Global exact controllability of the bilinear Schrödinger potential type models on compact quantum graphs

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In quantum mechanics any pure state of a system is mathematically represented by a wave function ψ in the unit sphere of a Hilbert space $\mathscr{H}$. The dynamics of a particle constrained in a compact graph structure $\mathscr{G}$ and excited by a controlled field is represented by the Cauchy problem in $\mathscr{H} = L^2(\mathscr{G}, \mathbb{C})$

$$\begin{cases} i\partial_t \psi(t) = A\psi(t) + u(t)B\psi(t), \\ \psi(0) = \psi^0, \qquad t \in (0, T). \end{cases} \tag{1}$$

The operator B is bounded symmetric, u is a control function and ψ^0 is the initial state of the system. The operator $A = -\Delta$ is the Laplacian equipped with self-adjoint type boundary conditions into the vertices of the graph.

In the work [1], we study how the boundary conditions and the structure of the graph affect the controllability of the bilinear Schrödinger equation (1). Provided the well-posedness of (1), we present assumptions on B and on the spectrum of A implying the global exact controllability in suitable subspaces of $\mathscr{H}$.
When the previous assumptions fail, we introduce a weaker notion of controllability. We define the so-called "energetic controllability" which allows to provide interesting results also when $\mathscr{G}$ is a complex structure and we are not able to verify the spectral assumptions for the global exact controllability.

References

[1] Alessandro Duca. Global exact controllability of the bilinear Schrödinger potential type models on quantum graphs. *http://arxiv.org/abs/1710.06022.*