REPORT ON THE PAPER BY JEAN-PIERRE DEMAILLY "BERGMAN BUNDLES AND APPLICATIONS TO THE GEOMETRY OF COMPACT COMPLEX MANIFOLDS" SUBMITTED TO PURE APPL. MATH. Q.

The author introduces an interesting infinite dimensional "very ample" bundle over an arbitray compact complex manifold X. The main observation is that X can be embedded diagonally in $X \times \bar{X}$ as a totally real submanifold, which possesses a Stein neighborhood U. Then Berndtsson's result implies that the associated bundle of L^2-holomorphic functions on the fibers is "Nakano–positive" in the sense that the associated metric locally can be written as the limit of a family of "Nakano–positive" metrics associated to the product families. In case the fibration from U to X is smoothly trivial up to the boundary then Wang's result implies that, using Lie-derivatives, one may further obtain the curvature formula for the above non-product family along Berndtsson's approach for the product case.

In this paper, the author gives an elementary approach to the curvature computations. The idea is to use the fiber-holomorphic part of the exponential map, which gives a special Stein neighborhood U as the total space of a real analytically trivial ball-bundle over X. Then one may first compute the precise curvature for the $\mathbb{C}^n$ case then use Taylor expansion of the exponential map to get the general case.

The author also mentioned two interesting potential applications: Kähler invariance of plurigenera and the transcendental Morse inequalities. The paper contains the author's main idea for first application and some missing estimates to be further studied.

The paper is also well written so I recommend it to Pure Appl. Math. Q.

A minor suggestion:

page 19, Proposition 4.6: it seems to me that the Ohsawa–Takegoshi extension with singular metric for the general weakly pesudoconvex Kähler case is more difficult to prove than the essential-Stein case. One (hard) proof that I know is due to Junyan Cao in:

—- *Ohsawa-Takegoshi extension theorem for compact Kähler manifolds and applications, arXiv:1404.6937v2*

It might be better if the author could say a little bit more there.