The mysteries of the famous Golden Ratio

The golden number (φ) has always fascinated mathematicians. Both by the variety of ways of defining it, and its omnipresence in extremely varied fields, not necessarily related to mathematics. The origin of this mysterious number goes back to 2600 years before J-C and it is still today at the heart of many mathematical problems.

Definition

- **Algebraic definition**

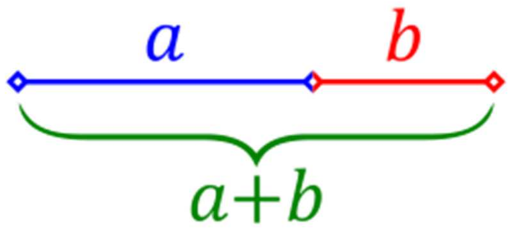
$$x^2 - x - 1 = 0$$

$$\Delta = 5 \qquad x_1 = \frac{-1-\sqrt{5}}{2} \;;\; x_2 = \frac{-1+\sqrt{5}}{2}$$

The golden ratio is one of root of this polynomial (the larger of the two)

$$\varphi = x_2 \approx 1.6180339887$$

- **Geometric definition**



φ is the ratio between a and b when $\frac{a+b}{a} = \frac{a}{b}$

The cutting of this segment is called by Euclide division in "extreme and average reason"

- **Arithmetic definition**

Everybody know the Fibonacci Sequence :

$$F_1 = F_2 = 1 \qquad F_{n+2} = F_{n+1} + F_n$$

And φ is the limit of the ratio between two consecutive terms of this sequence.

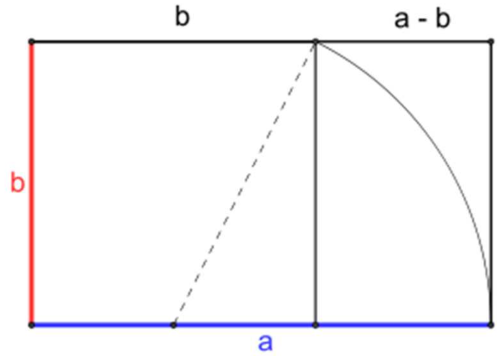
- **Other definition**

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}.$$

We can also approximate the golden ratio by iterating a very large number of times this process.
We start with a number A (A = 1), then we add 1 and we take the opposite of this result (which gives 1/2) and we start again with the last result obtained.

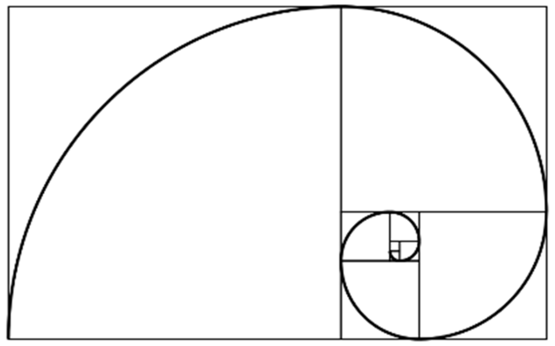
Applications

Golden rectangle



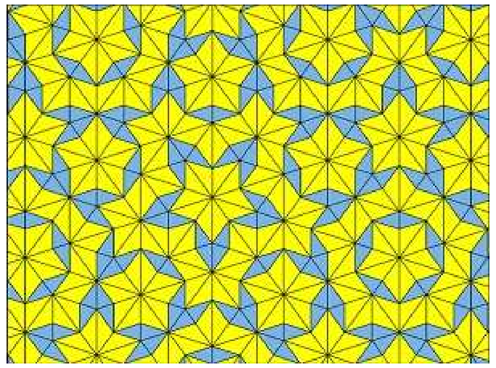
To draw a golden rectangle of length a and width b, the easiest way is to draw a square of side b. Taking the middle of the base as a center, draw a circle through opposite vertices. The intersection of the line extending the base of the square and the circle determines the end of the base of the golden rectangle.

Golden spiral



Our previous golden rectangle is composed of a square as well as another smaller rectangle which is also a golden rectangle. We repeat the process in this new rectangle and we start again ... We get at the end to have a good approximation of a gold spiral.

Penrose paving



A golden triangle is an isosceles triangle whose side lengths are in the ratio of the golden ratio. A silver triangle is an isosceles triangle whose side lengths are in the ratio of the inverse of the golden ratio. With these two triangles, it is possible to completely pave a Euclidean plane non-periodically.

Examples

Golden ratio is everywhere

The golden rectangle is in the art or in the architecture :

The golden spiral is in the nature.

For example, in a pine cone :

The number of scales in a spiral and the number of spirals correspond to two consecutive numbers in the following Fibonacci

The golden ratio is in human body

The ratio between the distance between the fingertips and the elbow & the distance between the wrist and the elbow.

Of course, the number of gold is present in various areas, but we must be careful not to see it everywhere. Even though this golden number is subject to many mathematical problems, people should not be obsessed with this one …