

a metrically *convex* Boolean metric space over a Boolean algebra that contains the original one as a sub-algebra. The congruence is, in fact, an isomorphism, and since metric convexity of B is easily seen to be equivalent to lack of atoms in the underlying algebra, it follows that every Boolean algebra may be embedded isomorphically in an atom-free Boolean algebra. Assuming from now on that B is atom-free (metrically convex) and lattice-complete, concepts involving continuity, based on the introduction of the Birkhoff-Kantorovich order topology (sequential or directed set) are defined. It is seen that B is also metrically complete. Arcs are defined as homeomorphs of maximal chains. Since every motion (congruence of B with itself) is a homeomorphism and every congruence between any two subsets of B is extendible to a motion, segments are arcs. One seeks to determine what metric and topological properties of arcs and segments in ordinary metric spaces are valid in the very different environment provided by a Boolean metric space. Homeomorphisms between maximal chains with the same end-elements, connectedness properties of chains, characterizations of segments among arcs (by having a length equal to the distance of the end-elements, for example) are considered. A theory of continuous curves is also begun.

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BEMERKUNGEN ZUR HOMOTOPIETHEORIE

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Kleinere Bemerkungen über den Zusammenhang zwischen Homologie- und Homotopieinvarianten der Abbildungen eines n -dimensionalen Polyeders K in einen Raum, der in den Dimensionen $< n$ sphärisch ist, unter Benutzung der von Eilenberg-MacLane (Ann. of Math. 43 (1942)) gegebenen Beschreibung der Kohomologiegruppen von K durch die ganzzahligen Homologiegruppen.

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THE SPACE OF KÄHLER METRICS

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Let M^n be a closed, n -dimensional complex manifold. We assume that M^n admits at least one Kähler metric $g_{\alpha\beta}$; its associated closed exterior form $\omega = \sqrt{-1} g_{\alpha\beta} dz^\alpha \wedge dz^{\beta*}$ determines a real cohomology class, called the principal class of the metric. Consider the space Ω of all infinitely differentiable

Kähler metrics in M^n with the same principal class; the topology of Ω is defined by the L^2 topology of the tensorial components of metrics in Ω in compact subregions of co-ordinate domains. If $R_{\alpha\beta^*}$ is the Ricci tensor of any metric in Ω , then the Ricci form $\sqrt{-1} R_{\alpha\beta^*} dz^\alpha \wedge dz^{\beta^*}$ is closed and its cohomology class is $2\pi C^{(1)}$ ($C^{(r)} = r$ th Chern class).

Theorem 1. Given in M^n any real, closed, infinitely differentiable exterior form Σ of type $(1, 1)$ and cohomologous to $2\pi C^{(1)}$, there exists exactly one Kähler metric in Ω whose Ricci form equals Σ .

The proof proceeds by joining the Ricci form of one metric in Ω with Σ by a differentiably parametrized arc in the same linear space of forms (for example by linear interpolation) and constructing a corresponding parametrized path in Ω , then by proving that it is unique, and that its terminal point is independent of the path.

One can introduce in Ω a generalized Riemannian structure by defining the metric form $ds^2 = \left(\frac{\partial\omega(t)}{\partial t}, \frac{\partial\omega(t)}{\partial t} \right) dt^2$, where $\omega(t)$ represents a differentiable path in Ω . One shows that between any two points in Ω there exists exactly one geodesic, that the topology induced by the geodesic distance coincides with the chosen one, and that Ω with this metric is isometric with a geodesically convex subset of a Hilbert sphere.

If $g_{\alpha\beta^*}$ belongs to Ω and Σ is its Ricci form, then (Σ, Σ) is stationary with respect to first order variations in Ω , if and only if the contravariant components of the gradient of the scalar curvature generate a group of complex analytic transformations of M^n . If M^n has the property that each one-parameter complex transformation group of M^n (if any exist) leaves no point fixed, or if it is the Gaussian sphere, then this implies that the scalar curvature is constant, that Σ is harmonic, and that (Σ, Σ) is minimized.

Theorem 2. If M^n has the restrictive property described above, then there exists a metric in Ω which minimizes (Σ, Σ) , unique but for analytic transformations of M^n , and characterized by the property that the scalar curvature is constant.

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ON TWO DIMENSIONAL ASPHERICAL COMPLEXES

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Let L be a connected two dimensional C. W. complex. Let K denote the complex $L \cup \{e_i^2\}$, where $\{e_i^2\}$, $i = 1, 2, \dots$, is a set of 2-cells which are adjoined