A Shape Optimization Overview and Introduction to Robust Optimization

Charles Dapogny , Julien Prando *, Boris Thibert

Shape optimization is all about finding the best design of a mechanical structure according to a performance criteria. This optimal design challenge arises whether it is for the conception of a thermal device, a pipe conveying fluids or a pedestrian bridge. Those physical situations are described with the help of physical parameters such as material coefficients or viscosity which are difficult to estimate. It's an issue because the optimal design for a set of parameters may worsen under a slight perturbation of these parameters.

Several models have been developed to handle this type of instability starting from worst case approaches [1] where we consider that the uncertain parameters are known up to a certain amplitude. The main issue of these approaches is the poor performance with respect to the reference parameters mainly because worst-case parameters will usually be unlikely. That's why, stochastic approaches have been considered [4] by assuming that these parameters suffer from randomness. The addition of weights allows to balance the phenomenon of unrealistic worst-case realisations. However, these stochastic models rely on the precise knowledge of the probability distribution of the uncertain parameters which is usually inaccessible.

In the context of convex optimization, methods have been developed to overcome the lack of knowledge of this probability distribution. In the distributionally robust optimization approach, the optimization is usually done with respect to a worst case distribution close to the empirical law built from observations. The Wasserstein distance has been used to characterize the notion of proximity [5] leading to an efficient tractable formulation under reasonable assumptions [2]. Another approach is to consider an ambiguity set of distributions that are close in term of moments.

In this talk, we will take a look at the domain of shape optimization from their motivations to the key tools used in practice. We will also explore the use of distributionally robust optimization approaches in the context of shape and topology optimization [3]. After that, we will observe the behavior of this model on numerical examples such as cantilever or mast optimization. This will illustrate that optimal distributionally robust solutions indeed yield good performances with respect to the reference parameters while handling likely worst-case realisations.

References

[1] Grégoire Allaire and Charles Dapogny. A linearized approach to worst-case design in parametric and geometric shape optimization. *Mathematical Models and Methods in Applied Sciences*, 24(11):2199–2257, 2014.

[2] Waïss Azizian, Franck Iutzeler, and Jérôme Malick. Regularization for wasserstein distributionally robust optimization. *arXiv preprint arXiv:2205.08826*, 2022.

[3] Charles Dapogny, Franck Iutzeler, Andrea Meda, and Boris Thibert. Entropy-regularized wasserstein distributionally robust shape and topology optimization. *Structural and Multidisciplinary Optimization*, 66(3):42, 2023.

[4] Subhayan De, Jerrad Hampton, Kurt Maute, and Alireza Doostan. Topology optimization under uncertainty using a stochastic gradient-based approach, 2019. arXiv:1902.04562.

[5] Daniel Kuhn, Peyman Mohajerin Esfahani, Viet Anh Nguyen, and Soroosh Shafieezadeh-Abadeh. Wasserstein distributionally robust optimization: Theory and applications in machine learning. In *Operations research & management science in the age of analytics*, pages 130–166. Informs, 2019.