Notes, 38.
Princeton University Press, *Princeton*, *NJ*, 1993. viii+701 *pp*. $39.50. *ISBN* 0-691-08579-X

Contents: H. Alexander, Removable sets for CR functions (1–7); Frederick J. Almgren, Jr., What can geometric measure theory do for several complex variables? (8–21); É. Amar, $\overline{\partial}_b$-equation and nonfactorization (22–33); Mats Andersson, L_p estimates for the $\overline{\partial}$-equation in analytic polyhedra in Stein manifolds (34–47); Eric Bedford, Survey of pluri-potential theory (48–97); Jay Belanger, Hölder estimates for $\overline{\partial}$ in $\mathbf{C}^2$ (98–125); S. Bell [Steven R. Bell], Algebraic mappings of circular domains in $\mathbf{C}^n$ (126–135); Bo Berndtsson, Levi-flat surfaces with circular sections (136–159); Bo Berndtsson, Weighted integral formulas (160–187); Urban Cegrell, Peak sets and representing measures in the spectrum of $H^\infty(\Omega)$ (188–193); C. H. Chang [Ch'ing Hui Chang] and H. P. Lee [Hsüan Pei Li], Explicit solutions for some extremal analytic discs of the domain $D = \{z \in \mathbf{C}^n |\ \sum_1^n |z_j|^{2m_j} < 1,\ m_j \in \mathbf{N}\}$ (194–204); Jacques Chaumat and Anne-Marie Chollet, Noyaux pour résoudre l'équation $\overline{\partial}$ dans des classes ultradifférentiables sur des compacts irreguliérs de $\mathbf{C}^n$ [Kernels for solving the $\overline{\partial}$ equation in ultradifferentiability classes on irregular compact sets in $\mathbf{C}^n$] (205–226); John P. D'Angelo, The structure of proper rational holomorphic maps between balls (227–244); Jean-Pierre Demailly, Holomorphic Morse inequalities on q-convex manifolds (245–257); Pierre Dolbeault, On the structure of residual currents (258–273); Avner Dor, Properties of smooth proper holomorphic maps between balls (274–296); Franc Forstnerič, Proper holomorphic mappings: a survey (297–363); Bernard Gaveau, Masami Okada and Tatsuya Okada, Explicit heat kernels on graphs and spectral analysis (364–388); Robert E. Greene and Steven G. Krantz, Techniques for studying automorphisms of weakly pseudoconvex domains (389–410); Andrei Iordan, Peak sets in domains with weak pseudoconvexity along nonintegral curves (411–415); Christine Laurent-Thiébaut and Jürgen Leiterer, The Andreotti-Vesentini separation theorem with C^k estimates and extension of CR-forms (416–439); László Lempert, Elliptic and hyperbolic tubes (440–456); Ingo Lieb, A survey of the $\overline{\partial}$-problem (457–472); Ingo Lieb and R. Michael

Range, The $\overline{\partial}$-Neumann kernel for codimension-one forms on strictly pseudoconvex domains (473–482); Qi Keng Lu and Yi Hong, The heat kernel of the classical domain $\mathfrak{R}_I(m,n)$ (483–506); Guido Lupacciolu and Edgar Lee Stout, Removable singularities for $\overline{\partial}_b$ (507–518); Per E. Manne, Carleman approximation on totally real submanifolds of a complex manifold (519–528); Alan Noell, Peak functions for pseudoconvex domains in $\mathbf{C}^n$ (529–541); Mikael Passare, On the support of residue currents (542–549); Juhani Riihentaus, Nullsets for BMO functions (550–562); Jean-Pierre Rosay and Walter Rudin, Holomorphic embeddings of $\mathbf{C}$ in $\mathbf{C}^n$ (563–569); Rita Saerens and William R. Zame, The local automorphism group of a CR hypersurface (570–572); A. G. Sergeev, On matrix Reinhardt and circled domains (573–586); Nessim Sibony, On Hölder estimates for $\overline{\partial}$ (587–599); Edgar Lee Stout, Removable singularities for the boundary values of holomorphic functions (600–629); S. M. Webster, An implicit function theorem of the Nash-Moser type (630–639); H. Wu [Hung Hsi Wu], Old and new invariant metrics on complex manifolds (640–682); Ahmed Zériahi, Inegalités de Markov et développement en série de polynômes orthogonaux des fonctions C^∞ et A^∞ [Markov inequalities and orthogonal polynomial series expansions of C^∞ and A^∞ functions] (683–701).

{The papers are being reviewed individually.}

MR1206442 (94m:32044) 32J20 (32C10 32C17 32C30 32L07)
Ji, Shanyu (1-HST)
Currents, metrics and Moishezon manifolds. (English summary)
Pacific J. Math. **158** (1993), *no. 2,* 335–351.

In this paper, the author proves the following theorem: Let M be a compact complex manifold of dimension n; then M is Moishezon (bimeromorphic to a projective variety) if and only if there exists a positive definite integral d-closed (1,1)-current (a singular (1,1)-form) ω on M such that ω is smooth outside a proper analytic subset. This is a generalization of Kodaira's embedding theorem. In another paper by the author and B. Shiffman [J. Geom. Anal. **3** (1993), no. 1, 37–61; MR1197016 (93m:32014)] it was proved by a combination of this theorem and a smoothing theorem by J.-P. Demailly [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)] that the result is true without the condition on smoothness, which was a conjecture of Shiffman. The proof is based on Demailly's L^2-estimate of the $\overline{\partial}$-operator on complete Kähler manifolds with noncomplete Kähler metric and with singular Hermitian metric on a line bundle by observing that the open set of smooth points of the current has a complete Kähler metric (Lemma 4.1). The author also proves that M is Moishezon if and only if there is a proper subset of M such that its complement in M admits a complete Kähler-Einstein metric with negative Ricci curvature

(Theorem 1.4).

Reviewed by *Zhuang Dan Guan*

MR1205448 (94d:14007) 14C20 (14E25 32J25 32L10)
Demailly, Jean-Pierre (F-GREN)
A numerical criterion for very ample line bundles. (English summary)
J. Differential Geom. **37** (1993), *no. 2,* 323–374.

Let X be a smooth projective variety of dimension n defined over the field of complex numbers. A divisor (or, equivalently, a line bundle) L on X is ample if, by definition, some positive multiple of L is very ample, that is, it is isomorphic to the restriction of $\mathcal{O}(1)$ for some embedding $X \subset \mathbf{P}^N$. The notion of very ampleness has a definite geometric meaning while, as it follows from well-known Kleiman and Nakai criteria, ampleness has a very numerical character. Therefore, it is natural to ask for a numerical criterion asserting very ampleness of an ample divisor. One observes easily that for curves the answer depends not only on the degree of the divisor L but also on the geometry of X; in particular, it depends on the genus of the curve X. A uniform answer for curves is as follows: if L is an ample divisor on a curve X then $K_X + 3L$ is very ample, where K_X denotes the canonical divisor of X. For surfaces, a result of I. Reider implies very ampleness of $K_X + 4L$. On the other hand, the theory of extremal rays of S. Mori implies ampleness of $K_X + (n+2)L$ for an ample divisor L on a smooth n-fold X; T. Fujita conjectured that $K_X + (n+2)L$ is actually very ample.

The main result of the paper under review is a significant step towards the conjecture of Fujita. Namely, the main theorem of the paper gives numerical conditions for an ample (or, more generally, big and nef) divisor L to imply spannedness or very ampleness (or, more generally, separation of s-jets of sections) of the adjoint divisor $K_X + L$. From the theorem it follows that $2K_X + mL$ is very ample if only $m \geq 12n^n$. This in turn yields an effective version of T. Matsusaka's big theorem [cf. Amer. J. Math. **94** (1972), 1027–1077; MR0337960 (49 #2729); Y. T. Siu, "An effective Matsusaka big theorem", Preprint, 1993; per revr.], and combined with results of Catanese and of Green and Morrison implies an effective bound on the number of irreducible families of n-dimensional smooth polarised varieties (X, L) depending only on the intersection numbers L^n and $K_X \cdot L^{n-1}$ [J. Kollár, Math. Ann. **296** (1993), no. 4, 595–605]. Although the main result of the paper and its applications are formulated in terms of algebraic geometry, the proof of the theorem is analytic and it applies Hörmander L^2 estimates for the operator $\overline{\partial}$, the Aubin-Calabi-Yau theorem and the theory of positive currents and Lelong numbers.

{Reviewer's remarks: Among most recent developments following the paper under review there are the above-mentioned papers of Siu and Kollár as well as a paper of L. M. H. Ein and R. K.

Lazarsfeld [J. Amer. Math. Soc. **6** (1993), no. 4, 875–903; MR1207013 (94c:14016)]. The Kollár and Ein-Lazarsfeld papers involve algebraic settings (Kollár works with varieties having Kawamata log terminal singularities) but, in the end, they depend on the transcendental Kodaira-Kawamata-Viehweg vanishing.}

Reviewed by *Jarosław A. Wiśniewski*

MR1197016 (93m:32014) 32C30 (32J27)

Ji, Shanyu (1-HST); **Shiffman, Bernard** (1-JHOP)

Properties of compact complex manifolds carrying closed positive currents. (English summary)

J. Geom. Anal. **3** (1993), *no. 1,* 37–61.

The authors give an interesting characterization of Moishezon manifolds via currents; it is similar to a previous characterization due to J.-P. Demailly using differential forms. Let M be a compact complex manifold; the authors define a Kählerian current as follows: A current T of bidegree $(1,1)$ is Kählerian if T is d-closed and if there is a strictly positive differential form ω of the same bidegree such that $T' = T - \omega$ is a semipositive current (in the following sense: T' is real and $T'(\eta)$ is nonnegative for any $(n-1, n-1)$ semipositive form η on M). The authors prove M is Moishezon if and only if there exists a Kählerian current in $H^2(M, \mathbf{Z})$; they give another equivalent condition using "singular metrics" on holomorphic line bundles. Moreover, they obtain a sufficient condition so that the intersection of two d-closed semipositive currents of complementary degrees is a positive current.

Reviewed by *Salomon Ofman*

MR1205883 (94e:32051) 32L20 (14F17)

Manivel, Laurent (F-GREN-F)

Théorèmes d'annulation pour les fibrés associés à un fibré ample. (French) [Vanishing theorems for vector bundles associated with an ample vector bundle]

Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **19** (1992), *no. 4,* 515–565.

In this paper the author generalizes a vanishing theorem of J.-P. Demailly [Invent. Math. **91** (1988), no. 1, 203–220; MR0918242 (89d:32066)]. More precisely, the main theorem proved here is the following: Let E be a holomorphic vector bundle of rank d and let L be a line bundle over an n-dimensional compact complex manifold X such that E is ample and L is nef or E is nef and L is ample. Then if $a \in (\mathbf{N}^d_{\geq})$ and $p \geq n-20$, $H^{p,q}(X, \Gamma^a E \otimes (\det\ E)^l \otimes L) = 0$ for $l \geq h(a) + n - p$ and $p+q > n$. Here $\Gamma^a(E)$ is the bundle associated to E and to the representation of $\mathrm{GL}(d, \mathbf{C})$ of weight $a \in (\mathbf{N}^d_{\geq})$, $(\mathbf{N}^d_{\geq})$ is the set of decreasing d-tuplets of natural numbers and $h(a)$ is the number of nonzero components of a. The methods used are similar to those in the above-mentioned paper of Demailly.

Reviewed by *Antonella Nannicini*

MR1195080 (93m:32024) 32F20

Langenbruch, Michael (D-MUNS)

Splitting of the $\overline{\partial}$-complex in weighted spaces of square integrable functions. (English summary)

Rev. Mat. Univ. Complut. Madrid **5** (1992), *no. 2-3,* 201–223.

The author continues the work of Meise, Taylor and collaborators on the splitting of the $\overline{\partial}$-complex in weighted L^2-spaces. Let Ω be a pseudoconvex set in $\mathbf{C}^m$, $\mathcal{B}$ an increasing sequence $\{W_n\}$ of weight functions in Ω, $L^2(\mathcal{B}, \Omega) = \{f \in L^2_{\mathrm{loc}}(\Omega)\colon \int_\Omega |f|^2 e^{-2W_n} < \infty$ for some $n \geq 1\}$. It is shown that the splitting of the $\overline{\partial}$-complex is equivalent to the existence of plurisubharmonic functions Φ_t in Ω $(t \in \Omega)$ and $I\colon \mathbf{N}^* \to \mathbf{N}^*$, $I(n) \geq n$, $A\colon \mathbf{N}^* \to \mathbf{N}^*$, such that, for every $z, t \in \Omega$, $\Phi_t(t) \geq 0$, $\Phi_t(z) \leq W_{I(n)}(z) - W_n(t) + A(n)$. He also provides explicit estimates for the splitting (right inverse) operators R. Applications to the existence of a linear extension from interpolating manifolds are given (this complements results of the reviewer and B. A. Taylor [in *Séminaire Pierre Lelong-Henri Skoda (Analyse), Années 1980/1981, et Colloque de Wimereux, Mai 1981*, 1–25, Lecture Notes in Math., 919, Springer, Berlin, 1982; MR0658877 (83k:32004)] and J.-P. Demailly [ibid., 77–107; MR0658880 (83j:32019)]).

Reviewed by *Carlos A. Berenstein*

MR1178721 (93g:32044) 32L10 (14J60 32L05)

Demailly, Jean-Pierre (F-GREN-F)

Singular Hermitian metrics on positive line bundles. (English summary)

Complex algebraic varieties (*Bayreuth*, 1990), 87–104, *Lecture Notes in Math.*, 1507, *Springer*, *Berlin*, 1992.

In this paper, the author introduces the notion of singular Hermitian metric on a holomorphic line bundle on a complex manifold. A singular Hermitian metric on a holomorphic line bundle L on a complex manifold M is a product of the form $h_0 e^{-\varphi}$, where h_0 is a smooth Hermitian metric on L and φ is a locally L^1-function. The beauty of the definition is that we can take the curvature of a singular Hermitian metric in the sense of a closed current. This extended definition of Hermitian metrics enables us to characterize the line bundle to be pseudoeffective, big or nef in terms of the positivity properties of the curvature of the singular metrics. For the applications of these metrics, the author shows the relation between the Seshadri constant and the singularity of singular Hermitian metrics. This is related to the global generation of nef line bundles. Finally, he uses the singular metrics to prove a new asymptotic estimate for the dimension of the cohomology groups with values in high multiples $\mathcal{O}(kL)$ of a big line bundle L.

{For the entire collection see MR1178715 (93d:14006)}

Reviewed by *Hajime Tsuji*

MR1178715 (93d:14006) 14-06

★**Complex algebraic varieties.**

Proceedings of the conference held in Bayreuth, April 2–6, 1990.
Edited by K. Hulek, T. Peternell, M. Schneider and F.-O. Schreyer.
Lecture Notes in Mathematics, 1507.
Springer-Verlag, *Berlin*, 1992. vi+179 *pp*. $25.00. *ISBN* 3-540-55235-9

Contents: Arnaud Beauville, Annulation du H^1 pour les fibrés en droites plats [Vanishing of H^1 for flat line bundles] (1–15); Mauro C. Beltrametti, Andrew J. Sommese and Jarosław A. Wiśniewski, Results on varieties with many lines and their applications to adjunction theory (16–38); Guntram Bohnhorst and Heinz Spindler, The stability of certain vector bundles on $\mathbf{P}^n$ (39–50); F. Catanese and F. Tovena, Vector bundles, linear systems and extensions of π_1 (51–70); Olivier Debarre, Vers une stratification de l'espace des modules des variétés abéliennes principalement polarisées [Toward a stratification of the moduli space of principally polarized abelian varieties] (71–86); Jean-Pierre Demailly, Singular Hermitian metrics on positive line bundles (87–104); Takao Fujita, On adjoint bundles of ample vector bundles (105–112); Yujiro Kawamata, Moderate degenerations of algebraic surfaces (113–132); Ulf Persson, Genus two fibrations revisited (a preliminary report) (133–144); Th. Peternell, M. Szurek and J. A. Wiśniewski [Jarosław A. Wiśniewski], Numerically effective vector bundles with small Chern classes (145–156); C. A. M. Peters, On the rank of nonrigid period maps in the weight one and two case (157–165); A. N. Tyurin, The geometry of the special components of moduli space of vector bundles over algebraic surfaces of general type (166–175).

{Most of the papers are being reviewed individually.}

MR1175540 (93f:32012) 32C30 (32-01)

Demailly, Jean-Pierre (F-GREN-F)

Courants positifs et théorie de l'intersection. (French) [Positive currents and intersection theory]

Gaz. Math. No. 53 (1992), 131–159.

This article is a brief exposition of intersection theory from the point of view of positive currents.

Let Y be a complex analytic manifold of dimension n. To any subvariety X of Y, we can associate a current denoted $[X]$ defined by $\langle [X], \varphi\rangle = \int_X \varphi$. After recalling all the requisite definitions, the author gives in the first part of this article the relation between the intersection number of two analytic subvarieties A and B of Y $(\dim A + \dim B = n)$ and the product $[A] \wedge [B]$. The main tools are two theorems of P. Lelong, for which short proofs are given; as an application, Example 5.2 calculates the intersection number of the curves Γ_1 and Γ_2 of $\mathbf{C}^2$ defined, respectively, by the equation $z^2 = w^3$ and $z^3 = w^5$, where (z, w) are the coordinate functions on $\mathbf{C}^2$.

The second part (the last two sections) deals with the "Lelong numbers": to any positive d-closed (p, p)-current T and any $y \in Y$ is associated a number $\nu(T, y) \in \mathbf{R}_+$ called the "Lelong number of T at y", which is related to the "regularity" of T at the point $y \in Y$. Its definition is recalled and generalized in Section 6. The last section deals with more recent results: by using a theorem of Siu, the author states interesting estimations for the degree of the irreducible components of $E_c(T) = \{y \in Y\colon\ \nu(T, y) \geq c\}$.

This article uses mainly elementary results of analytic geometry in the proofs. It is an excellent introduction to the subject.

Reviewed by *Salomon Ofman*

MR1158622 (93e:32015) 32C30 (32C17 32J25)
Demailly, Jean-Pierre (F-GREN)
Regularization of closed positive currents and intersection theory.
J. Algebraic Geom. **1** (1992), *no. 3,* 361–409.

Let X be a complex manifold of dimension n and let T be a closed current of bidegree $(1,1)$ on X. T is said to be almost positive if there exists a smooth form v of bidegree $(1,1)$ such that $T+v\geq 0$. If T is closed and almost positive, let (T,x) denote the Lelong number of T at x. For every $c>0$, let $E_c(T)=\{x\in X\colon \nu(T,x)\geq c\}$. By Siu's theorem $E_c(T)$ is an analytic subset of X. The main result of the paper under review is an "approximation-regularization" theorem for closed almost positive currents T. It says that T can be approximated, for every $c>0$, by T_c which are smooth outside $E_c(T)$, and such that $\nu(T_c,x)=(\nu(T,x)-c)_+$ at every point $x\in X$. The T_c can be chosen so that they satisfy certain estimates from below. Although the precise statement of the theorem is too long to be described here, it includes the good old regularization theorem of Richberg as a special case, where $T=\partial\overline{\partial}\psi$ for finite and continuous ψ. The proof of the result is based on a combination of three different types of L^2-techniques. Among other things, the reader will be delighted to find an elegant proof of a local approximation theorem for plurisubharmonic functions by logarithms of holomorphic functions. Interesting applications are also given. Some of them are described below. Let $H^{p,q}_{\partial\overline{\partial}}(X)=\{d\text{-closed }(p,q)\text{-forms}\}/\{\overline{\partial}\text{-exact }(p,q)\text{-forms}\}$. A cohomology class $\{\alpha\}\in H^{1,1}_{\partial\overline{\partial}}(X)$ is said to be pseudo-effective (psef) if it can be represented by a closed positive $(1,1)$-current, and nef if for some fixed Hermitian metric ω on X and for every $\varepsilon>0$ there is a smooth form $\alpha_\varepsilon\in\{\alpha\}$ such that $\alpha_\varepsilon\geq-\varepsilon\omega$. Denote respectively by $H^{1,1}_{\mathrm{psef}}(X)$ and $H^{1,1}_{\mathrm{nef}}(X)$ the cones of pseudo-effective and nef cohomology classes. Then, for compact X, the main result of this article implies that $H^{1,1}_{\mathrm{nef}}(X)=H^{1,1}_{\mathrm{psef}}(X)$ if the tangent bundle TX of X is nef. Here one says that a vector bundle E is nef if $C_1(\mathcal{O}_E(1))$ is nef over the projectivized bundle $P(E^*)$ of hyperplanes of E. A related new result is that X is Kahler if TX is nef and X is in the Fujiki class. A self-intersection inequality proved in an earlier work of the author's is extended here to an arbitrary closed positive $(1,1)$-current T on a Kahler manifold X. It may be worthwhile to note that this work was strongly motivated by the question of classifying compact Kahler varieties with nef tangent bundle, which is of course of current research interest.

Reviewed by *Takeo Ohsawa*

MR1222208 (94j:32025) 32L20 (14F17 32L10)

Demailly, J.-P. (F-GREN-F)

Transcendental proof of a generalized Kawamata-Viehweg vanishing theorem. (English summary)

Geometrical and algebraical aspects in several complex variables (*Cetraro*, 1989), 81–94, *Sem. Conf.*, 8, *EditEl*, *Rende*, 1991.

Let L be a numerically effective line bundle on a projective algebraic manifold X of dimension n. The main purpose of this paper is to show a vanishing theorem for the cohomology group $H^q(X, \Omega^n(L \otimes D))$ for the effective **Q**-divisor D which may have nonnormal crossings, under a certain natural integrability condition for D. As a corollary, the Kawamata-Viehweg vanishing theorem follows, i.e. if L is as above and $\bigwedge^k c_1(L) \neq 0$, then $H^q(X, \Omega^n(L)) = 0$ for $q > n - k$. The author's proof is analytic in the sense that his method is based on a vanishing theorem of L^2 cohomology for $\overline{\partial}$ on complete Kähler manifolds for line bundles provided with a singular metric which yields a positive curvature in the sense of currents. This theorem is induced from an L^2 estimate for $\overline{\partial}$ by the Bochner-Kodaira-Nakano curvature inequality and a smooth procedure for plurisubharmonic functions on Kähler manifolds by the author. To show the vanishing theorem the problem is reduced to the case that L is ample by standard slicing arguments and a trick of Kawamata in algebraic geometry. Finally, the theorem is shown by using a vanishing theorem on compact Kähler manifolds which follows from the vanishing theorem of L^2 cohomology.

{For the entire collection see MR1222200 (93m:32002)}

Reviewed by *Kensho Takegoshi*

MR1222200 (93m:32002) 32-06 (00B25)

★**Geometrical and algebraical aspects in several complex variables.**

Papers from the conference held in Cetraro, June 1989.
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Seminars and Conferences, 8.
Editoria Elettronica, *Rende*, 1991. x+376 *pp*.

Contents: T. Aoki [Takashi Aoki], Characteristic sets of differential operators of infinite order (1–11); U. Backlund and A. Fällström, A pseudoconvex domain with nonschlicht H^∞-envelope (13–

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{The papers are being reviewed individually.}

MR1153703 (93f:32032) 32L10 (58E05 58G25)
Bouche, Thierry (F-GREN-F)
Sur les inégalités de Morse holomorphes lorsque la courbure du fibré en droites est dégénérée. (French) [Holomorphic Morse inequalities when the curvature of the line bundle is degenerate]
Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **18** (1991), *no. 4,* 501–523.

Demailly's holomorphic Morse inequalities for the d''-cohomology of $E^k \otimes G$, E a holomorphic line bundle over a complex manifold X with nondegenerate curvature form, G a holomorphic vector bundle of rank g ([J.-P. Demailly, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)], hereafter referred to as [1]), are extended to the d''-cohomology of $E^k \otimes F^l \otimes G$. Here E is a line bundle with degenerate curvature form and F is a line bundle. Denoting by $c(E)$ the curvature of the Chern connection of E, the author assumes that $\operatorname{rank} ic(E)_x \leq r$ and the kernel fibration of $ic(E)$ is well foliated, that is, there is a codimension r foliation Y of X such that its tangent fibre at any point $x \in X$ is included in $\ker ic(E)_x$ [cf. R. Bott, Ann. of Math. (2) **60** (1954), 248–261; MR0064399 (16,276f)]. Setting $\gamma = ic(E)|NY \oplus ic(F)|TY$, he defines the q-index sets $X(q)$ and $X(\leq q)$ to be $\{x \in X\colon\ \operatorname{ind}\gamma_x = q\}$ and $\bigcup_{1\leq j\leq q} X(j)$, respectively. Then the following inequalities are proved. (1) The asymptotic Morse inequality:

$$\dim H^q(X, E^k \otimes F^l \otimes G) \leq \\ g\frac{k^r}{r!}\frac{l^{n-r}}{(n-r)!}\int_{X(q)}(-1)^q\Big(\frac{i}{2\pi}c(E)\Big)^r \wedge \Big(\frac{i}{2\pi}c(F)\Big)^{n-r} + o(k^r l^{n-r}).$$

(2) The strong Morse inequality (Theorem 0.1):

$$\sum_{\nu=0}^{q}\dim H^\nu(X, E^k \otimes F^l \otimes G) \leq \\ g\frac{k^r}{r!}\frac{l^{n-r}}{(n-r)!}\int_{X(\leq q)}\Big(\frac{i}{2\pi}c(E)\Big)^r \wedge \Big(\frac{i}{2\pi}c(F)\Big)^{n-r} + o(k^r l^{n-r}).$$

(3) There exists a constant C such that $\dim H^q(X, E^k \otimes G) \leq Ck^r$, $k \to \infty$ (Corollary 0.2). Here $H^q(X, E^k \otimes F^l \otimes G)$ denotes $H^q(X, \mathcal{O}(E^k \otimes F^l \otimes G))$ and $\mathcal{O}(E^k \otimes F^l \otimes G)$ is the sheaf of germs of holomorphic sections of $E^k \otimes F^l \otimes G$. To show these inequalities, the author uses the metric $\omega_{k,l} = k\eta + l\xi$, where η [resp. ξ] is a C^∞-class Hermitian metric on NY [resp. on TY]. By using $\omega_{k,l}$, Demailly's Hermitian Bochner-Kodaira-Nakano identity is extended to the antiholomorphic Laplacian $\Delta''_{k,l}$ of $E^k \otimes F^l \otimes G$ (Lemma 1.2). Thus the method of [1] can be applied to $\Delta''_{k,l}$ and the inequalities are obtained. As an application, the existence of infinitely many tensor products Q^k which admit nontrivial sections, where $Q = F/E$, E is the same as above and the Kodaira dimension of a fibre F is larger than $r+1$, is shown (Theorem 4.4). An alternative proof of these inequalities using heat kernels and an evaluation of the constant C in (3) was given by the author [Bull. Sci. Math. (2) **116** (1992), no. 2, 167–183].

Reviewed by *Akira Asada*

MR1150978 (93h:32021) 32F07 (32F05 35J60)
Klimek, Maciej (IRL-DBLN)
★**Pluripotential theory.**
London Mathematical Society Monographs. New Series, 6.
Oxford Science Publications.
The Clarendon Press, *Oxford University Press*, *New York*, 1991. xiv+266 *pp*. $59.95.
ISBN 0-19-853568-6

This book is the first relatively comprehensive book ever written on the subject of the title. Pluripotential theory is, loosely speaking, the study of plurisubharmonic (psh) functions; in this text, all psh functions are defined on domains in $\mathbf{C}^n$. An uppersemicontinuous function $u\colon\Omega\subset\mathbf{C}^n\to[-\infty,\infty)$ is psh on a domain Ω provided $u\not\equiv-\infty$ and its restriction to every complex line l which intersects Ω is either subharmonic or $\equiv-\infty$ on each component of $\Omega\cap l$. P. Lelong's 1969 text [*Plurisubharmonic functions and positive differential forms*, Gordon and Breach, New York, 1969] is classic but much has been accomplished since then; the text of U. Cegrell [*Capacities in complex analysis*, Vieweg, Braunschweig, 1988; MR0964469 (89m:32001)] dealt mainly with capacities in $\mathbf{C}^n$; and P. Lelong and L. Gruman [*Entire functions of several complex variables*, Springer, Berlin, 1986; MR0837659 (87j:32001)] concentrated on growth properties of entire functions. The complex Monge-Ampère operator, $(dd^c(\cdot))^n$, is a nonlinear operator which, in $\mathbf{C}^n$, $n>1$, is the natural replacement for the Laplacian. If $u\colon\Omega\subset\mathbf{C}^n\to\mathbf{R}$ belongs to the class $C^2(\Omega)$, then $(dd^cu)^n=c_n\det H(u)dV_n$ where $H(u)=(\partial^2u/\partial z_j\partial\overline{z}_k)_{j,k=1,\cdots,n}$ is the complex Hessian of u, c_n is a dimensional constant, and $dV_n=$ Lebesgue measure on $\mathbf{C}^n$. More generally, E. Bedford and B. A. Taylor [Invent. Math. **37** (1976), no. 1, 1–44; MR0445006 (56 #3351)] have shown that $(dd^cu)^n$ can be defined as a positive measure (precisely, a positive (n,n) current) for any locally bounded psh function u. Such functions which satisfy the homogeneous complex Monge-Ampère equation $(dd^cu)^n=0$ in Ω are the so-called maximal psh functions in Ω and they play the role of harmonic functions in classical potential theory. This is the point of view of this book. A priori, u is maximal on Ω if u is psh and for any relatively compact subdomain G, if v is psh and $v\le u$ on ∂G, then $v\le u$ in G. A large part of the text is aimed at showing that locally bounded maximal functions satisfy the homogeneous complex Monge-Ampère equation. After a motivational discussion of maximal functions in the preface and a short (essentially notational) chapter on complex differentiation, the author gives all the necessary background material on subharmonic and psh functions in the second (and longest) chapter of the book. No prior knowledge of several complex variables or even classical potential theory is assumed; indeed, the reader who completes the exercises at the end of each of these chapters will have no trouble continuing through the rest of the material.

In Section II of the book (beginning with Chapter 3), the reader is introduced to the complex Monge-Ampère operator. Chapter 3 begins with a discussion of currents à la Lelong, and the precise definition of the positive (k,k) current $dd^cu_1\wedge dd^cu_2\wedge\cdots\wedge dd^cu_k$ for locally bounded psh functions $u_1, \cdots, u_k$ is given. Properties of this operator, such as continuity under decreasing limits and comparison theorems, are proved, following another paper of Bedford and Taylor [Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)]; one goal is to show that locally bounded solutions to $(dd^c u)^n = 0$ are maximal. Given a Borel set E in a domain Ω, the notion of the relative capacity of E in Ω, $C(E,\Omega) \equiv \sup\{\int_E (dd^c u)^n\colon\ 0 < u < 1, u \text{ psh in } \Omega\}$ is introduced to prove the important quasicontinuity property of psh functions: a psh u on Ω is continuous off an open set whose relative capacity can be made arbitrarily small. At this stage in the text, the quasicontinuity theorem is proved for locally bounded psh functions; the general case is deferred until later. In Chapter 4, the author proves existence and uniqueness of the solution to the generalized Dirichlet problem for the complex Monge-Ampère operator for a ball; i.e., given f continuous, real-valued on the boundary ∂B of a ball B, find u psh and locally bounded in B satisfying $(dd^c u)^n = 0$ in B and $u = f$ on ∂B. He states certain potential-theoretic and measure-theoretic preliminaries (Riesz decomposition for subharmonic functions, Poisson-Jensen formula, smoothing by using integral averages of locally integrable functions) and then follows the proof of Bedford and Taylor [op. cit., 1976] (in that paper they solved the inhomogeneous equation but the author does not need this, which makes the exposition a bit simpler). The hard part is verifying maximality of the Perron-Bremermann upper envelope $u(z) \equiv \sup\{v(z)\colon\ v \text{ psh in } B,\ \limsup_{\xi\to\zeta} v(\xi) \le f(\zeta),\ \zeta \in \partial B\}$. The rest of the chapter discusses the relative extremal function $u_{E,\Omega}(z) \equiv \sup\{v(z)\colon\ v \text{ psh in } \Omega,\ v \le 0,\ v|_E \le -1\}$ for a subset E of Ω. This is used, again following Bedford and Taylor [op. cit., 1982], to relate the notions of negligible sets, pluripolarity, and thinness. A set $E \subset \mathbf{C}^n$ is pluripolar if for each point $a \in E$ there exists a neighborhood U of a and a psh function u in U with $E \cap U \subset \{z \in U\colon\ u(z) = -\infty\}$. If A is a family of psh functions on a domain Ω which are locally bounded above, the function $u(z) = \sup\{v(z)\colon\ v \in A\}$ may not be psh because it may not be upper semicontinuous (usc). Define $u^*(z) \equiv \limsup_{\xi\to z} u(\xi)$, the usc regularization of u. The set $N \equiv \{z \in \Omega\colon\ u(z) < u^*(z)\}$ is called a negligible set. Using the relative extremal function $u_{E,\Omega}(z)$ and the notion of relative capacity $C(E,\Omega)$, the author shows, following arguments of Bedford and Taylor, that a pluripolar set can be defined by one global psh function (B. Josefson's theorem [Ark. Mat. **16** (1978), no. 1, 109–115; MR0590078 (58 #28669)]) and that negligible sets are pluripolar; indeed, these sets are characterized as having outer capacity $C^*(E,\Omega) = 0$.

Other than the background material on psh functions and currents, most of the content of Chapters 3 and 4 can be found in two or three papers of Bedford and Taylor; however, the material discussed in the last two chapters is scattered throughout the literature. In Chapter 5, the author studies maximal functions of logarithmic growth; i.e., functions in the class

$$L \equiv \{u \text{ psh in } \mathbf{C}^n\colon\ u(z) - \log|z| \le O(1),\ |z| \to \infty\}.$$

Given a set $E \subset \mathbf{C}^n$, the pluricomplex Green function of E with pole at infinity is the function $V_E \equiv \sup\{u(z)\colon\ u \in L,\ u \le 0 \text{ on } E\}$. The Monge-Ampère measure $\mu_E \equiv (dd^c V_E)^n$ associated with V_E is called the equilibrium measure for E. If $n = 1$ and E is nonpolar and compact, V_E is the usual Green function for the unbounded component of $\widehat{\mathbf{C}} - E$ with logarithmic pole at infinity and μ_E is the usual equilibrium measure for E. Following J. Siciak [Ann. Polon. Math. **39** (1981), 175–211; MR0617459 (83e:32018)], it is shown that if K is compact, then $V_K(z) =$

$$\sup\{(1/\deg p)\log|p(z)|\colon\ p\ \text{polynomial},\ \|p\|_K \le 1,\ \deg p \ge 1\}.$$

Thus the pluricomplex Green function can be used in studying problems involving polynomial approximation. The author proves some fundamental properties of these functions and even gives explicit computational examples of V_E for certain compact subsets $E \subset \mathbf{R}^n \subset \mathbf{C}^n$, following M. Baran [Ann. Polon. Math. **48** (1988), no. 3, 275–280; MR0978678 (90j:32019); see also M. Lundin, Michigan Math. J. **32** (1985), no. 2, 197–201; MR0783573 (86h:32030)].

Finally, in the last chapter, a pluricomplex Green function with a logarithmic singularity at a finite point is discussed. Given a domain $\Omega \subset \mathbf{C}^n$ and a point $a \in \Omega$, we call $g_\Omega(z, a) \equiv$

$$\sup\{u(z)\colon\ u\ \text{psh in}\ \Omega,\ u < 0, u(z) - \log|z - a| \le O(1),\ z \to a\}$$

the pluricomplex Green function of Ω with pole at a. If Ω is bounded, then $g_\Omega(z, a)$ is maximal in $\Omega - \{a\}$. If $n = 1$, $g_\Omega(z, a)$ is the classical Green function with pole at a (if it exists). After extending the definition of the Monge-Ampère operator so as to include psh functions with logarithmic singularities, the author proves J.-P. Demailly's theorem [Math. Z. **194** (1987), no. 4, 519–564; MR0881709 (88g:32034)] that, at least for hyperconvex domains (pseudoconvex domains with bounded psh exhaustions), $g_\Omega(z, a)$ is the unique solution of the degenerate Monge-Ampère equation $(dd^c u)^n = (2\pi)^n \delta_a$, u is psh in Ω, continuous in $\Omega - \{a\}$, $u(z) - \log|z - a| = O(1)$, $z \to a$, $u(z) \to 0$ on $\partial\Omega$. The book ends (modulo a brief appendix on foliations) with some applications of this pluricomplex Green function, including a relationship between $g_\Omega(z, a)$ and the Carathéodory and Kobayashi pseudodistances, and with a pluricomplex Poisson/Riesz decomposition theorem, due to Demailly, for a psh function u on a hyperconvex domain with u continuous up to the boundary.

The beginner should find the book very readable, especially with the long introductory Chapter 2 on subharmonic and plurisubharmonic functions. The experts will be pleased to have the material in Section II of the book all in one place for the first time and will find this book to be a valuable reference tool, especially with a 12–13 page bibliography. There are several exercises at the end of the introductory chapters in Section I. Perhaps it would have been desirable to include exercises at the end of the chapters in Section II which could be designed to encourage the reader to seek out some of the references; on the other hand, the author liberally sprinkles remarks with bibliographic references throughout the text. Certain topics, such as the differential-geometric aspect of the complex Monge-Ampère equation, are not discussed but references are given. All in all, this is a very welcome and much overdue book on an important topic in several complex variables.

Reviewed by *Norman Levenberg*

MR1143476 (93a:32021) 32E30 (32F05)
Zériahi, Ahmed (F-TOUL3-AV)
Fonction de Green pluricomplexe à pôle à l'infini sur un espace de Stein parabolique et applications. (French) [Pluricomplex Green function with pole at infinity on a parabolic Stein space, and applications]
Math. Scand. **69** (1991), *no. 1,* 89–126.

Let X be a complex space of dimension n. E. Bedford [in *Séminaire Pierre Lelong-Henri Skoda (Analyse), Années 1980/1981, et Colloque de Wimereux, Mai 1981*, 294–323, Lecture Notes in Math., 919, Springer, Berlin, 1982; MR0658889 (83i:32025)], J.-P. Demailly [Mém. Soc. Math. France (N.S.) No. 19 (1985), 124 pp.; MR0813252 (87g:32030)] and J. E. Fornæss and R. Narasimhan [Math. Ann. **248** (1980), no. 1, 47–72; MR0569410 (81f:32020)] established basic results on plurisubharmonic (psh) and weakly psh functions and the complex Monge-Ampère operator $(dd^c)^n$. The author begins the article under review by discussing some related results and then defines the natural notions of equilibrium potential $h^*_{K,\Omega}$ and equilibrium measure $\mu_c \equiv (dd^c h^*_{K,\Omega})^n$ associated to a compact subset K contained in a hyperconvex open subset Ω of a pure n-dimensional Stein space X [see Bedford and B. A. Taylor, Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)]. If X is a parabolic Stein space, i.e., if there exists a continuous, psh exhaustion function $g\colon X \to [-\infty, +\infty)$ satisfying $(dd^c g)^n = 0$ on $X - g^{-1}(-\infty)$, one can define a pluricomplex Green function $g_E(x)$ with pole at infinity associated to any $E \Subset X$ by setting $g_E(x) \equiv \sup\{v(x)\colon\ v \in L_g(X),\ v|_E \le 0\}$, where $L_g(X)$ is the class of psh functions v on X satisfying $v(x) \le c_v + g^+(x)$ on X for some constant c_v. In the case $X = \mathbf{C}^n$, this is the usual Siciak extremal function [J. Siciak, Ann. Polon. Math. **39** (1981), 175–211; MR0617459 (83e:32018)].

The author develops the standard properties of g_E à la Siciak and gives many examples. The final two chapters are devoted to applications. He shows that the existence of a parabolic potential (a function g as above) on a Stein space X imposes restrictions on psh functions of minimal increase on X. For example, the space of "generalized" polynomials $P^d_g(X)$ consisting of holomorphic functions f on X satisfying an estimate of the form $|f(x)| \le c_f[1 + \exp g(x)]^d$, $x \in X$, has dimension at most $\binom{n+dN}{n}$ for some $N > 1$ independent of d. Finally, if X is an algebraic variety of pure dimension n imbedded in $\mathbf{C}^N$, the Siciak theory carries over nicely in the sense that there exists a "natural" parabolic potential g on X satisfying $c_1 + \log^+|x| \le g(x) \le c_2 + \log^+|x|$, $x \in X$, where $|x|$ is the Euclidean norm on $\mathbf{C}^N$ restricted to X. He then shows that for $K \subset X$ compact, $g_K(x) = \sup\{(1/d)\log|f(x)|\colon\ f \in A_d(X),\ \|f\|_K \le 1,\ d \ge 1\}$, where $f \in A_d(X) =$ regular functions on X such that $\sup(1+|x|)^{-d}|f(x)| < \infty$ (note $A_d(X) \subset P^g_d(X)$). This characterization is used to get a Bernstein-Walsh type theorem using approximation by the spaces $A_d(X)$. See also related results by the author [Ann. Inst. Fourier (Grenoble) **37** (1987), no. 2, 79–104; MR0898932 (88k:32047)].

Reviewed by *Norman Levenberg*

MR1139755 (92k:32055) 32L15 (32J20 32L10 58E05 58G25)
Marinescu, George [Marinescu, Gheorghe]
Asymptotic Morse inequalities. (Romanian. English summary)
Stud. Cerc. Mat. **43** (1991), *no. 5-6,* 243–297.

This survey paper gives an introduction to and a proof of J.-P Demailly's proof [Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147); in *Séminaire d'Analyse P. Lelong–P. Dolbeault –H. Skoda, Années 1985/1986*, 24–47, Lecture Notes in Math., 1295, Springer, Berlin, 1987; MR1047720 (91h:32025)] of Witten's asymptotic Morse inequalities for d''-cohomology and of the Grauert-Riemenschneider conjecture (for a compact irreducible analytic space to be a Moishezon space it is necessary and sufficient that there exist a desingularization $\pi\colon X \to Y$ of Y and that X admit a quasipositive holomorphic line bundle $E \to X$). The last section contains the author's own results on asymptotic Morse inequalities for compact manifolds with isolated cone-like singularities.

Reviewed by *J. S. Joel*

MR1128538 (93b:32048) 32L10 (58G11 58G25)
Demailly, Jean-Pierre (F-GREN)
Holomorphic Morse inequalities.
Several complex variables and complex geometry, *Part* 2 (*Santa Cruz, CA*, 1989), 93–114, *Proc. Sympos. Pure Math.*, 52, *Part* 2, *Amer. Math. Soc.*, *Providence*, *RI*, 1991.

A simple heat equation proof of the author's holomorphic Morse inequalities is presented. For this purpose, the asymptotic eigenvalue distribution of $\Box_k = (1/k)D_k^* D_k - V$ is studied. Here D_k is the associated connection on $E^k \otimes F$, where E, F are complex vector bundles over a smooth manifold M equipped with Hermitian connections, and $V = V \otimes \mathrm{id}_{E^k}$ is a Hermitian endomorphism of F. The asymptotic estimates of the heat kernel $e^{-t\Box_k}$ are purely local and the explicit form of the heat kernel in the case of connections with constant curvature, assuming M has a flat metric, is obtained by using Mehler's formula. From these facts, the asymptotic estimate of the heat kernel of $\Box_k$ is expressed in terms of $c(E)$, the curvature form of E (Theorem 3.1). To obtain holomorphic Morse inequalities from this estimate, $(2/k)\Delta_k'' = (1/k)\nabla_k^*\nabla_k - V + (1/k)\Theta$ is shown. Here Δ_k''

and ∇_k are the Dolbeault Laplacian and Chern connection on $E^k \otimes F \otimes \bigwedge^{0,q} T^*X$, X a compact complex manifold, E and F are holomorphic Hermitian bundles over X of ranks 1 and r, Θ a Hermitian form independent of k; the eigenvalues of V are easily counted (reviews of complex geometry and Hodge theory are given in Section 2). Thus the asymptotic formula of the heat kernel $e^{-(2t/k)}\Delta_k''$ in bidegree $(0,q)$ is obtained from Theorem 3.1 (Theorem 4.4). Then, according to Witten's idea, a finite-dimensional subcomplex of the Dolbeault complex on $E^k \otimes F$ with the same cohomology is constructed by using the eigenspaces of $(1/k)\Delta_k''$. This allows one to use linear algebraic considerations, and combining these considerations and Theorem 4.4, the strong holomorphic Morse inequality follows. We have $\dim\, H^0(X, E^k) \geq O(k^n)$ if $ic(E) \geq 0$ and $ic(E) > 0$ at at least one point by this inequality. Hence we have an alternative proof of Siu's theorem (the Grauert-Riemenschneider conjecture, Section 5). In Section 6, a generalization of the holomorphic Morse inequality to q-convex manifolds and its application to an a priori estimate for the Monge-Ampère operator are given (Theorem 6.1 and Corollary 6.6, the author states, were obtained by Bouche and Siu). The case rank $E \geq 2$ is discussed in Section 7 [cf. E. Getzler, C. R. Acad. Sci. Paris Sér. I Math. **304** (1987), no. 16, 475–478; MR0894572 (88j:32040)]. Related open problems are discussed in Section 8, the last section.

{For the entire collection see MR1128530 (92d:32002)}

Reviewed by *Akira Asada*

MR1128530 (92d:32002) 32-06

★**Several complex variables and complex geometry. Part 2.**

Proceedings of the Thirty-seventh Annual Summer Research Institute held at the University of California, Santa Cruz, California, July 10–30, 1989.
Edited by Eric Bedford, John P. D'Angelo, Robert E. Greene and Steven G. Krantz.
Proceedings of Symposia in Pure Mathematics, 52, Part 2.
American Mathematical Society, *Providence*, *RI*, 1991. xvi+625 *pp*. $219.00 the three volume set. *ISBN* 0-8218-1490-7

Contents: Robert E. Greene, The geometry of complex manifolds: an overview (pp. 1–22); Marco Abate, Angular derivatives in strongly pseudoconvex domains (pp. 23–40); Yukinobu Adachi and Masakazu Suzuki, Degeneracy points of the Kobayashi pseudodistances on complex manifolds (pp. 41–51); Takao Akahori, On the construction of the moduli space for strongly pseudoconvex domains (pp. 53–58); Andrew Balas, On the holomorphic sectional curvature of complete domains in $\mathbf{C}^n$ that are not Stein (pp. 59–63); J. Bland and T. Duchamp, Normal forms for convex domains (pp. 65–81); Ciprian Borcea, Homogeneous vector bundles and families of Calabi-Yau threefolds. II (pp. 83–91); Jean-Pierre Demailly, Holomorphic Morse inequalities (pp. 93–114);

Pierre Dolbeault, Some problems (pp. 115–116); Michael G. Eastwood and C. Robin Graham, Invariants of CR densities (pp. 117–133).

Yakov Eliashberg [Ya. M. Èliashberg] and Mikhael Gromov, Convex symplectic manifolds (pp. 135–162); H. R. Fischer and R. J. Fisher, Jr., Simple partial connections and the Einstein condition (pp. 163–182); Sidney Frankel, Applications of affine geometry to geometric function theory in several complex variables. I. Convergent rescalings and intrinsic quasi-isometric structure (pp. 183–208); Akito Futaki, A Lie algebra character and Kähler-Einstein metrics of positive scalar curvature (pp. 209–215); Bruce Gilligan, On the ends of complex manifolds homogeneous under a Lie group (pp. 217–224); James F. Glazebrook and Franz W. Kamber, Determinant line bundles for Hermitian foliations and a generalized Quillen metric (pp. 225–232); Ian Graham, Sharp constants for the Koebe theorem and for estimates of intrinsic metrics on convex domains (pp. 233–238); Chong-Kyu Han, Rigid immersions of G-structures and analyticity of CR mappings into spheres (pp. 239–249); J. William Helton and Orlando Merino, Optimal analytic disks (pp. 251–262); Marek Jarnicki and Peter Pflug, Some remarks on the product property for invariant pseudometrics (pp. 263–272).

Shanyu Ji, Smoothing of currents and Moišezon manifolds (pp. 273–282); Kang Tae Kim, Biholomorphic mappings between quasicircular domains in $\mathbf{C}^n$ (pp. 283–290); Akio Kodama, Characterizations of certain weakly pseudoconvex domains in $\mathbf{C}^n$ from the viewpoint of biholomorphic automorphism groups (pp. 291–296); Claude LeBrun, Complete Ricci-flat Kähler metrics on $\mathbf{C}^n$ need not be flat (pp. 297–304); Steven Shin-Yi Lu, On meromorphic maps into varieties of log-general type (pp. 305–333); Ngaiming Mok, Aspects of Kähler geometry on arithmetic varieties (pp. 335–396); J. Noguchi, Moduli spaces of holomorphic mappings into hyperbolic spaces and its applications (pp. 397–401); Salomon Ofman, The analytic Radon transform (pp. 403–412); Takeo Ohsawa, Applications of the $\overline{\partial}$ technique in L^2 Hodge theory on complete Kähler manifolds (pp. 413–425); Marius Overholt, Bounds on the derivatives of holomorphic endomorphisms (pp. 427–433).

Giorgio Patrizio and Pit-Mann Wong, Monge-Ampère functions with large center (pp. 435–447); Mathias Peternell, A characterization of affine varieties (pp. 449–453); Thomas Peternell and Michael Schneider [Michael Hellmut Schneider], Compactifications of $\mathbf{C}^n$: a survey (pp. 455–466); T. Ratiu and A. Todorov, An infinite-dimensional point of view on the Weil-Petersson metric (pp. 467–476); Min Ru and Wilhelm Stoll, The Cartan conjecture for moving targets (pp. 477–508); Morihiko Saito, On Kollár's conjecture (pp. 509–517); Leslie Saper and Steven Zucker [Steven M. Zucker], An introduction to L^2-cohomology (pp. 519–534); Vo Van Tan, On the compactification problems for Stein 3-folds (pp. 535–542); Gang Tian, On one of Calabi's problems (pp. 543–556); T. Tonev, Multi-dimensional analytic structures and uniform algebras (pp. 557–561).

J. Varouchas, The $\partial\overline{\partial}$-equation on complex spaces (pp. 563–577); Jean-Pierre Vigué, Fixed points of holomorphic mappings in a bounded convex domain in $\mathbf{C}^n$ (pp. 579–582); L. Waelbroeck, Holomorphic functions taking their values in a q-space and the Cauchy-Fantappiè formula (pp. 583–591); B. Wong [Bun Wong], Schwarz's lemma and Hermitian manifolds with constant holomorphic curvature (pp. 593–600); H. Wu [Hung Hsi Wu], Polynomial functions on complete Kähler manifolds (pp. 601–610); Kichoon Yang, The Chern numbers of projective algebraic hypersurfaces (pp. 611–617); Shing-Tung Yau, A review of complex differential geometry (pp. 619–625).

{Most of the papers are being reviewed individually.}

MR1100993 (92c:32012) 32C30 (32A25)
Berndtsson, Bo (S-CHAL)
Cauchy-Leray forms and vector bundles.
Ann. Sci. École Norm. Sup. (4) **24** (1991), *no. 3,* 319–337.

In this paper the author obtains integral formulas of Cauchy-Leray-Koppelman type for differential forms on a complex manifold. From the introduction: "The original motivation for our paper was to generalize the constructions of G. M. Khenkin and J. Leiterer [*Theory of functions on complex manifolds*, Birkhäuser, Basel, 1984; MR0774049 (86a:32002)] to forms of arbitrary bidegree. Such a generalization has already been found by J.-P. Demailly and C. Laurent-Thiébaut [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)], but they only gave the leading terms in the expansion of the kernels. Still, the idea of Demailly and Laurent-Thiébaut, to use a connection on a bundle, is of fundamental importance in this paper as well."

Let X be a complex manifold and let Y be a complex submanifold of codimension p. Suppose $\pi\colon E \to X$ is a holomorphic vector bundle of rank p, with a holomorphic section $\eta\colon X \to E$, that defines Y, i.e. $Y = \{\eta = 0\}$. The main part of this paper consists in finding a large family of explicit solutions K to the equation $dK = [Y] - c_p[\Theta]$, where $c_p[\Theta]$ is the pth Chern form of the curvature Θ of some connection on E (Theorem 2.4).

Then, setting $X = M \times M$, and $Y = \Delta = \{(\zeta, z) \in M \times M\colon \zeta = z\}$, the author obtains the Cauchy-Leray-Koppelman formula (Theorem 4.1) on the complex manifold M.

Reviewed by *Alexandr M. Kytmanov*

MR1087191 (92f:32023) 32F07 (32J20 35J30)
Foote, Robert L. (1-TXT)
Homogeneous complex Monge-Ampère equations and algebraic embeddings of parabolic manifolds.
Indiana Univ. Math. J. **39** (1990), *no. 4,* 1245–1273.

Let M be an n-dimensional, connected complex manifold with a strictly plurisubharmonic exhaustion $\tau\colon M \to [0, R^2)$ such that the complex homogeneous Monge-Ampère equation $(dd^c \log \tau)^n = 0$ holds on $M \setminus K$, where K is a compact subset of M. This situation has been studied extensively in case K is small and τ (and $\log \tau$) is well behaved near K. For example, it is known that if $\tau \in C^\infty(M)$ and $K = \tau^{-1}(0)$, then, up to biholomorphic maps, M is the ball of radius R in $\mathbf{C}^n$ and τ is the squared Euclidean norm [W. Stoll, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **7** (1980), no. 1, 87–154; MR0577327 (81h:32028)]. The author studies the general setting and his main result is a criterion which guarantees that the Stein manifold M is an affine algebraic submanifold of $\mathbf{C}^{2n+1}$. The result is obtained under the additional hypothesis that τ is at least of class C^6 and locally Reinhardt, i.e., in a neighborhood of every point of M there are holomorphic coordinates $z = x + iy$ such that $\tau(x + iy) = \tau(x)$. This assumption allows the author to study the differential geometry of M with respect to the metric $dd^c\tau$ using the interplay, possible in this case, between the real and the complex homogeneous Monge-Ampère equations. Using previous results of the author on the real equation, an estimate on the Ricci curvature of the metric $dd^c\tau$ is achieved which is the key ingredient for applying and obtaining the proof of the theorem by means of Demailly's algebraicity criterion.

Reviewed by *Giorgio Patrizio*

MR1084013 (92f:32017) 32C30
Blel, Mongi (TN-TUNISM); **Demailly, Jean-Pierre** (F-GREN-F);
Mouzali, Mokhtar (F-GREN-F)
Sur l'existence du cône tangent à un courant positif fermé. (French) [Existence of the tangent cone of a closed positive current]
Ark. Mat. **28** (1990), *no. 2,* 231–248.

Summary: "Let T be a closed positive current in a neighbourhood of 0 in $\mathbf{C}^n$. We show here that T admits a tangent cone (limit of the family of its homotheties) when the projective mass $\nu_T(r)$ converges to $\nu_T(0)$ rapidly enough for the function $(\nu_T(r) - \nu_T(0))/r$ to be locally integrable at 0. This sufficient condition is optimal: we build $(1,1)$ currents without tangent cone such that the integral at $r = 0$ of $(\nu_T(r) - \nu_T(0))/r$ has as small a divergence as one likes. When T is given

by integration on an analytic set, we show that $\nu_T(r) - \nu_T(0) = O(r^\varepsilon)$ and that this recovers the Thie-King theorem on the existence of the tangent cone."

Reviewed by *Daniel Barlet*

MR1077088 (92e:14014) 14F17 (14M15)
Manivel, Laurent (F-GREN-F)
Sur la cohomologie des fibrés associés au fibré quotient universel sur la grassmannienne. (French. English summary) [On the cohomology of fiber bundles associated with the universal quotient bundle on the Grassmannian]
Bull. Soc. Math. France **118** (1990), *no. 1,* 67–84.

Summary: "Using a suitable filtration of the inverse image of the tangent bundle of the Grassmannian of a complex vector space, on the variety of its complete flags, we determine, as a continuation of one of J. P. Demailly's papers, certain cohomology groups of the vector bundles associated to the universal quotient bundle on the Grassmannian. In particular, we obtain a vanishing property of the cohomology of the tensor powers of that bundle tensored with large enough powers of its determinant. Finally, the case of symmetric powers leads us to new counterexamples to a conjecture of Faltings and a question raised by Le Potier, both proved false by Peternell, Le Potier and Schneider."

MR1062924 (91e:14016) 14F17 (18G40 32L20)
Manivel, Laurent (F-GREN-F)
Un exemple de non-dégénérescence en E_2 de la suite spectrale de Borel-Le Potier. (French. English summary) [An example of E_2 nondegeneracy of the Borel-Le Potier spectral sequence]
C. R. Acad. Sci. Paris Sér. I Math. **311** (1990), *no. 1,* 31–36.

Summary: "The Borel-Le Potier spectral sequence and its possible degeneracy properties appear as an important tool for the study of the cohomology of the bundles associated with a holomorphic vector bundle on a complex compact variety. We give the example of an ample line bundle on a

variety of incomplete flags, projecting on a Grassmannian, such that the associated Borel-Le Potier spectral sequence does not degenerate at E_2: this allows us to answer in the negative a question raised by J.-P. Demailly."

Reviewed by *Jürgen Leiterer*

MR1056776 (91e:32025) 32J25 (32C30 32F05 32S25)

Azhari, Abdelhak

Sur la conjecture de Chudnovsky-Demailly et les singularités des hypersurfaces algébriques. (French. English summary) [On the Chudnovsky-Demailly conjecture and singularities of algebraic hypersurfaces]

Ann. Inst. Fourier (Grenoble) **40** (1990), *no. 1,* 103–116.

For a finite subset $S \subseteq \mathbf{C}^n$, let $\omega_t(S)$ be the least degree of a hypersurface in $\mathbf{C}^n$ having at least t-fold points at each point of S. It is known that $\omega_{t_1}(S)/(t_1+n-1) \le \omega_{t_2}(S)/t_2$. The conjecture of Chudnovsky and Demailly is that $(\omega_{t_1}(S)+n-1)/(t_1+n-1) \le \omega_{t_2}(S)/t_2$. Here it is shown that $(\omega_{t_1}(S)+n-1-a_{t_2})/(t_1+n-1) \le \omega_{t_2}(S)/t_2$, where a_{t_2} is the minimal dimension of the locus of worse than normal crossings singularities among all hypersurfaces of degree ω_{t_2} having t_2-fold points at S. Some related results are also obtained. The methods are complex analytic and depend, via a theorem of Demailly, on L^2 estimates due to Hörmander.

Reviewed by *G. K. Sankaran*

MR1055992 (91e:32014) 32F10 (32L10)

Demailly, Jean-Pierre (F-GREN-F)

Cohomology of q-convex spaces in top degrees.

Math. Z. **204** (1990), *no. 2,* 283–295.

Highly elegant proofs of the following three theorems are given. Theorem 1: Let Y be an analytic subvariety in a complex space X. If Y is strongly q-complete, then Y has a fundamental family of strongly q-complete neighbourhoods in X. Theorem 2: Let X be a complex space such that all irreducible components have dimension $\le n$. Then: (a) X is always strongly $(n+1)$-complete.

(b) If X has no compact irreducible component of dimension n, then X is strongly n-complete. (c) If X has only finitely many irreducible components of dimension n, then X is strongly n-convex. Theorem 3: Let (M,ω) be an n-dimensional Kähler manifold. Suppose that M is absolutely q-convex, i.e. admits a smooth plurisubharmonic exhaustion function that is strongly q-convex on $M \smallsetminus K$ for some compact set K in M. Set $\Omega^r = \mathcal{O}(\Lambda^r T^* M)$. Then the de Rham cohomology groups with arbitrary [resp. compact] support have decompositions $H^k(M,\mathbf{C}) \simeq \bigoplus H^s(M,\Omega^r)$, $H^r(M,\Omega^s) \simeq H^s(M,\Omega^r)$, $k \geq n+q$, $H^k_c(M,\mathbf{C}) \simeq \bigoplus H^s_c(M,\Omega^r)$, $H^r_c(M,\Omega^s) \simeq H^s_c(M,\Omega^r)$, $k \leq n-q$, and these groups are finite-dimensional. Moreover, there is a Lefschetz isomorphism $\omega^{n-r-s} \wedge \cdot \colon H^s_c(M,\Omega^r) \to H^{n-r}(M,\Omega^{n-s})$, $r+s \leq n-q$.

Reviewed by *Takeo Ohsawa*

MR662440 (84k:32011) 32C30
Demailly, Jean-Pierre
Sur les nombres de Lelong associés à l'image directe d'un courant positif fermé. (French. English summary) [On the Lelong numbers associated with the direct image of a closed positive current]
Ann. Inst. Fourier (Grenoble) **32** (1982), *no. 2,* ix, 37–66.

Generalized Lelong numbers with respect to a logarithmically plurisubharmonic weight are defined for closed positive currents. The invariance properties of these numbers with respect to analytic morphisms give precise bounds for Lelong numbers of direct images.

Reviewed by *Jürgen Leiterer*

MR679762 (84f:32007) 32C30 (14C30)
Demailly, Jean-Pierre
Courants positifs extrêmaux et conjecture de Hodge. (French) [Extremal positive currents and the Hodge conjecture]
Invent. Math. **69** (1982), *no. 3,* 347–374.

On a complex manifold X, the convex cone $\mathrm{SPC}^p(X)$ of strongly positive closed (p,p)-currents is defined (for $0 \le p \le n = \dim_{\mathbf{C}} X$), and it is known that the current $[Z]$ corresponding to integration on an irreducible p-dimensional subvariety Z of X is an element of the set $E^p(X)$ of extremal elements of $\mathrm{SPC}^p(X)$. It has been conjectured [cf., e.g., P. Lelong, *Séminaire Pierre Lelong (Analyse), Année 1971–1972*, 112–131, Lecture Notes in Math., 332, Springer, Berlin, 1973; MR0412474 (54 #600)] that, for a Stein manifold X, every $T \in E^p(X)$ is a scalar multiple of such a $[Z]$. In this paper the author shows that the conjecture is false for $X = \mathbf{C}^n$ or $\mathbf{P}^n$ and $1 \le p \le n-1$. In fact, he proves that, if $C_d \subset \mathbf{P}^2$ is the curve $Z_0^d + Z_1^d + Z_2^d = 0$, then $((1/d)[C_d])$ converges (in the weak topology on currents) to a counterexample, and he passes to the case of $\mathbf{C}^n$ and $\mathbf{P}^n$ by using an extension theorem for closed positive currents due to H. Skoda [Invent. Math. **66** (1982), 361–376].

In the rest of the paper, the author indicates why the above conjecture is too optimistic (even for Stein or projective manifolds). When X is projective, let $\mathrm{SPC}^p_{\mathbf{Z}}(X)$ consist of those $T \in \mathrm{SPC}^p(X)$ whose cohomology class belongs to the $\mathbf{R}$-span of (Image $H^{2q}(X,\mathbf{Z}) \cap H^{q,q}(X,\mathbf{C})$), $q = n-p$. Then, the conjecture that the convex cone generated by irreducible p-dimensional subvarieties of

X is dense in $\mathrm{SPC}^p_{\mathbf{Z}}(X)$ already implies the Hodge conjecture that $H^{q,q}(X,\mathbf{C})\cap H^{2q}(X,\mathbf{Q})$ is generated by algebraic p-cycles, while the original conjecture (density in all of $\mathrm{SPC}^p(X)$) implies a stronger condition which is obviously false in general. The author proves the new conjecture (and its analogue when X is Stein) for $p=n-1$.

Reviewed by *R. R. Simha*

MR662130 (84d:32014) 32C30 (10F35)

Demailly, J.-P.

Formules de Jensen en plusieurs variables et applications arithmétiques. (French. English summary) [Jensen formulas in several variables and arithmetic applications]

Bull. Soc. Math. France **110** (1982), *no. 1,* 75–102.

Let F be an entire function on $\mathbf{C}^n$ and $P_1,\cdots,P_N$ be polynomials of degree δ, the maximal homogeneous parts $Q_1,\cdots,Q_N$ of which have a unique common zero at $0\in\mathbf{C}^N$. Let $\varphi(z)=\sum_{j=1}^N|P_j(z)|^2$, $\beta=i\partial\overline{\partial}\varphi$, $T=(i/\pi)\partial\overline{\partial}\mathrm{Log}\,|F|$ and $|F|_r=\sup_{|z|\le r}|F(z)|$.

Using an appropriate generalization of the Poisson-Jensen formula, the author proves the following new variant of the Schwarz lemma in $\mathbf{C}^n$: There exists a constant $C\in(0,1]$, depending only on $P_1,\cdots,P_N$, such that for all $R\ge r\ge 1$ we have

$$\int_{r^{2\delta}}^{CR^{2\delta}}\frac{dt}{t^n}\int_{\varphi(z)<t}T\wedge\beta^{n-1}\le(2\delta)^n\pi^{n-1}\mathrm{Log}\frac{|F|_R}{|F|_r}.$$

This inequality permits the author to give a new proof of E. Bombieri's theorem on algebraic values of meromorphic maps without using L^2-estimates for the $\overline{\partial}$-operator.

Some new results concerning zero sets of polynomials in $\mathbf{C}^n$ are also given.

Reviewed by *G. M. Khenkin*

MR691167 (85d:46004) 46A06 (32F05 46G20)
Lelong, Pierre

A class of Fréchet complex spaces in which the bounded sets are C-polar sets.

Functional analysis, holomorphy and approximation theory (*Rio de Janeiro*, 1980), *pp.* 255–272, *North-Holland Math. Stud.*, 71, *North-Holland*, *Amsterdam*, 1982.

Let $P_b(\Omega)$ be the set of plurisubharmonic (p.s.h.) functions bounded above in a domain Ω of a complex topological vector space E; a subset A of Ω is a control for $P_b(\Omega)$ if there exists a strictly positive function $\gamma(A,x)$ such that (1) $f(x)\le\gamma(A,x)\cdot\sup\{f(x)\colon x\in A\}$ for all $f\in P_b(\Omega)$.

Such a control is useful for improving bounds for p.s.h. functions. Let f be a given p.s.h. function, less than M in Ω, and less than m in A; then $f\le M(1-\gamma)+m\gamma<M$. Define g_A by $g_A(x)=\sup\{f(x)\colon f\le 0 \text{ in } \Omega,\ f\equiv -1 \text{ in } A,\ f \text{ is p.s.h. in } \Omega\}$.

The author establishes the equivalence of the following properties: (i) A is a control for $P_b(\Omega)$; (ii) $g_A<0$; (iii) A is not strictly **C**-polar in Ω. They imply that g_A is the best control associated with A.

To obtain uniform bounds in a neighbourhood of a point, the author looks for an upper semicontinuous control. Let g_A^* be the upper regularization of g_A. There are only two possibilities: (I) $g_A^*\equiv 0$ and a semicontinuous control does not exist; (II) $g_A^*\not\equiv 0$, in which case g_A^* is the best semicontinuous control.

Whenever E is a Frechet space, one finds by using the polycylinders studied by the reviewer [Ann. Inst. Fourier (Grenoble) 20 (1970), no. 1, 361–432; MR0274804 (43 #564)] that A is a semicontinuous control if and only if A is not strictly **C**-polar in Ω.

From this result and an extension to infinite-dimensional space of the author's inverse function theorem [Seminaire Pierre Lelong-Henri Skoda (Analyse), Annee 1976/77, 172–195, Lecture Notes in Math., 694, Springer, Berlin, 1978; MR0522476 (80i:58009)], bounds can be found for a function $M(x,|z|)$ p.s.h. in $E\times\mathbf{C}$. Bounds of this kind have been used by H. Skoda and J. P. Demailly to find a counterexample to Serre's conjecture.

The last section is devoted to the construction of Frechet spaces in which bounded sets are **C**-polar.

{For the entire collection see MR0691157 (84b:46001)}.

{For the entire collection see MR0691157 (84b:46001)}

Reviewed by *G. Coeuré*

MR690650 (85d:32057) 32L15 (32L20)

Demailly, Jean-Pierre

Estimations L^2 pour l'opérateur $\bar\partial$ d'un fibré vectoriel holomorphe semi-positif au-dessus d'une variété kählérienne complète. (French) [L^2-estimates for the $\bar\partial$-operator of a semipositive holomorphic vector bundle over a complete Kähler manifold]

Ann. Sci. École Norm. Sup. (4) **15** (1982), *no. 3,* 457–511.

Author's review: Let E be a Hermitian vector bundle of rank r over an n-dimensional Kahler manifold X. The bundle E is said to be s-positive if its curvature tensor $K(E)$ identified with a Hermitian form on $TX \otimes E$ takes positive values on tensors of rank $\leq s$ and $\neq 0$. For example, if E is Griffiths positive (i.e. 1-positive) of rank $r \geq 2$, we show that $E^* \otimes (\det E)^s$ is s-positive and that $E \otimes \det E$ is Nakano positive (i.e. n-positive). In connection with these results, we prove the following vanishing theorem: If E is s-positive and X is weakly pseudoconvex, then $H^q(X, \bigwedge^n T^*X \otimes E) = 0$ for $q \geq \sup(1, n - S + 1)$. Given a surjective morphism $E \to Q \to 0$ of Hermitian bundles, we also obtain curvature conditions which imply the surjectivity of the map $H^q(X, E \otimes L) \to H^q(X, Q \otimes L)$, $0 \leq q < n$, where L is a line bundle. All these results are proved in quantitative versions using L^2 estimates and plurisubharmonic weights. In order to get rid of continuity assumptions for weights or exhaustion on X, a smoothing method is developed for psh functions involving the exponential map $TX \to X$. In particular, if X has an upper semicontinuous exhaustive psh function, then it can be endowed with a complete Kahler metric.

Reviewed by *Autorreferat* (Zbl 507:32021)

MR658880 (83j:32019) 32D15 (32L05)

Demailly, J.-P.

Scindage holomorphe d'un morphisme de fibrés vectoriels semi-positifs avec estimations L^2. (French) [Holomorphic splitting of a morphism of semipositive vector bundles with L^2 estimates]

Seminar Pierre Lelong-Henri Skoda (Analysis), 1980/1981, *and Colloquium at Wimereux*, *May* 1981, *pp.* 77–107, *Lecture Notes in Math.*, 919, *Springer*, *Berlin-New York*, 1982.

Let $0 \to S \to E \xrightarrow{g} Q \to 0$ be an exact sequence of holomorphic Hermitian vector bundles over the weakly pseudoconvex complex Kählerian manifold X and let M be a line bundle over X. Using a result of H. Skoda [Ann. Sci. École Norm. Sup. (4) **11** (1978), no. 4, 577–611; MR0533068 (80j:32047)] the author gives geometric conditions in terms of the curvature of the bundles which are sufficient to find for every $f \in \Gamma(X, \mathrm{Hom}(Q, Q \otimes M))$ a holomorphic preimage

$h \in \Gamma(X, \mathrm{Hom}(Q, E \otimes M))$ with L^2-estimates. In the natural situation of the exact sequence $0 \to TX \to T\Omega|_X \to NX \to 0$, where $\Omega \subset \mathbf{C}^n$ is a pseudoconvex domain and X is a closed submanifold in Ω, the above results are applied to prove extension theorems for holomorphic functions on X with precise estimates. Similar results have been obtained before by B. Jennane [*Séminaire Pierre Lelong-Henri Skoda (Analyse), Année 1976/77* (French), pp. 126–133, Lecture Notes in Math., 694, Springer, Berlin, 1978; MR0522474 (80m:32016)], C. A. Bernstein and B. A. Taylor [J. Analyse Math. **38** (1980), 188–254; MR0600786 (82h:32002)] and T. Yoshioka [Proc. Japan Acad. Ser. A Math. Sci. **57** (1981), no. 3, 181–184; MR0618087 (82f:32029)].

{For the entire collection see MR0658876 (83d:32001)}

Reviewed by *Peter Pflug*

MR658879 (83j:32032) 32L20 (32L05)

Demailly, J.-P.

Relations entre les différentes notions de fibrés et de courants positifs. (French) [Relations between the different notions of positive vector bundles and currents]

Seminar Pierre Lelong-Henri Skoda (Analysis), 1980/1981, *and Colloquium at Wimereux*, *May* 1981, *pp.* 56–76, *Lecture Notes in Math.*, 919, *Springer*, *Berlin-New York*, 1982.

The author discusses relations among various positivity concepts for Hermitian forms Θ on vector spaces of the form $T \otimes E$. The main result states: If Θ is semipositive in the sense of P. A. Griffiths, i.e., $\Theta(x, x) \geq 0$ for the decomposable vectors $x \in T \otimes E$, then for a scalar product φ on E the form $\Theta + \mathrm{Tr}_E\,\Theta \otimes \varphi$ is strictly semipositive, i.e., for all $x \in T \otimes E$ one has: $(\Theta + \mathrm{Tr}_E\,\Theta \otimes \varphi)(x, x) = \sum |x_i^*(x)|^2$, where x_i^* are finitely many decomposable linear forms on $T \otimes E$. In particular, it follows that $(\Theta + \mathrm{Tr}_E\,\Theta \otimes \varphi)(x, x) \geq 0$, i.e., this form is semipositive in the sense of S. Nakano.

This result is then applied to vector bundles and the positivity concepts valid there [cf. the author and H. Skoda, *Séminaire Pierre Lelong- Henri Skoda* (*Analyse*), *Années 1978/79* (French), pp. 304–309, Lecture Notes in Math., 822, Springer, Berlin, 1980; MR0599033 (82h:32028)].

In the last section the author compares a weakly positive (p, p)-form $\alpha \in \Lambda^{p,p}\,\mathrm{Hom}_{\mathbf{R}}(T, \mathbf{C})$ (i.e., a form α for which, with every (k, k)-form $\beta = \sum_j \varepsilon_k \cdot \beta_j \wedge \overline{\beta}_j$, where the β_j's are decomposable $(k, 0)$-forms, the form $\alpha \wedge \beta$ is a positive (n, n)-form) with forms of the type $L^{p-1} \bigwedge^{p-1} \alpha$. Here, with the existing scalar product on T the operators L, $\bigwedge$ are modeled after the usual Kähler operators. For example,

$$C(n, p) \cdot \frac{1}{p!^2} \cdot L^{p-1} \bigwedge{}^{p-1}\alpha - \alpha$$

is a sum of forms $\varepsilon_p \cdot \alpha_j \wedge \overline{\alpha}_j$ with decomposable $(p, 0)$-forms α_j and exactly definable constants

$C(n,p)$. An essential tool in the proofs is a summation formula for the qth roots of unity.
{For the entire collection see MR0658876 (83d:32001)}

Reviewed by *Peter Pflug*

MR658876 (83d:32001) 32-06

★**Séminaire Pierre Lelong-Henri Skoda (Analyse), Années 1980/1981, et Colloque de Wimereux, Mai 1981. (French) [Seminar Pierre Lelong-Henri Skoda (Analysis), 1980/1981, and Colloquium at Wimereux, May 1981]**

Colloquium on "Plurisubharmonic Functions in Finite or Infinite Dimensions" in honor of Pierre Lelong held in Wimereux, May 1981.
Edited by Lelong and Skoda.
Lecture Notes in Mathematics, 919.
Springer-Verlag, *Berlin-New York*, 1982. vii+386 *pp*. ar20.00. *ISBN* 3-540-11482-3

{The previous seminar has been reviewed [MR0599013 (81j:32003)].}

Contents: Part I. Séminaire d'Analyse (Paris) [Analysis Seminar (Paris)]: Carlos A. Berenstein and B. A. Taylor, On the geometry of interpolating varieties (pp. 1–25); Mongi Blel, Fonctions plurisouharmoniques et idéal définissant un ensemble analytique [Plurisubharmonic functions and the ideal defining an analytic set] (pp. 26–55); J.-P. Demailly, Relations entre les différentes notions de fibrés et de courants positifs [Relations between the different notions of positive vector bundles and currents] (pp. 56–76); J.-P. Demailly, Scindage holomorphe d'un morphisme de fibrés vectoriels semi-positifs avec estimations L^2 [Holomorphic splitting of a morphism of semipositive vector bundles with L^2 estimates] (pp. 77–107); Bernard Gaveau, Intégrales de courbure et potentiels sur les hypersurfaces analytiques de $\mathbf{C}^n$ [Curvature integrals and potentials on analytic hypersurfaces of $\mathbf{C}^n$] (pp. 108–122); B. Gaveau and G. Laville, Fonctions holomorphes et particule chargée dans un champ magnétique uniforme [Holomorphic functions and a charged particle in a uniform magnetic field] (pp. 123–130); Bernard Gaveau and Julian Ławrynowicz, Intégrale de Dirichlet sur une variété complexe. I [The Dirichlet integral on a complex manifold I] (pp. 131–166); Pierre Lelong, Calcul du nombre densité $\nu(x,f)$ et lemme de Schwarz pour les fonctions plurisousharmoniques dans un espace vectoriel topologique [Calculation of the density number $\nu(x,f)$ and Schwarz's lemma for plurisubharmonic functions in a topological vector space] (pp. 167–176); R. Michael Range, Boundary regularity for the Cauchy-Riemann complex (pp. 177–186).

Part II. Colloque de Wimereux, Mai 1981 [Colloquium at Wimereux, May 1981]. Gérard Coeuré, En l'honneur du Professeur Pierre Lelong [In honor of Professor Pierre Lelong] (pp. 189–191); V. Avanissian, Sur les fonctions harmoniques d'ordre quelconque et leur prolongement analytique dans $\mathbf{C}^N$ [On harmonic functions of arbitrary order and their analytic continuation in $\mathbf{C}^N$] (pp. 192–281); D. Barlet, Développements asymptotiques des fonctions obtenues par intégration sur les fibres [Asymptotic expansions of functions obtained by integration over the fibers] (pp. 282–293); Eric Bedford, The operator $(dd^c)^n$ on complex spaces (pp. 294–323); Christer O. Kiselman, Stabilité du nombre de Lelong par restriction à une sous-variété [Stability of the Lelong number under restriction to a subvariety] (pp. 324–336); Robert E. Molzon and Bernard Shiffman, Capacity, Tchebycheff constant, and transfinite hyperdiameter on complex projective space (pp. 337–357); V. S. Vladimirov, Several complex variables in mathematical physics (pp. 358–386).

{Most of the papers of mathematical interest are being reviewed individually.}

MR633888 (83m:32002) 32A22 (10F35 14J17)

Chudnovsky, G. V.

Singular points on complex hypersurfaces and multidimensional Schwarz lemma.

Seminar on Number Theory, *Paris* 1979–80, *pp.* 29–69, *Progr. Math.*, 12, *Birkhäuser*, *Boston*, *Mass.*, 1981.

This is mainly an expository paper concerning the so-called Schwarz lemma in two complex variables, and the related problem of the degree of a hypersurfaces with given singularities. Many results are stated, either as problems or as theorems. Here is an example. Let S be a finite subset of $\mathbf{C}^n$. For each integer $t \geq 1$, let $\omega_t(S)$ be the minimal degree of hypersurface in $\mathbf{C}^n$ having at each point of S a singularity of order at least t (see Chapter 7 of the reviewer's book, *Nombres transcendants et groupes algébriques* [Astérisque, 69–70, Soc. Math. France, Paris, 1979; MR0570648 (82k:10041)]). The author states the inequality $\omega_t(S)/t \geq (\omega_1(S)+n-1)/n$ as a conjecture for general $n \geq 1$, and as a theorem for $n = 2$. Work in this direction, including a proof of this claim for $n = 2$, has been done by D. W. Masser using linear algebra ["A note on multiplicities of polynomials", *Groupe d'étude sur les problèmes diophantiens 1980/81*, Publ. Math. Univ. Pierre et Marie Curie, no. 43, 1981], by G. Wüstholz using commutative algebra ["On the degree of algebraic hypersurfaces with given singularities", ibid.; see also Wüstholz, *Séminaire de Théorie des Nombres Paris 1980–1981*, 359–362, Birkhäuser, Boston, 1982], by J.-P. Demailly using analytic methods [Bull. Soc. Math. France **110** (1982), no. 1, 75–102], by H. Esnault and E. Viehweg using algebraic geometry [Math. Ann. **263** (1983), no. 1, 75–86].

The last part of this paper deals with the study of algebraic values of meromorphic functions of several complex variables (see the reviews of the author's previous works [*Séminaire Delange-Pisot-Poitou, 19é année; 1977–78*, Fasc. 2, Exp. No. 45, Secrétariat Math., Paris, 1978; MR0520333 (80c:10038); Ann of Math. (2) **109** (1979), no. 2, 353–376; MR0528967 (80j:10040)].)

{For the entire collection see MR0633885 (82j:10004)}

Reviewed by *Michel Waldschmidt*

MR641821 (83f:32014) 32E10 (30F15 32L05)
Mok, Ngaiming
The Serre problem on Riemann surfaces.
Math. Ann. **258** (1981/82), *no. 2,* 145–168.

This paper is devoted to the famous Serre problem [J. P. Serre, *Colloque sur les fonctions de plusieurs variables* (Brussels, 1953), pp. 57–68; Masson, Paris, 1953; MR0064155 (16,235b)]: If E is a holomorphic fiber bundle with a Stein base and a Stein fiber, is E Stein? Many positive partial results have been obtained, but H. Skoda's counterexample [C. R. Acad. Sci. Paris Sér. A-B **284** (1977), no. 19, A1199-A1202; MR0437802 (55 #10724); Invent. Math. **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] shows that the answer is, in general, negative [cf. also J.-P. Demailly, *Séminaire Pierre Lelong-Henri Skoda* (*Analyse*) (année 1976/1977), pp. 15–41, Lecture Notes in Math., 694, Springer, Berlin, 1978; MR0522471 (80e:32008)]. The author solves the Serre problem when the fiber is an open Riemann surface X. If X is hyperbolic (in the sense that it admits the Green kernel), the author constructs a continuous strictly subharmonic exhaustion function φ on X such that $\varphi - \varphi \circ g$ is bounded for every automorphism g of X. Then there exists a continuous strictly plurisubharmonic exhaustion function on E by a result of J.-L. Stehlé on patching plurisubharmonic functions [*Séminaire Pierre Lelong* (*Analyse*) (année 1973–1974), pp. 155–179, Lecture Notes in Math., 474, Springer, Berlin, 1975; MR0399524 (53 #3368)]. Thus E is Stein by the R. Narasimhan theorem [Math. Ann. **146** (1962), 195–216; MR0182747 (32 #229)]. If X does not admit a Green kernel it is said to be parabolic. In this case the author constructs a continuous subharmonic exhaustion function s on X such that $s - s \circ g$ is bounded for any automorphism x of X. Now the Stehlé theorem only gives a continuous (but not necessarily strictly) plurisubharmonic exhaustion function Ψ on the total space E. Then $E_c = \{z \in E\colon \Psi(z) < c\}$ is an increasing continuous one-parameter family of manifolds with union E. In view of the Docquier-Grauert theorem [F. Docquier and H. Grauert, ibid. **140** (1960), 94–123; MR0148939 (26 #6435)], to prove that E is Stein it is enough to prove that each E_c is Stein. The function $(c - \Psi)^{-1}$ is a continuous plurisubharmonic exhaustion function on E_c. Thus, applying the Narasimhan theorem, it suffices to construct a continuous strictly plurisubharmonic function on E_c. This is done by a partition of unity argument (following J. Brun [Manuscripta Math. **14** (1974), 217–222; MR0364686 (51 #940)]).

A characterisation of the irregular boundary of certain hyperbolic Riemann surfaces and an improvement of Stehlé's theorem are also given.

Reviewed by *J. T. Davidov*

MR636112 (83d:32026) 32L05
Skoda, H.
Morphisme surjectif de fibrés vectoriels semi-positifs. (French) [Surjective morphism of semipositive vector bundles]
Conference on Complex Analysis, Nancy 80 (*Nancy*, 1980), *pp.* 26–32, *Inst. Élie Cartan*, 3, *Univ. Nancy*, *Nancy*, 1981.

From the text: "Because most of the results in this article have appeared in an earlier publication [the author, Ann. Sci. École Norm. Sup. (4) **11** (1978), no. 4, 577–611; MR0533068 (80j:32047)], we limit ourselves to a brief summary and refer the reader elsewhere for the proofs [J.-P. Demailly and the author, *Séminaire Pierre LeLong-Henri Skoda (Analyse), Années 1978/79*, pp. 304–309, Lecture Notes in Math., 822, Springer, Berlin, 1980; MR0599033 (82h:32028); the author, ibid., pp. 259–303; MR0599032 (82h:32027); the author, op. cit.; MR0533068 (80j:32047)]."
{For the entire collection see MR0636110 (82i:32004)}

MR599033 (82h:32028) 32L20 (32J25)
Demailly, J.-P.; **Skoda, H.**
Relations entre les notions de positivités de P. A. Griffiths et de S. Nakano pour les fibrés vectoriels. (French)
Séminaire Pierre Lelong-Henri Skoda (Analyse). Années 1978/79 (*French*), *pp.* 304–309, *Lecture Notes in Math.*, 822, *Springer*, *Berlin*, 1980.

La différence entre les deux notions en question est que celle de Nakano se teste sur tous les tenseurs alors que celle de Griffiths se teste sur les tenseurs décomposables. Les auteurs démontrent que si E est un fibré semi-positif au sens de Nakano, alors $E \otimes \det E$ est semi-positif au sens de Griffiths. Cet énoncé, dont la démonstration très simple relève de l'algèbre multilinéaire, est suffisamment précis par exemple pour réduire le théorème d'annulation de Griffiths à celui de Nakano. Il permet aussi, à partir des résultats de Skoda [Ann. Sci. École Norm. Sup. (4) **11** (1978), no. 4, 577–611; MR0533068 (80j:32047)] concernant la notion de Nakano, d'obtenir des énoncés concernant celle de Griffiths, sensiblement plus fins que ceux obtenus directement par Skoda [see MR0599032 (82h:32027)above].

{For the entire collection see MR0599013 (81j:32003)}

Reviewed by *A. Hirschowitz*

MR597024 (82f:32007) 32A15 (32C25)

Demailly, Jean-Pierre

Construction d'hypersurfaces irréductibles avec lieu singulier donné dans $\mathbf{C}^n$. (French)

Ann. Inst. Fourier (Grenoble) **30** (1980), *no. 3,* 219–236.

The author obtains the following interesting result. Theorem: If $S = \{f_1 = \cdots = f_k = 0\} \subset \mathbf{C}^n$ is an analytic subvariety of codimension ≥ 2, then there exist entire functions $g_1, \cdots, g_k$ of slow growth such that the variety $X = \{\sum f_j g_j = 0\}$ is irreducible, and the singular set of X is contained in S. (If f_j is replaced by f_j^2, then the singular set of X may be taken to be exactly S.)

This theorem then is used to construct two noteworthy examples. The first is an irreducible algebraic curve in $\mathbf{C}^2$ of order zero such that the number of singular points in the ball of radius R is larger than any preassigned function $\psi(R)$. The reason for giving the example is that an irreducible algebraic curve of degree n can have at most $\frac{1}{2}(n-1)(n-2)$ double points. Thus this example is related to an earlier example of M. Cornalba and B. Shiffman [Ann. of Math. (2) **96** (1972), 402–406; MR0311937 (47 #499)].

Next, the author considers the Fourier transforms of functions in $\mathcal{D}(\mathbf{R}^n)$ and $\mathcal{E}'(\mathbf{R}^n)$, $n \geq 2$. The elements of $\widehat{\mathcal{D}(\mathbf{R}^n)}$ and $\widehat{\mathcal{E}'(\mathbf{R}^n)}$ are entire functions on $\mathbf{C}^n$ with certain growth conditions. By an application of the theorem above, there exists a function $V = \sum u_j * v_j$, where $u_j, v_j \in \mathcal{D}(\mathbf{R}^n)$, and V is irreducible in $\mathcal{E}'(\mathbf{R}^n)$. As a consequence, it follows that $\mathcal{D}(\mathbf{R}^n) * \mathcal{D}(\mathbf{R}^n) \neq \mathcal{D}(\mathbf{R}^n)$ for $n \geq 2$, a result which was obtained for $n \geq 3$ by L. A. Rubel, W. A. Squires and B. A. Taylor [ibid. (2) **108** (1978), no. 3, 553–567; MR0512433 (80d:32003)] and for $n = 2$ by J. Dixmier and P. Malliavin [Bull. Sci. Math. (2) **102** (1978), no. 4, 307–330; MR0517765 (80f:22005)].

Reviewed by *Eric Bedford*

MR599013 (81j:32003) 32-06

★**Séminaire Pierre Lelong-Henri Skoda (Analyse). Années 1978/79. (French) [Seminar Pierre Lelong-Henri Skoda (Analysis). 1978/79]**

Edited by Pierre Lelong and Henri Skoda.

Lecture Notes in Mathematics, 822.
Springer, *Berlin*, 1980. viii+356 *pp*. $23.00. *ISBN* 3-540-10241-8

Contents: Avant-propos (pp. iii-v); D. Barlet, Majoration du volume des fibres génèriques et forme géométrique du théorème d'aplatissement (pp. 1–17); Jorge Alberto Barroso, Comparaison de topologies sur des espaces d'applications holomorphes (pp. 18–32); Urban Cegrell, On product capacities with application to complex analysis (pp. 33–45); J. F. Colombeau, Holomorphy in locally convex spaces and operators on the Fock spaces (pp. 46–60); Hassine El Mir, Fonctions plurisousharmoniques et ensembles polaires (pp. 61–76); Roger Gay, Division des fonctionnelles analytiques. Applications aux fonctions entières de type exponentiel moyennes-périodiques (pp. 77–89); Lawrence Gruman, La géométrie globale des ensembles analytiques dans $\mathbf{C}^n$ (pp. 90–99); B. Jennane, Groupes de cohomologie d'un fibré holomorphe à base et à fibre de Stein (pp. 100–108); Bruno Kramm, $(\mathcal{D}FN)$-analytic spaces, Stein algebras and a "universal" holomorphic functional calculus (pp. 109–128); R. Langevin, Singularités complexes, points critiques et intégrales de courbure (pp. 129–143); Pierre Lelong, Potentiels canoniques et comparaison de deux méthodes pour la résolution du $\partial\overline{\partial}$ à croissance (pp. 144–168); I. Lieb, L'opérateur $\overline{\partial}$ sur une variété q-concave (pp. 169–173); Jean-Charles Moreau, Lemmes de Schwarz en plusieurs variables et applications arithmétiques (pp. 174–190); Laurent Schwartz, Martingales conformes sur une variété analytique complexe (pp. 191–198); A. Sebbar, Prolongement des solutions holomorphes de certains opérateurs différentiels d'ordre infini à coefficients constants (pp. 199–220); Nessim Sibony and Pit Mann Wong, Some results on global analytic sets (pp. 221–237); H. Skoda, Diviseurs d'aire bornée dans la boule de $\mathbf{C}^2$: réflexions sur un article de Bo Berndtsson (pp. 238–251); H. Skoda, Remarques à propos des théorèmes d'annulation pour les fibrés semi-positifs (pp. 252–258); Henri Skoda, Relèvement des sections globales dans les fibrés semi-positifs (pp. 259–303); J.-P. Demailly and H.Skoda, Relations entre les notions de positivités de P. A. Griffiths et de S. Nakano pour les fibrés vectoriels (pp. 304–309); M. Valdivia, On certain infinitely differentiable function spaces (pp. 310–316); Jean-Pierre Vigué, Sur la convexité des domaines bornés cerclés homogènes (pp. 317–331); Michel Waldschmidt, Propriétés arithmétiques de fonctions de plusieurs variables, III (pp. 332–356).

{The papers are being reviewed individually.}

MR533896 (81c:32049) 32H30 (32A10)

Demailly, Jean-Pierre

Fonctions holomorphes à croissance polynomiale sur la surface d'équation $e^x + e^y = 1$. (French. English summary)

Bull. Sci. Math. (2) **103** (1979), *no. 2,* 179–191.

Let $S \subset \mathbf{C}^2$ be the surface $e^x + e^y = 1$, $(x, y) \in \mathbf{C}^2$. The author proves that if $f(x, y)\colon S \to \mathbf{C}$ is a holomorphic function on S with polynomial growth, then $f(x, y)$ is the restriction of a polynomial on $\mathbf{C}^2$. As a result he deduces that if $f\colon S \to \mathbf{P}^1(\mathbf{C})$ is a meromorphic function on S with finite fibres, then f is constant. In particular, if $f\colon S \to \mathbf{C}$ is a bounded holomorphic function then f is constant. ($\mathbf{P}^1(\mathbf{C})$ is the complex projective "plane".)

Reviewed by *Adib A. Fadlalla*

MR549547 (80j:32027) 32E10

Jennane, B.

Groupes de cohomologie d'un fibré holomorphe à base et à fibre de Stein. (French)

Invent. Math. **54** (1979), *no. 1,* 75–79.

Let X and Ω be complex spaces and $\Pi\colon X \to \Omega$ be a surjective holomorphic map such that Ω admits a covering by Stein open subsets whose inverse images under Π are Stein. The author proves that if Ω is Stein and has bounded dimension and $\mathcal{F}$ is a coherent sheaf on X, then $H^p(X, \mathcal{F})$ vanishes for $p \geq 2$. The proof consists of choosing a suitable Stein covering of Ω and applying the Mayer-Vietoris sequence. The result is of interest in view of the counterexamples of H. Skoda [same journal **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] and J.-P. Demailly [ibid. **48** (1978), no. 3, 293–302; Séminaire Pierre Lelong-Henri Skoda (Analyse), Année 1976/77, pp. 15–41, Lecture Notes in Math., Vol. 694, Springer, Berlin, 1978; MR0522471 (80e:32008)] to the question posed by Serre whether a holomorphic fiber bundle with Stein base and Stein fiber is Stein.

Reviewed by *Y.-T. Siu*

MR522015 (80e:32002) 32A10 (32H99)
Demailly, Jean-Pierre
Fonctions holomorphes bornées ou à croissance polynomiale sur la courbe $e^x + e^y = 1$. (French. English summary)
C. R. Acad. Sci. Paris Sér. A-B **288** (1979), *no. 1,* A39–A40.

Author's summary: "We prove a very precise extension theorem for holomorphic functions on the curve $e^x + e^y = 1$, and deduce from it that bounded holomorphic functions are constant, or more generally, that every holomorphic function with polynomial growth extends to a polynomial in $\mathbf{C}^2$. This result immediately applies to meromorphic functions, and can also be used to study some hypersurfaces of $\mathbf{C}^n$."

MR508989 (81m:32036) 32L05 (32E10)
Demailly, Jean-Pierre
Un exemple de fibré holomorphe non de Stein à fibre $\mathbf{C}^2$ ayant pour base le disque ou le plan. (French)
Invent. Math. **48** (1978), *no. 3,* 293–302.

The author provides yet another example of a holomorphic fiber space with Stein base, Stein fiber and non-Stein total space in the mainstream originated by H. Skoda's brilliant counterexample to Serre's problem [same journal **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)]. Here the base is any nonempty connected open subset of $\mathbf{C}$ with $\mathbf{C}^2$ as fiber and with transition automorphisms of exponential type, while in Skoda's example the base is multiply connected and the transition automorphisms are locally constant with exponential growth. Moreover the holomorphic functions on such a bundle are constant on each fiber and its Dolbeault group $H^{0,1}$ is non-Hausdorff of infinite dimension.

Reviewed by *Alessandro Silva*

MR522471 (80e:32008) 32E10 (32L05)
Demailly, J.-P.
Différents exemples de fibrés holomorphes non de Stein. (French)
Séminaire Pierre Lelong-Henri Skoda (Analyse), Année 1976/77, *pp.* 15–41, *Lecture Notes in Math.*, 694, *Springer, Berlin*, 1978.

L'auteur simplifie notablement l'exemple de H. Skoda [Invent. Math. **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] et construit un fibré de base $\mathbf{C}^*$ et de fibre $\mathbf{C}^2$ dont l'espace total n'est pas de Stein. L'automorphisme de transition est constant et polynomial (dans l'exemple de Skoda, les automorphismes de transition étaient à croissance exponentielle). L'exemple peut être décrit en une ligne comme quotient de $\mathbf{C}\times\mathbf{C}^2$ par le groupe cyclique engendré par α avec $\alpha(x,z_1,z_2)=(x+2i\pi,z_1^k-z_2,z_1)$, $k\geq 2$.

L'auteur montre plus précisément que ce fibré X est de Stein au-dessus de la couronne $\rho_1<|x|<\rho_2$ si et seulement si $\mathrm{Log}(\rho_2|\rho_1)\leq 2\pi^2/\mathrm{Log}k$; dans le cas contraire, $H^1(X,\mathcal{O}_X)$ est grossier. Toujours en utilisant l'inégalité de Lelong sur la croissance des fonctions plurisousharmoniques, l'auteur construit aussi un fibré de base le disque, de fibre $\mathbf{C}^2$ et dont l'espace total n'est pas de Stein (en particulier, ce fibré n'est pas trivial!).

{For the entire collection see MR0522469 (80a:32001)}

Reviewed by *A. Hirschowitz*

MR512433 (80d:32003) 32A15 (30D30)
Rubel, L. A.; **Squires, W. A.**; **Taylor, B. A.**
Irreducibility of certain entire functions with applications to harmonic analysis.
Ann. of Math. (2) **108** (1978), *no. 3,* 553–567.

The authors show that if $f_1,\cdots,f_n$ $(n\geq 3)$ are nonconstant meromorphic functions of one variable and $g(z_1,\cdots,z_n)=f_1(z_1)+\cdots+f_n(z_n)$ then the local variety V of g, $V=\{z\in\mathbf{C}^n\colon g$ is analytic at z and $g(z)=0\}$, is irreducible. An immediate corollary is that if $f_1,\cdots,f_n$ are entire functions, then g is irreducible in the ring of entire functions in $\mathbf{C}^n$; this generalizes a known result for polynomials [cf. J. W. S. Cassels, Proceedings of the Fifteenth Scandinavian Congress (Oslo, 1968), pp. 1–17, Lecture Notes in Math., Vol. 118, Springer, Berlin, 1970; MR0268161 (42 #3060); erratum, MR **42**, p. 1825].

The following result in harmonic analysis follows also from the above theorem: $(*)$ $C_0^\infty(\mathbf{R}^n) * C_0^\infty(\mathbf{R}^n) \neq C_0^\infty(\mathbf{R}^n)$ if $n \geq 3$, which answers a question of L. Ehrenpreis [Amer. J. Math. **82** (1960), 522–588; MR0119082 (22 #9848)]. Recently, J. Dixmier and P. Malliavin have proved $(*)$ when $n = 2$ by a different method [Bull. Sci. Math. (2) **102** (1978), no. 4, 307–330; MR0517765 (80f:22005)]. The case $n = 1$ is still open.

Recent work of J.-P. Demailly [ibid. (2) **103** (1979), no. 2, 179–191] on analysis on the variety V defined by the functions $f_1(z) = f_2(z) = e^z$, $f_3(z) = 1$, is of related interest.

Reviewed by *Carlos A. Berenstein*

MR522469 (80a:32001) 32-06

★Séminaire Pierre Lelong-Henri Skoda (Analyse). Année 1976/77. (French) [Seminar Pierre Lelong-Henri Skoda (Analysis). Year 1976/77]

Edited by Pierre Lelong and Henri Skoda.
Lecture Notes in Mathematics, 694.
Springer, *Berlin*, 1978. iii+334 *pp*. $17.60. *ISBN* 3-540-09101-7

From the foreword: "The present volume of the 1976–1977 seminar continues the series of volumes previously published as Lecture Notes [71 (1968), 116 (1969), 205 (1970), 275 (1971), 332 (1972), 410 (1973), 474 (1974), 524 (1975), 578 (1976)]. Certain articles have been edited, we must admit, with a certain delay and in fact several were not in final form before the beginning of 1978."

Table of Contents: Pierre Lelong and Henri Skoda, Préface (p. i); Exposés faits au Séminaire d'Analyse dont les résultats paraitront ailleurs (p. iii); Jacek Bochnak, Sur le 17ème problème de Hilbert pour les fonctions de Nash (pp. 1–4); J.-P. Demailly, Différents exemples de fibrés holomorphes non de Stein (pp. 15–41); G. Dloussky, Prolongements d'applications analytiques (pp. 42–95); François Gramain, Fonctions entières arithmétiques (pp. 96–125); B. Jennane, Extension d'une fonction définie sur une sous-variété avec contrôle de la croissance (pp. 126–133); Paul Krée, Méthodes fonctorielles en analyse de dimension infinie et holomorphie anticommutative (pp. 134–171); Pierre Lelong, Un théorème de fonctions inverses dans les espaces vectoriels topologiques complexes et ses applications à des problèmes de croissance en analyse complexe (pp. 172–195); Leopoldo Nachbin, Sur la densité des sous-algèbres polynomiales d'applications continûment différentiables (pp. 196–202); Philippe Noverraz, Sur la mesure gaussienne des ensembles polaires en dimension infinie (pp. 203–213); P. Raboin, Le problème du $\overline{\partial}$ sur un espace de Hilbert (pp. 214–227); J.-P. Ramis, Géométrie analytique et géométrie algébrique (variations sur le thème "GAGA") (pp. 228–289); Henri Skoda, Morphismes surjectifs et fibrés linéaires semi-positifs (pp. 290–324); Hiroshi Yamaguchi, Fonctions entières paraboliques dans $\mathbf{C}^2$ (pp. 325–334).

{The papers are being reviewed individually.}

MR0508091 (58 #22657) 32E10

Skoda, H.

Fibrés holomorphes à base et à fibre de Stein. (French)

Invent. Math. **43** (1977), *no. 2,* 97–107.

Author's review: There exist holomorphic locally trivial fiber spaces, with Stein base and Stein fiber, which are not Stein manifolds. Therefore we give a negative answer to a problem of J.-P. Serre [*Colloque sur les fonctions de plusieurs variables* (Brussels, 1953), pp. 57–68, Thone, Liège, 1953; MR0064155 (16,235b)]. In this example, the base is an open set in $\mathbf{C}$, the fiber is $\mathbf{C}^2$, the transition automorphisms are locally constant. It is possible to choose the fundamental group of the base free with two generators. The counterexample is based on a Lelong type inequality concerning the growth of a holomorphic function on a fiber space with fiber $\mathbf{C}^n$. Using this inequality, we prove that holomorphic functions on this fiber space are constant on the fiber. Recently, J.-P. Demailly [cf. *Séminaire Pierre Lelong-Henri Skoda (Analyse), Année 1976/77*, pp. 15–41, Lecture Notes in Math., Vol. 694, Springer, Berlin, 1978] has given another example of a fiber space, with fiber $\mathbf{C}^2$, base the open unit disc, but which is not a Stein space.