converging to a solution u that is L^2 with respect to ω. In order to get rid of the global L^2 condition for v, one can likewise observe that $X_c = \{z \in X\,;\ \psi(z) < c\}$ is relatively compact in X and weakly pseudoconvex with psh exhaustion $\psi_c(z) = 1/(c - \psi(z))$. One then gets a solution u_c on X_c, and finally a global solution $u = \lim u_{c_k}$ as a weak limit for some subsequence $c_k \to +\infty$.

Corollary 2. *Let X be a Kähler weakly pseudoconvex manifold and (E, h) be a hermitian holomomorphic line bundle such that $i\Theta_{E,h} > 0$. Then $H^{p,q}(X, E) = 0$ for $p + q \geq n + 1$.*

Proof. Let ψ be a psh exhaustion. By replacing h with $h_\chi = h\,e^{-\chi\circ\psi}$ where $\chi : \mathbb{R} \to \mathbb{R}$ is a fast increasing convex function, and taking

$$\omega = \omega_\chi = i\theta_{E,h_\chi} = i\theta_{E,h} + i\partial\overline{\partial}\chi \circ \psi,$$

we can at the same time obtain that ω_χ is complete, and achieve the convergence of the integral

$$\int_X |v|^2_{h_\chi,\omega_\chi} dV_{\omega_\chi} \leq \int_X |v|^2_{h_\chi,\omega} dV_\omega = \int_X |v|^2_{h,\omega} e^{-\chi\circ\psi} dV_\omega$$

for any given $v \in C^\infty(X, \Lambda^{p,q}T^*_X \otimes E)$ with $\overline{\partial}_E v = 0$ (here the eigenvalues are equal to 1 and $A^{p,q} = (p + q - n)\,\mathrm{Id}\,)$.