The main goal of this chapter is to investigate a famous conjecture of Shoshichi Kobayashi (1970), according to which a generic algebraic hypersurface of dimension n and of sufficiently large degree $d \geq d_n$ in the complex projective space $\mathbb{P}^{n+1}$ is hyperbolic; in other words, by a classical characterization due to Brody, such a variety does not possess nonconstant entire holomorphic curves. As is well-known since the work of Green and Griffiths, one crucial ingredient is the geometric structure of certain jet bundles and their associated jet differentials. More precisely, one makes use of the so-called Semple tower, which is a twisted tower of projective bundles; it is related to jet differentials that are invariant by reparametrization. According to a fundamental vanishing theorem, global jet differentials with values in negative line bundles provide algebraic differential equations that all entire curves must satisfy; if the base locus of these differential equations is small enough, in other words, if there are enough independent differential equations, then all entire curves must be constant. In the early 2000's, Yum-Tong Siu proposed a somewhat different strategy that ultimately led to a proof in 2015. Siu's proof is based on Nevanlinna theory arguments combined with a use of slanted vector fields; it appears to be long and delicate. In 2016, the conjecture has been settled in a different way by Damian Brotbek, making a direct use of Wronskian differential operators and associated multiplier ideals. Shortly afterwards, Ya Deng showed how the approach could be completed to yield an explicit value of d_n. We give here a short proof based on a drastic simplification of their ideas, along with a further improvement of Deng's bound, namely $d_n = \lfloor (en)^{2n+2}/5 \rfloor$. We show that the same technique provides examples of smooth algebraic hypersurfaces of $\mathbb{P}^{n+1}$ of low degree $d = O(n^2)$, following an approach due to Shiffman and Zaidenberg.