Analytic extension from non-pseudoconvex domains and A(D)-convexity [*†‡]

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1 Introduction

Let $D \subset\subset \mathbb{C}^n$ be a domain with smooth boundary. A compact $K \subset \partial D$ is called removable if every continuous CR-function f on $\partial D \setminus K$ has a holomorphic extension to all of D. In this paper we are interested in the link between removability and convexity properties of K. For any compact $L \subset \overline{D}$, we define its $A(D)$-convex hull as

$$A(D)\text{-hull}(L) = \{z \in \overline{D} : |f(z)| \leq \max_L |f|, \forall f \in A(D)\},$$

where $A(D)$ denotes as usual the space of holomorphic functions which are continuous up to the boundary. If a compact $K \subset \partial D$ satisfies $A(D)\text{-hull}(K) \cap \partial D = K$, we call K CR-convex.

The main result of this paper is the following.

Theorem 1 *Let D be a bounded domain in $\mathbb{C}^n, n \geq 2$, with boundary of class $\mathcal{C}^2$ and $K \subset \partial D$ be a compact CR-convex set such that $\partial D \setminus K$ is connected. Then each continuous CR-function u on $\partial D \setminus K$ admits a holomorphic extension $\tilde{u} \in \mathcal{O}(D \setminus A(D)\text{-hull}(K)) \cap \mathcal{C}((D \setminus A(D)\text{-hull}(K)) \cup (\partial D \setminus K))$.*

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We may notice that the connectedness of $\partial D \backslash K$ and the CR-convexity of K imply, by the maximum principle, that $D \backslash A(D)$-hull(K) is connected. Secondly we observe that Theorem 1 is essentially a result about domains with connected boundary. Otherwise the CR-convexity of K and the connectedness of $\partial D \backslash K$ imply that K contains all inner components of ∂D. If ∂D has several components, Theorem 1 and the Hartogs-Bochner theorem imply that $\tilde{u}$ extends holomorphically to all relatively compact components of $\mathbb{C}^n \backslash D$.

The following removability result is an immediate consequence.

Corollary 2 *Let D be a bounded domain in $\mathbb{C}^n, n \geq 2$, with connected boundary of class $\mathcal{C}^2$ and $K \subset \partial D$ be a compact CR-convex set such that $\partial D \backslash K$ is connected. If K is $A(D)$-convex, then K is removable.*

The theorem is best commented in its historical context.

For the case that $\overline{D}$ is a Stein compact, analogous results were proved by E. L. Stout and G. Lupacciolu but with hypotheses formulated with respect to the $\mathcal{O}(\overline{D})$-hull, which can be larger than the $A(D)$-hull in the situation of Theorem 1. However in these papers more general results were stated, in [14] for weakly pseudoconvex domains, in [9] even for non-pseudoconvex domains. Later J. M. Ortega [12] discovered that the construction of integral kernels used in [14], [9] are only possible if $\overline{D}$ possesses a Stein neighborhood basis. We stress that all statements appearing in [9] and [14] are true without change and confirmed by Theorem 1.

In [8], the first author gave a complete proof of the following theorem, which we state for later reference.

Theorem 3 *Let Ω be a Stein manifold of complex dimension $n \geq 2$, $K \subset \Omega$ an $\mathcal{O}(\Omega)$-convex compact subset, and $D \subset \Omega$ a relatively compact domain such that $\partial D \backslash K$ is a connected hypersurface of class $\mathcal{C}^1$. Then every continuous CR-function u on $\partial D \backslash K$ admits a holomorphic extension $\tilde{u} \in \mathcal{O}(D \backslash K) \cap \mathcal{C}((D \backslash K) \cup (\partial D \backslash K))$.*

Note that in this result the convexity condition is given with respect to functions holomorphic on a uniform neighborhood of $\overline{D}$.

In [5], B. Jöricke was able to treat the weakly pseudoconvex case completely and with sharper hypotheses on $A(D)$-convex hulls. Following a suggestion of Stout [14], she was the first to attack the problem by a global version of the continuity principle, in contrast to the integral formula methods in the preceeding works. The reader may consult [5] for explanations of additional features in the pseudoconvex case. In particular, no assumption on connectedness of $\partial D \backslash K$ is needed.

As a starting point to the non-pseudoconvex setting, let us look at two examples illustrating the assumptions in Theorem 1.

Example 4 a) Consider $D = B(0,1)\backslash B(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, where $B(z,r) \subset \mathbb{C}^2$ denotes the ball of radius r centered at z. The compact $K = \{(z_1, z_2) \in \partial D : x_1 \geq \frac{1}{2}\}$ is $A(D)$-convex. Now $\partial D\backslash K$ has two components, whereas $D\backslash A(D)$-hull(K) is connected. Hence CR functions do not extend analytically from $\partial D\backslash K$ to $D\backslash A(D)$-hull(K) in general. By rounding off the corner we get an example with smooth boundary.
b) If we drop CR-convexity, we run into monodromy problems: Consider $D = Q\backslash(\{|z_1| \leq \epsilon, |x_2 - \frac{1}{2}| \leq \epsilon\} \cup \{|z_1| \leq \epsilon, |x_2 + \frac{1}{2}| \leq \epsilon, y_2 \leq 1-\epsilon\})$, where $Q = (-1,1) \times i(-1,1) \times (-1,1) \times i(-1,1)$ and $\epsilon > 0$ is a small constant. If $K = \partial Q \cap \{y_2 = 0\}$, then $\partial D\backslash K$ is connected, whereas $\partial D\backslash A(D)$-hull$(K)$ has three components. We can define a univalent holomorphic function u near $\partial D\backslash K$ which coincides near every $z \in \partial D\backslash K$ with some branch of $\log(z_2)$. But we cannot extend u to $\partial D\backslash A(D)$-hull$(K)$ without loosing the coincidence near some part of the boundary $\square$

Theorem 1 was first proved by the second author in his thesis [13]. His method combines the continuity principle with special constructions relying on geometrical properties of Stein manifolds. The essential point both in [5] and in [13] are monodromy problems of increasing difficulty, which are typical, if one tries to construct hulls by direct application of the continuity principle. Therefore techniques designed to handle this aspect may be of independent interest. This provided the motivation for further joint research of B. Jöricke and the second author [7], which led to a new version of the proof of Theorem 1, where all essential steps are executed by extension along suitable families of complex curves.

The argument presented in the present paper goes back to the observation of the first author how to adapt the integral formula constructions of [8] to the case at hand. However this would require considerable repetitions of the material of [8]. At last we found it preferable to reduce Theorem 1 to Theorem 2 by exhaustion techniques. In the preparational steps (cf. Section 2), we use a device of [5] in order to reduce the problem to the extension of holomorphic boundary values.

Finally a comment on dimensions is in order. Some authors ([14],[5],[7]) prefer to state results of the type of Theorem 1 only for domains in complex dimension 2. The reason is that it is known that in dimension $n \geq 3$ additional extension phenomena occur, which are principally overlooked by convexity assumptions of the above type: The continuity principle tells how families of complex *curves* give rise to holomorphic extension. Now the proofs based on the continuity principle [5], [7] exhibit Theorem 1 clearly as a result on extension along families of complex *hypersurfaces*. The difference between $n = 2$ and $n \geq 3$ gets very transparent in the characterization of removability for strongly pseudoconvex domains, where convexity is adequat in dimension 2 (cf. [3]), whereas the characterizing properties in higher dimension are of cohomological nature (cf. [10]).

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2 Extension to one-sided neighborhoods

In this section we reduce the proof of Theorem 1 to an analogous statement with holomorphic boundary data.

Let $H \subset \mathbb{C}^2$ be a hypersurface. A one-sided neighborhood of H is an open set V such that for every $z \in H$ and for every euclidean ball B of sufficiently small radiuscentered at z contains at least one of the two components of $B \backslash H$ is contained in V.

We shall use the following Lemma from [5].

Lemma 5 *Let D, K be as in Theorem 1. Then there is a connected one-sided neighborhood V of $\partial D \backslash K$ such that every continuous CR-function on $\partial D \backslash K$ extends to V.*

As the result is only stated in dimension 2 in [5], we give a short proof, which makes intensive use of the concept of CR-orbits. We shall use without further comment their elementary properties, for which the reader can consult [15], [6].

Proof of Lemma 5: We observe that it is enough to show that $\partial D \backslash K$ has only one CR-orbit. Indeed, in this case $\partial D \backslash K$ contains a point z in which $\partial D \backslash K$ is minimal in the sense of Tumanov, i.e. there is no local holomorphic hypersurface $X \subset \partial D$ passing through z (otherwise the complex tangent bundle $T^c\partial D \backslash K$ would be Frobenius integrable, and all orbits would be lower-dimensional). The theorem of Trépreau [16] yields one-sided analytic extension near z, and this property propagates along CR-curves to all points of $\partial D \backslash K$.

Assume that $\partial D \backslash K$ has several CR-orbits. It is well known that CR-orbits of a real hypersurface are either subdomains or injectively immersed smooth holomorphic hypersurfaces. The union of all lower-dimensional CR-orbits is relatively closed and forms a lamination. Hence the connectedness of $\partial D \backslash K$ implies the existence of a lower-dimensional CR-orbit $\mathcal{O}$. By general properties of laminations the relative closure L of $\mathcal{O}$ in $\partial D \backslash K$ is a union of lower-dimensional orbits, and each of these is dense in L.

Pick some $z \in L$ and a function $f \in A(D)$ with $f(z) = 1$ and $\max_K |f| < 1/2$. Then the modulus of $f|L$ attains a maximum in some $z' \in L$. By the maximum principle, f is constant on $\mathcal{O}_{z'}$ and, by density, also on L. So L has positiv distance from K and must be compact. But a non-void compact union of holomorphic subvarieties of positive dimension is impossible. Indeed, for such an L one finds a closed euclidean ball B containing L such that $L \cap \partial B \neq \emptyset$. But this leads to a contradiction to the maximum principle on the varieties passing through points of $L \cap \partial B$ (cf. [4], §V, Lemma 5) □

Remark 6 As the referee pointed out, the original proof of [5], which was given for $n = 2$, can be extended to boundaries of class $\mathcal{C}^1$ by using techniques from [2]. For $n > 2$, some additional slicing arguments would be necessary □

Let V be as in the lemma. After shrinking V conveniently, we can suppose that (i) $D' = \operatorname{int}\overline{(D \cup V)}$ is a domain which contains K in its boundary, and that (ii) every $f \in A(D)$ extends to a function in $A(D')$. For (ii), we have to observe that f extends holomorphically through a point $z \in \partial D \backslash K$ if V contains near z the exterior side of ∂D. By (ii), $A(D)$ can be identified with $A(D')$, and we have $A(D)\text{-hull}(K) = A(D')\text{-hull}(K)$.

Near every point $z \in \partial D \backslash K$, V contains at least one side of ∂D. Slightly deforming $\partial D \backslash K$ into V, we can construct a third domain D'' such that $\partial D''$ is of class $\mathcal{C}^2$ and $\partial D'' \backslash K \subset D'$. As $A(D) = A(D')$, we can choose $\partial D'' \backslash K$ so close to $D \backslash K$ that

$$A(D')\text{-hull}(K) \cap \partial D'' = K$$

holds true. Observe that the analytic extension to V of a given CR-function on $D \backslash K$ induces holomorphic data on $\partial D'' \backslash K$. Writing again D instead of D'', the proof of Theorem 1 is now reduced to the following intermediate statement.

Proposition 7 *Let D be a bounded domain in $\mathbb{C}^2$ with boundary of class $\mathcal{C}^2$ and $K \subset \partial D$ be a compact set with $\partial D \backslash K$ connected. Let $D' \supset D$ be a bounded domain satisfying $K \subset \partial D'$, $K = A(D')\text{-hull}(K) \cap \partial D$, and $\overline{D} \backslash K \subset D'$. Then each function u which is holomorphic in a neighborhood U of $\partial D \backslash K$ has (after shrinking U if necessary) a holomorphic extension to $D \backslash A(D')\text{-hull}(K)$.*

3 Proof of Theorem 1

As explained in Section 2, it is enough to prove Proposition 7.

First we observe that we cannot immediatly apply Theorem 3 by taking Ω as the envelope of holomorphy of D'. Of course D need not be relatively compact in Ω. In what follows we will derive Proposition 7 by an exhaustion argument.

For every $w \in D\backslash A(D')\text{-hull}(K)$, there is a function $f_w \in A(D')$ with $f_w(w) = 1$ and $\max_{z\in K} |f_w(z)| \leq 1/4$. By a standard covering argument, we select a subsequence $f_1, f_2, \ldots$, such that

$$D\backslash A(D')\text{-hull}(K) = \bigcup_{i=1}^{\infty} \{z \in D : |f_i(z)| > 3/4\}.$$

Note that a given compact subset of $D\backslash A(D')\text{-hull}(K)$ is already contained in the finite union $\bigcup_{i=1}^{k} \{z \in D : |f_i(z)| > 3/4\}$, if k is sufficiently large.

Let $\rho \in \mathcal{C}^2(\overline{D})$ be a defining function of D, i.e. a function with $\nabla\rho \neq 0$ on ∂D and $D = \{\rho < 0\}$. We can choose a strictly decreasing sequence of positive numbers $r_j \downarrow 0$ and domains $D_j = \{\rho < -r_j\}$ so that for every j the intersection $\partial D_j \cap \bigcup_{i=1}^{j} \{z \in D : |f_i(z)| > 1/2\}$ is contained in U (the neighborhood of $\partial D\backslash K$ where u is defined). If Ω denotes the envelope of holomorphy of D', we can consider each D_j as a relatively compact subdomain of Ω. Define

$$K_j = \mathcal{O}(\Omega)\text{-hull}(\bigcap_{i=1}^{j} \{z \in \overline{D_j} : |f_i(z)| \leq 3/4\}).$$

As Ω is holomorphically convex, K_j is compact in Ω. By construction, we have $K_j \cap \overline{D_j} = \bigcap_{i=1}^{j} \{z \in \overline{D_j} : |f_i(z)| \leq 3/4\}$ and $\partial D_j \backslash K_j \subset \partial D_j \cap \bigcup_{i=1}^{j} \{z \in D : |f_i(z)| > 1/2\} \subset U$. For later use we remark $\overline{D_j}\backslash K_j \subset D_{j+1}\backslash K_{j+1}$. Observe that we cannot immediately apply Theorem 3 because (i) $D_j\backslash K_j$ may have several components, and (ii) the intersection of the closure of a component of $D_j\backslash K_j$ with ∂D_j need not be connected.

In order to meet (i), we choose subdomains $G_j \subset D_j$ in the following way: Let G_1 be an arbitrary component of $\{z \in D_1 : |f_1(z)| > 3/4\}$. By induction we choose, for every $j > 1$, G_j as the unique component of $\bigcup_{i=1}^{j} \{z \in D_j : |f_i(z)| > 3/4\}$ with $G_{j-1} \subset G_j$. Because $\partial D\backslash K$ is connected and $D\backslash A(D')\text{-hull}(K)$ has, by the maximum principle, no components which are relatively compact in D, $D\backslash A(D')\text{-hull}(K)$ is connected. As mentioned above, every compact arc in $D\backslash A(D')\text{-hull}(K)$ is contained in almost every finite union $\bigcup_{i=1}^{j} \{z \in D : |f_i(z)| > 3/4\}$. Since $D\backslash A(D')\text{-hull}(K)$ is connected, we deduce $D\backslash A(D')\text{-hull}(K) = \bigcup_{j=1}^{\infty} G_j$.

As indicated in (ii), there is no reason for $\partial G_j \backslash K_j$ to be connected. We shall use an elementary topological property of Stein manifolds to handle this difficulty.

Lemma 8 *Let Ω be a Stein manifold of dimension $n \geq 2$ and L a compact $\mathcal{O}(\Omega)$-convex subset. Let M be a connected, properly embedded, orientable real hypersurface of $\Omega \backslash L$ of class $\mathcal{C}^1$ such that $M \cup L$ is compact. Then $\Omega \backslash (M \cup L)$ contains exactly one relatively compact component.*

Of course Lemma 8 is a byproduct of E. M. Chirka's relative version of the Harvey-Lawson theorem ([1], Theorem 19.6.2). For the reader's convenience, we provide an elementary proof, based on an argument communicated by N. Sherbina. The assumption that M be orientable is not necessary, since our proof also works with $\mathbb{Z}/2\mathbb{Z}$-valued intersection numbers.

Proof: By the maximum principle, $\Omega \backslash L$ has no relatively compact components. Hence every component of $\Omega \backslash (M \cup L)$ has accumulation points on M. As M is connected, $\Omega \backslash (M \cup L)$ has at most two components. Because Ω has only one end and $M \cup L$ is compact, there is a unique unbounded component. Consequently the second component, if it exists, has to be relatively compact.

Assume that there is no relatively compact component of $\Omega \backslash (M \cup L)$. Then we can easily construct a smoothly embedded oriented loop $\gamma \subset \Omega \backslash L$ which intersects M transversely in only one point. Hence the intersection number of γ and M equals ± 1, the sign depending on the orientations we choose for γ and M. We shall obtain a contradiction to the homotopy invariance of intersection numbers by deforming γ within $\Omega \backslash L$ to a loop contained in $\Omega \backslash (M \cup L)$.

As L is $\mathcal{O}(\Omega)$-convex, an elementary construction gives a smooth plurisubharmonic non-negative exhaustion function ϕ of Ω such that $L = \{\phi = 0\}$ and ϕ is strictly plurisubharmonic on $\Omega \backslash L$ (being exhaustive means that $\{\phi < c\}$ is relatively compact for any $c \in \mathbb{R}$). Fix $c_1, c_2 > 0$ such that $M \cup L \subset \{\phi < c_2\}$ and $\gamma \subset \{c_1 < \phi < c_2\}$. After a slight modification, we can assume that ϕ is a Morse function on a neighborhood of $\{c_1 \leq \phi \leq c_2\}$ (for information on Morse theory we refer to [11]). Then there are finitely many critical points $q_1, \ldots, q_k$ in $\{c_1 \leq \phi \leq c_2\}$. It is a well-known consequence of strict plurisubharmonicity that the Morse-indices at the points $q_1, \ldots, q_k$ cannot exceed n. This implies that the associated stable manifolds

$$S_i = \{p \in \Omega : \lim_{t \to +\infty} \Phi_{\nabla\phi, t}(p) = q_i\}$$

are at most of dimension n, in particular of codimension at least $n \geq 2$. Here $\Phi_{X,t}$ denotes the time-t-map of a vectorfield X and $\nabla\phi$ the gradient with respect to some fixed riemannian metric on Ω. After an arbitrarily small deformation of γ, we can assume $\gamma \cap \bigcup_{i=1}^k S_i = \emptyset$. This means that, for every $p \in \gamma$, we have $\phi(\Phi_{\nabla\phi,t}(p)) > c_2$, if t is sufficiently large. Now a compactness argument yields that there exists $T > 0$ such that $\phi(\Phi_{\nabla\phi,t}(\gamma))$ is disjoint from $\{\phi \leq c_2\} \supset M \cup L$ for $t > T$, a contradiction to the homotopy invariance of intersection numbers $\square$

In the situation of Lemma 8, we shall call the relatively compact component of $\Omega\backslash(M\cup L)$ the inner domain of $M\cup L$. For j fixed, let $\mathcal{C}_j$ be the set of connected components of $\partial G_j\backslash K_j$. Since K_j is $\mathcal{O}(\Omega)$-convex, Lemma 8 associates to any $T \in \mathcal{C}_j$ the inner domain B_T of $T \cup K_j$. Hence we may introduce a partial order on $\mathcal{C}_j$ by writing $T_1 \prec T_2$ if $B_{T_1} \subset B_{T_2}$. Reflexivity and transitity are obvious. The following lemma contains antisymetry and the existence of a unique maximal element.

Lemma 9 *a) If $T_1 \prec T_2, T_2 \prec T_1$, for $T_1, T_2 \in \mathcal{C}_j$, then $T_1 = T_2$.*
b) There is a unique maximal component $M_j \in \mathcal{C}_j$, which is the unique element of $\in \mathcal{C}_j$ which belongs to the closure of the unbounded connected component of $\Omega\backslash(G_j \cup K_j)$. Moreover its inner domain B_{M_j} contains G_j and all the other inner domains $B_T, T \in \mathcal{C}_j$.

Proof: a) If we assume $T_1 \prec T_2, T_2 \prec T_1$, and $T_1 \neq T_2$, then the definition of $\prec$ implies $T_1 \subset B_{T_2}$ and $T_2 \subset B_{T_1}$. Hence $B_{T_1} \cup B_{T_1}$ is a relatively compact domain whose boundary is contained in K_j, a contradiction to the $\mathcal{O}(\Omega)$-convexity of K_j and the maximum principle.

b) Lemma 8 implies that there is at most one $M_j \in \mathcal{C}_j$ belonging to the closure of the unbounded connected component of $\Omega\backslash(G_j \cup K_j)$. If there were no such M_j, then G_j would be contained in a relatively compact component of $\Omega\backslash K_j$, which is in contradiction with the maximum principle. Hence M_j is uniquely defined, and we have $B_{M_j} \supset G_j$.

Let T be another component of $\partial G_j\backslash K_j$. Then $(G_j \cup T) \subset B_{M_j}$ by connectedness of G_j, and $B_T \subset B_{M_j}$ by definition of B_T and B_{M_j} $\square$

By Theorem 3, the restriction of u to a sufficiently small neighborhood of M_j extends to a function $u_j \in \mathcal{O}(B_{M_j}) \subset \mathcal{O}(G_j)$ which coincides with u near M_j. It is not yet clear, whether u_j coincides with u near all components of $\partial G_j\backslash K_j$. So we must carefully check that we can produce the desired extension of u by gluing the u_j.

For this purpose we fix some compact subset $L \subset D\backslash A(D')$-hull$(K)$. By construction, $L \subset G_j$ for sufficiently large j. According to the following lemma, the sequence $\{u_j\}$ gets stable near L thus suggesting a natural candidate for the final extension near L.

Lemma 10 *There is $k_L \in \mathbb{N}$ and a neighborhood V of L such that the functions u_j coincide on V, for all $j \geq k_L$.*

Proof: Choose $c \in \mathbb{C}, |c| > 3/4$, such that $C = \{p \in \Omega : f_1(p) = c\}$ is a smooth complex curve which intersects $\partial D \backslash K$ transversally. Since Ω does not contain compact complex curves, there is some $p_0 \in \partial D \backslash K$ which is an accumulation point of a non-relatively compact component of $C \backslash \overline{D}$.

Near p_0, C intersects the hypersurfaces ∂D and ∂D_j, for j sufficiently large, in a family of almost parallel short segments λ_j which are all contained in U. Hence for large j, the segments λ_j are adherent to an unbounded component of $C \backslash \overline{D_j}$ by transversality. Fix some j_0 for a moment. Then λ_{j_0} is contained in all G_j, for $j \geq j_1$, if j_1 is sufficiently large. For $j \geq j_1$, we deduce $\lambda_j \subset \partial G_j \backslash K_j$. As λ_j lies in the closure of an unbounded component of $C \backslash \overline{D_j}$, we even obtain $\lambda_j \subset M_j$.

If we take $j_2 \geq j_1$ so large that $L \subset G_j, j \geq j_2$, then all the functions $u_j, j \geq j_2$, coincide near L. Indeed we can connect a given point $p \in L$ with λ_{j_2} by an arc $\gamma \subset G_{j_2} \cup \lambda_{j_2}$ and compare the functions $u_j, j \geq j_2$, along γ. Hence we can take $k_L = j_2$. □

Proposition 7 follows from Lemma 10 without difficulties: Let us take some exhaustion of $D \backslash A(D')$-hull(K) by compact sets $L_1 \subset L_2 \subset \ldots$ satisfying $L_j \subset \text{int}(L_{j+1})$. By Lemma 10 we get near every L_j a natural candidate $\tilde{u}_j$ by taking the restriction of some u_k for $k \geq k_{L_j}$. Since the L_j are monotonuously increasing sets, it is clear from Lemma 10 that the $\tilde{u}_j$ glue to a well-defined function $\tilde{u} \in \mathcal{O}(D) \backslash A(D')$-hull$(K)$. Finally the connectedness of $\partial D \backslash K$ implies the coincidence of u and $\tilde{u}$ near $\partial D \backslash K$. Proposition 7 and Theorem 1 are proved □

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