REPORT ON THE CHAPTER "A SIMPLE PROOF OF THE KOBAYASHI CONJECTURE ON THE HYPERBOLICITY OF GENERAL ALGEBRAIC HYPERSURFACES" BY JEAN-PIERRE DEMAILLY

The chapter under review aims at describing some of the technics that can be used to study entire curves in a given complex space X, a central topic in the area of hyperbolicity related problems. As an illustration of those technics, the author then studies the case of general hypersurfaces of degree large enough. In particular, he provides new examples of hyperbolic hypersurfaces of low degree in any projective variety, and more importantly, he presents here a short self-contained proof of an old-standing conjecture of Kobayashi:

Conjecture 0.1. *A general hypersurface in $\mathbb{P}^{n+1}$ of degree large enough is hyperbolic.*

Let us give a brief overview of the content of this chapter.

In Section 1, the author gives a concise introduction to the main hyperbolicity notions and the main conjectures in this area of research. This part will naturally have some overlap with other chapters in the same book but it is nice to have all the material summarized in this section.

In Section 2 and Section 3, the author introduces the formalism of directed manifold and associated Semple towers as well as the theory of jet differentials in this setting. This point of view was developed by the author in different previous works and has already proven its importance. This self contained presentation is therefore very valuable in the context of this book.

In section 4, using the results of the previous sections, the author produces examples of hyperbolic hypersurfaces in $\mathbb{P}^{n+1}$ (or any projective variety), of relatively small degree, $d \geqslant 4n^2$. This bound is not the best bound currently known ($d \geqslant \lceil \frac{1}{4}(n+3)^2 \rceil$), but the proof is very short and is of pedagogical interest because it illustrates in an elementary fashion some ideas that will be used afterwards concerning Wronskian jet differentials. To the best of our understanding, this part is original.

The last section of the chapter is dedicated to a proof of the Kobayashi conjecture with an effective bound $\lfloor \frac{1}{2\pi}(n+4)(en)^{2n+1} \rfloor$ which appeared in a recent article of the author. Several proofs now exist in the literature, which yield different bounds of similar nature. While the bound given here is not the best currently known bound, the proof presented by the author is probably the shortest and most direct, therefore making it very suitable in the perspective of this book. This part ends with several interesting remarks which could motivate futur research.

The manuscript is extremely well written and could certainly be used as a first reference for anyone interested in these problems. It is remarkable that this chapter is almost self contained (up to the standard algebrogeometric background). The only missing point might be the statement of the Ahlfors-Schwarz lemma used in the proof of theorem 3.23, which is classical enough to be omitted, but it is unfortunate since it doesn't seem to appear anywhere else in the book.

Altogether, this chapter provides a very valuable presentation of some of the most powerful tools used nowadays to study entire curves in algebraic varieties and some of its most striking applications. Therefore it should absolutely appear in this book.

Here is a list of typos we've noticed and some suggestions.

(1) Page 1, first sentence of the introduction: "...asserts that hyperbolicity..." should be "asserts that the hyperbolicity...".
(2) Page 5, last sentence of Section 1:
"This would of course imply that the Kobayashi hyperbolicity is an open property with respect to the countable Zariski topology, a generalized form of the Kobayashi conjecture."
It is not clear which conjecture the author is referring to.
(3) Page 21 Lemma 4.11, line two of the statement, $\tau = [\tau_0 : \ldots : \tau_N]$ instead of $\tau = [\tau_0, \ldots : \tau_N]$.
(4) Page 21 Lemma 4.11, last line of the statement. It is not clear whether $\tau^{-1}(Y_{J,w})$ is the preimage under $\tau : Z \to \mathbb{P}^N$ or the preimage under its restriction $\tau : X \to \mathbb{P}^N$
(5) Page 21 Lemma 4.11, first formula in the proof.

$$\sum_{1\leqslant \ell \leqslant p} a_{j\ell}\sigma_\ell(x)$$

should be

$$\sum_{1\leqslant \ell \leqslant m} a_{j\ell}\sigma_\ell(x)$$

The same typo appears on the second line of page 22.

(6) Page 22 Proof of Lemma 4.11. The second part of the proof is unclear to us. In particular, we don't understand why, on line 8 of page 22, the author uses the condition

$$\tau(x_1) = \tau(x_2) \in Y_{J,w}$$

and not the condition

$$\tau(x_1) \in Y_{J,w}, \tau(x_2) \in Y_{J,w}.$$

From our understanding, it is this last condition that one has to study to prove the desired finiteness result. The dimension count is almost the same, but seems to yield the bound $N \geqslant 2n+1$ instead of $N \geqslant 2n$, hence the bound $d \geqslant (2n+1)^2$ instead of $d \geqslant 4n^2$ in Theorem 4.8.

(7) Page 24 Section 2.B line 9. "product ofs".

(8) Page 25 line 24: $g(\Phi(Z))$ instead of Z.

(9) Page 25 Section 5.C line 4: $|\complement J| = N + 1 - c$ instead of $|\complement J| = N - c$.

(10) Page 26 Formula (5.12) in the statement of Corollary 5.11. The last term of the formula should be

$$(N + 1 - c)(kb + \rho)$$

instead of

$$(N + 1 + c)(kb + \rho).$$

The same typo appears in the statement of Lemma 5.24 and seems also to have been used in the estimation of the bound in Section 5.F. Of course, this is not critical since the modification will actually improve the bound.

(11) Page 30 Statement of Corollary 5.26: "Zarisk i".

(12) Bibliography. The bibliography could be double checked. For instance:
- Some references are not in alphabetic order: [Bro78] should appear before [BrDa17]
- Both [BrDa17] and [Xie15] were published in 2018.
- The brackets are missing for [DeEG00].