Magnetohydrodynamics

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Magnetohydrodynamics (MHD) is the study of electrically conducting flux (salt water, plasma...) subject to an electromagnetic field. We model it with two coupled partial differential equations: the Navier-Stokes equations and the Maxwell equations. In this lecture, in a first time we will understand the building of this model and applications in different fields, and in a second time we will discuss about the existence and unicity of solutions, with the following mathematical framework:

Let Ω be a bounded simply connected domain in $\mathbb{R}^3$ of class $\mathcal{C}^{1,1}$ with a connected boundary Γ. We denote by $\boldsymbol{n}$ the unit vector normal to Γ and exterior to Ω. We consider the following magnetohydrodynamic system (MHD): find the velocity field $\boldsymbol{u}$, the dynamic pressure p, and the magnetic field $\boldsymbol{b}$ satisfying:

$$\begin{aligned} -\nu\,\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla p - \kappa(\mathbf{curl}\,\boldsymbol{b})\times\boldsymbol{b} = \boldsymbol{f} \qquad & \text{in}\,\Omega \\ \kappa\mu\;\mathbf{curl}\,\mathbf{curl}\,\boldsymbol{b} - \kappa\,\mathbf{curl}(\boldsymbol{u}\times\boldsymbol{b}) = \boldsymbol{g} \qquad & \text{in}\,\Omega \\ \operatorname{div}\boldsymbol{u} = 0, \qquad & \text{in}\,\Omega, \\ \operatorname{div}\boldsymbol{b} = 0, \qquad & \text{in}\,\Omega. \end{aligned}$$

Here, ν^{-1} is the hydrodynamic Reynolds number, μ^{-1} is the magnetic Reynolds number and κ is the coupling number. The functions $\boldsymbol{f}$ and $\boldsymbol{g}$ are given source terms. We will speak about the different boundary conditions (Dirichlet, Navier), and the difference of the functional framework when we change.