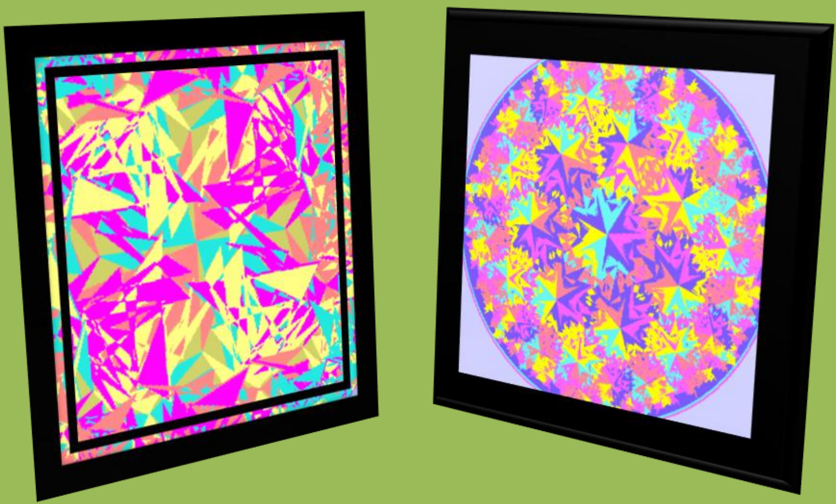
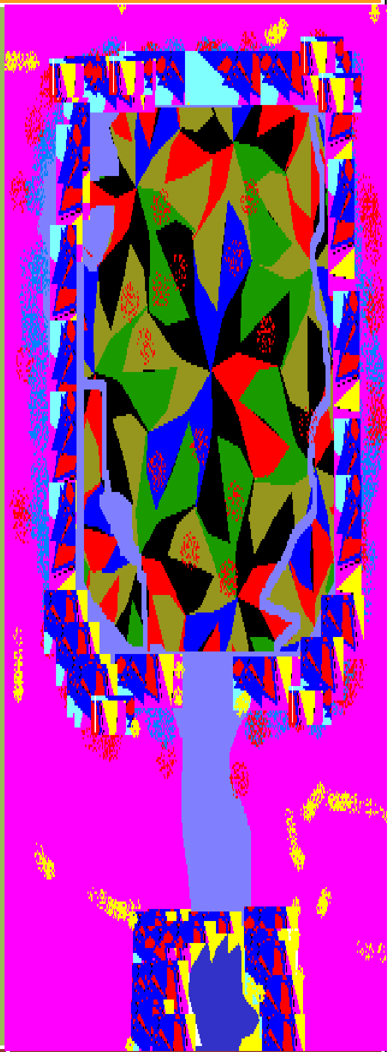
MARCEL MORALES

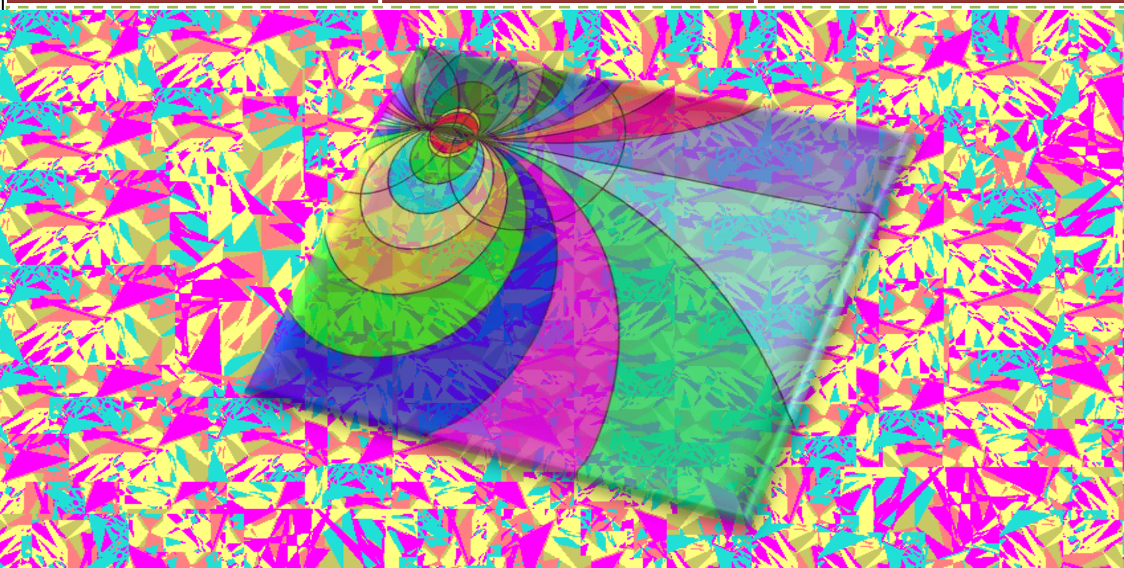
Tiling, tessellations by hand

Tessellations





2010



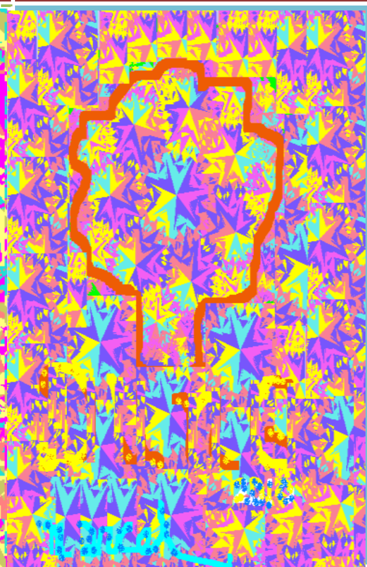


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Table des matières

Tiling, tessellation, periodic or non periodic tiling: fulfilling a picture by some figure was used by all the civilizations to decorate walls, carpets, potteries …

This book can help everybody to produces and drawn his own tiling, without any knowledge in mathematics.
After some introduction to the subject with elementary notions of geometry, we introduce the seventeen groups of tiling of the plane.
I have developed software that helps us to draw a picture for each group of tiling.
Our aim is to introduce people to:

1. Recognize a tiling, and the basic figure,
2. Describe the tiling group, i.e. the transformation used to fulfill the plane, which rotation, symmetries.
3. How realize a tiling by drawing, cutting and gluing without using difficult techniques.
4. For each tiling's group a complete sequence of realization is done, explaining and allowing realizing it quickly.
5. Every tiling group is illustrated by one picture.

The software has been developed by Marcel Morales, but the author has learned a lot from the high school class of Alice Morales. The software has been used by school students during many years, in all the degrees of schools. A joint work with the classroom of Alice Morales has been presented in the international exposition Exposciences International 2001 in Grenoble.
The software has been introduced in many scientific expositions in France and outside France: Mexico, Peru, Colombia, Iran, Turkey, and Vietnam.

Moreover, after the expositions and collaborations with high schools, it appears that doing tiling can be a kind of game, but also improves the knowledge in mathematics, without any formal course.
We introduce some tiling founded in old civilizations.

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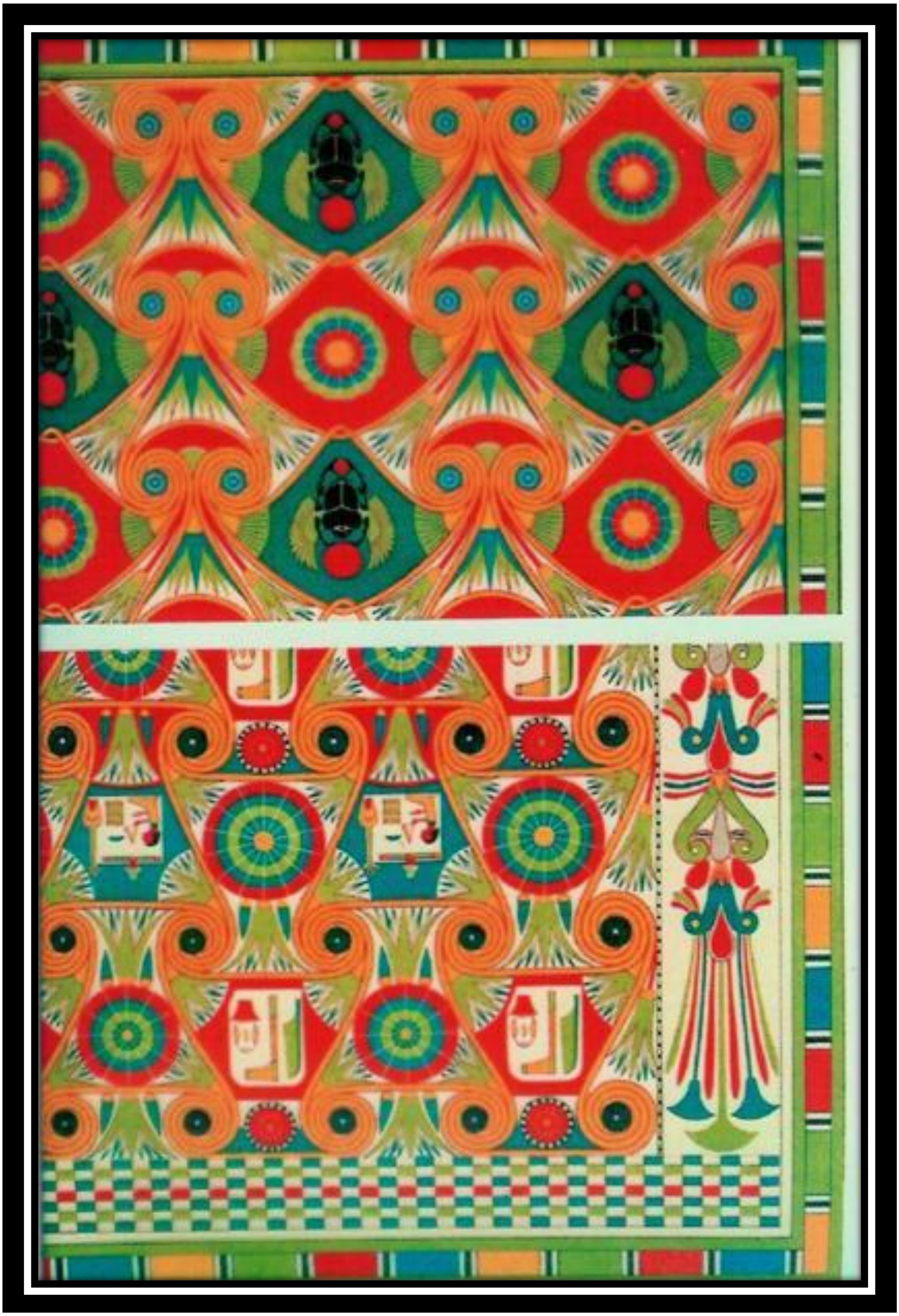
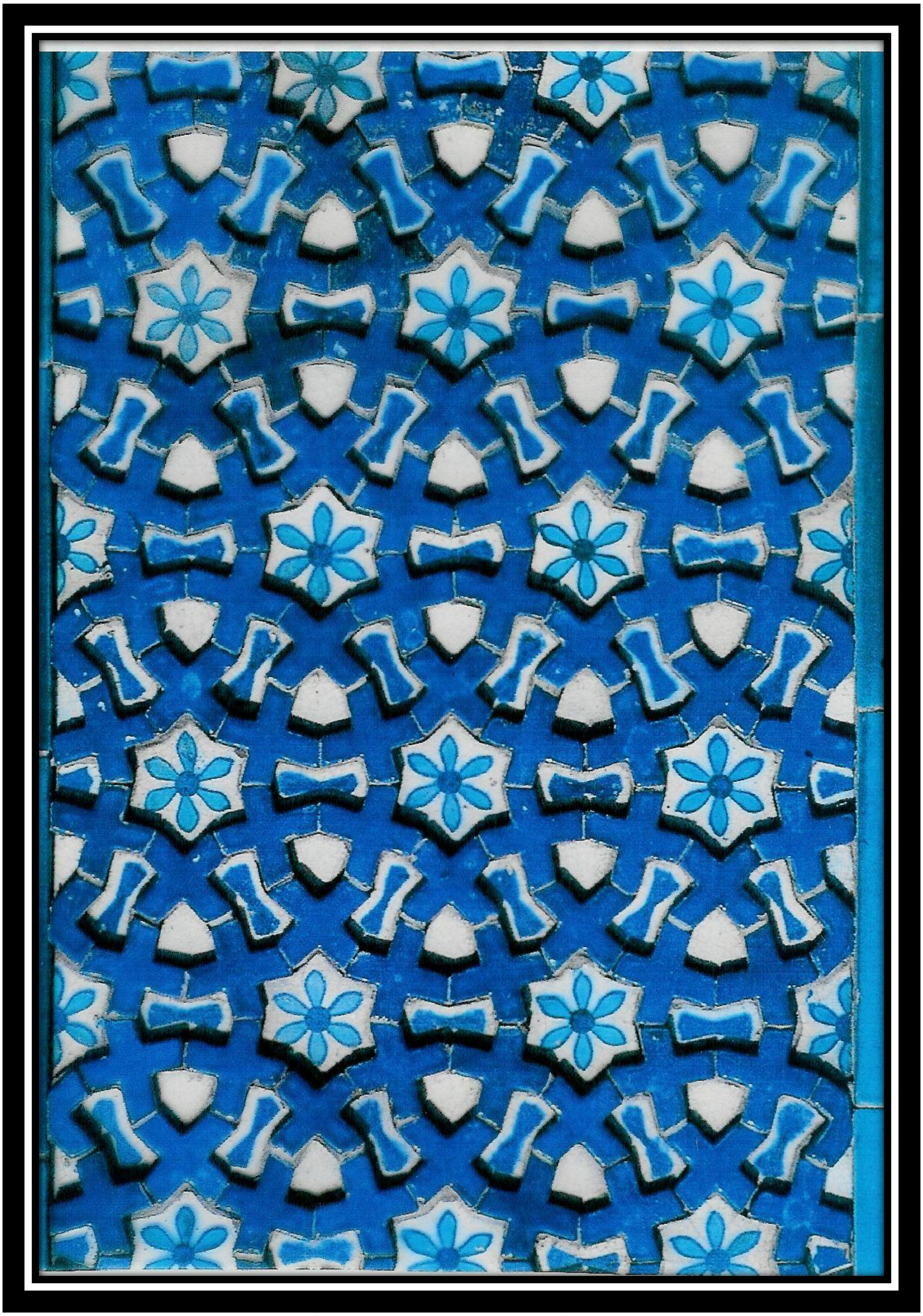
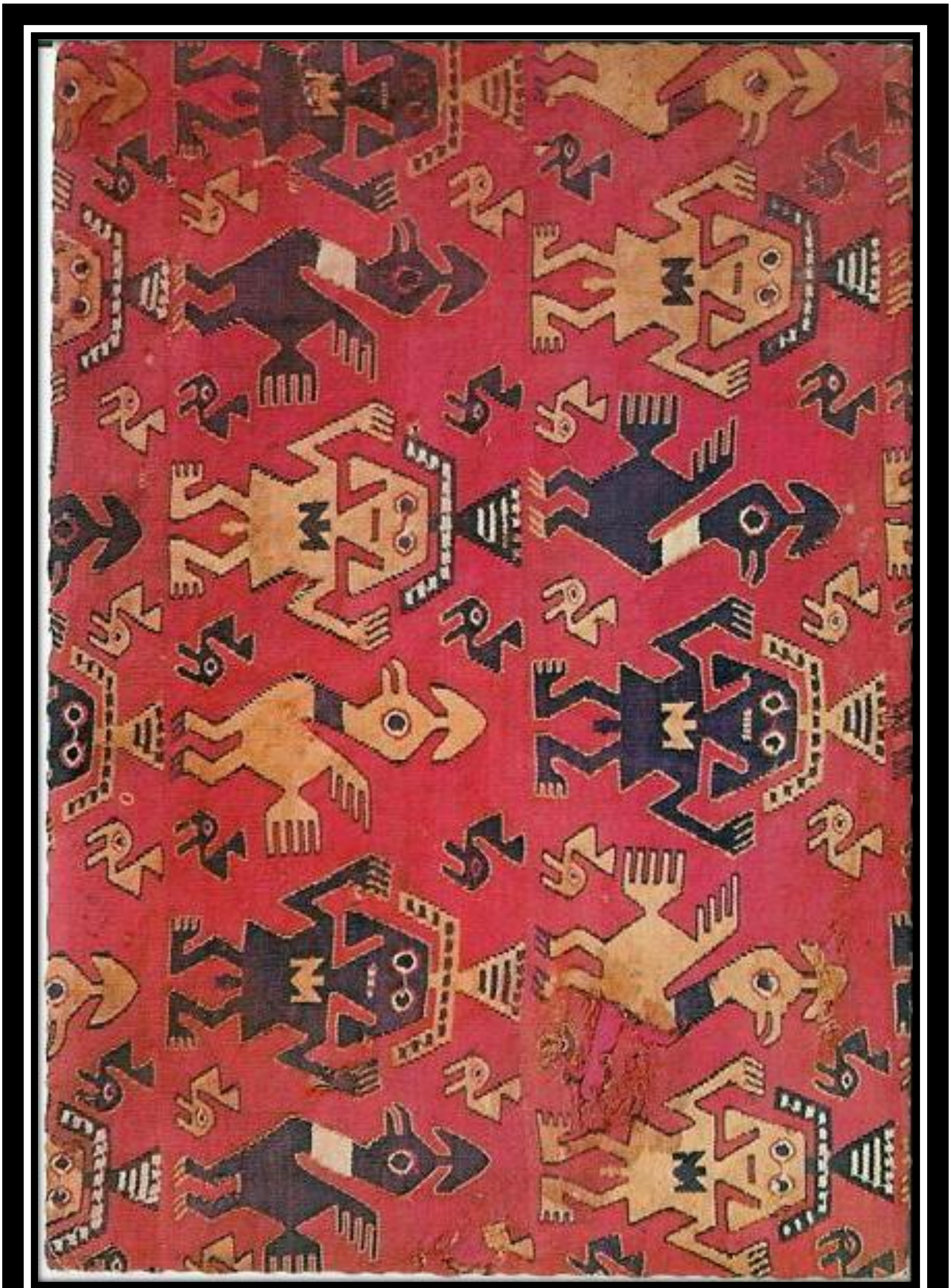
Institut Fourier
Université de Grenoble I



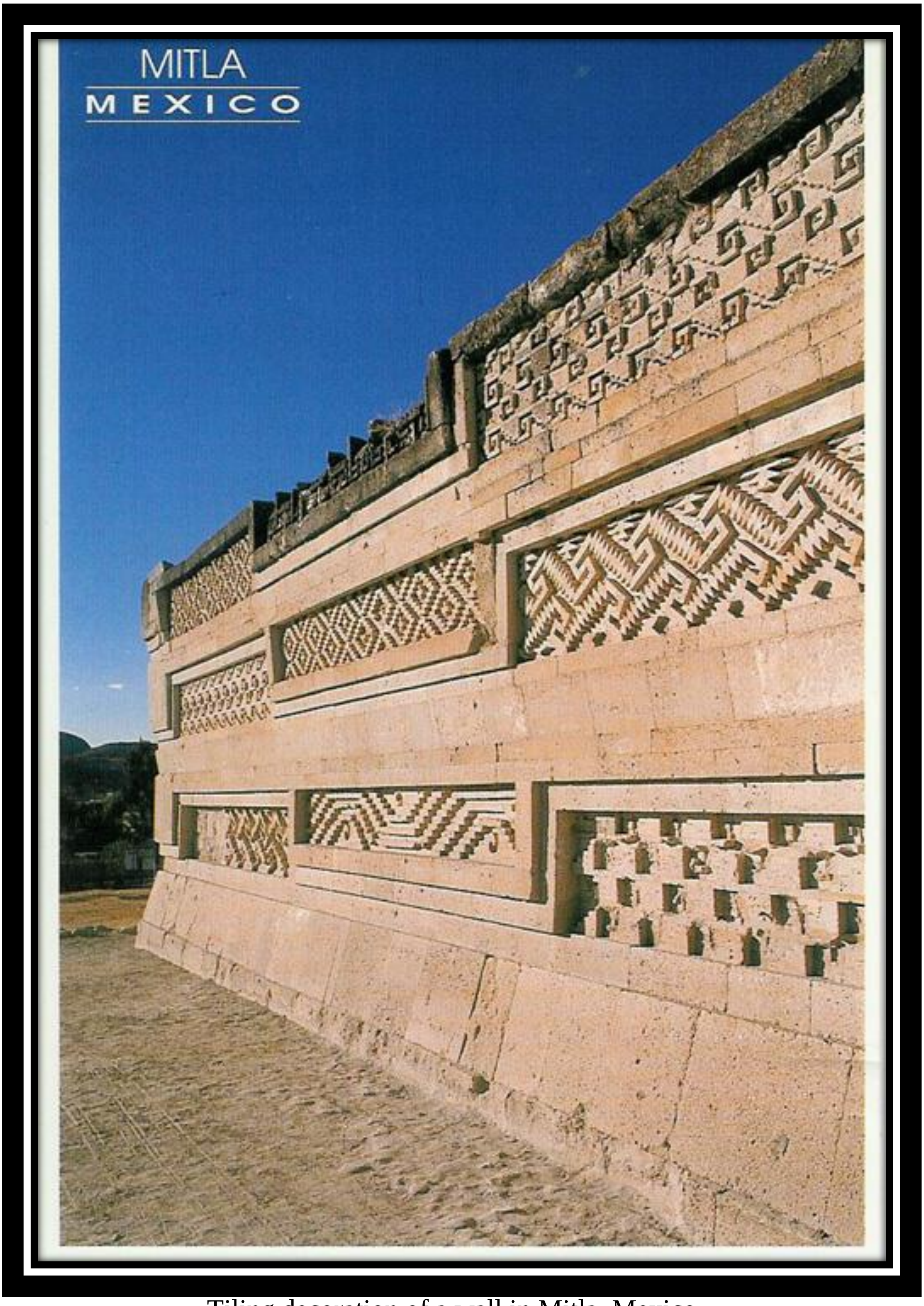
Tiling founded in the old Egypt.



Tiling founded on the wall of a mosque.

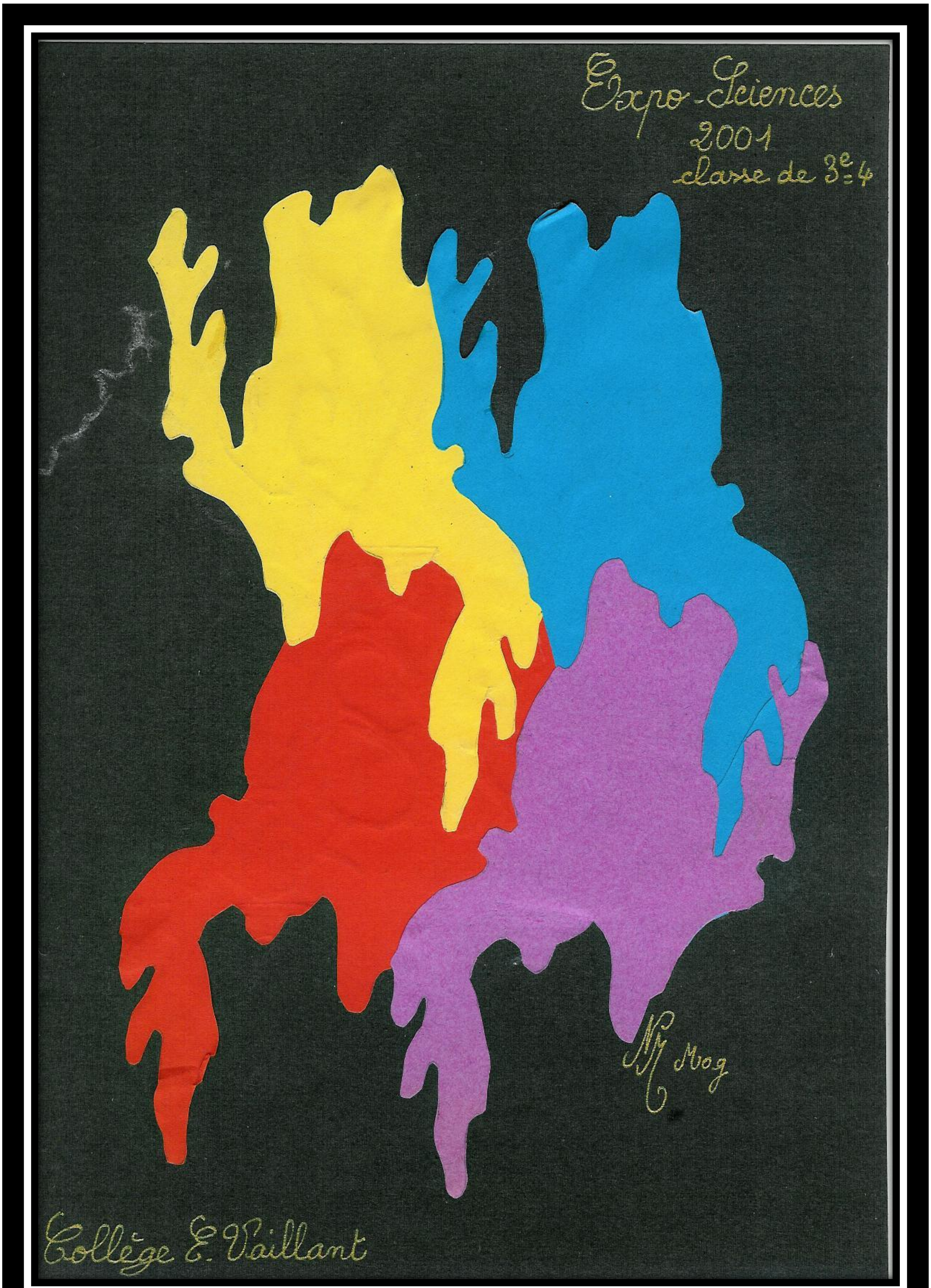


Tiling founded in a Peruvian carpet (Paracas).

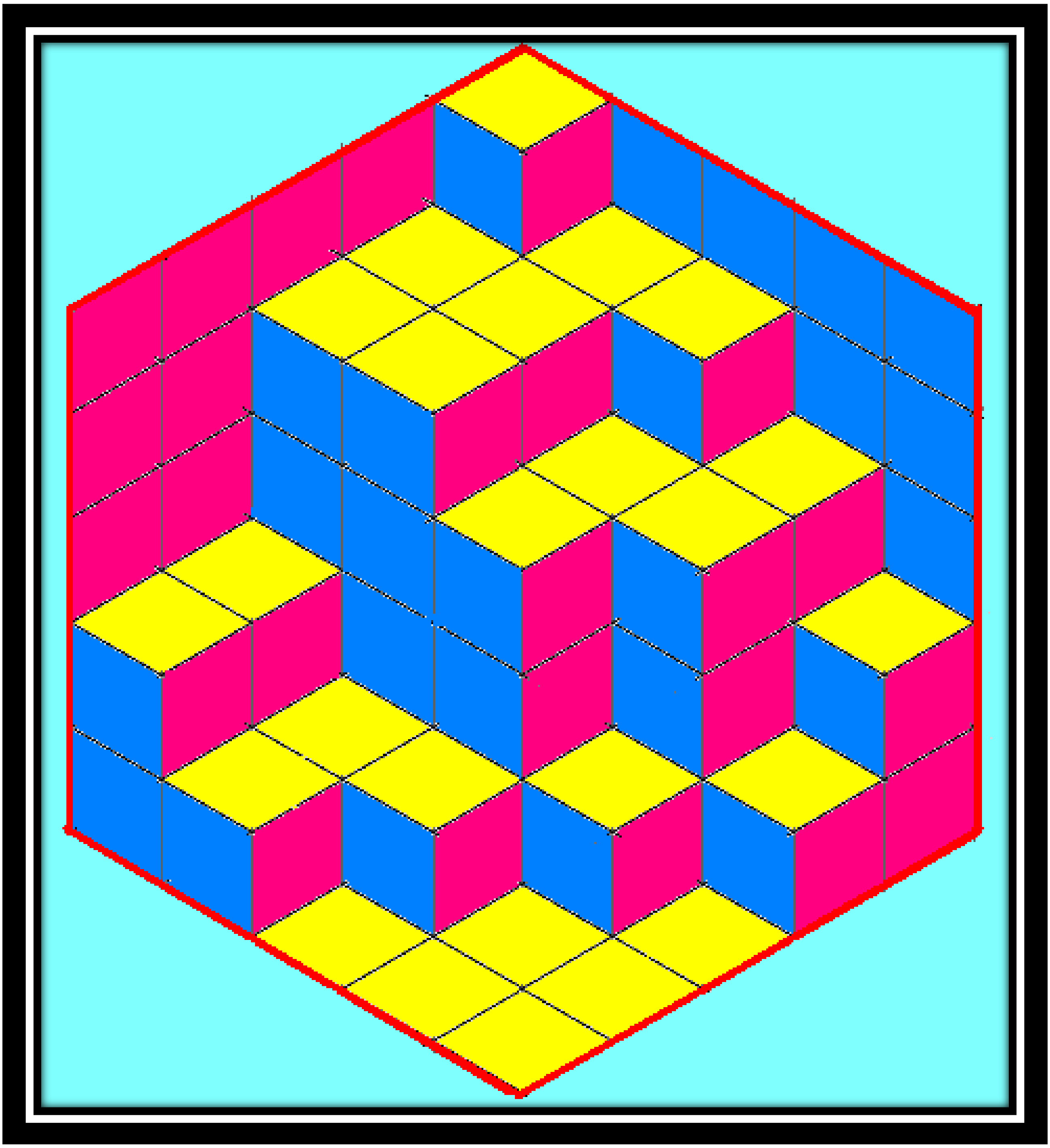


Tiling decoration of a wall in Mitla, Mexico

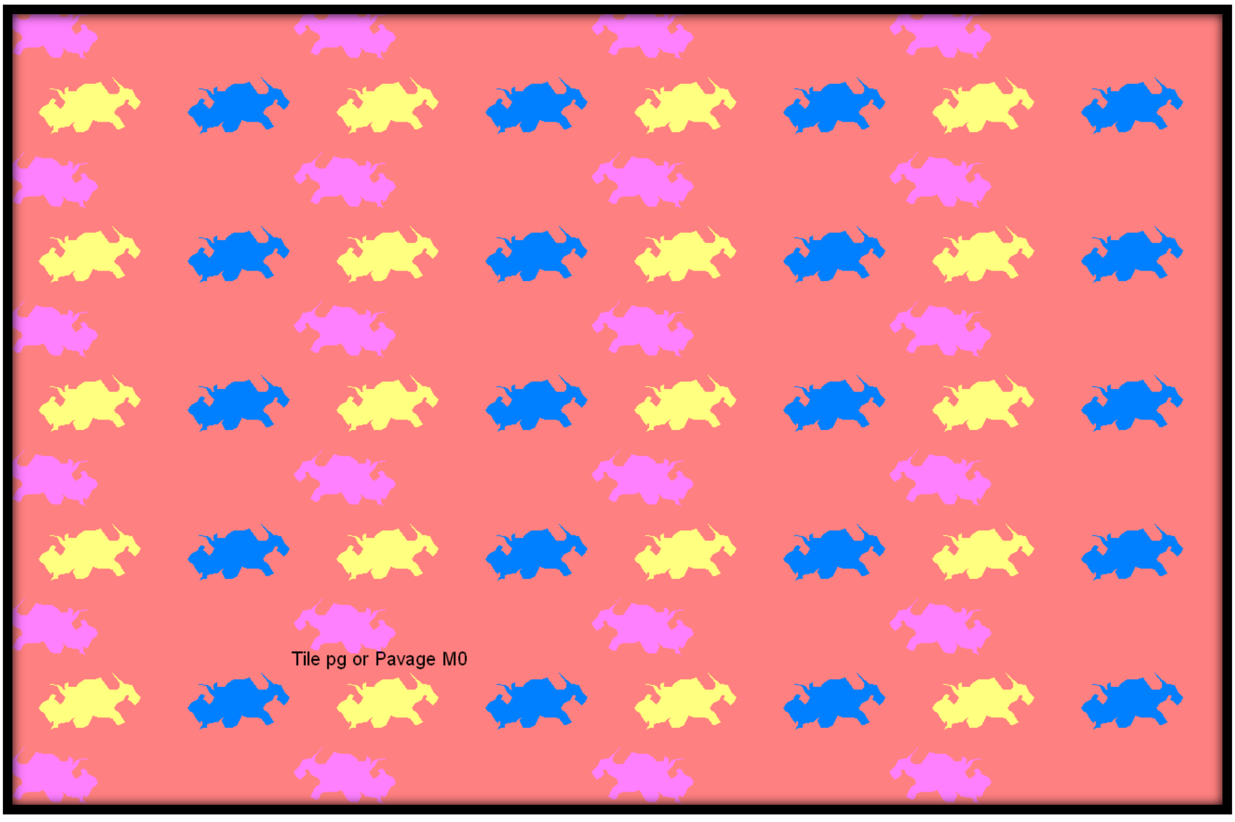
Potteries, Indians from North America.

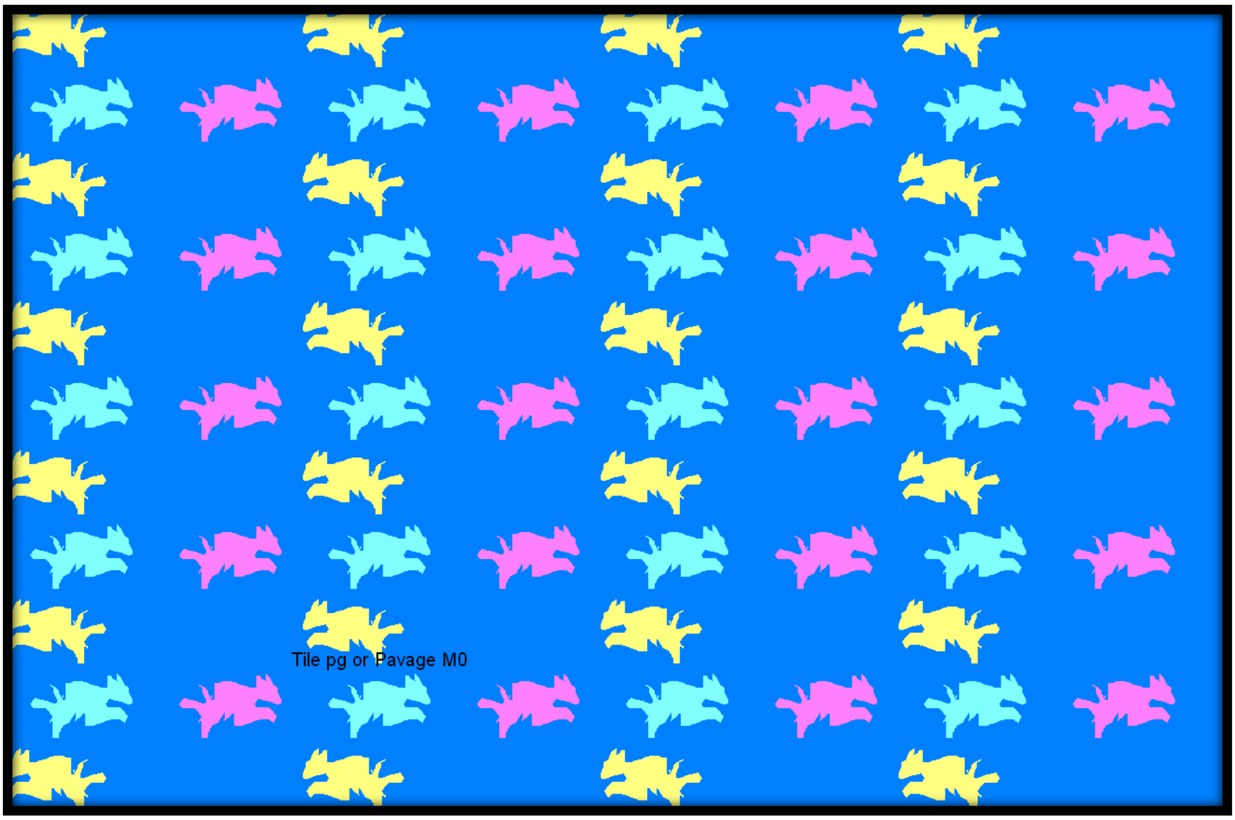


Tiling realized by a pupil in the High school.



Non periodic tiling of an hexagon, by using a rhomb (colored yellow), transformed by using thee transformations (see the cube up): by translations it remains yellow, by rotation of 120° and translations it is colored red, by rotation of 240° and translations it is colored blue.


Tile pg or Pavage M0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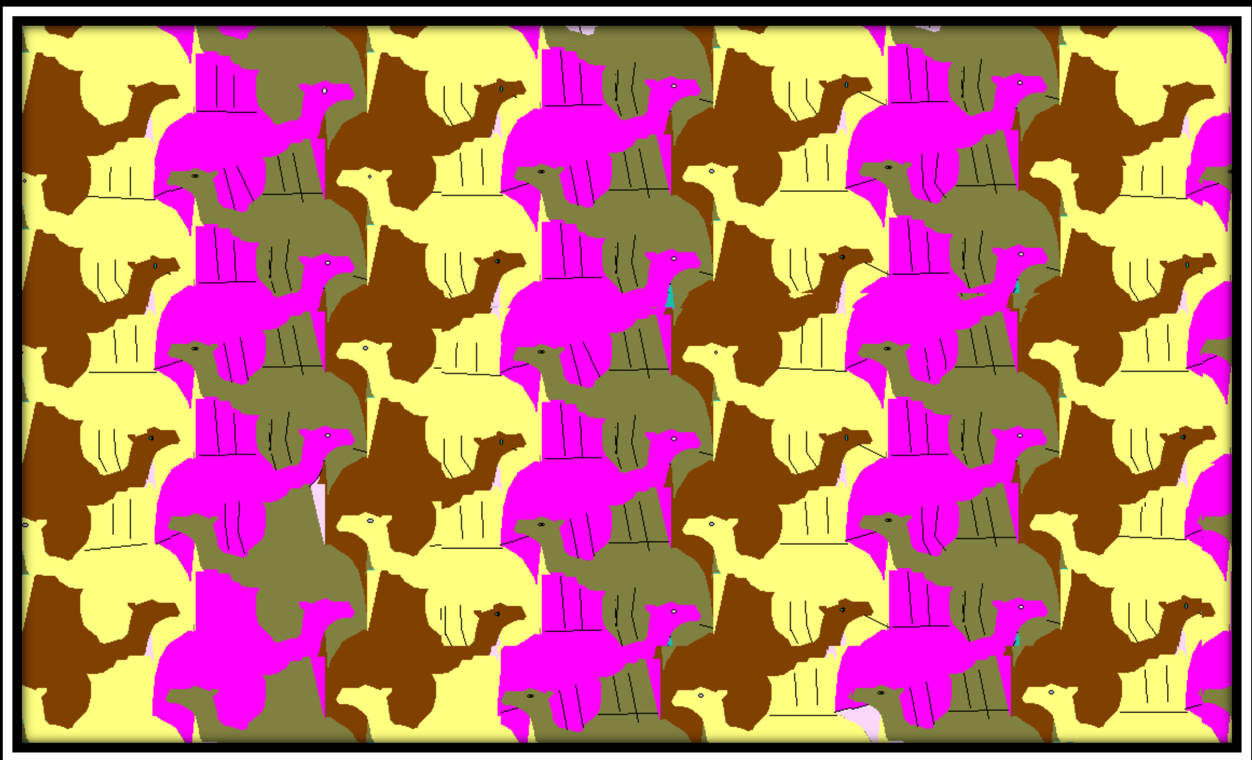
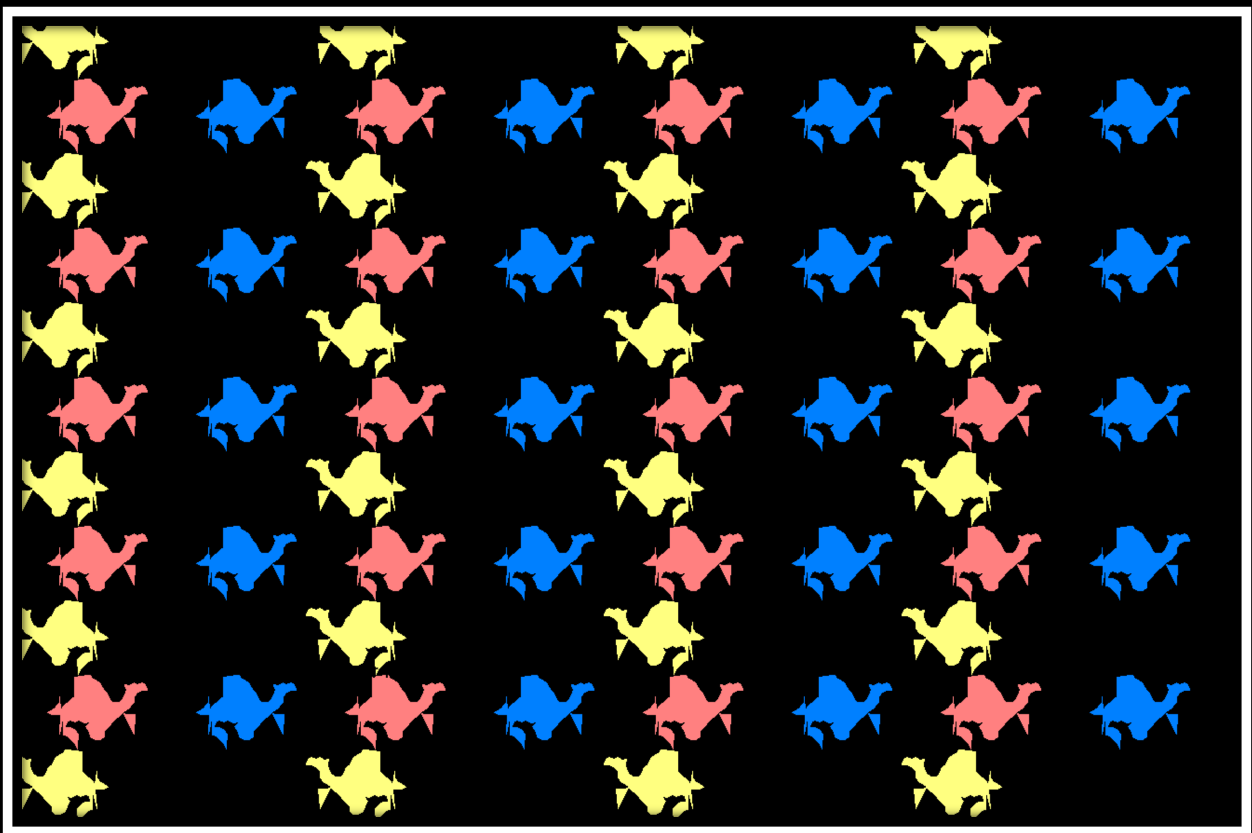
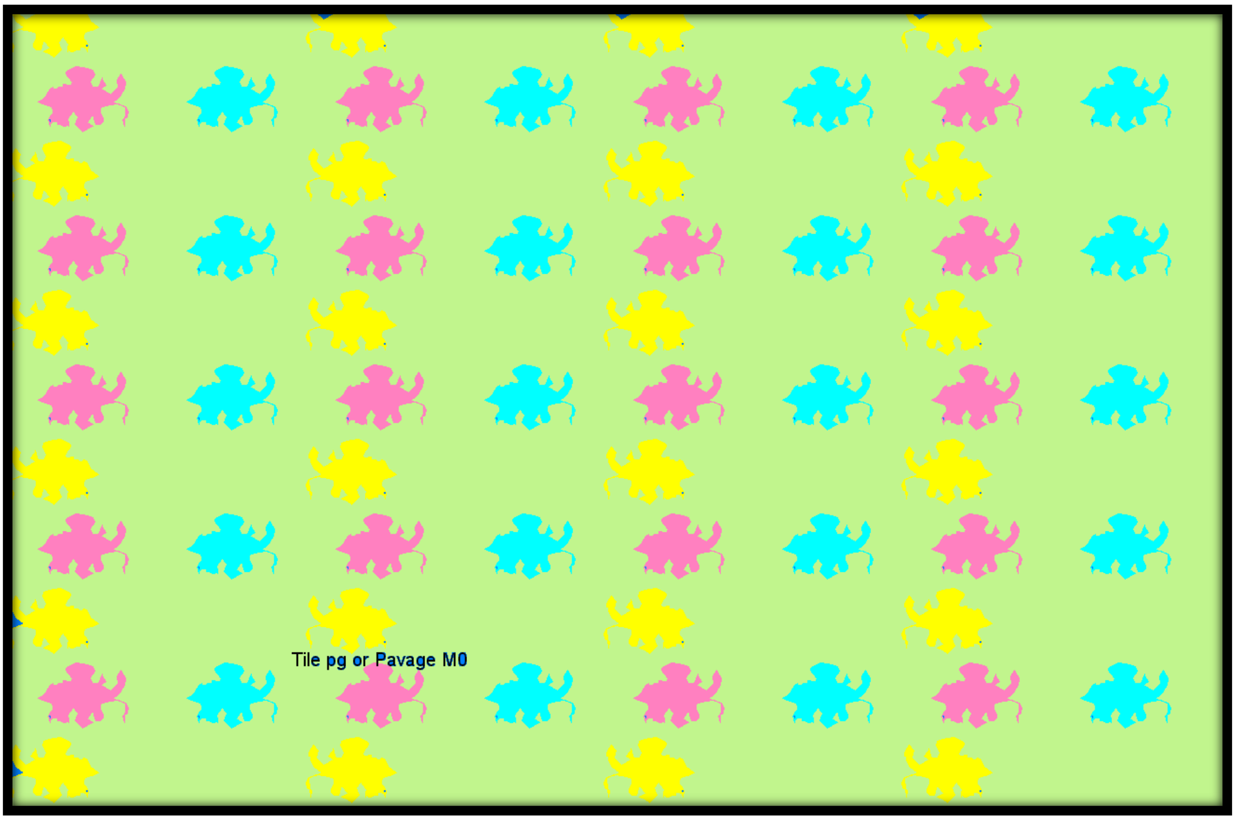
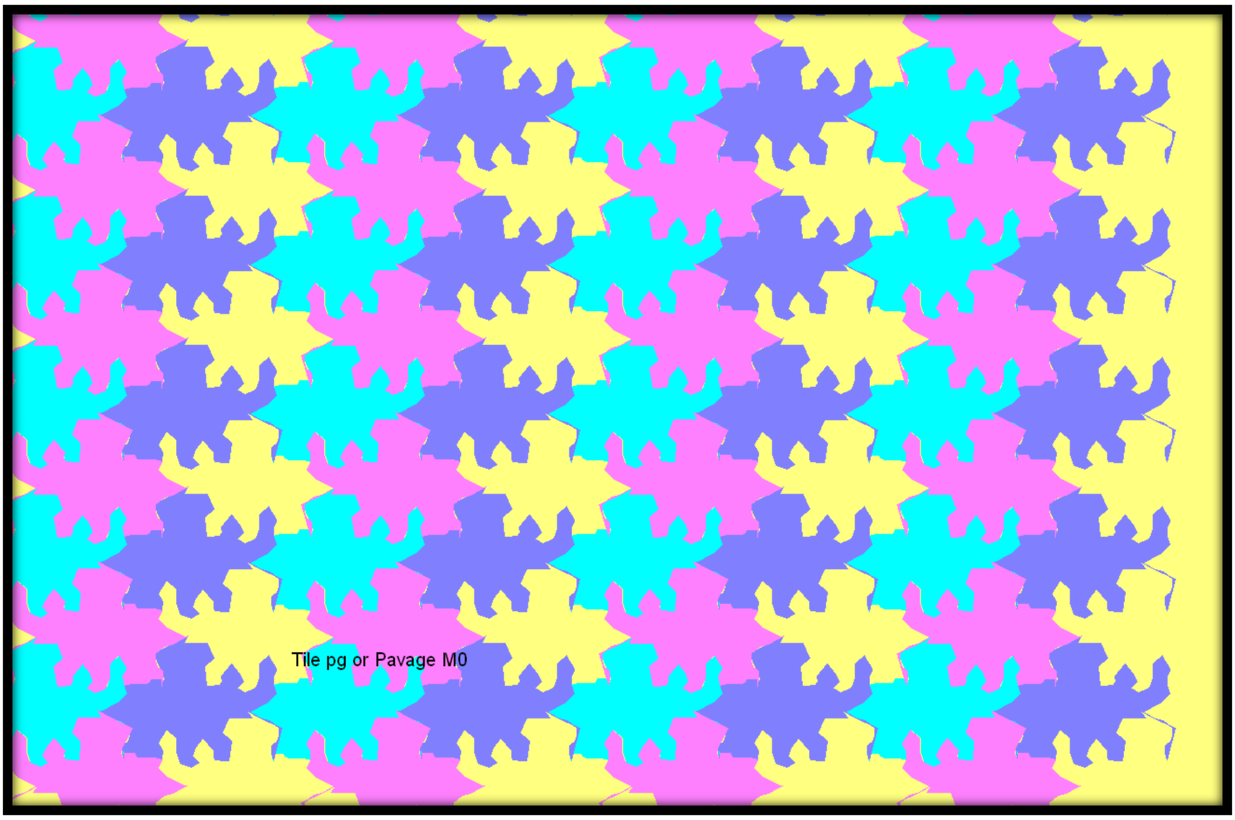
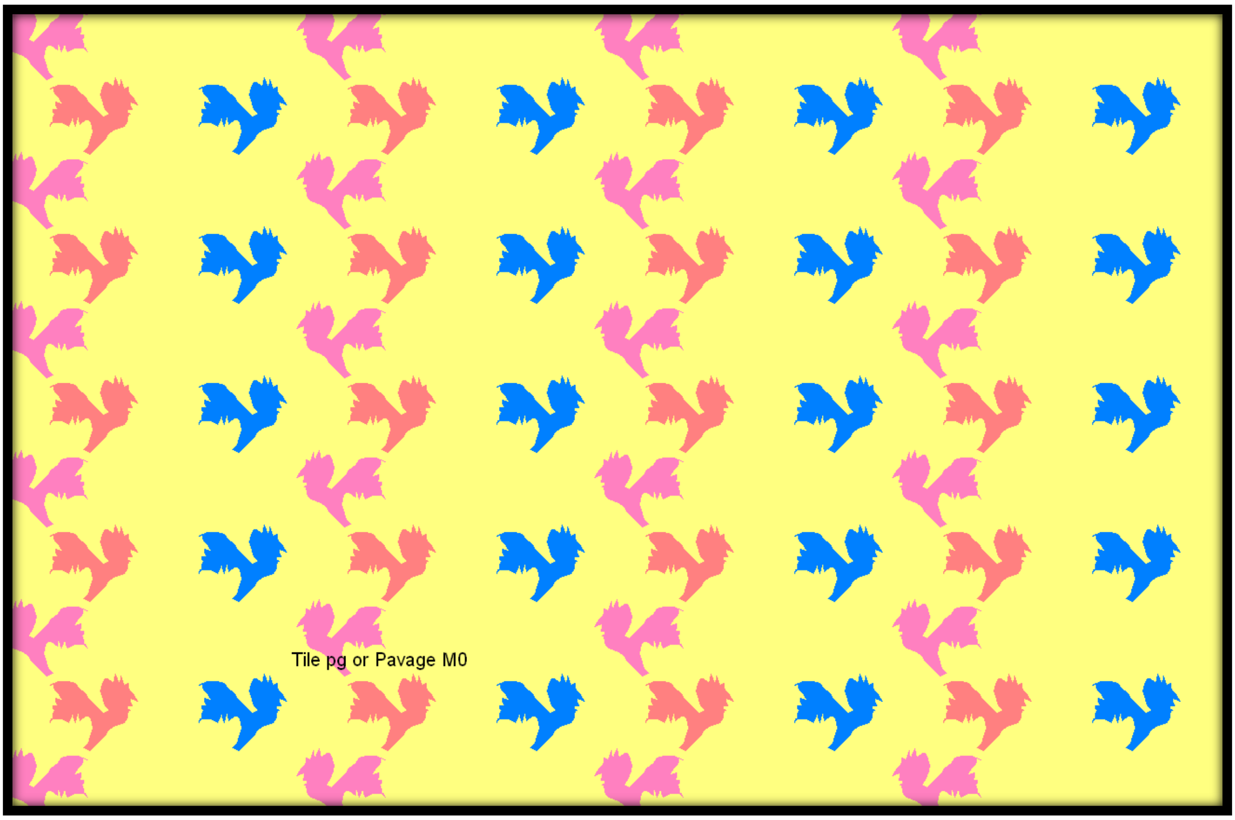
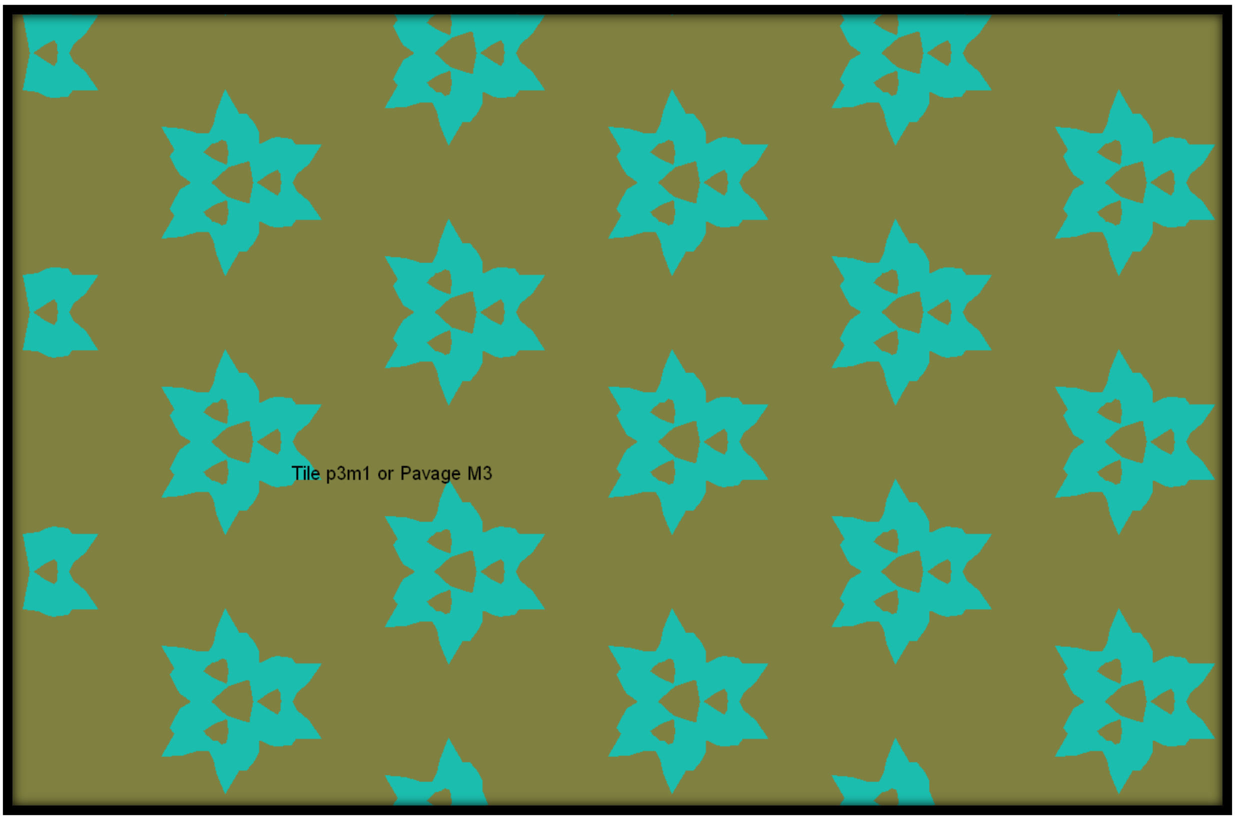


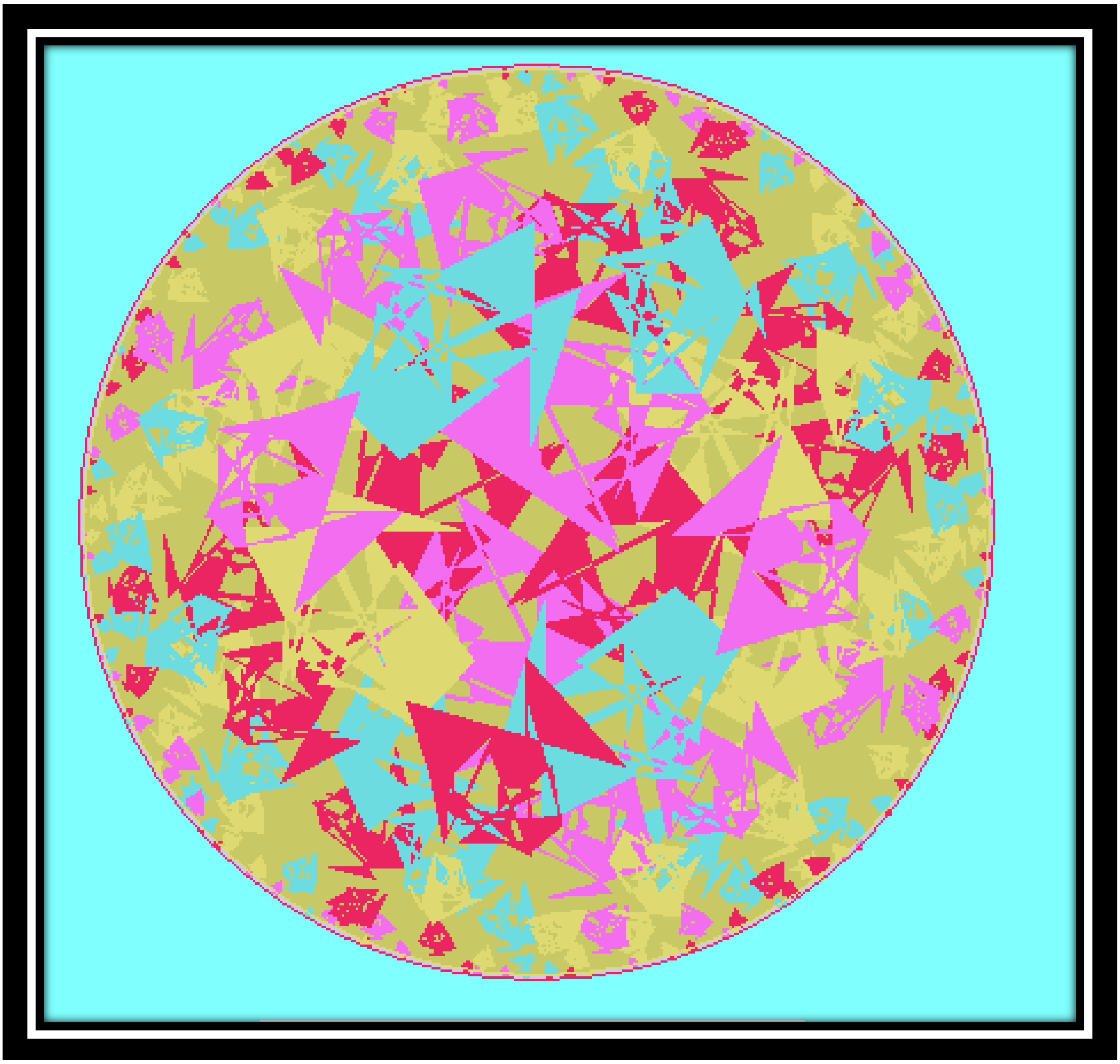


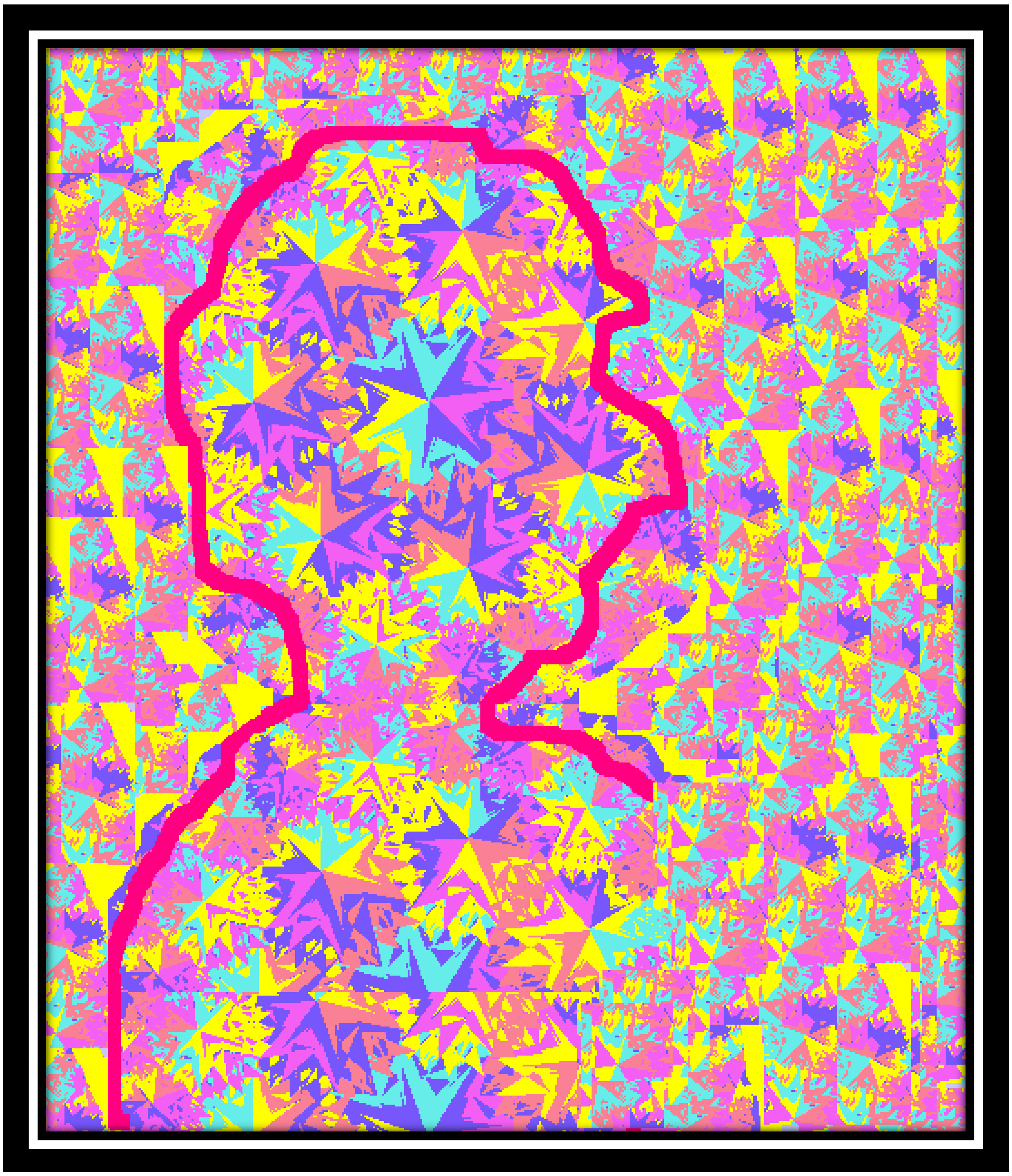

Tile pg or Pavage M0

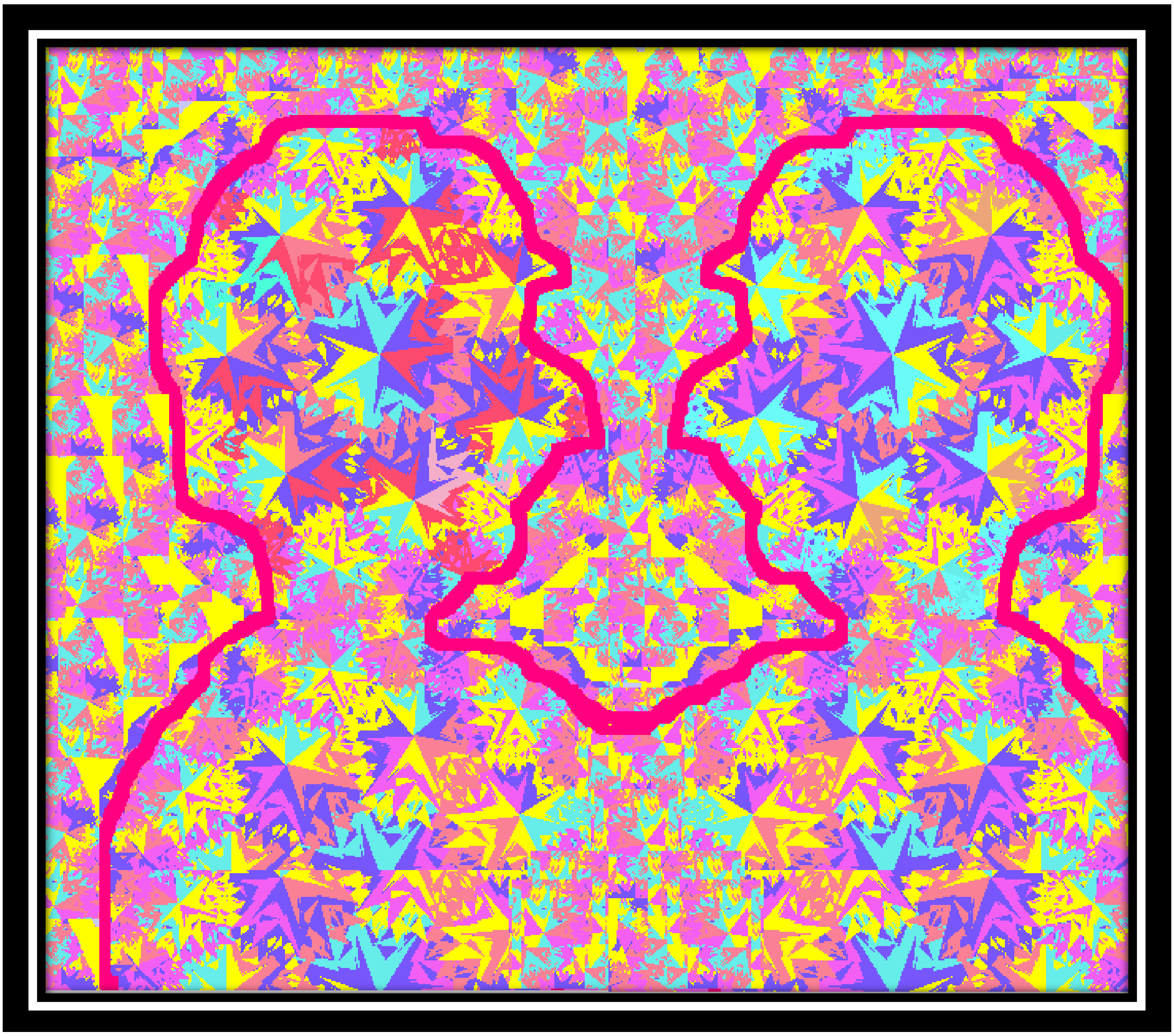

Tile pg or Pavage M0

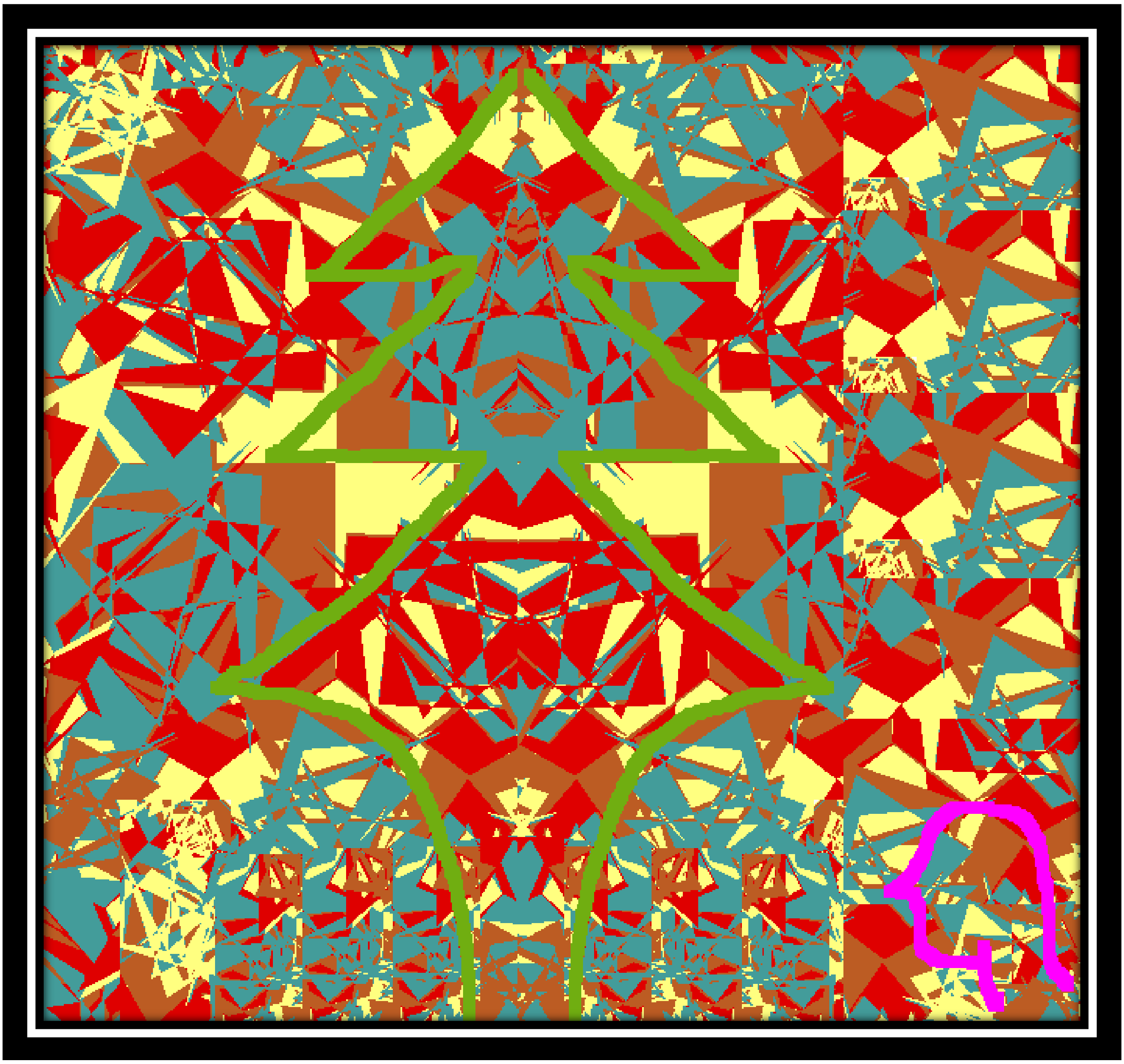

Tile pg or Pavage M0


Tile p3m1 or Pavage M3

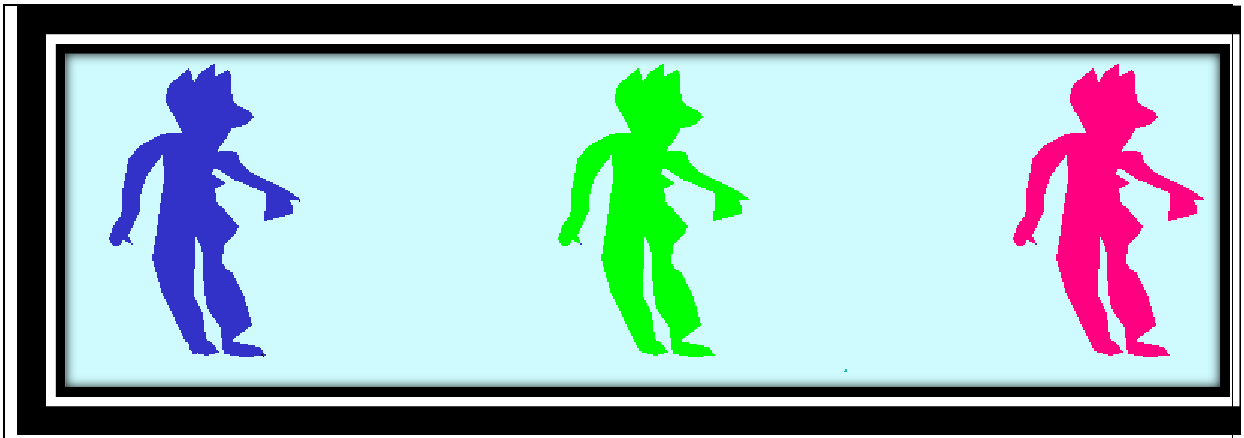




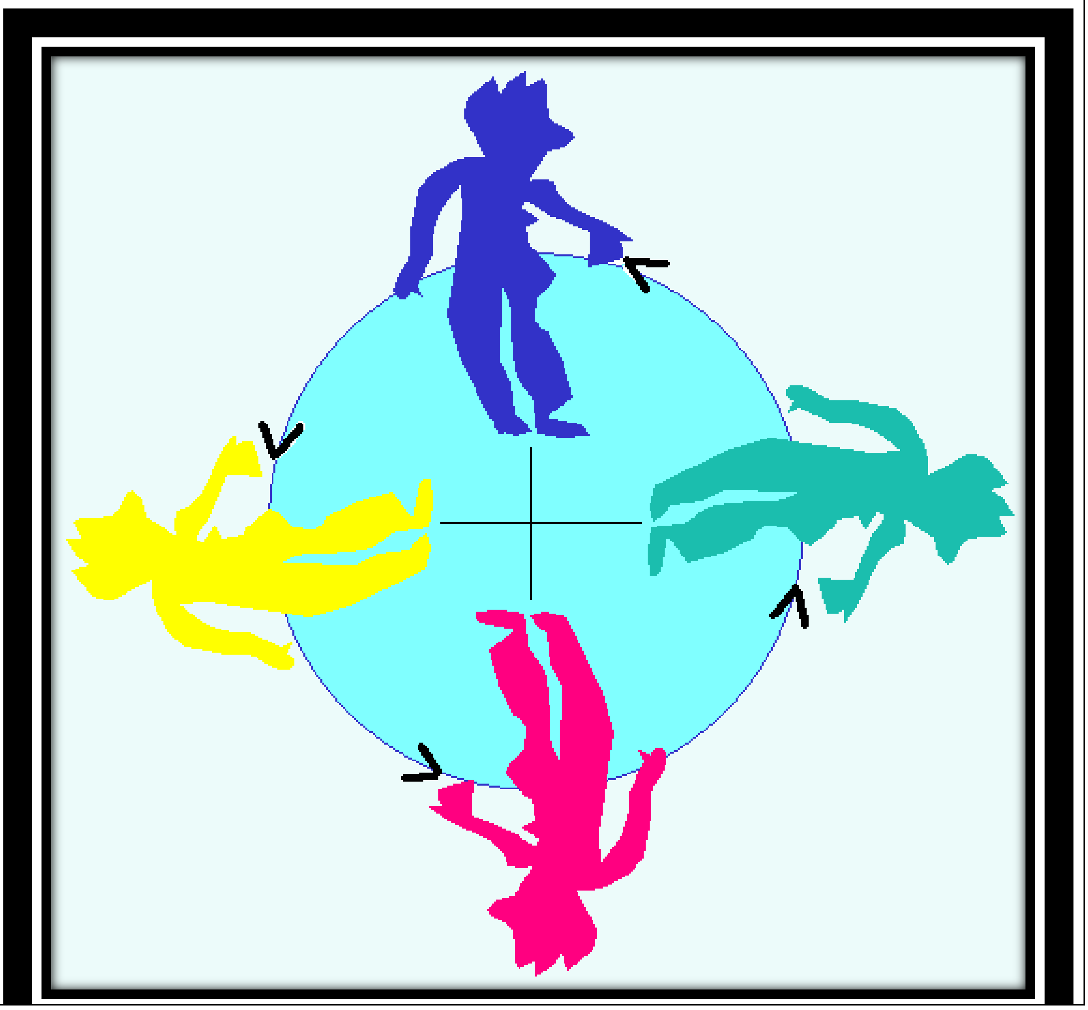




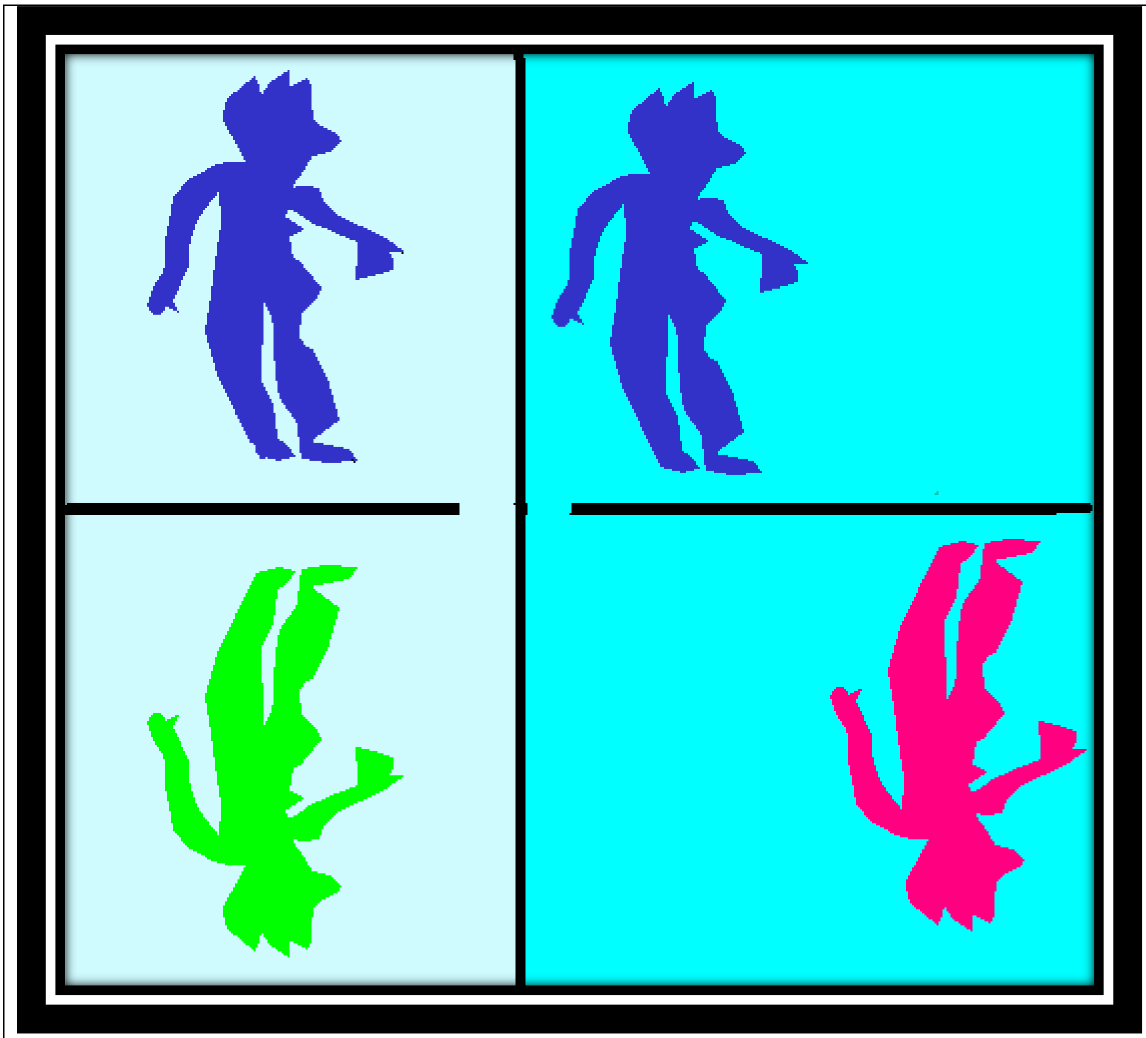
We give now a few on the transformations of the plane in Euclidian Geometry.
Translation, rotations, symmetry, and glide symmetry.



Translation



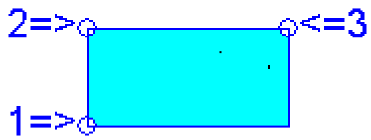
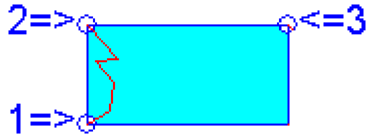
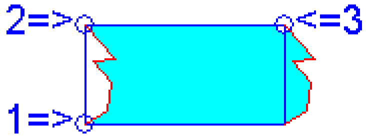
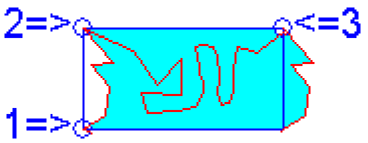
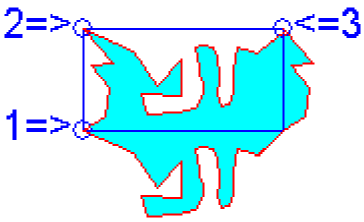
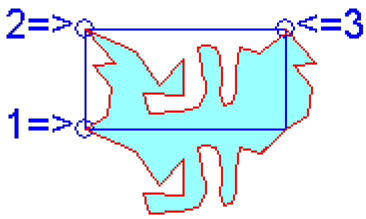
Rotations by 90°, 180°, 270°



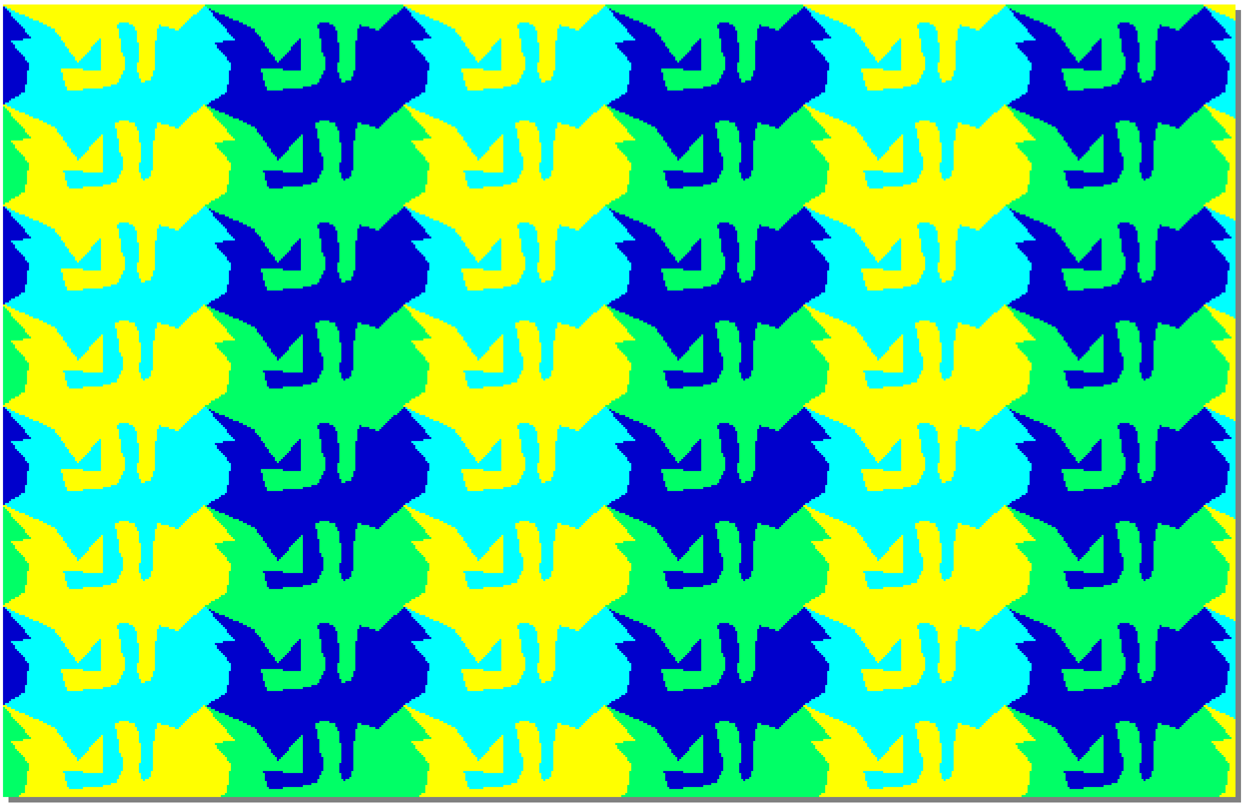
Symmetry and **glide symmetry**

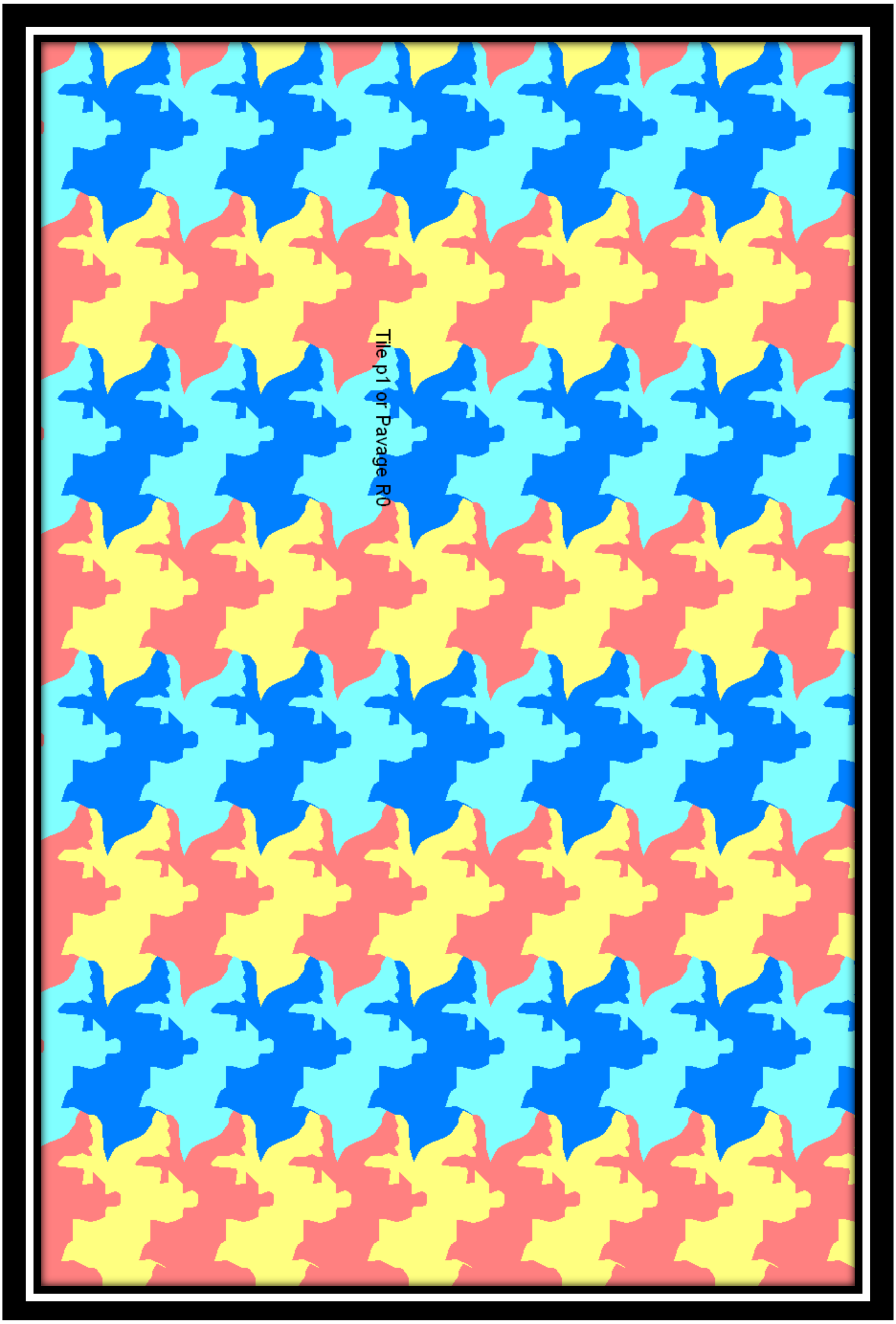
<u>Doing a pattern for tiling the plane by using a sheet of paper and scissors.</u>
We will describe the process for each one of the seventeen groups of tiling. We will use both notations English and French. In each one we have numbered some special points

Tiling p1or R0

 Take a rectangular piece of paper.	 Draw a simple curve starting in 1 and going to 2	 Cut along the curve and glue it on the right of your rectangle
 Draw a simple curve starting in 2 and going to 3	 Cut along the curve and glue it on the down of your rectangle	 This is your pattern for this tiling group.

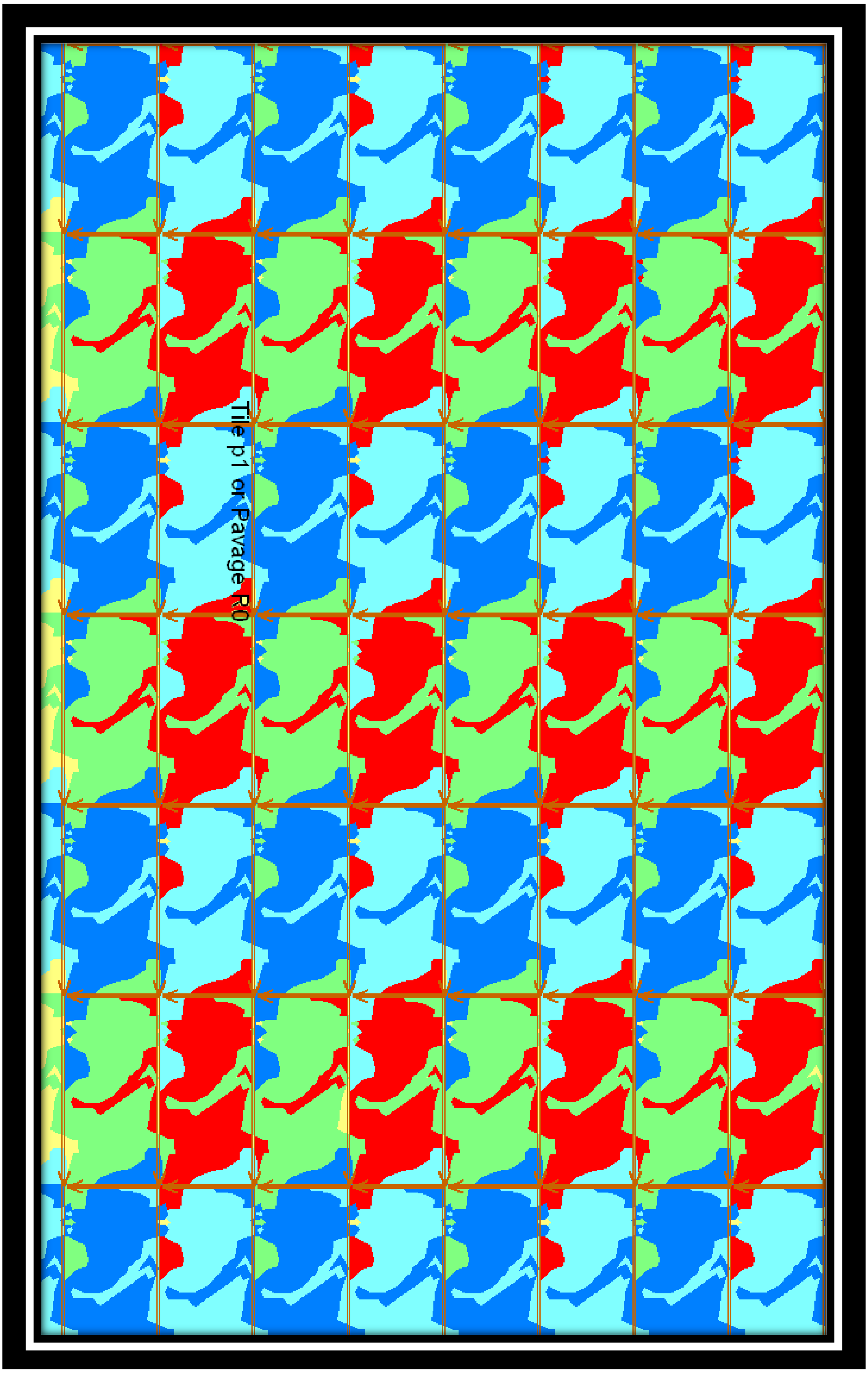
We fulfill the plane by using horizontal and vertical translations; we color it in order to distinguish them:





Lattice and fundamental region for p1 (R0)

The following picture is the lattice associated to the Tiling p1 (R0), any one of the rectangles is a fundamental region and as you can check the arrows are translations.



Tiling p2 or R2

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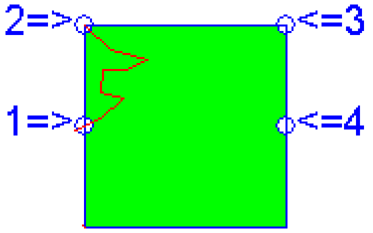
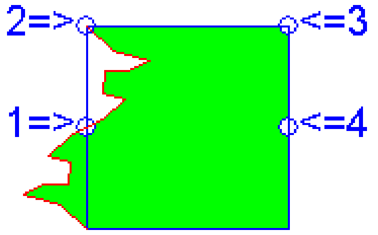
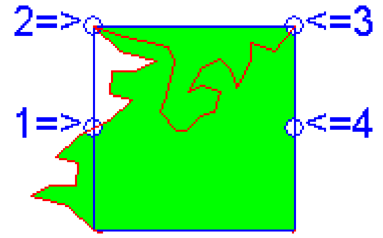
<td>

Take a square piece of paper. Draw a simple curve starting in 1 and going to 2.</td>
<td>

Cut along the curve and glue it after doing a half-tour with center in the point 1.</td>
<td>

Draw a simple curve starting in 2 and going to 3</td>
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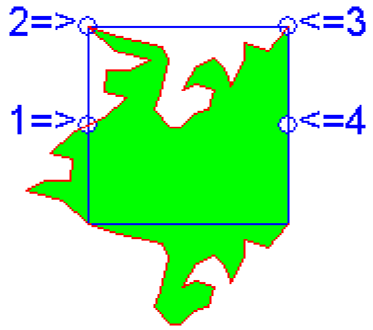
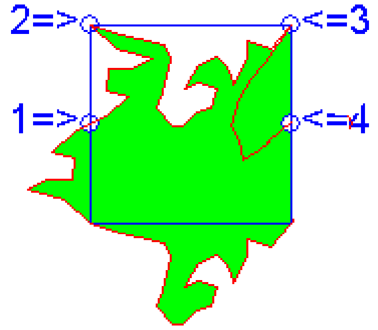
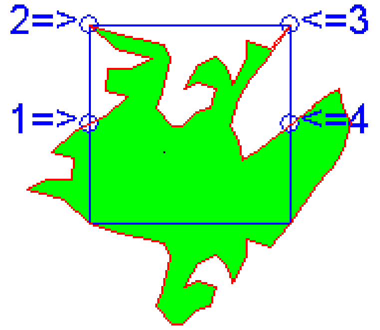
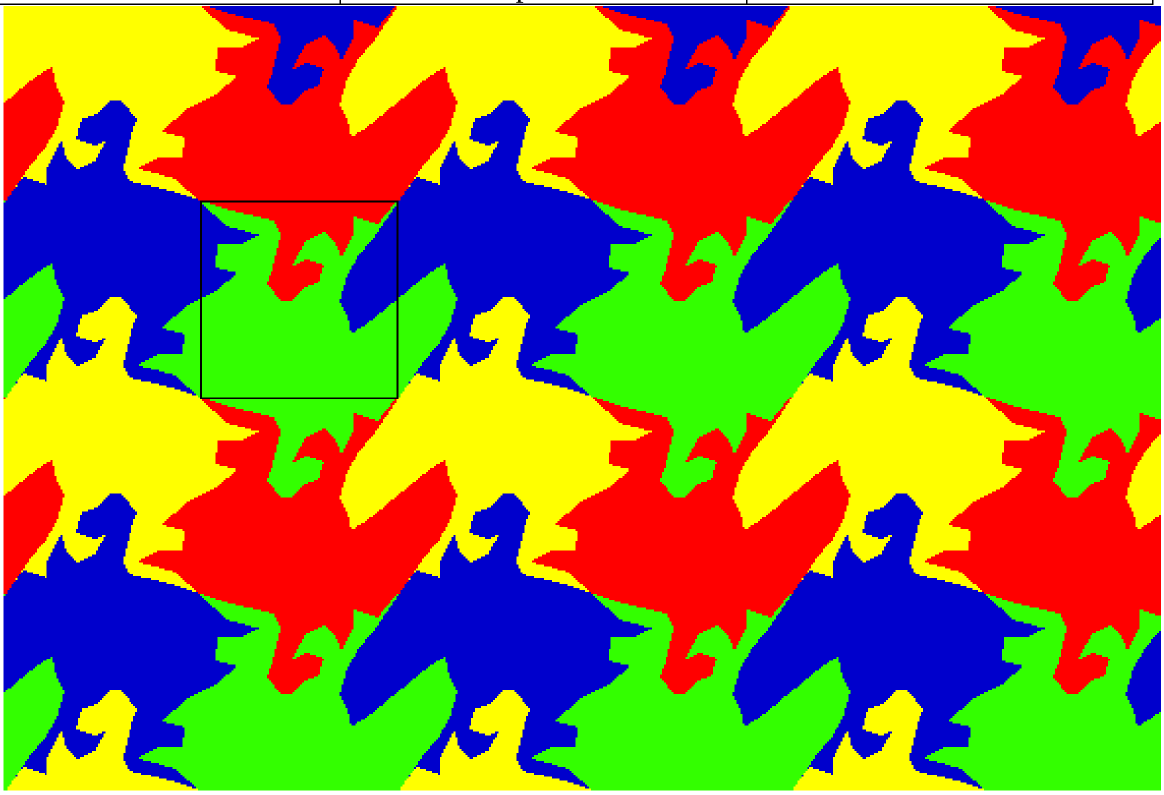
<tr>
<td>

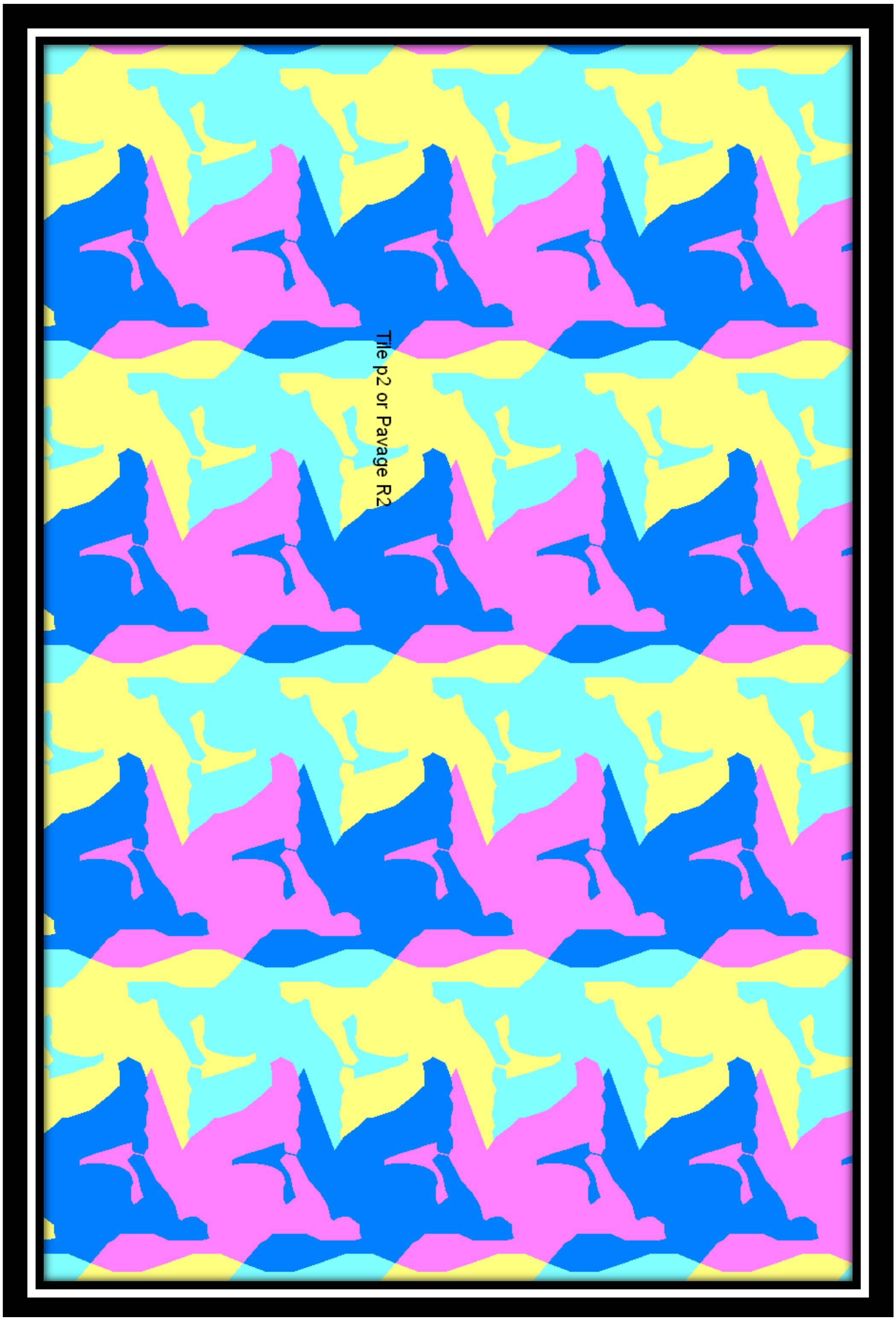
Cut along the curve, translate it vertically and glue.</td>
<td>

Draw a simple curve starting in 3 and going to 4. Cut along the curve and glue it after doing a half-tour with center in the point 4.</td>
<td>

This is your pattern for this tiling group.</td>
</tr>
</table>



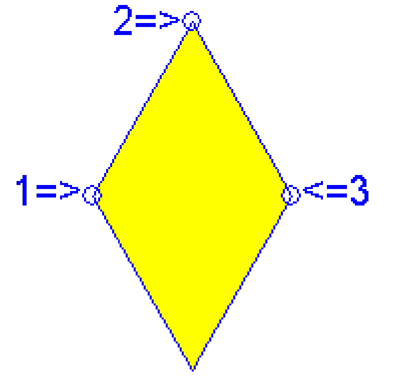
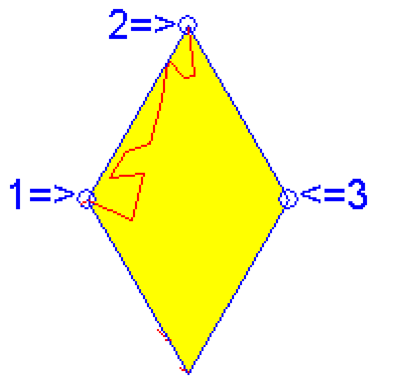
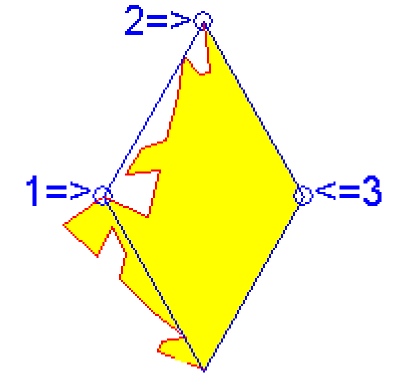
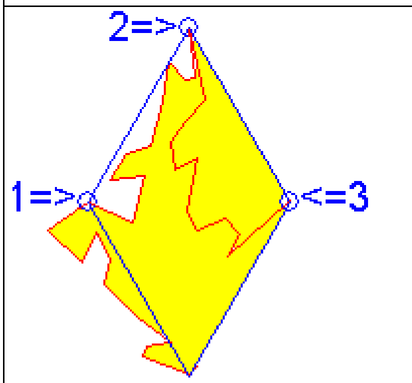
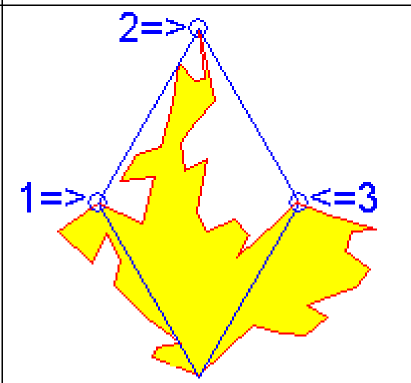
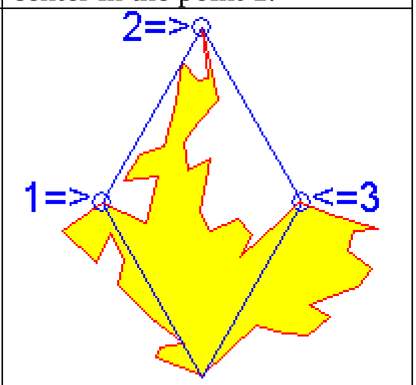
We fulfill the plane by using rotations of angle 180° and translations.



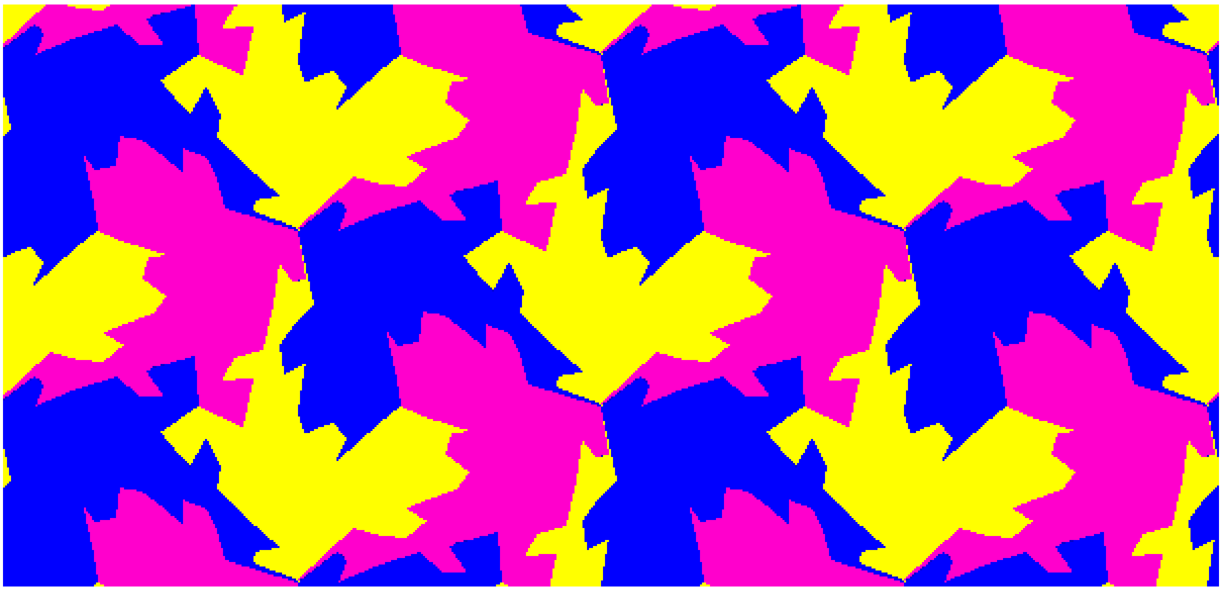
Lattice and fundamental region for p2 (R2)

The following picture is the lattice associated to the Tiling p2 (R2), any one of the squares is a fundamental region and as you can check the arrows are translations, and the circles are centers of rotation π , also called central symmetry.

Tiling p3 ou R3

 Take a rhomb piece of paper. (two equilateral triangles)	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve and glue after a rotation of 120° with center in the point 1.
 Draw a simple curve starting in 2 and going to 3.	 Cut along the curve and glue after a rotation of 120° with center in the point 3.	 This is your pattern for this tiling group.

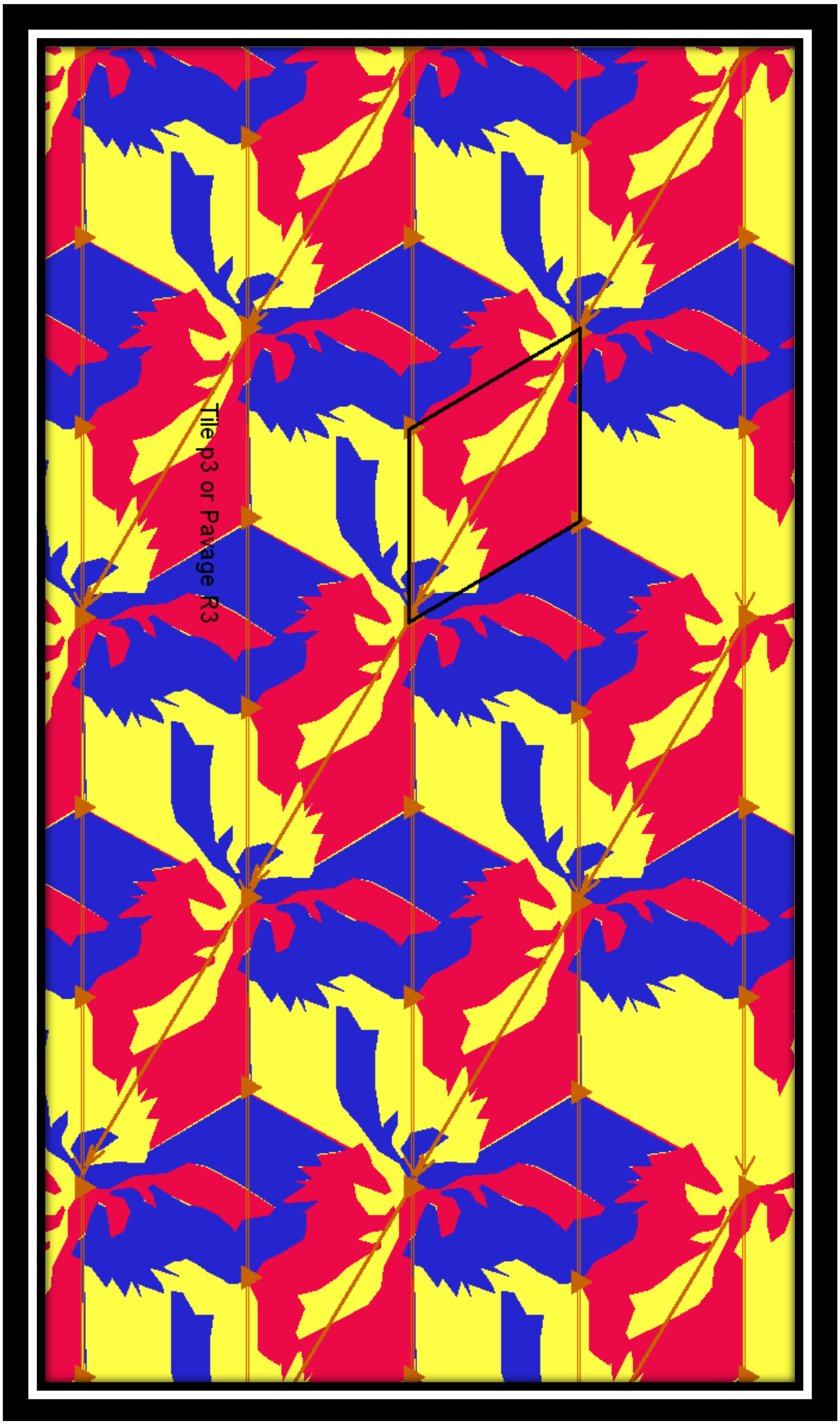
We fulfill the plane by using rotations of angle 120° and translations:



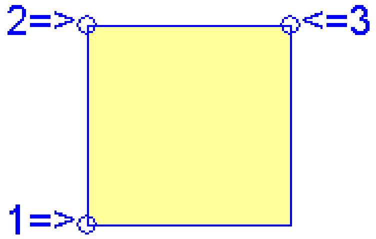
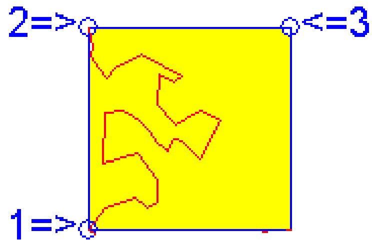
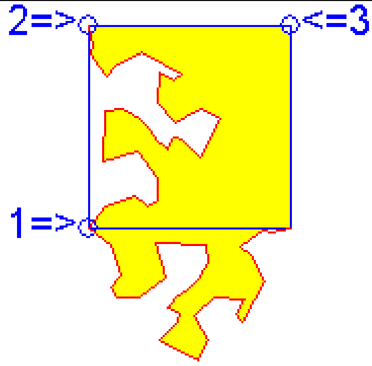
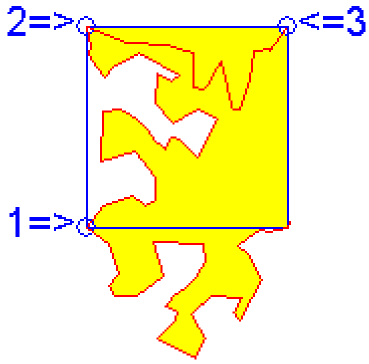
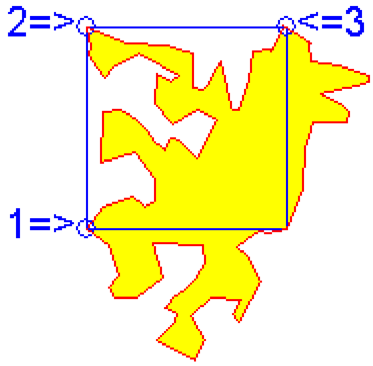
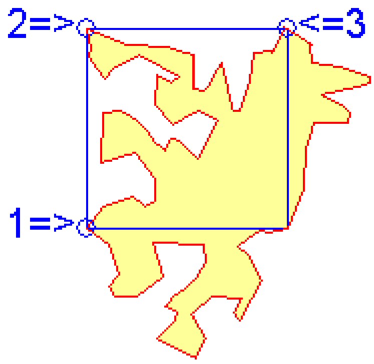

Tile p3 or Pavage R3

Lattice and fundamental region for p3 (R3)

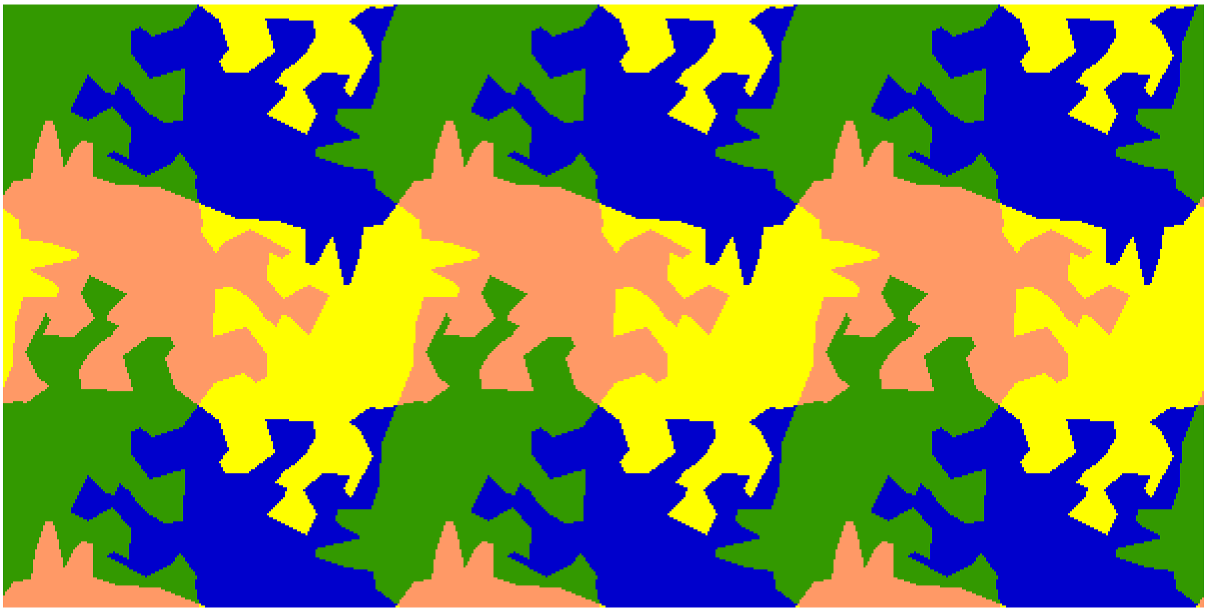
The following picture is the lattice associated to the Tiling p2 (R2), the rhombus drawn is a fundamental region and as you can check the arrows are translations, and the small triangles are centers of rotation $2\pi/3$.

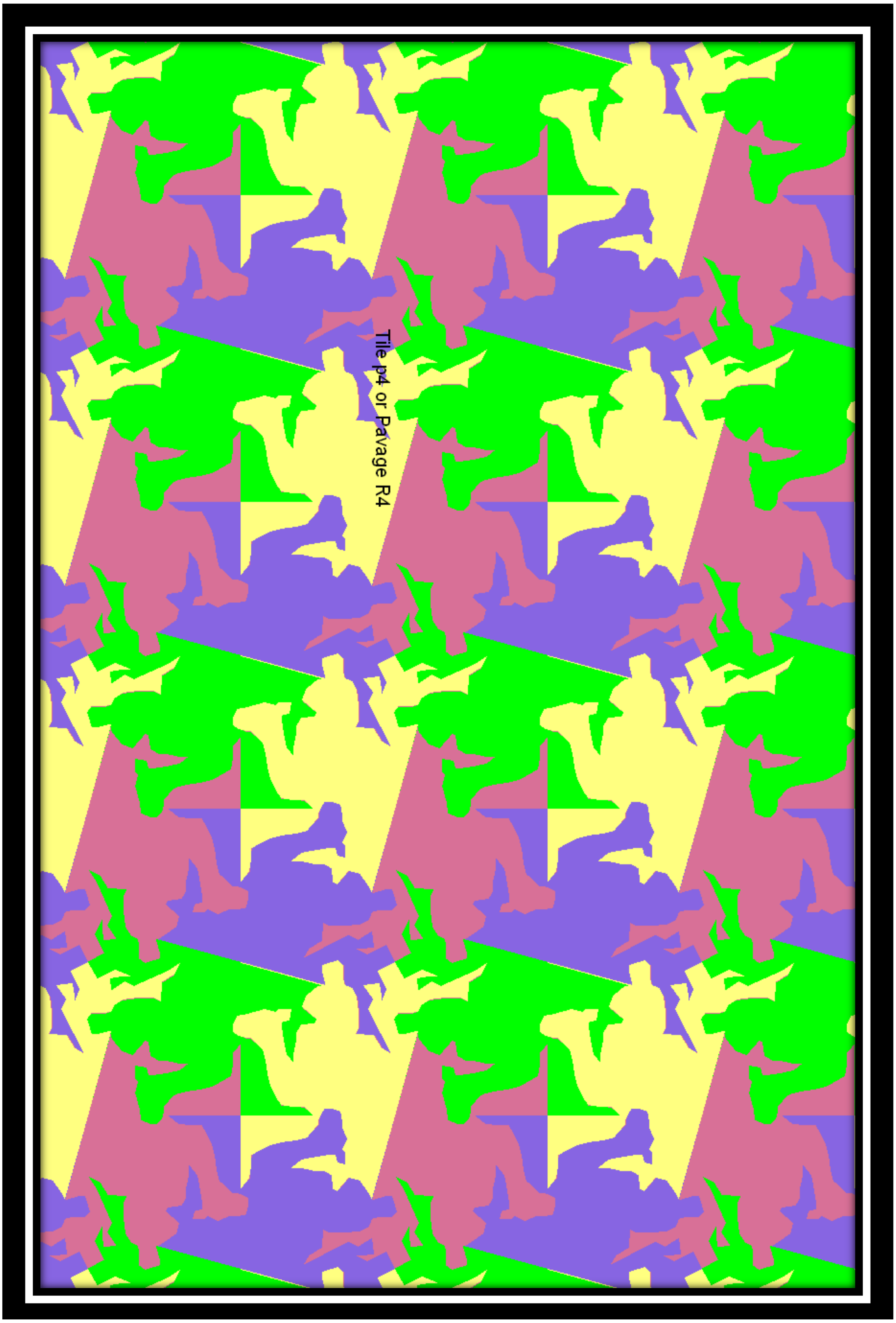


Tiling p4 ou R4

 Take a square piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve and glue after a rotation of 90° with center in the point 1.
 Draw a simple curve starting in 2 and going to 3.	 Cut along the curve and glue after a rotation of 120° with center in the point 3.	 This is your pattern for this tiling group.

We fulfill the plane by using rotations of angles 90°, 180°, 270° and translations:




Tile p4 or Pavage R4

Lattice and fundamental region for p4 (R4)

The following picture is the lattice associated to the Tiling p4 (R4), any square drawn is a fundamental region and as you can check the arrows are translations, the circles are centers of rotation π and the small squares are centers of rotation $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

Tiling p6 ou R6

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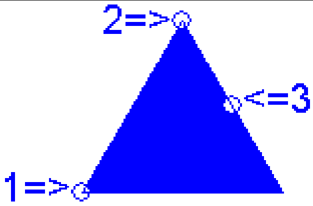
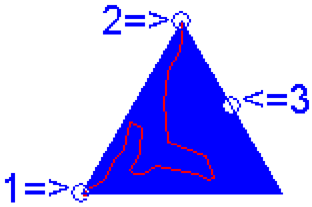
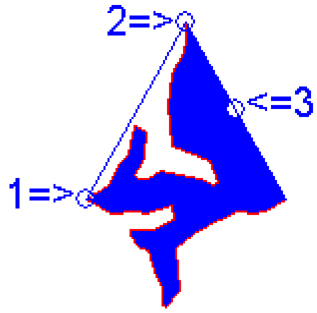
<td>

Take an equilateral triangle piece of paper.</td>
<td>

Draw a simple curve starting in 1 and going to 2.</td>
<td>

Cut along the curve and glue after a rotation of 60° with center in the point 1.</td>
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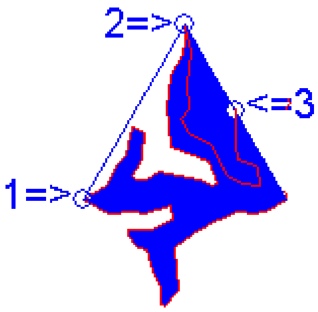
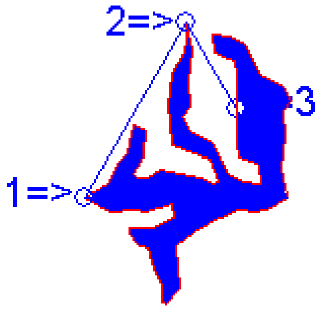
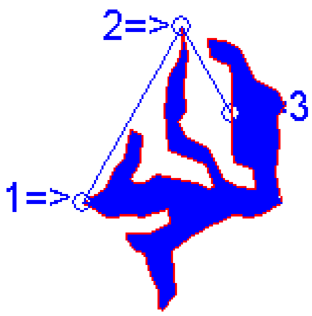
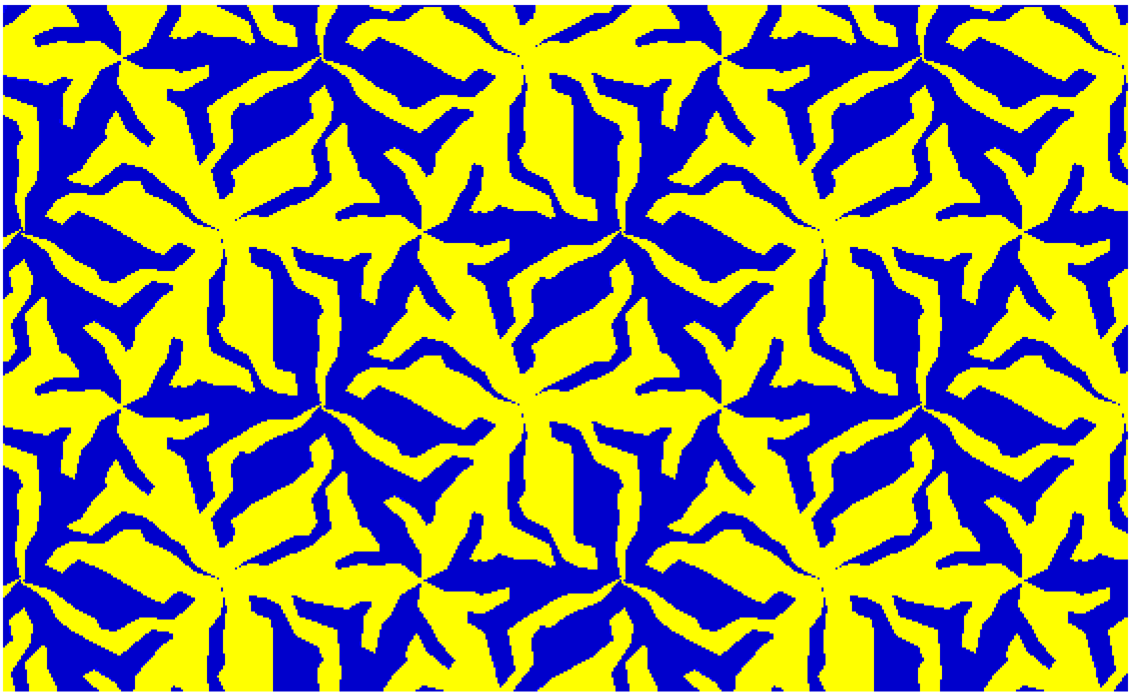
<td>

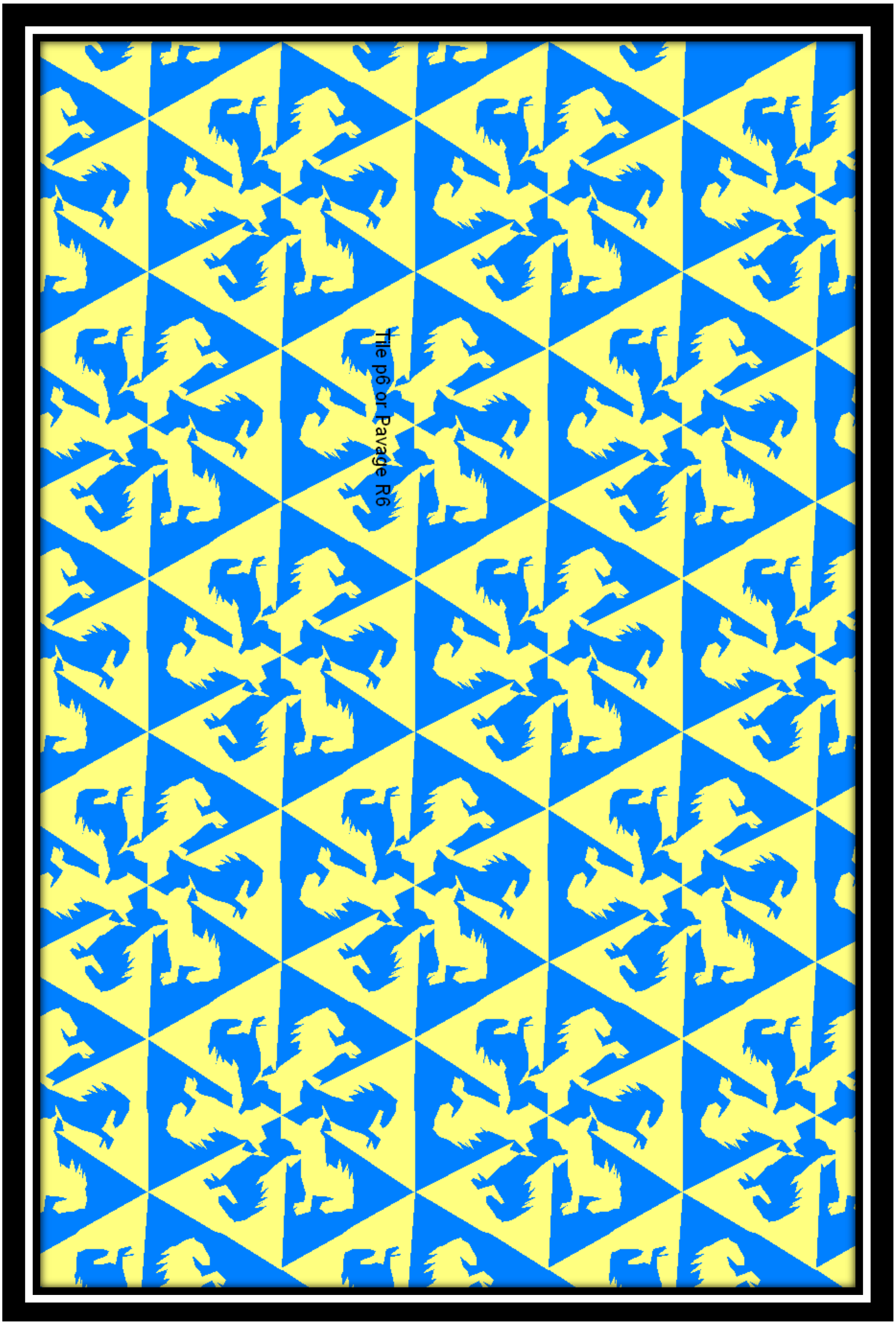
Draw a simple curve starting in 2 and going to 3.</td>
<td>

Cut along the curve and glue after a rotation of 180° with center in the point 3.</td>
<td>

This is your pattern for this tiling group.</td>
</tr>
</table>

We fulfill the plane by using rotations of angle 60°, 120°, 180° and translations:

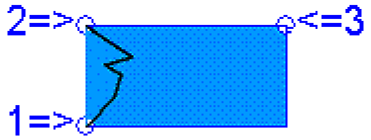
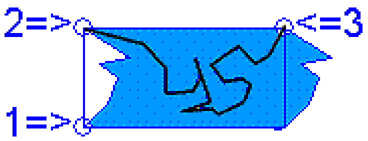
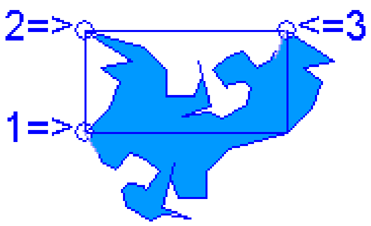
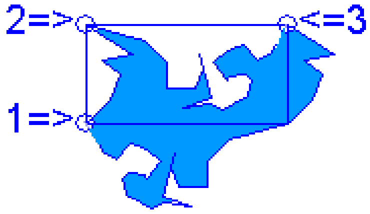



Tile p6 or Pavage R6

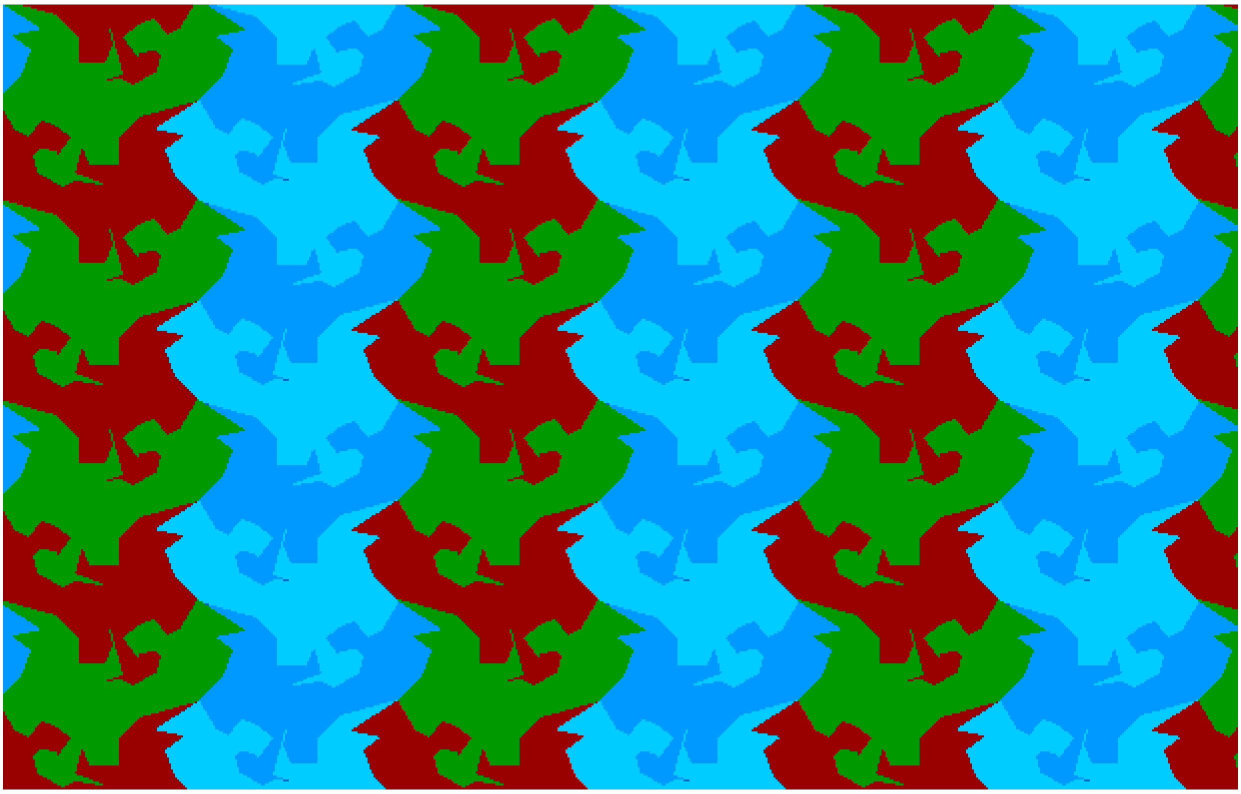
Lattice and fundamental region for p6 (R6)

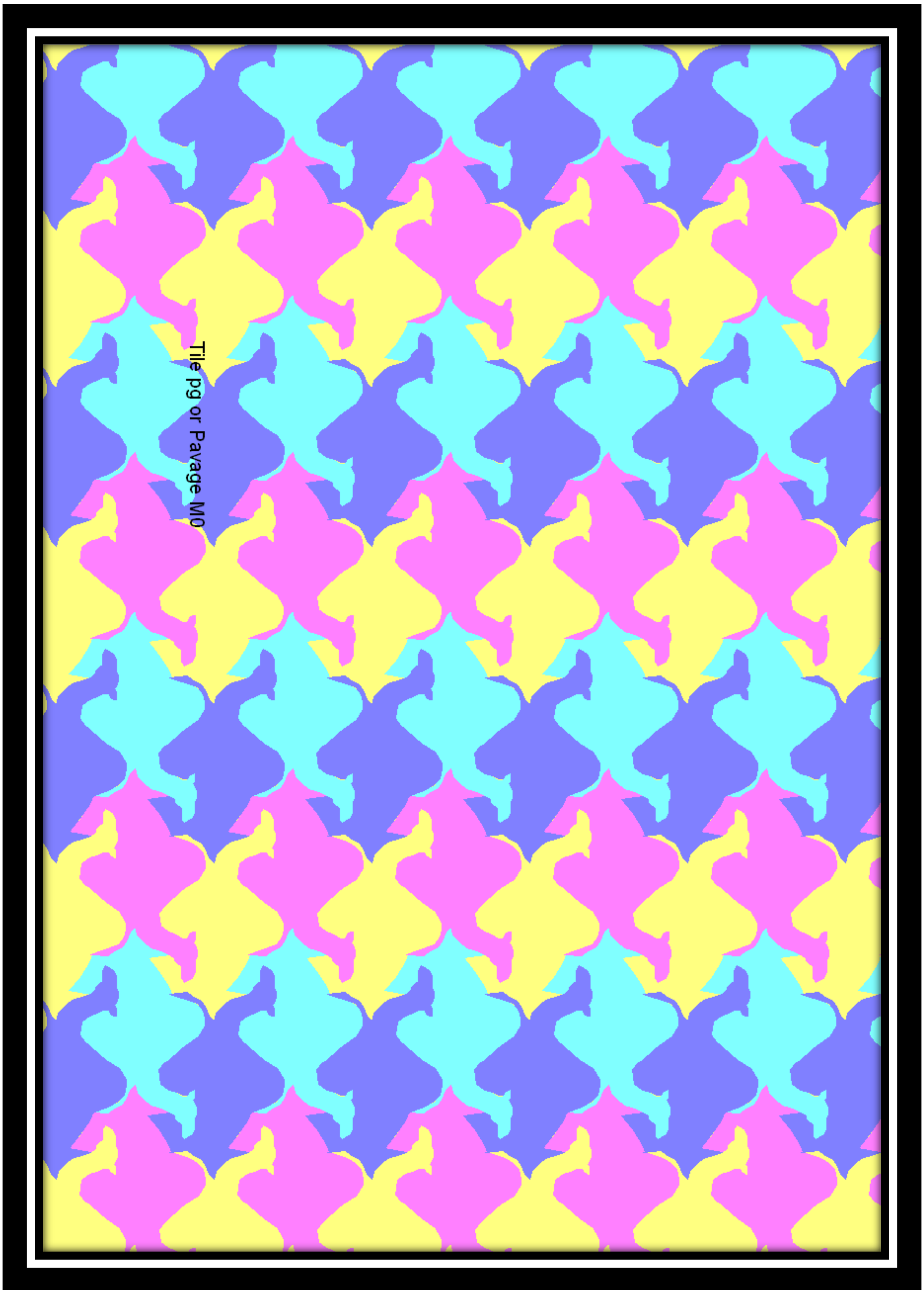
The following picture is the lattice associated to the Tiling p6 (R6), the triangle drawn is a fundamental region and as you can check the arrows are translations, the circles are centers of rotation π, the small hexagons are centers of rotations $\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$. And the small triangles are centers of rotation $\frac{2\pi}{3}, \frac{4\pi}{3}$.

Tiling pg ou M0

 Take a rectangular piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve and glue it on the right side.
 Draw a simple curve starting in 2 and going to 3.	 Cut along the curve, turn on and glue it on the top down side.	 This is your pattern for this tiling group.

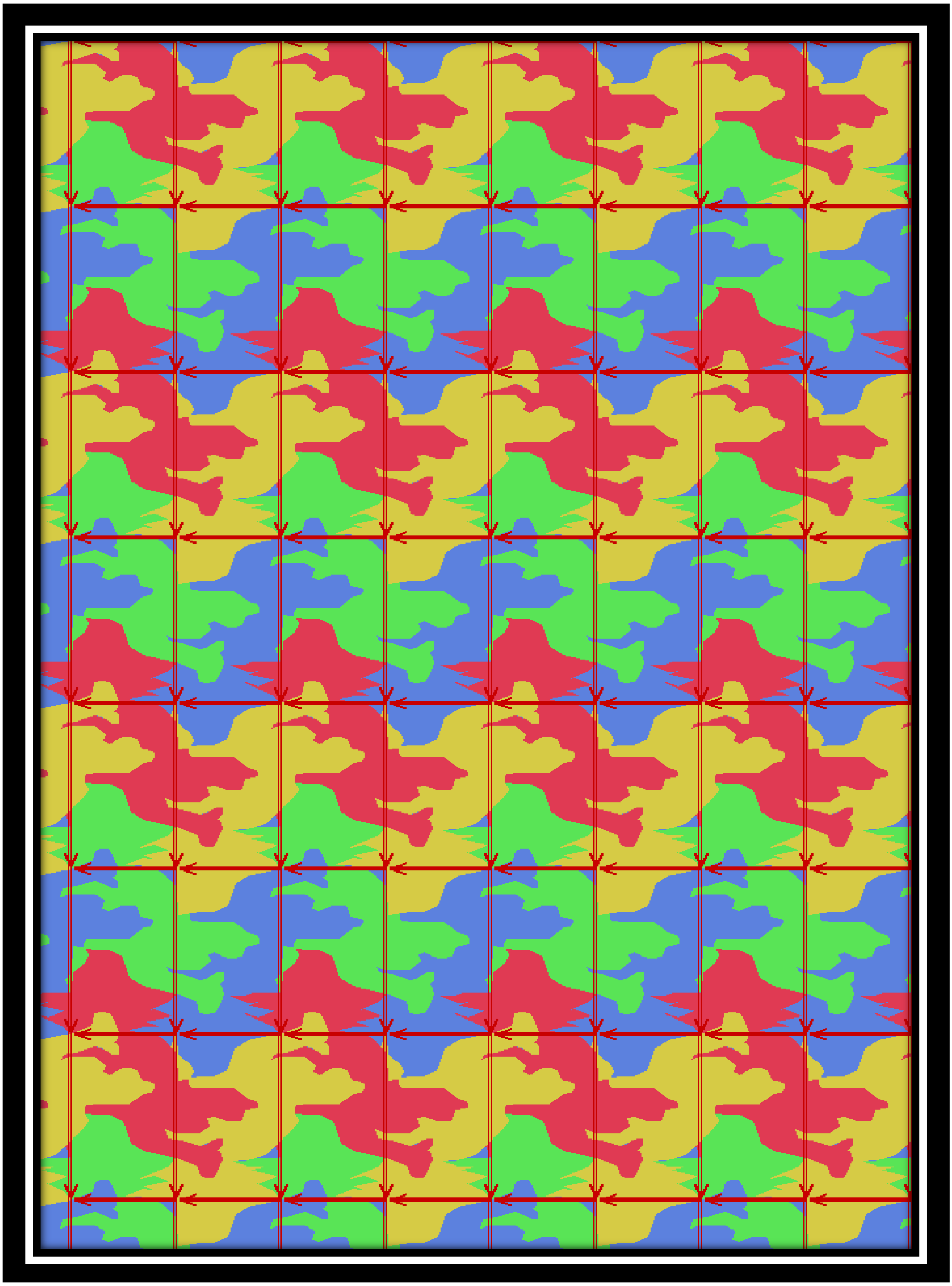
We fulfill the plane by using glides symmetries and translations:



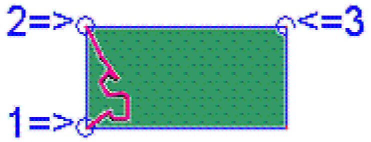
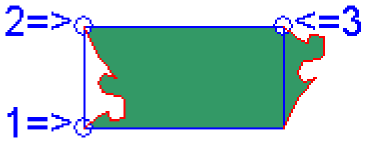
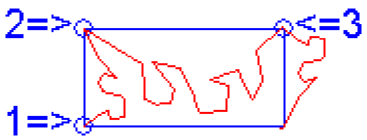
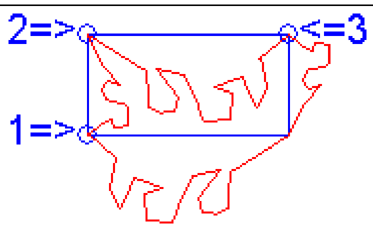
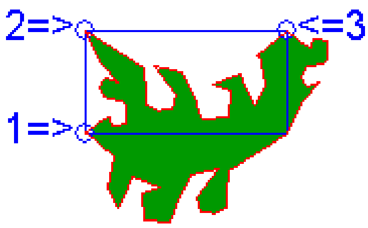


Lattice and fundamental region for pg (M0)

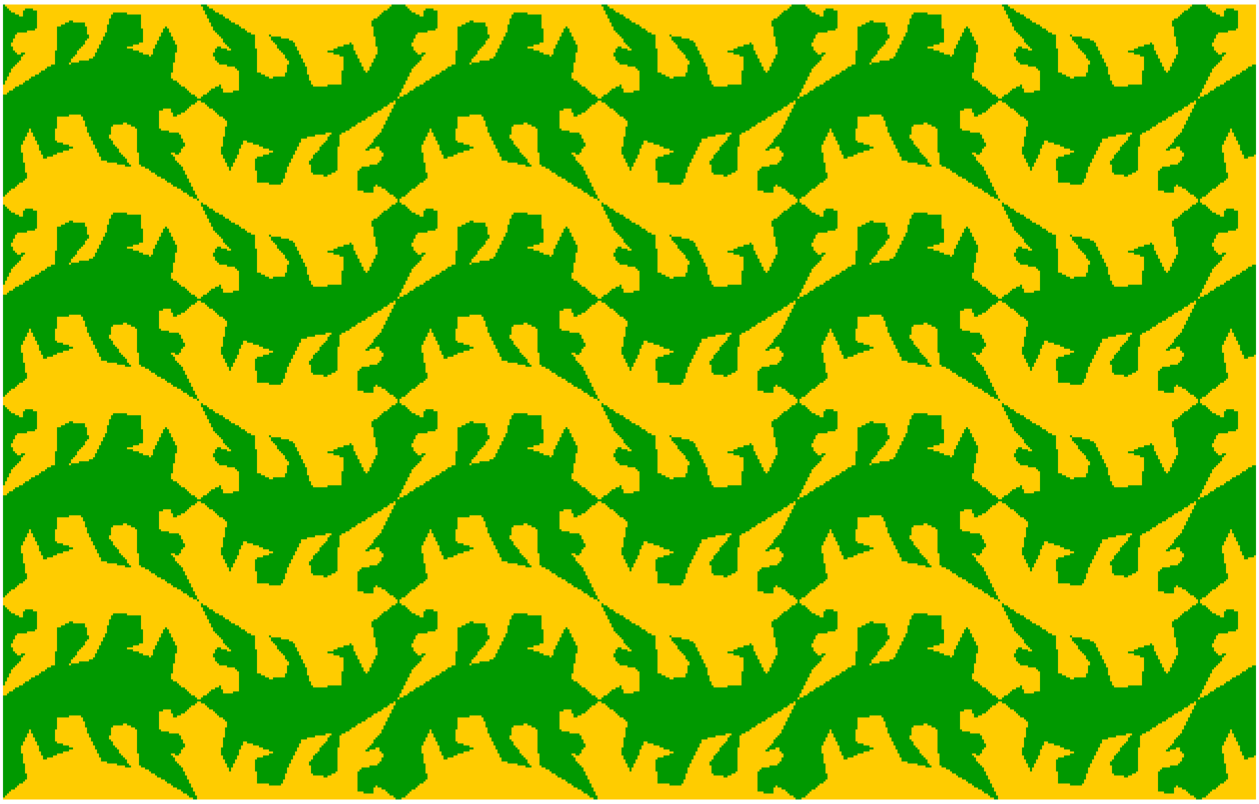
The following picture is the lattice associated to the Tiling pg (M0), any rectangle is a fundamental region and as you can check the arrows are translations. The lattice is the same as the tiling p0 (R0).

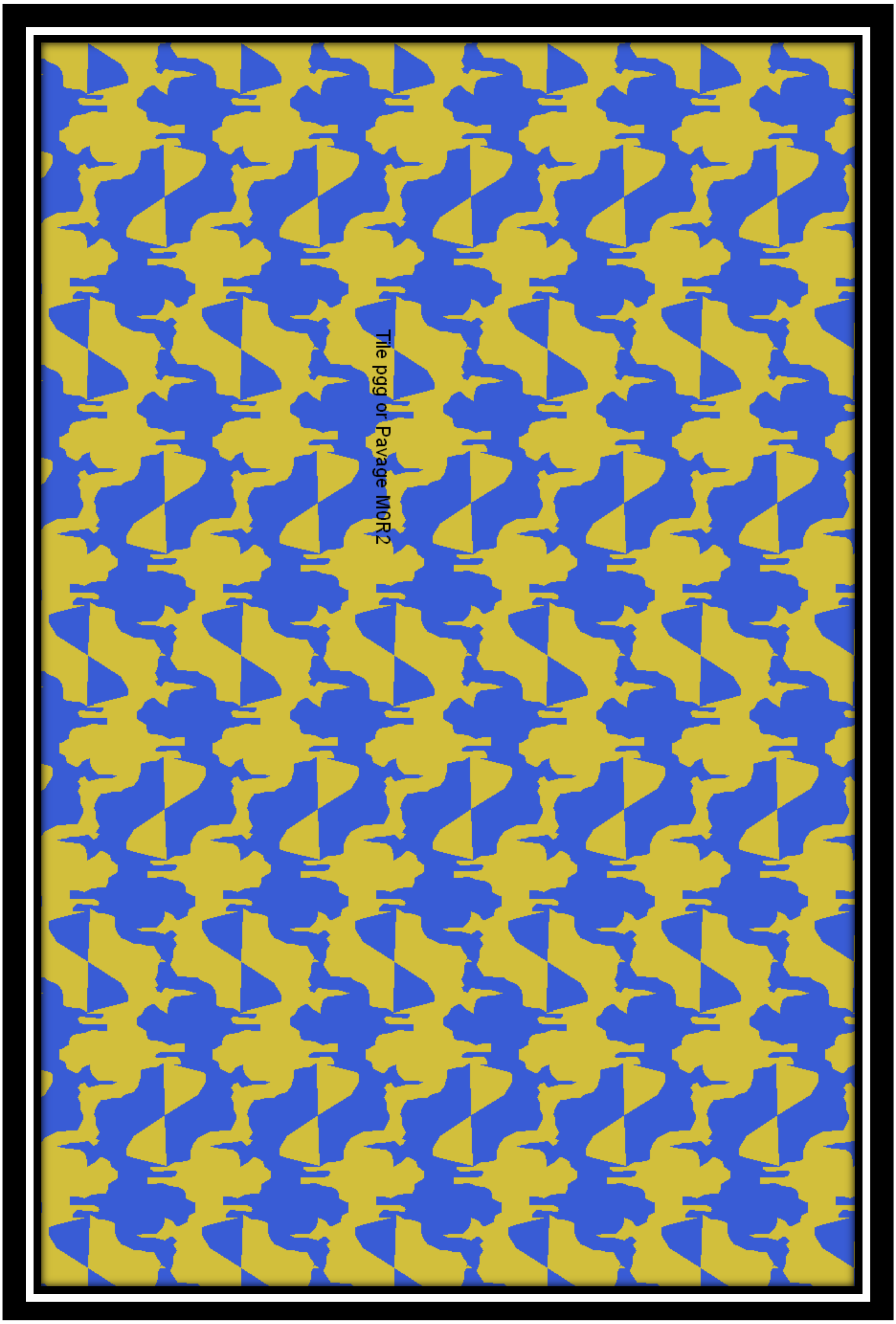


Tiling pgg ou M0R2

 Take a rectangular piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve, turn on and glue it on the right side.
 Draw a simple curve starting in 2 and going to 3.	 Cut along the curve, turn on and glue it on the top down side.	 This is your pattern for this tiling group.

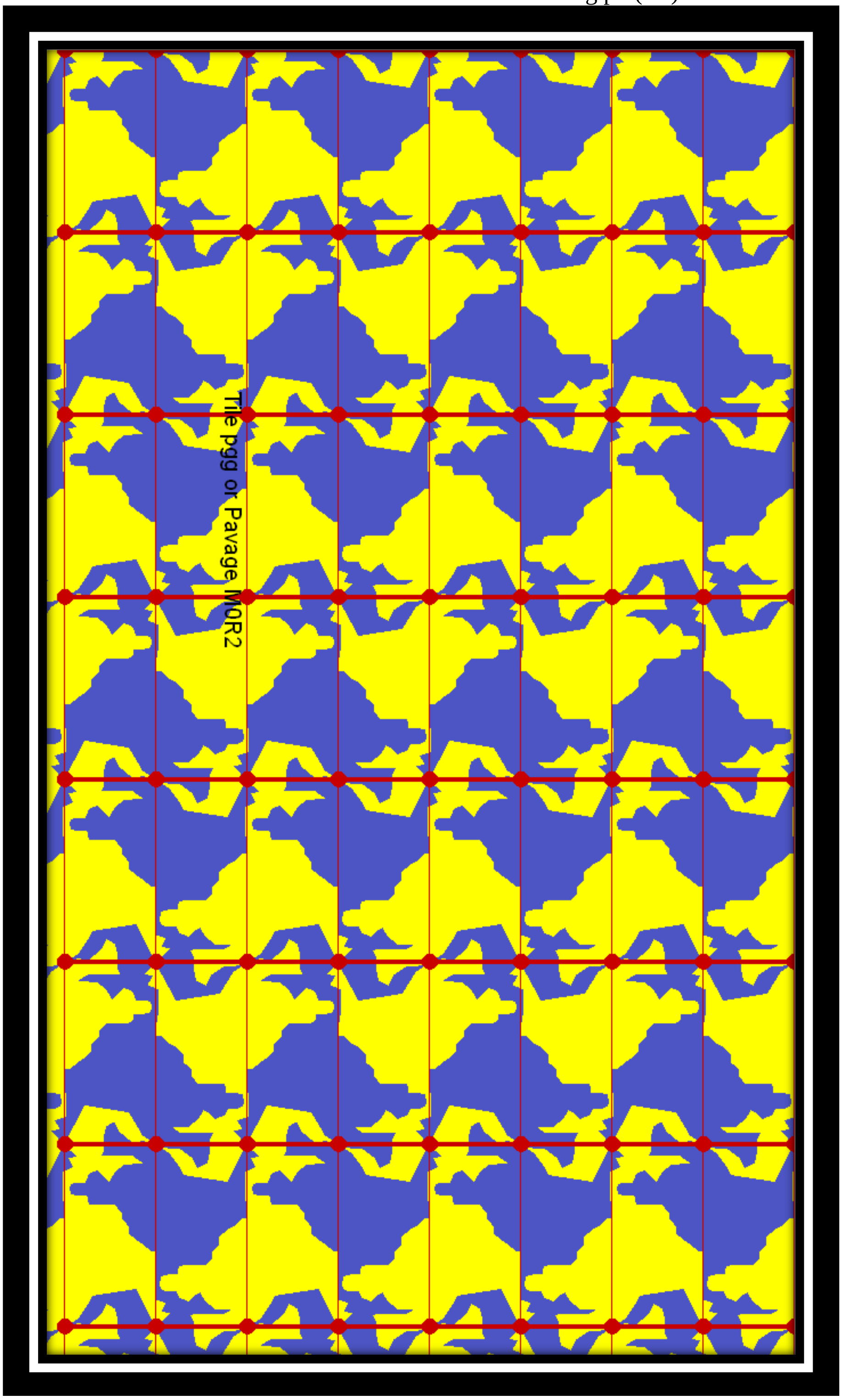
We fulfill the plane by using glides symmetries, rotations of angle 180° and translations:



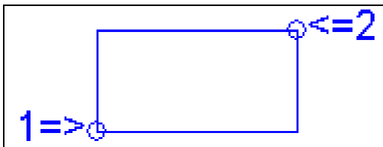
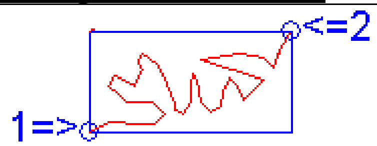
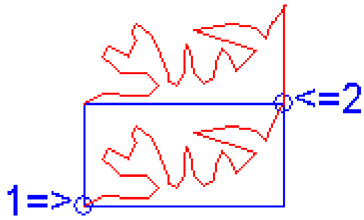
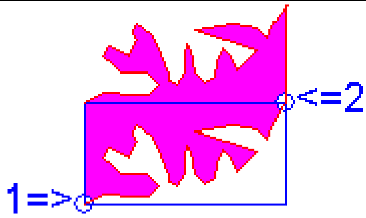

Tile pgg or Pavage M0R2

Lattice and fundamental region for pgg (M0R2)

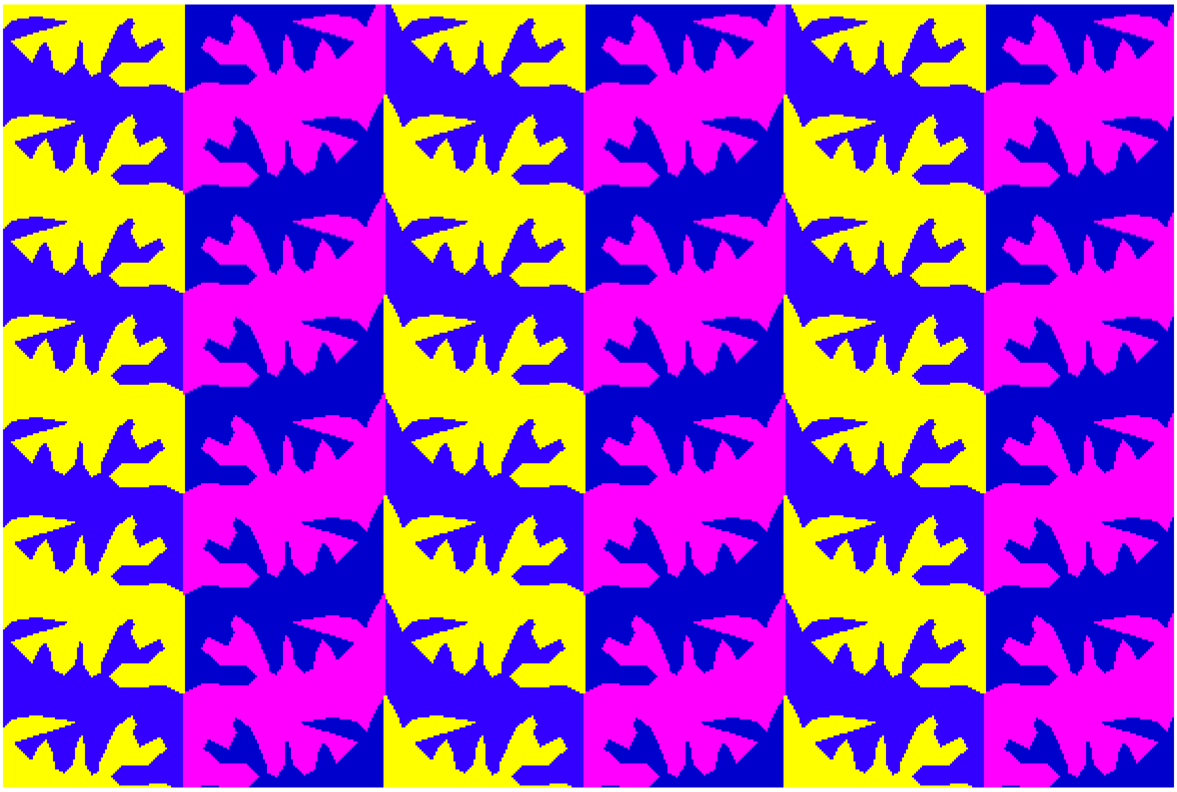
The following picture is the lattice associated to the Tiling pgg (M0R2), any rectangle is a fundamental region and as you can check the arrows are translations. The lattice is the same as the tiling p2 (R2).

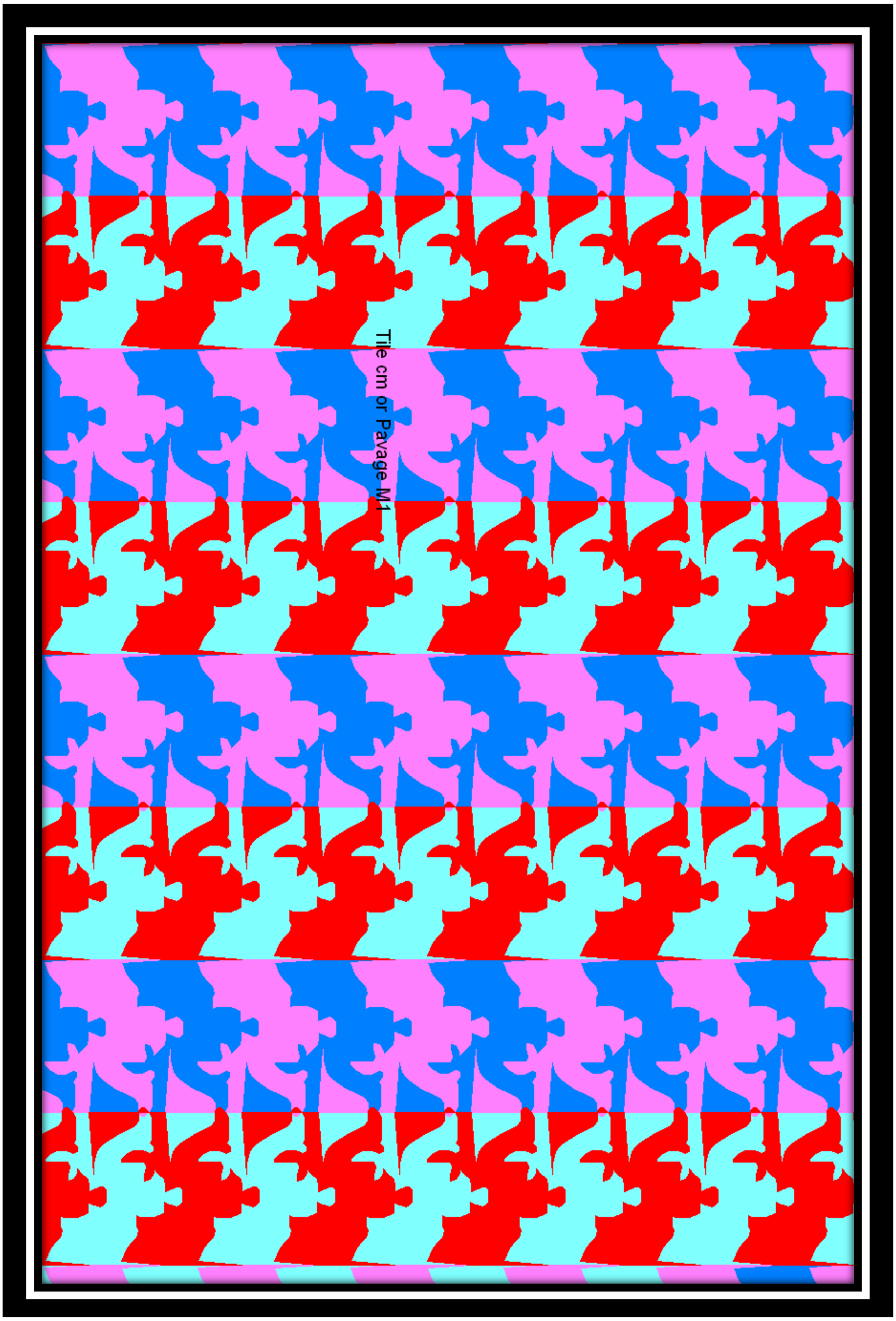


Tiling cm ou M1

 Take a rectangular piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve and glue it on the top up side.
	 This is your pattern for this tiling group.	

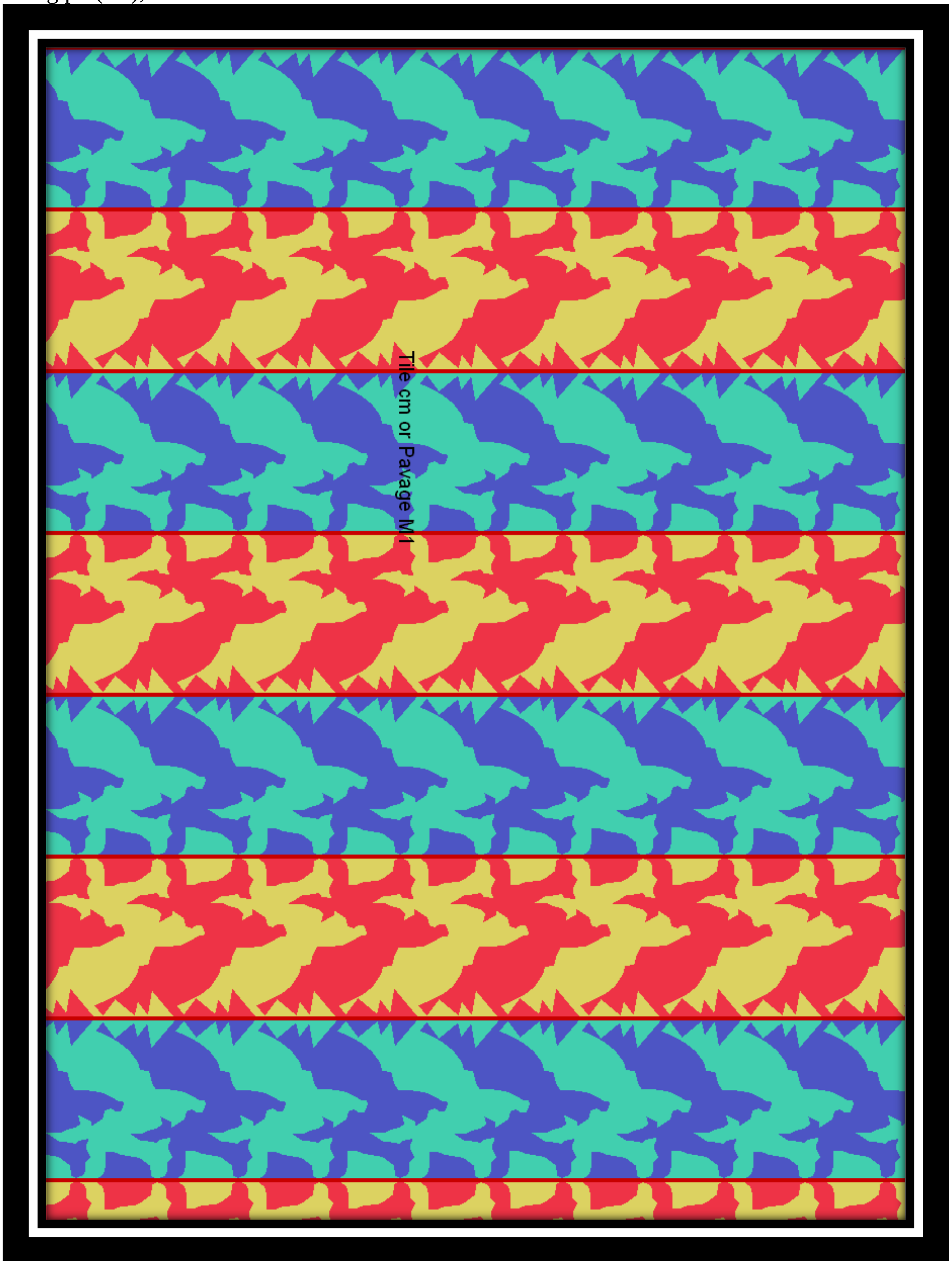
We fulfill the plane by using symmetries and translations:



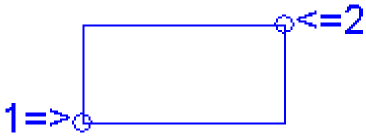
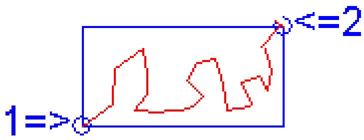
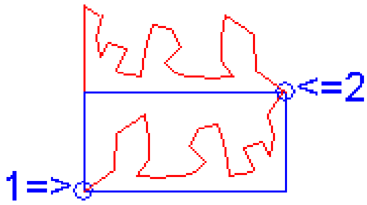
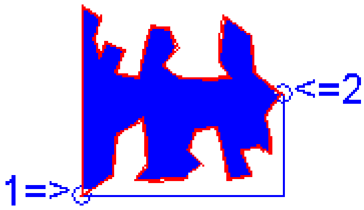

Tile cm or Pavage M1

Lattice and fundamental region for cm (M1)

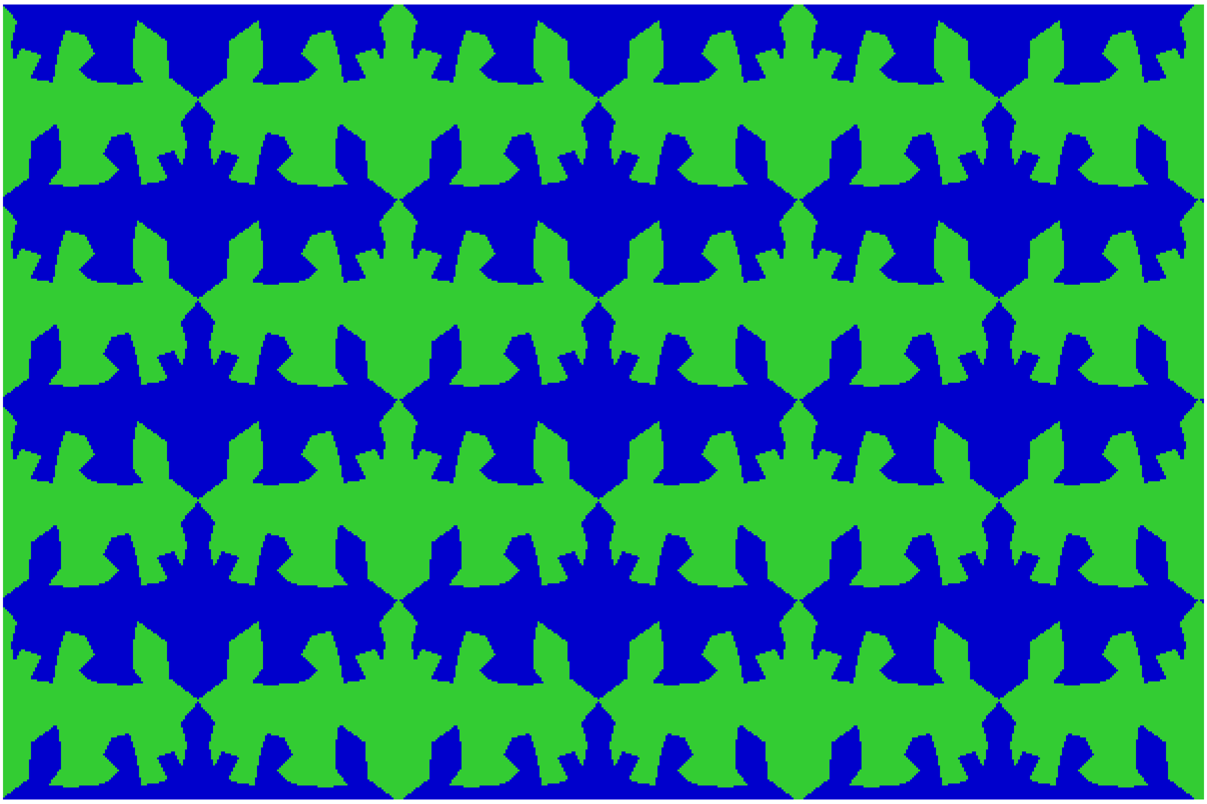
The following picture is the lattice associated to the Tiling cm (M1), the fundamental region is rectangle that you are invited to draw, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p0 (R0), and we must add the mirror horizontal lines.

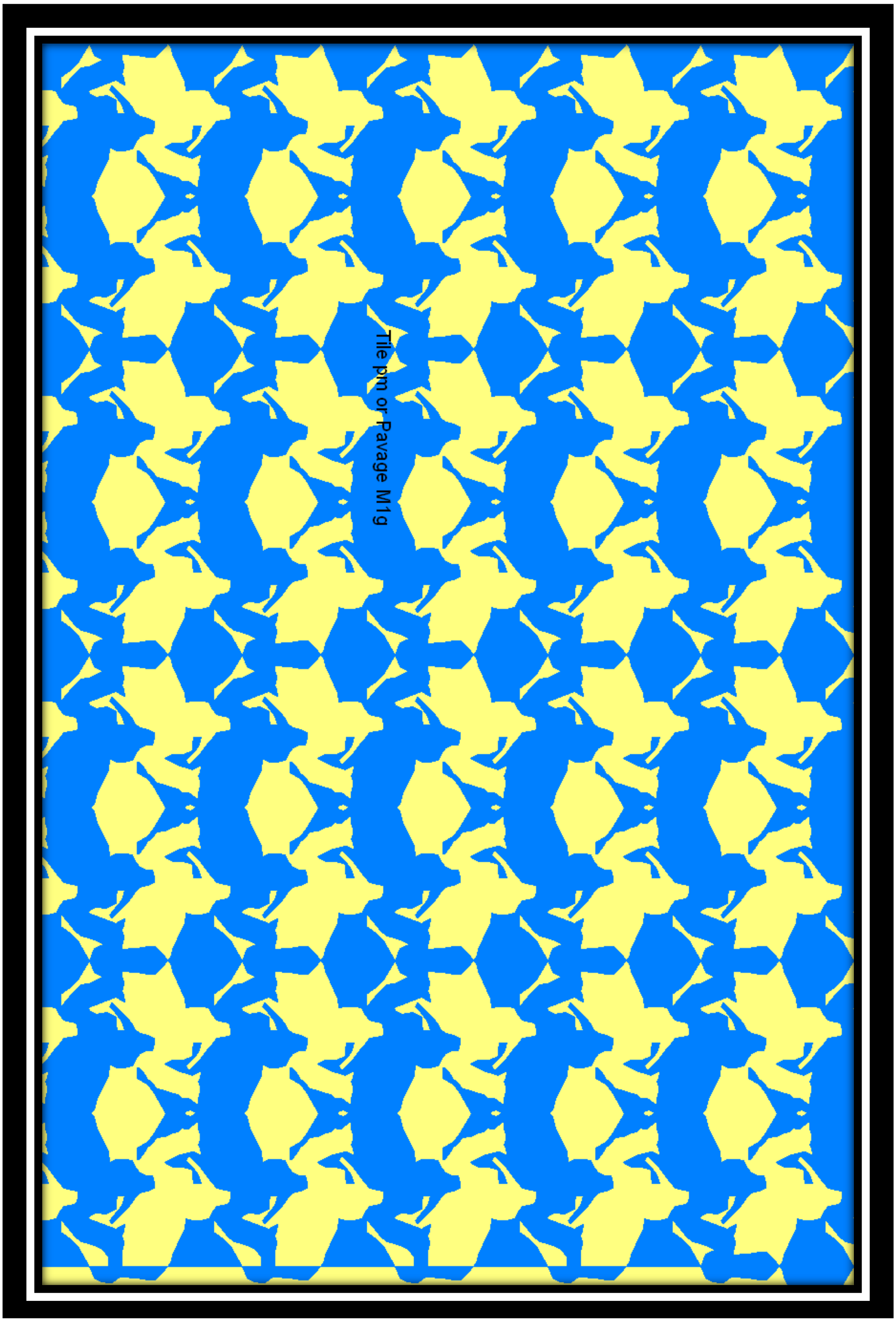


Tiling pm ou M1g

 Take a rectangular piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve, turn on and glue it on the top up side.
	 This is your pattern for this tiling group.	

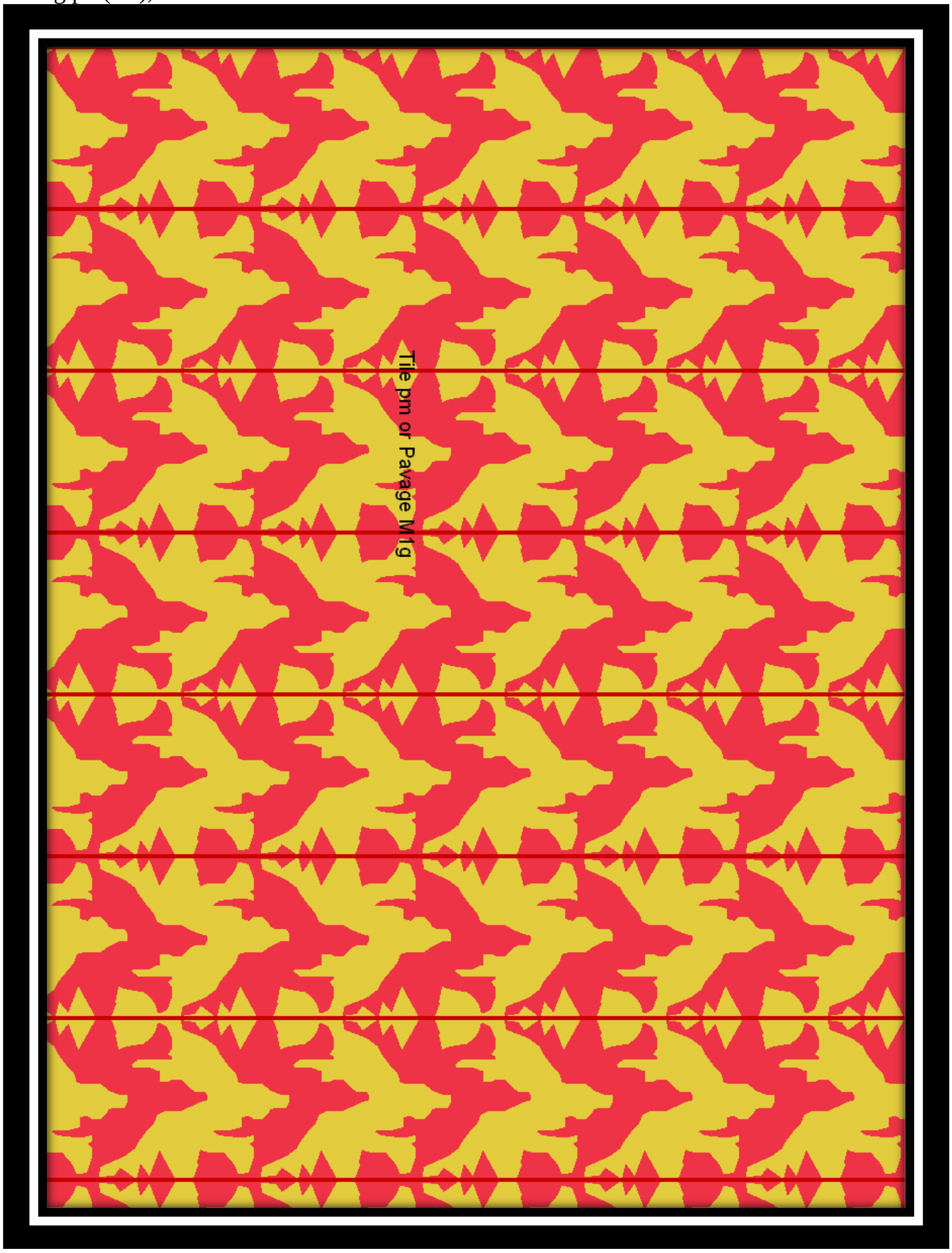
We fulfill the plane by using vertical symmetries and glide symmetries:



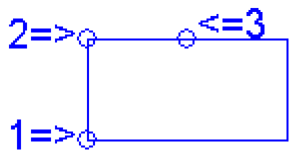
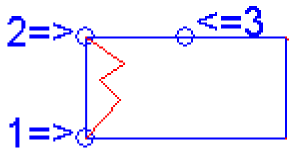
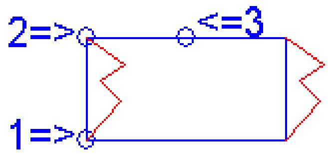
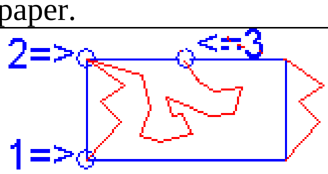
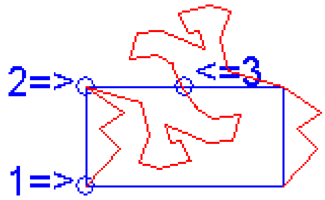
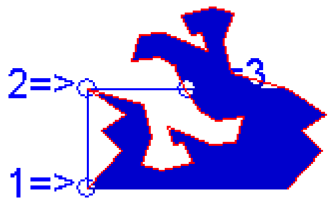

Tile pm or Pavage M1g

Lattice and fundamental region for pm (M1g)

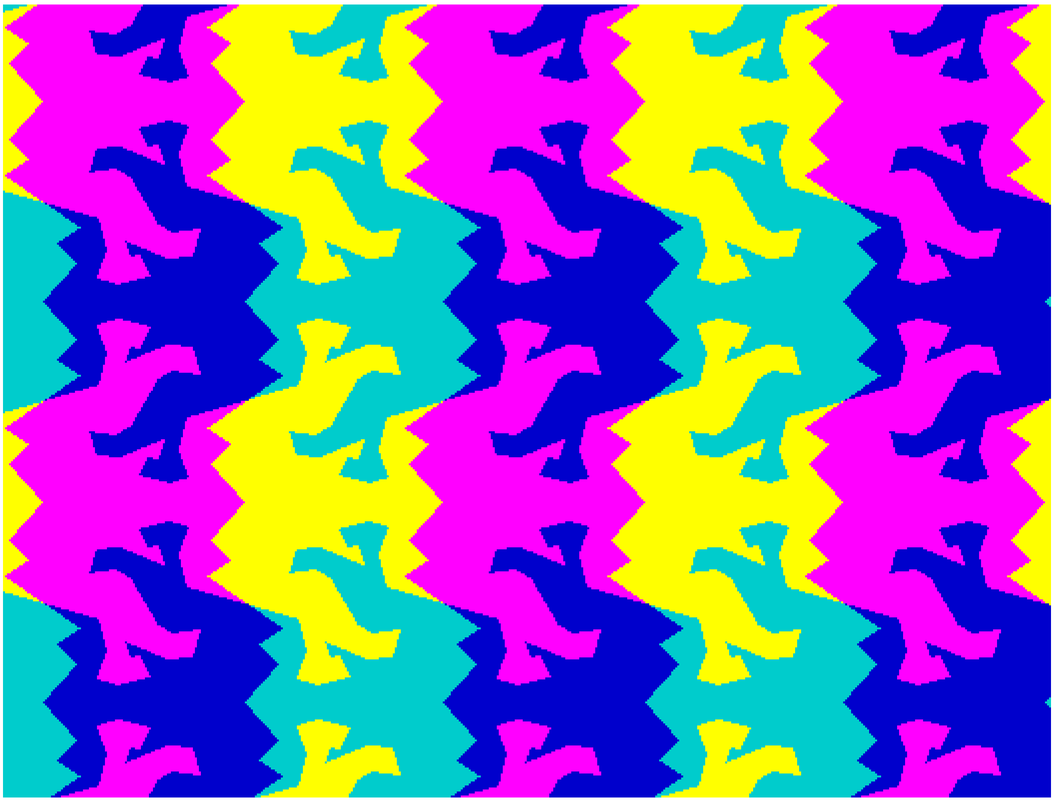
The following picture is the lattice associated to the Tiling pm (M1g), the fundamental region is a rectangle that you are invited to draw, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p0 (R0), and we must add the mirror horizontal lines.

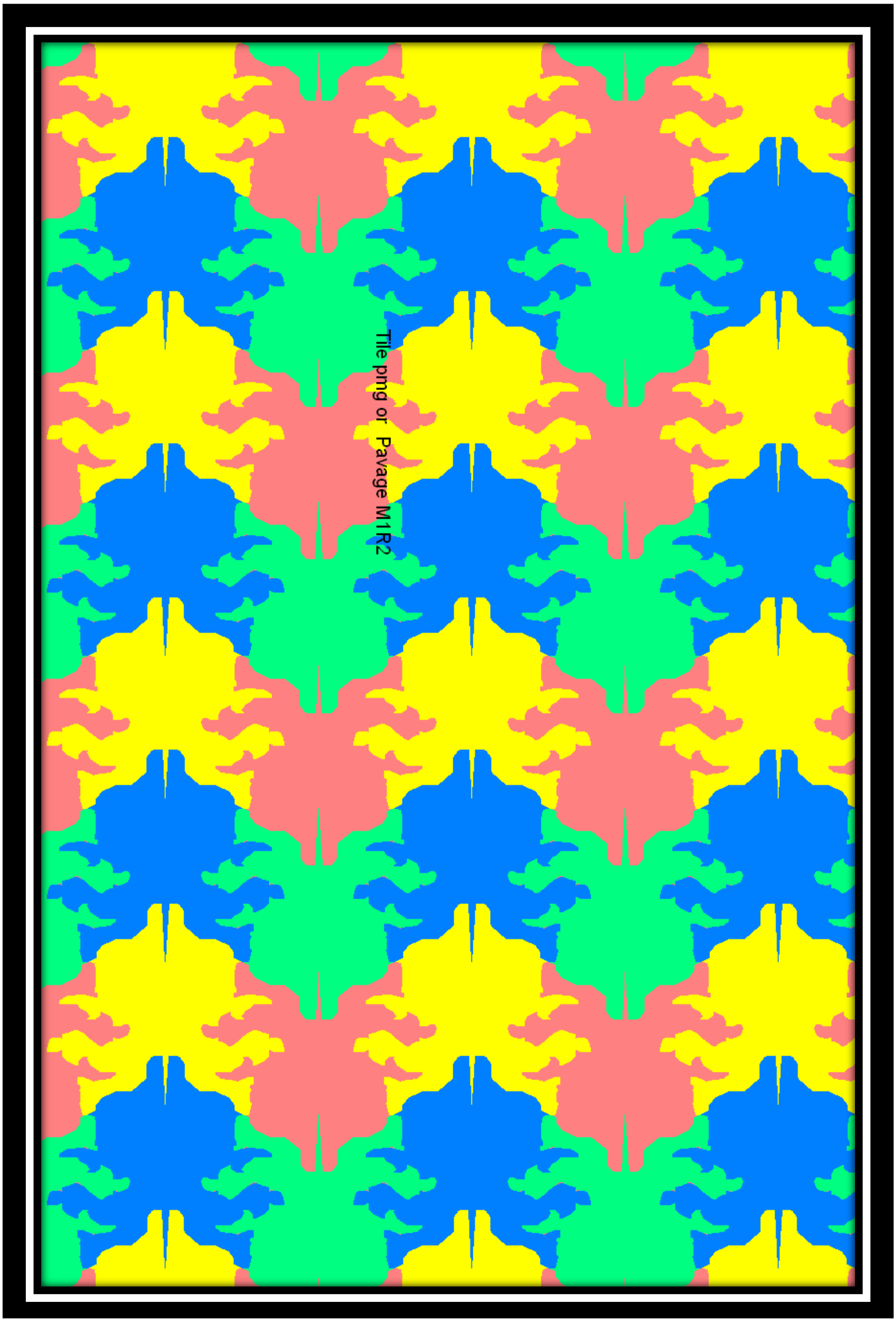


Tiling pmg ou M1R2

 Take a rectangular piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve and glue it on the right side.
 Draw a simple curve starting in 2 and going to 3.	 Cut along the curve, rotate it of 180° and glue it on the top up side.	 This is your pattern for this tiling group.

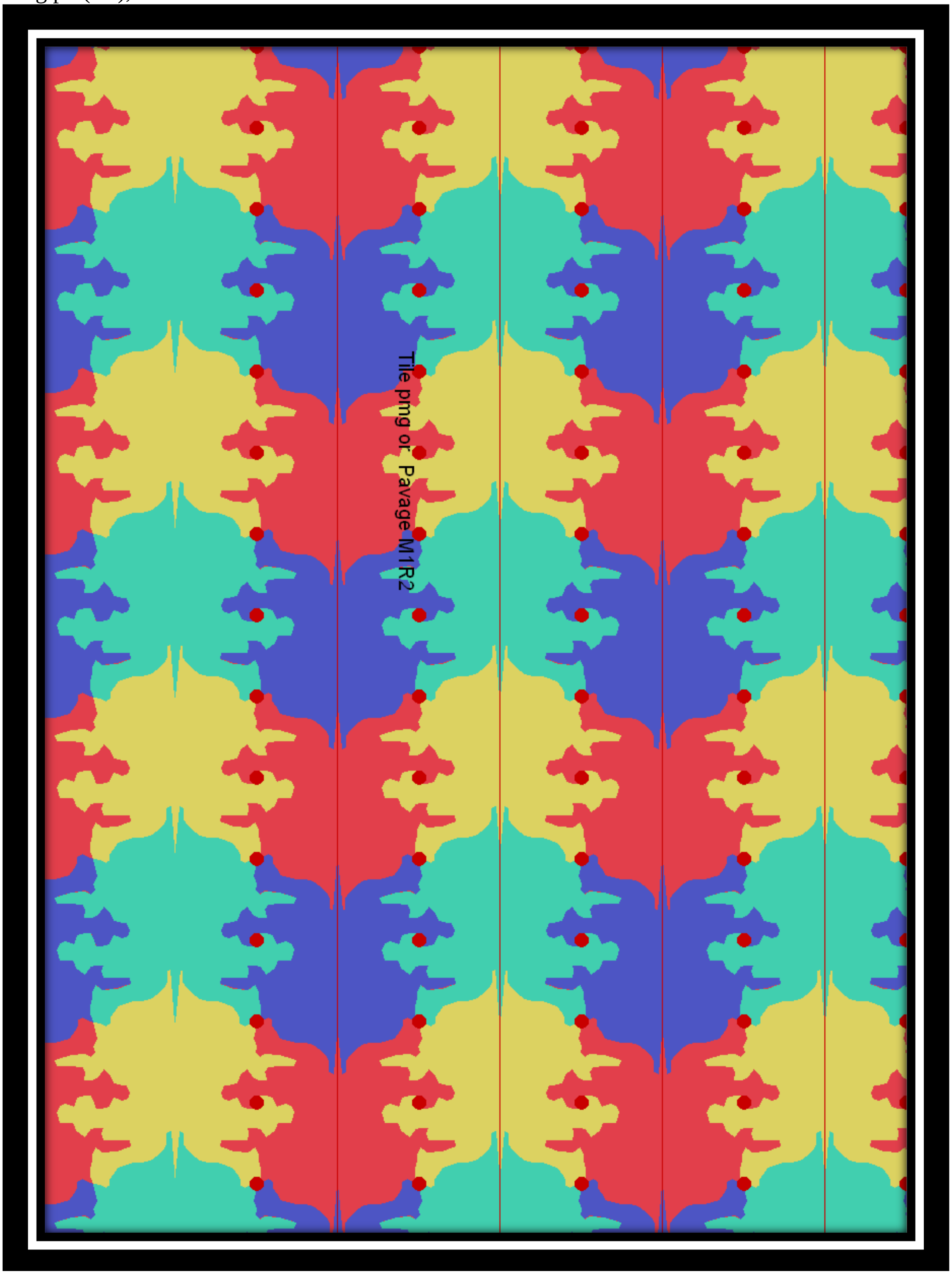
We fulfill the plane by using rotations of angle 180° and horizontal symmetries:



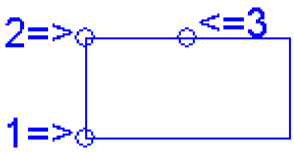
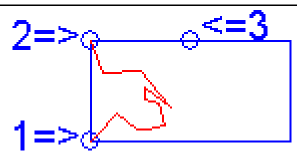
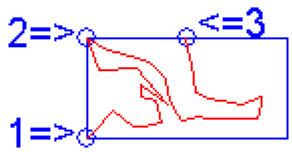
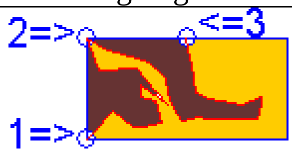

Tile png or Pavage M1R2

Lattice and fundamental region for pgm (M1R2)

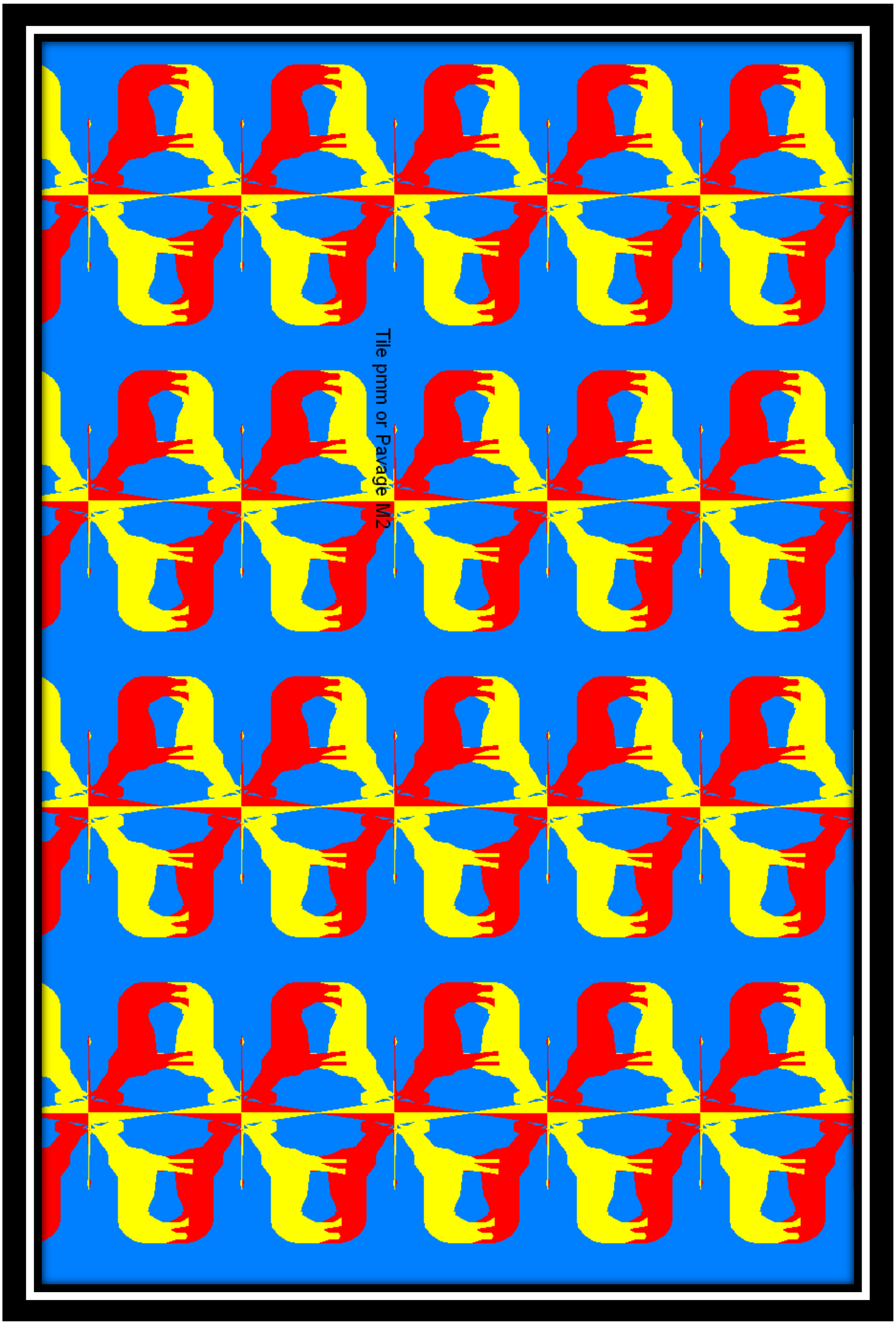
The following picture is the lattice associated to the Tiling pgm (M1R2), the fundamental region is a rectangle that you are invited to draw, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p2 (R2), and we must add the mirror vertical lines.



Tiling pmm ou M2

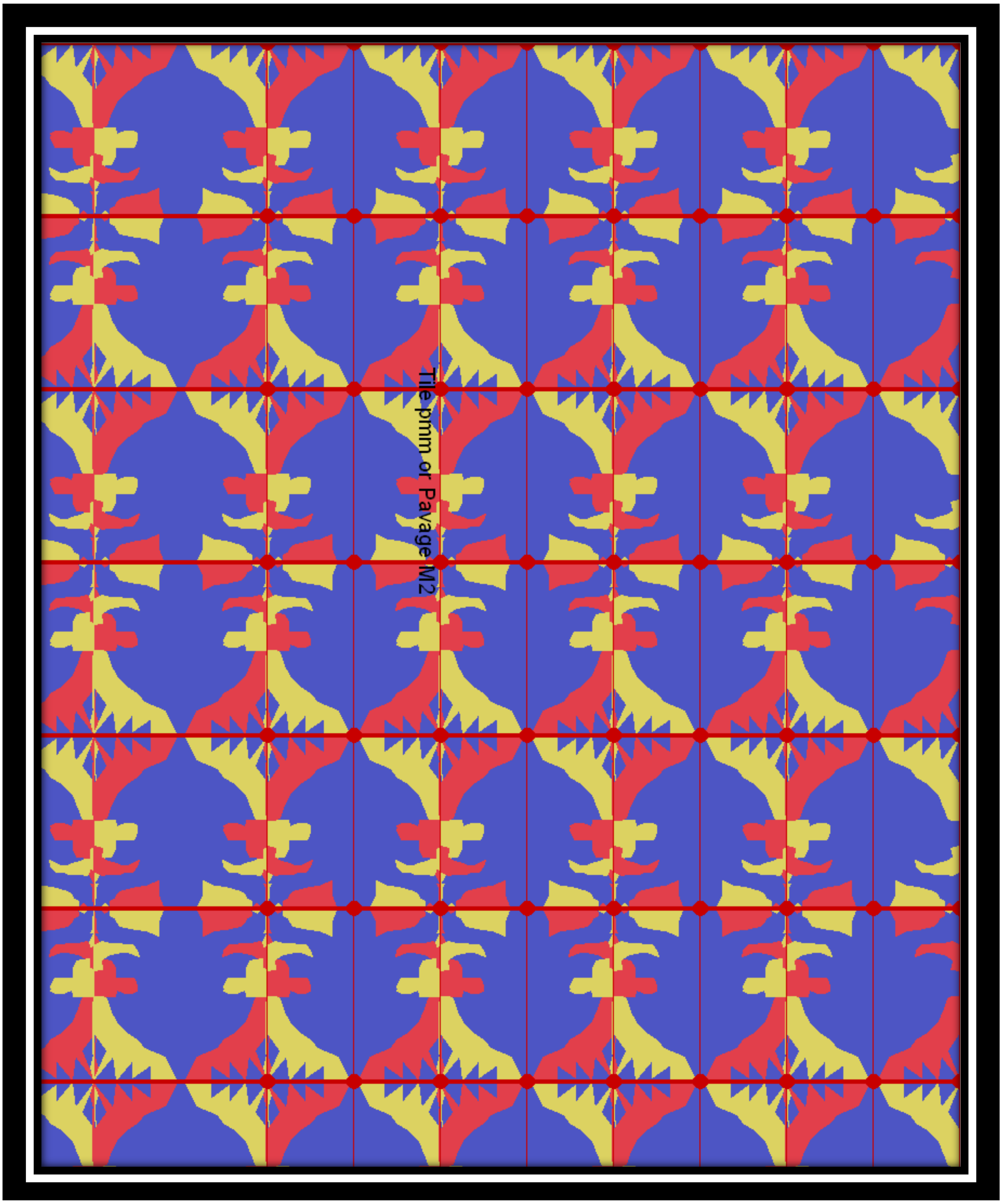
<table>
<tr>
<td>Take a rectangular piece of paper.</td>
<td>Draw a simple curve starting in 1 and going to 2.</td>
<td>Draw a simple curve starting in 2 and going to 3.</td>
</tr>
<tr>
<td></td>
<td>This is your pattern for this tiling group. (a draw rectangle)</td>
<td></td>
</tr>
</table>

We fulfill the plane by vertical and horizontal symmetries :

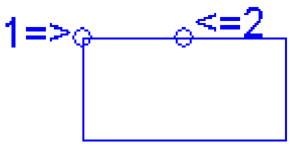
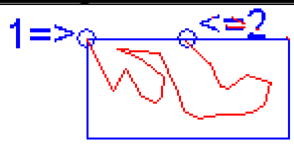
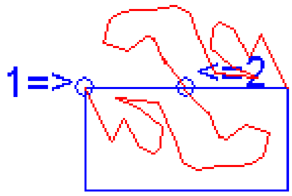
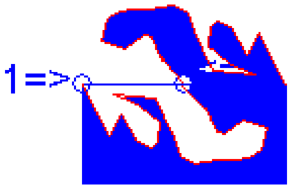

Tile pmm or Pavage M2

Lattice and fundamental region for pmm (M2)

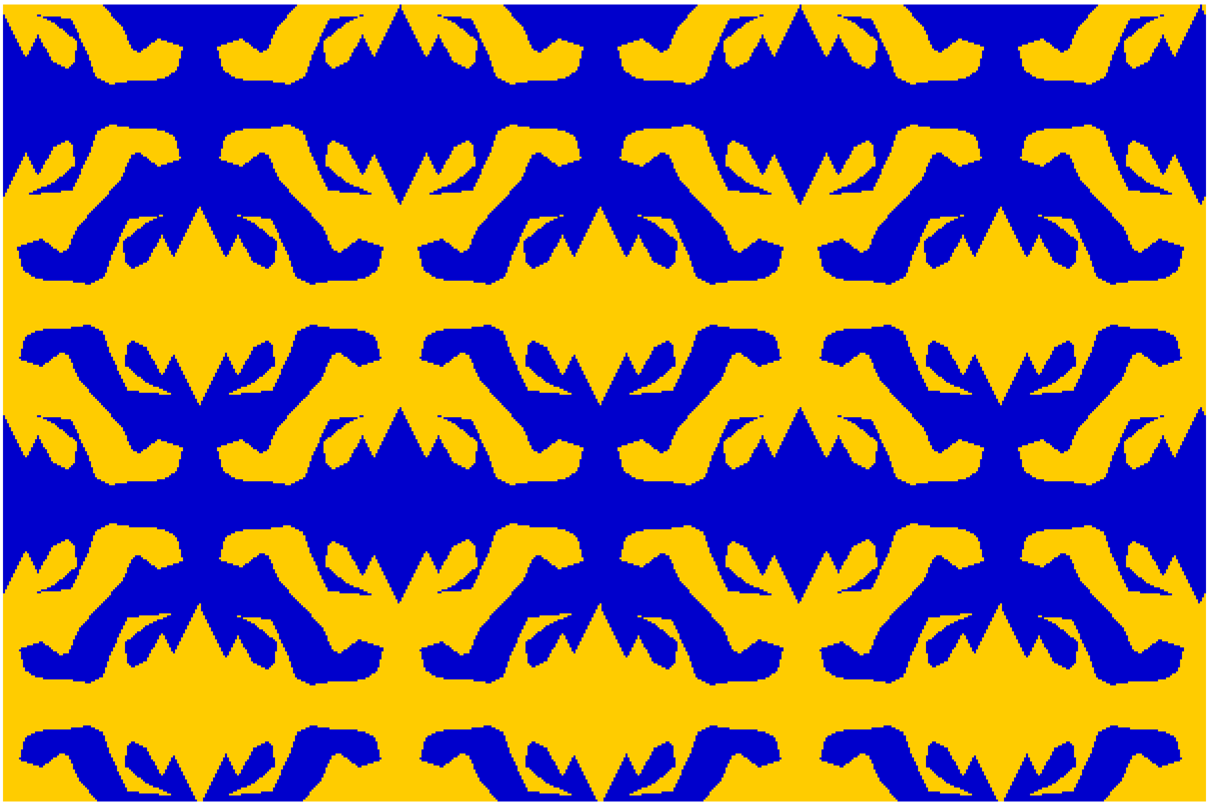
The following picture is the lattice associated to the Tiling cm (M1), the fundamental region is a rectangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p2 (R2), and we must add the mirror horizontal and vertical lines.

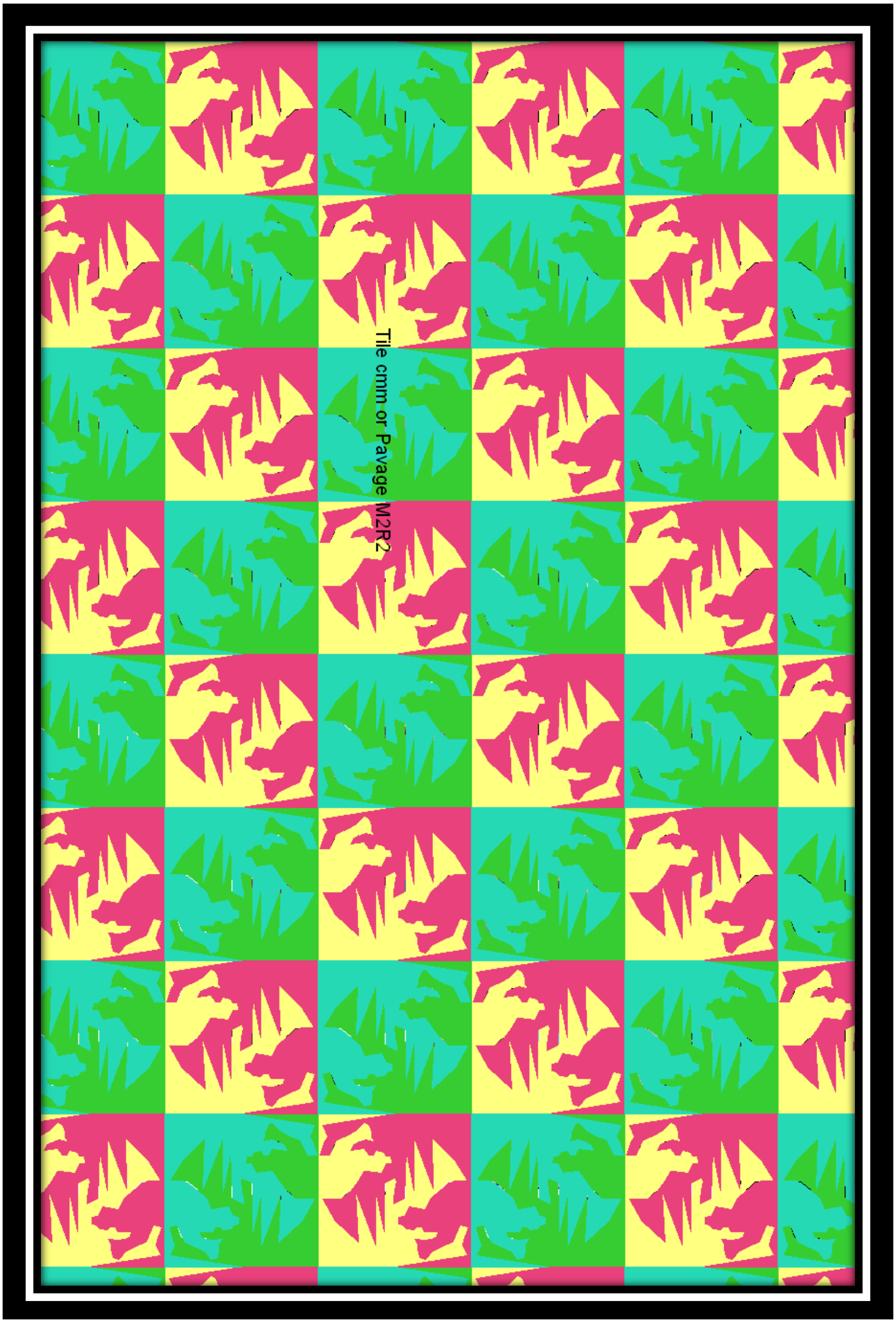


Tiling cmm ou M2R2

 Take a rectangular piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve, rotate it, and glue it on the top up side.
	 This is your pattern for this tiling group.	

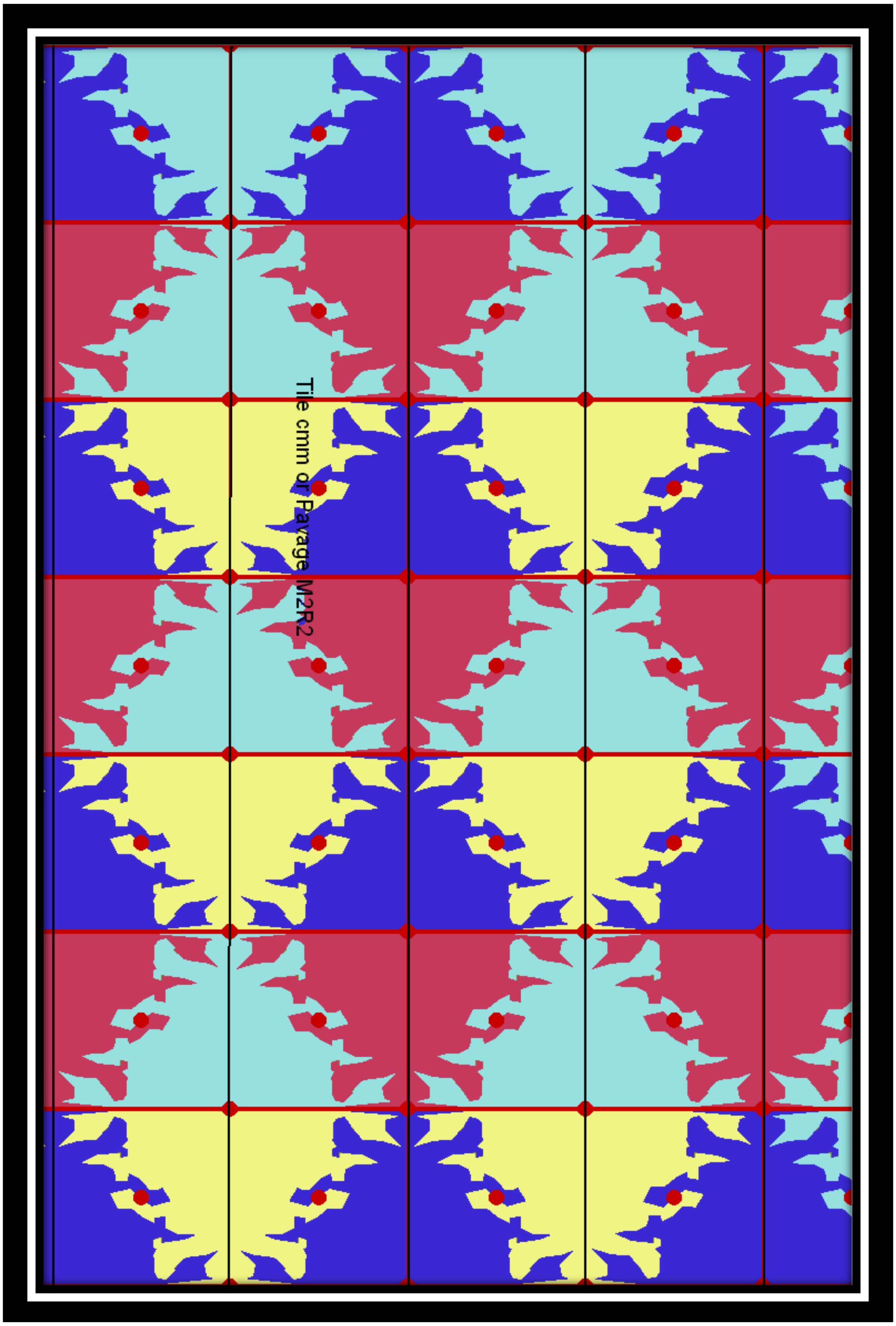
We fulfill the plane by using glides symmetries , rotations of angle 180° and translations:



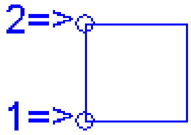
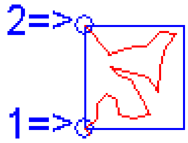
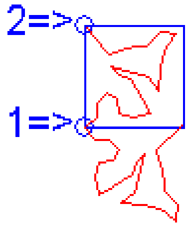
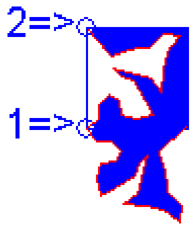

Tile cmm or Pavage M2R2

Lattice and fundamental region for cmm (M2R2)

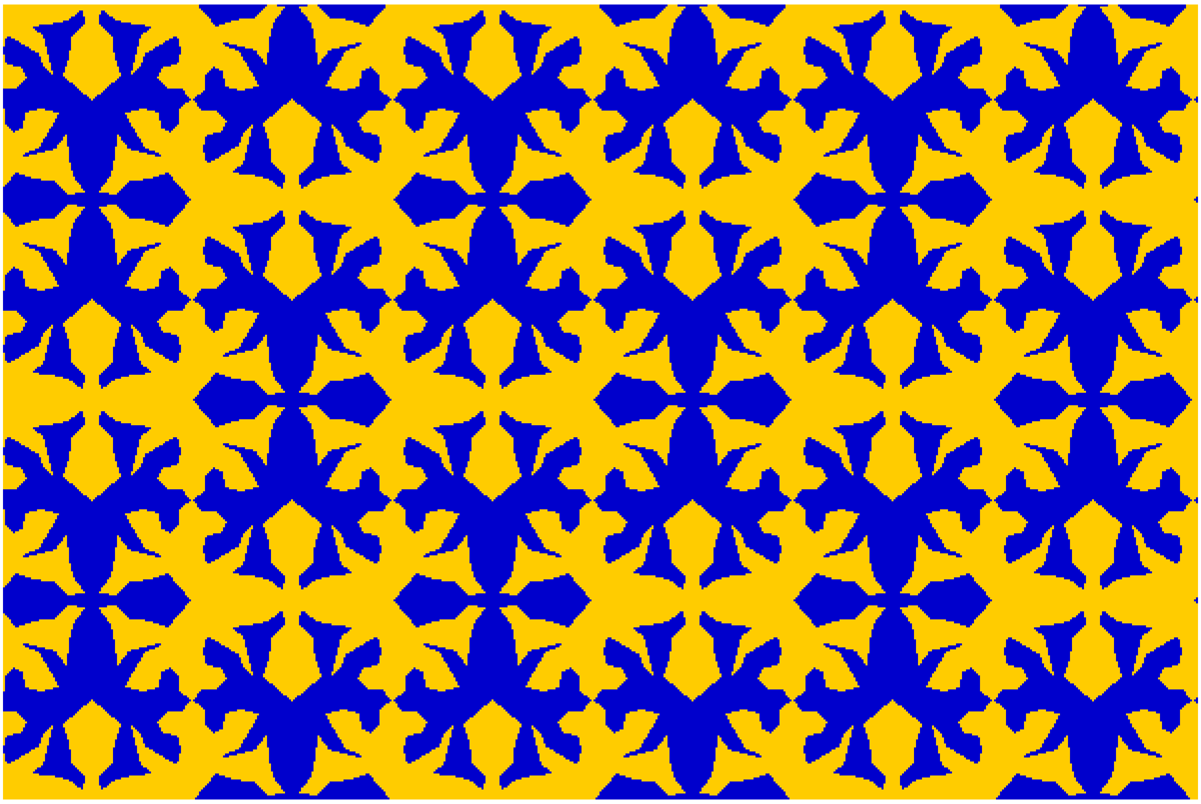
The following picture is the lattice associated to the Tiling cmm (M2R2), the fundamental region is a rectangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p2 (R2), and we must add the mirror horizontal and vertical lines.

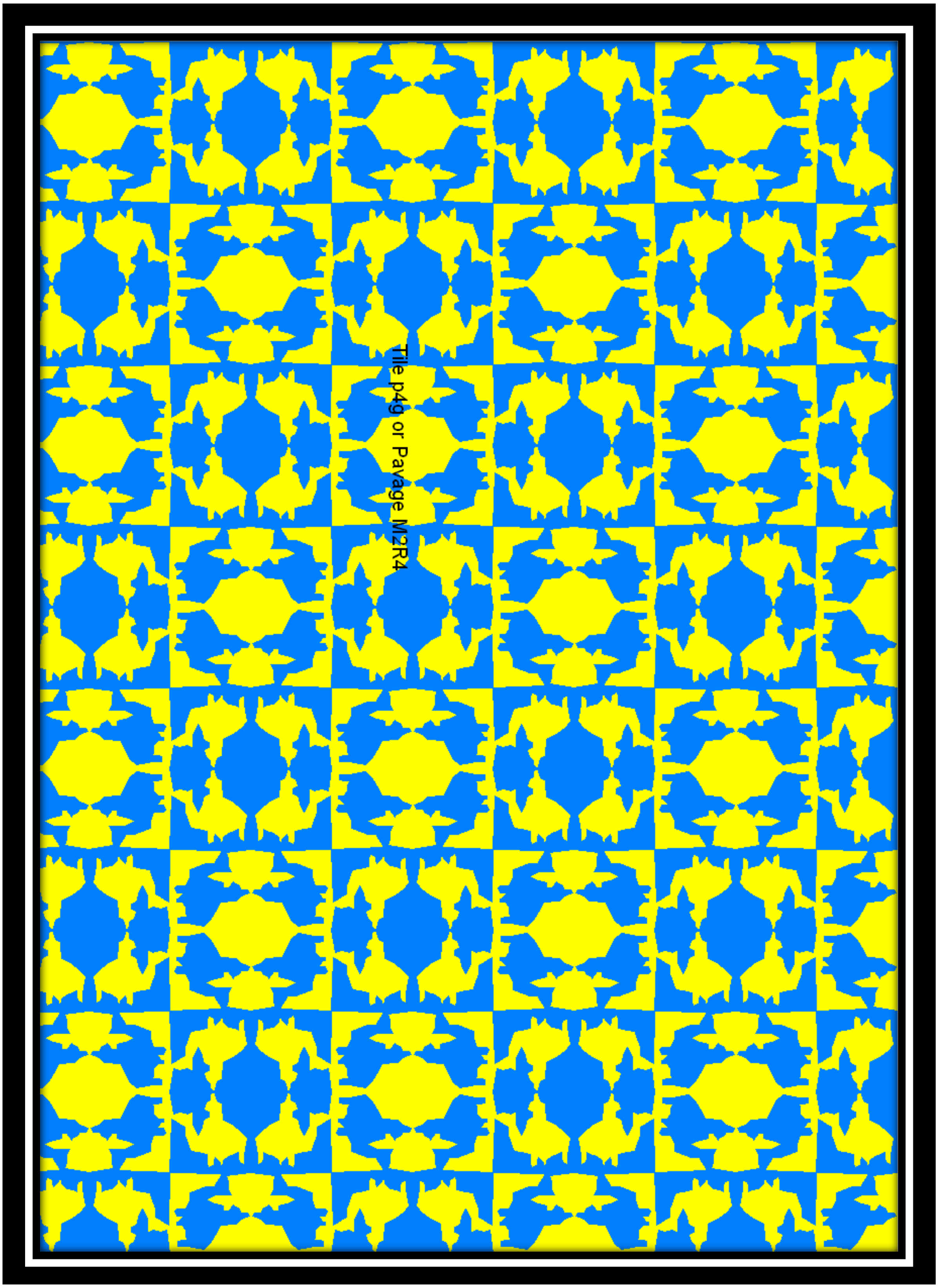


Tiling p4g ou M2R4

 Take a square piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Cut along the curve, rotate it, and glue it on the top down side.
	 This is your pattern for this tiling group.	

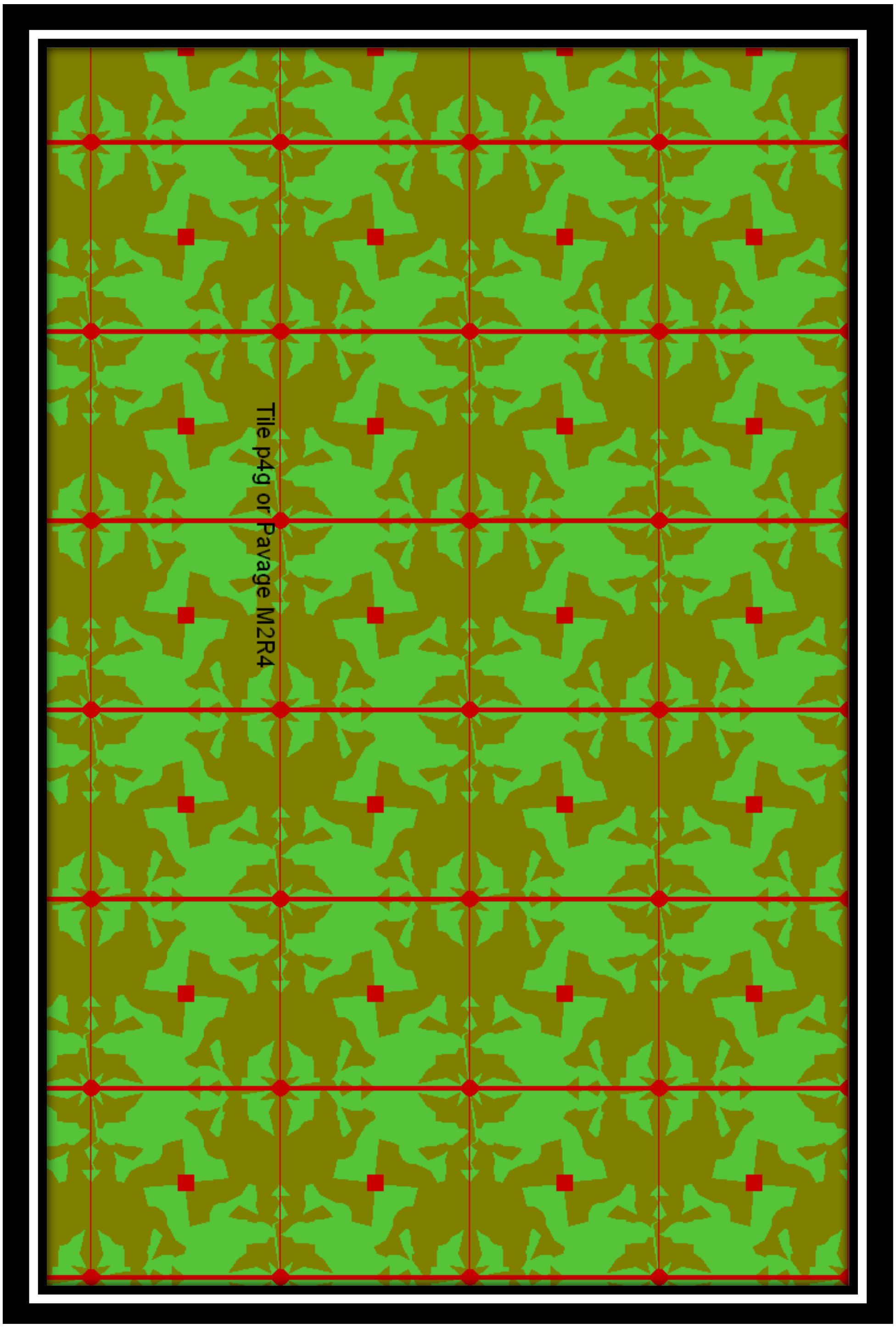
We fulfill the plane by using glides symmetries , rotations of angle 90°, 180°, 270 and translations:



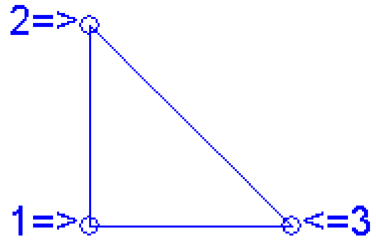
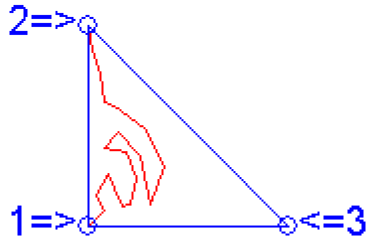
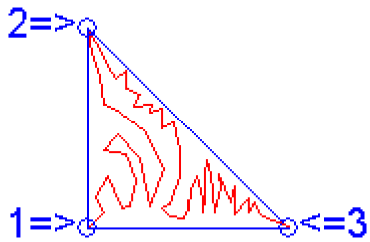
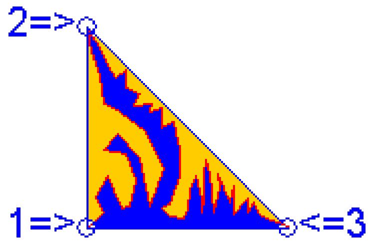

Tile p4g or Pavage M2R4

Lattice and fundamental region for p4g (M2R4)

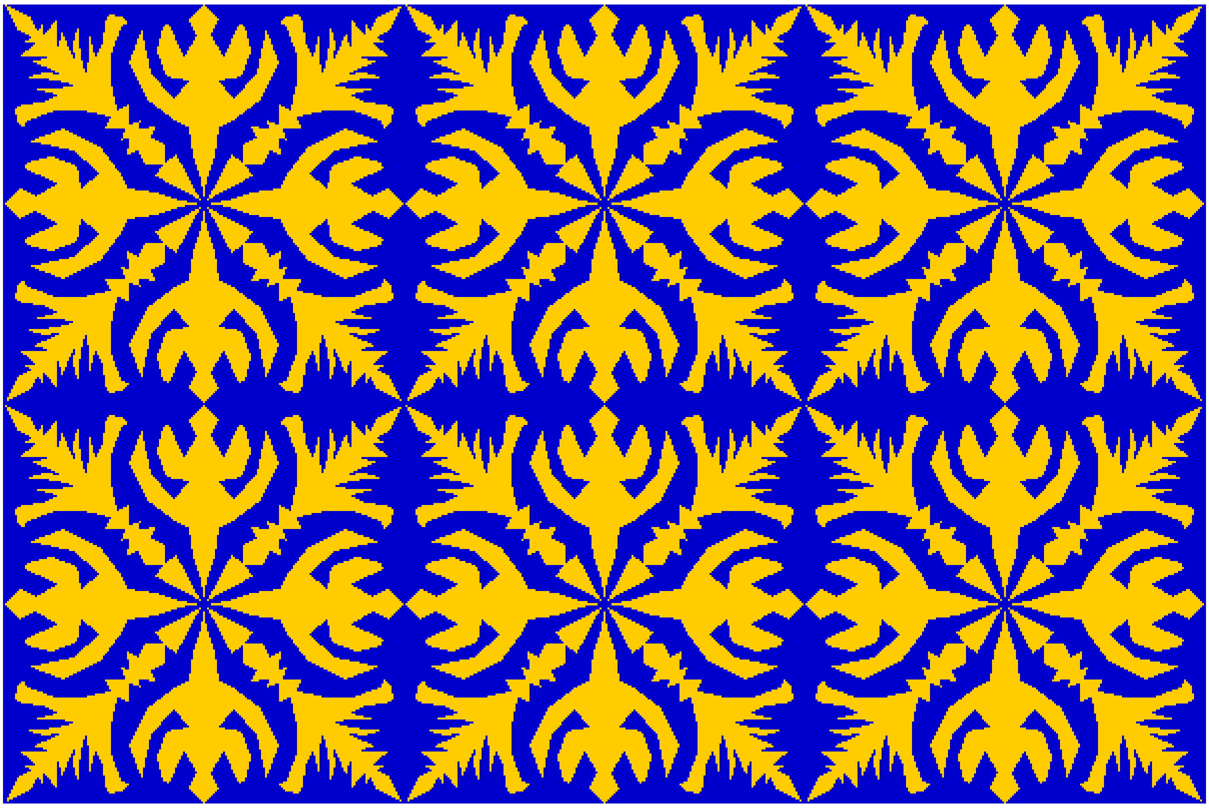
The following picture is the lattice associated to the Tiling p4g (M2R4), the fundamental region is a half of a square, that is a right isosceles triangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p4 (R4), and we must add the mirror horizontal and vertical lines.

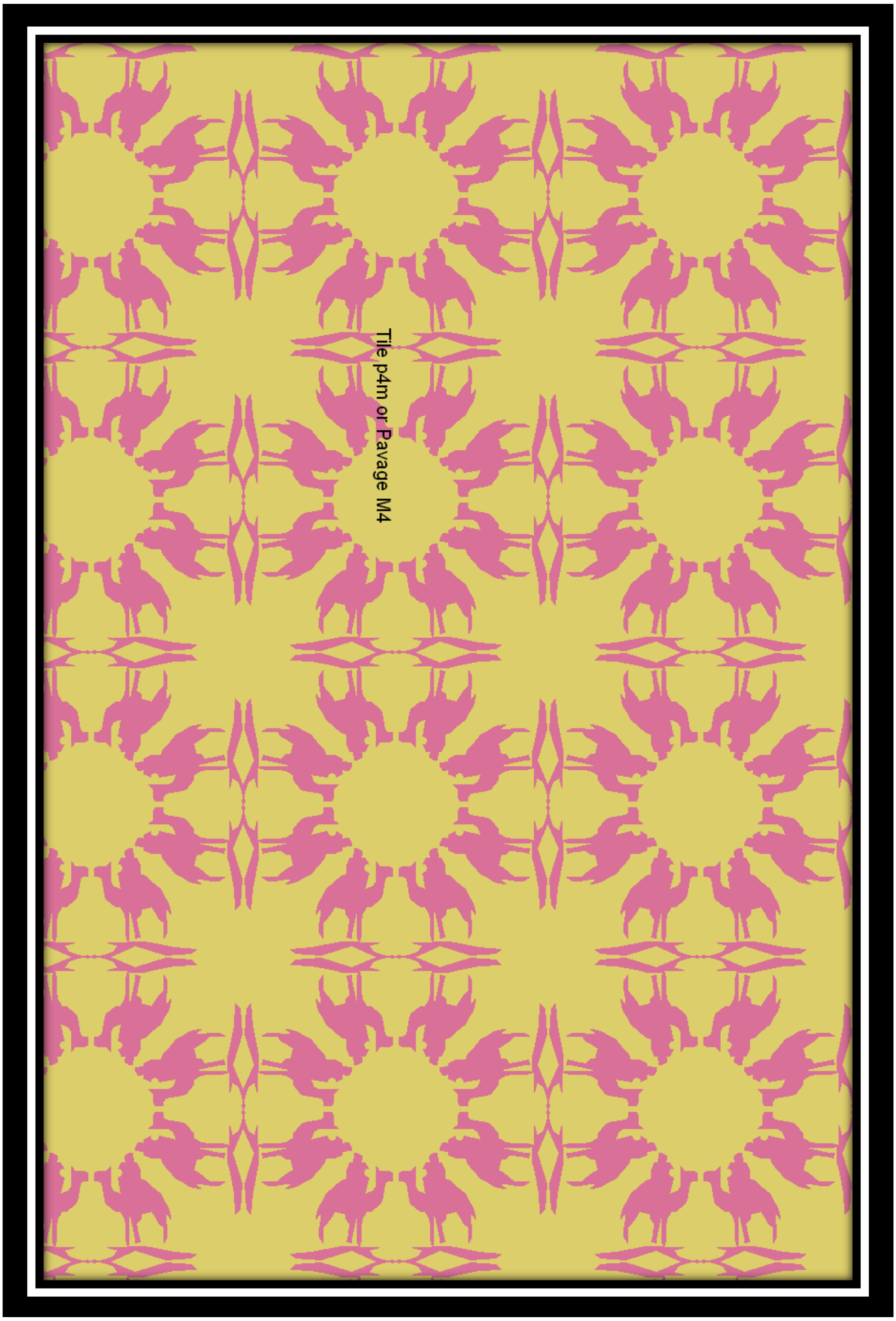


Tiling p4m ou M4

 Take a right isosceles triangular piece of paper.	 Draw a simple curve starting in 1 and going to 2.	 Draw a simple curve starting in 2 and going to 3.
	 This is your pattern for this tiling group. (a bicolor triangle)	

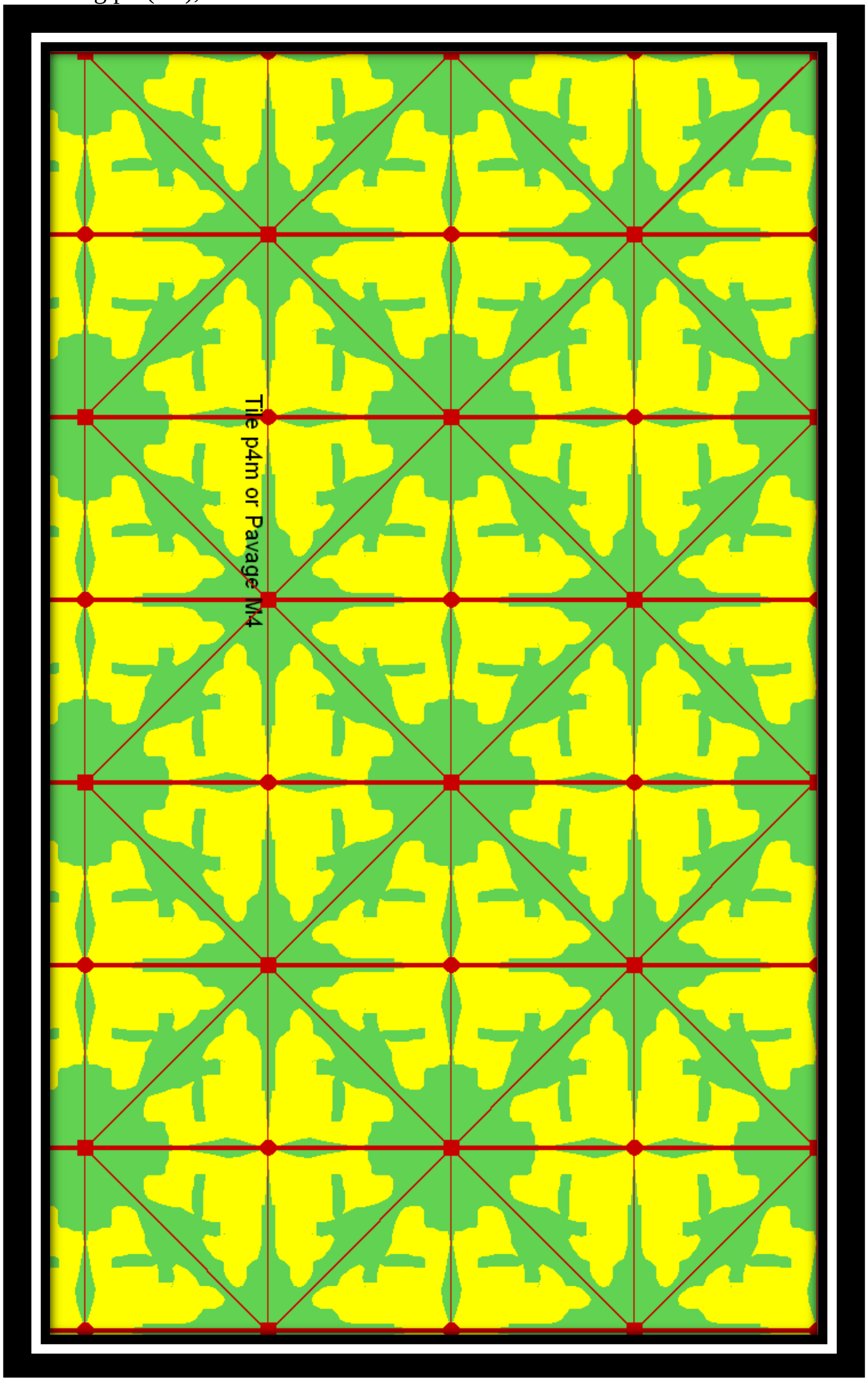
We fulfill the plane by using symmetries with axis the sides of the rectangle triangle:



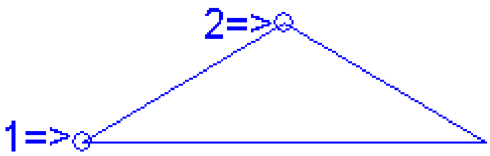

Tile p4m or Pavage M4

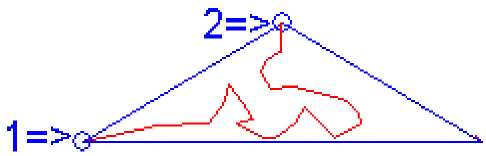
Lattice and fundamental region for p4m (M4)

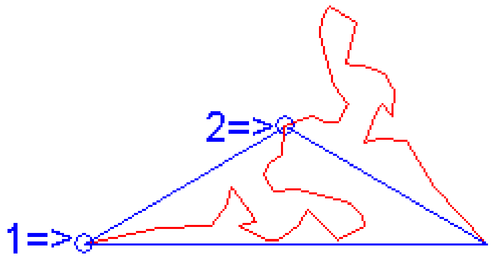
The following picture is the lattice associated to the Tiling p4m (M4), the fundamental region is a half of a square, that is a right isosceles triangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p4 (R4), and we must add the mirror horizontal and vertical lines.

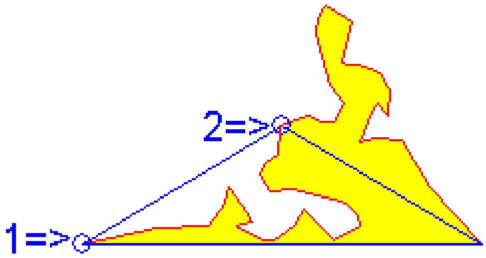


Tiling p31m ou M3R3

<table>
<tr>
<td>

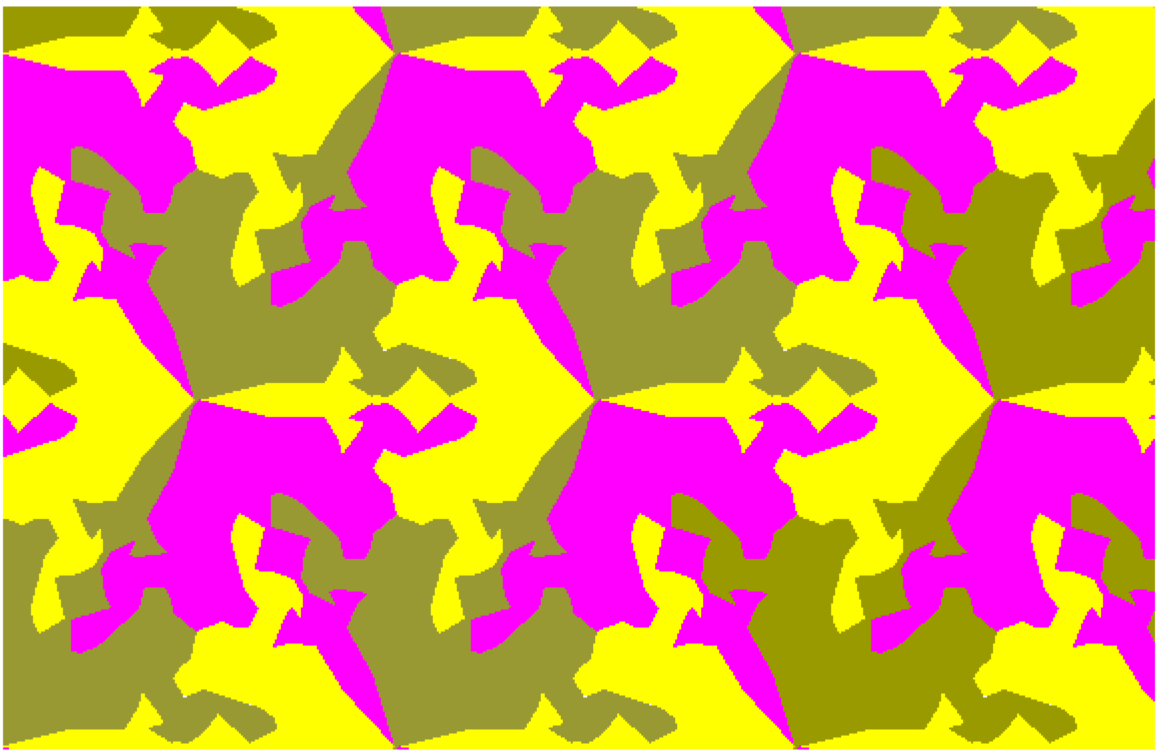
Take a triangular piece of paper. (isosceles having the principal angle 120°)</td>
<td>

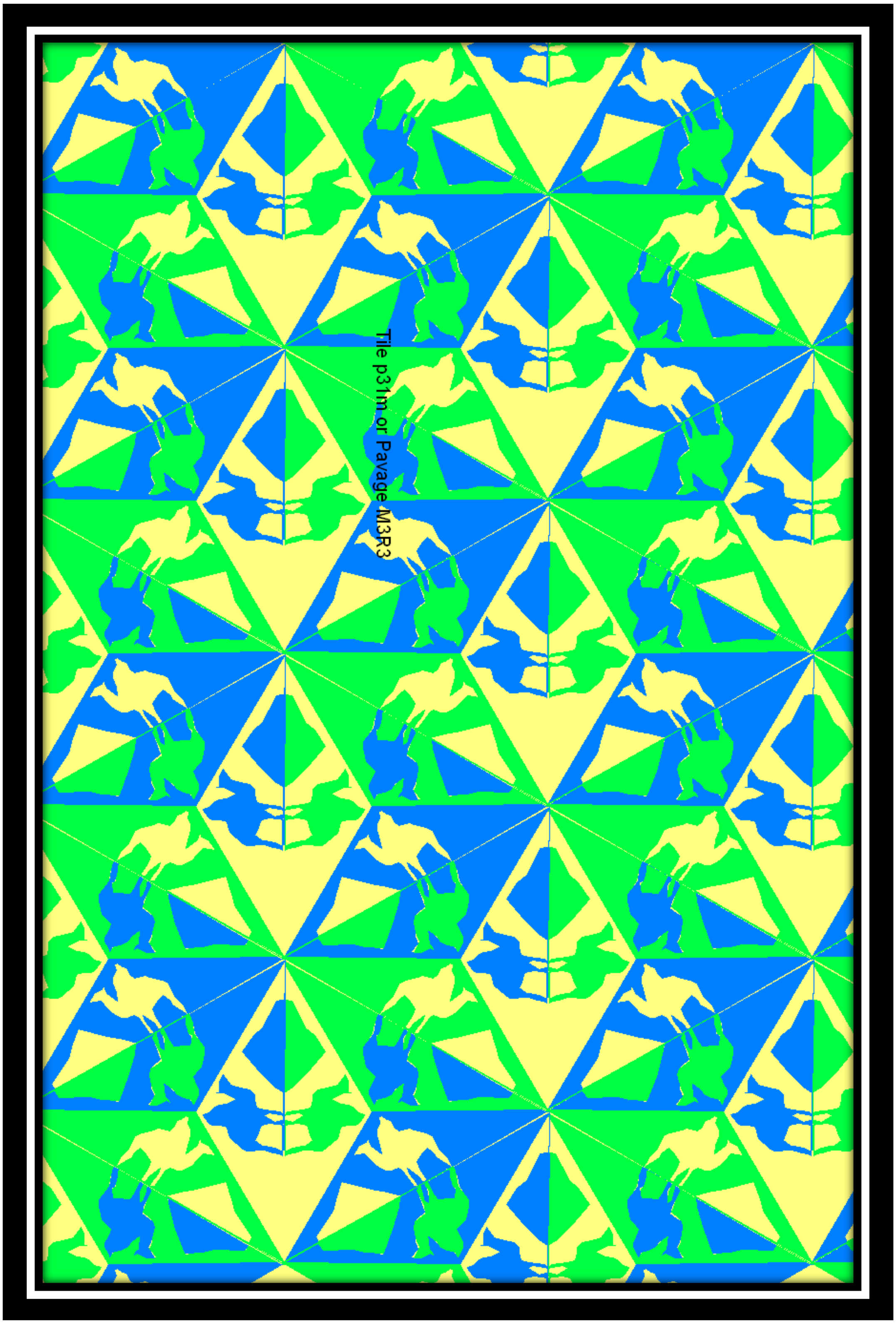
Draw a simple curve starting in 1 and going to 2.</td>
</tr>
<tr>
<td>

Cut along the curve, rotate it, and glue it on the right side.</td>
<td>

This is your pattern for this tiling group.</td>
</tr>
</table>

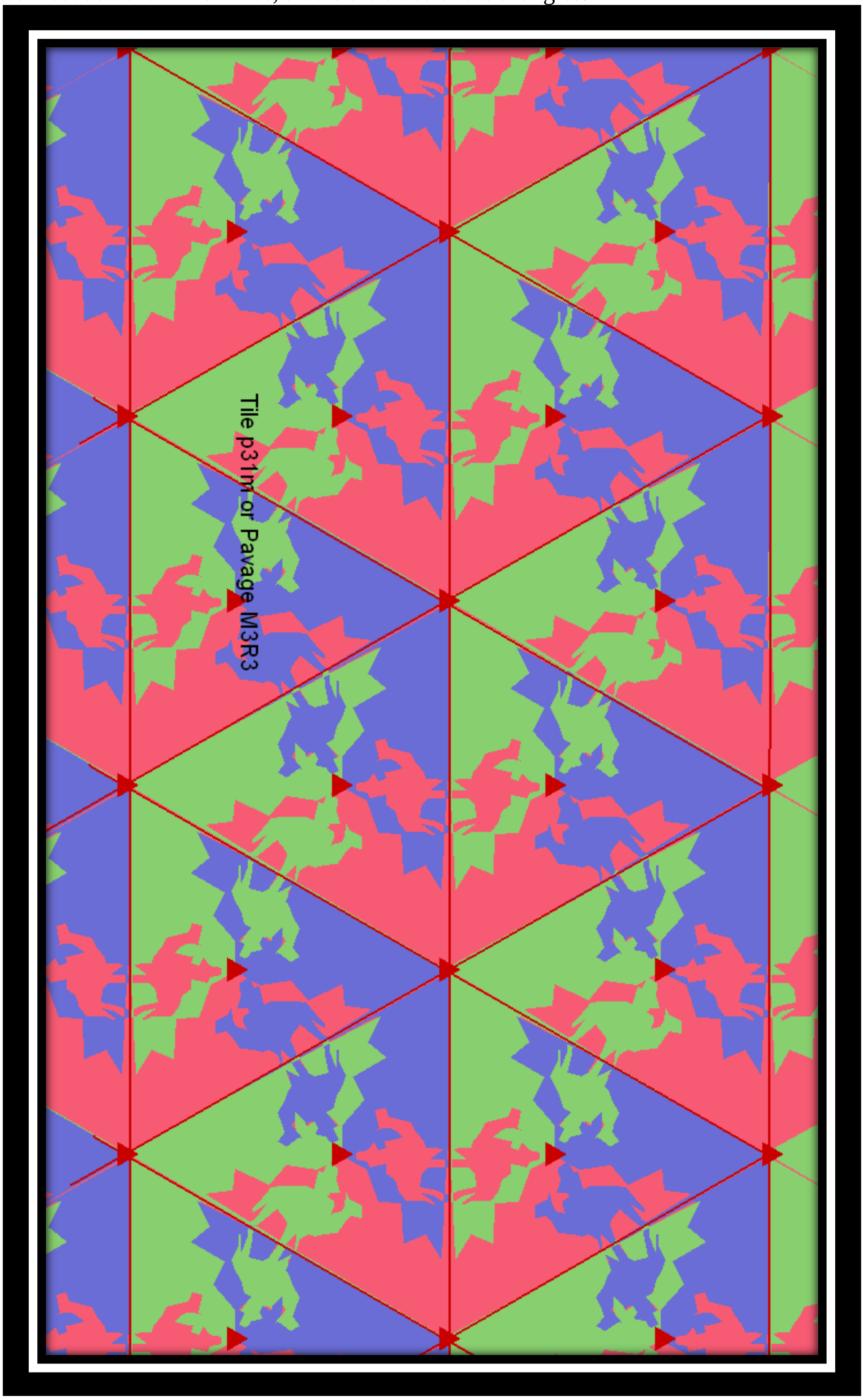
We fulfill the plane by using symmetries and rotations of angle 120°:




Tile p31m or Pavage M3R3

Lattice and fundamental region for p31m (M3R3)

The following picture is the lattice associated to the Tiling p31m (M3R3), the fundamental region is equilateral triangle, and as you can check there are translations following the sides of the triangles. The lattice is the same as the tiling p3 (R3), and we must add the mirror lines, that is the sides of the triangles.



Tiling p3m1 or M3

<table>
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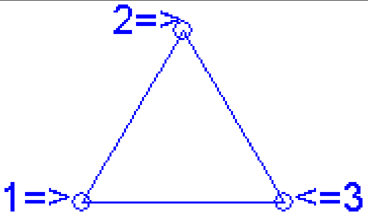
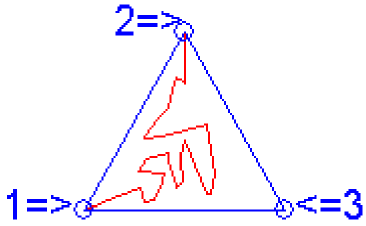
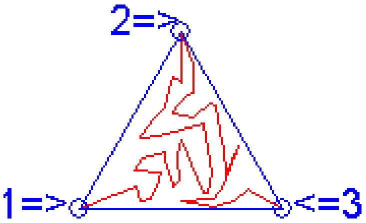
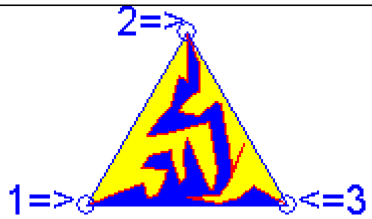
<td>

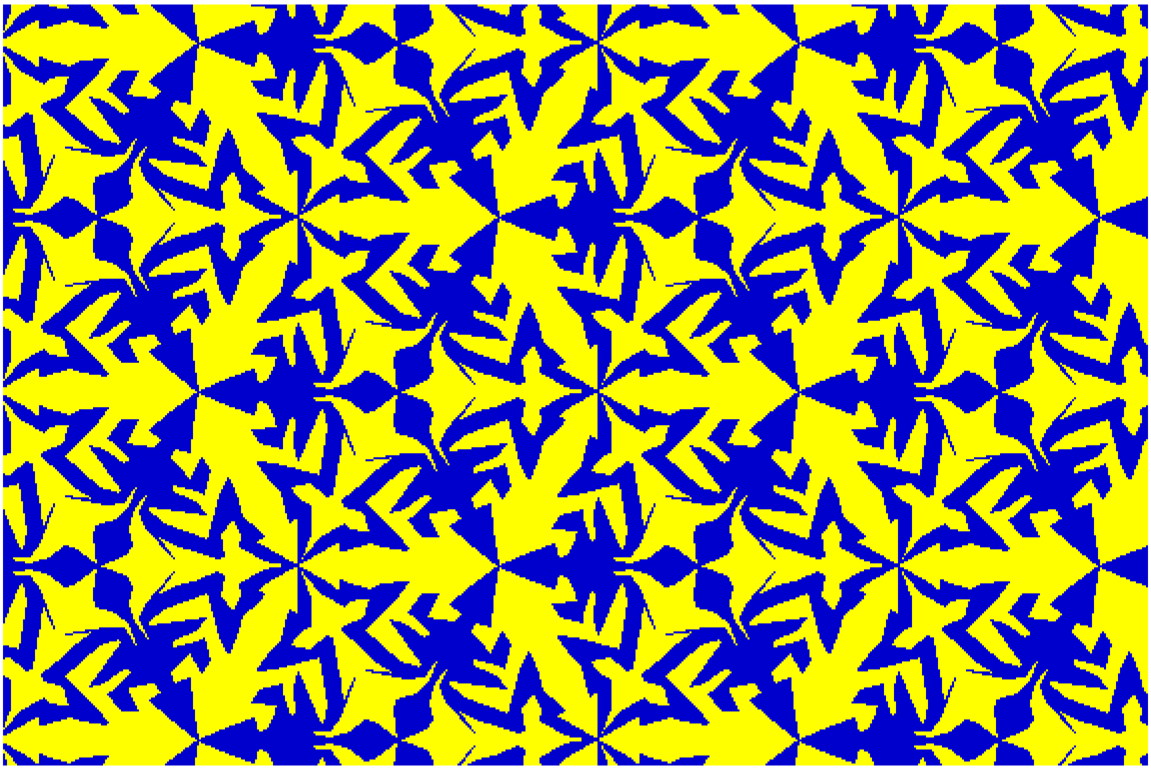
Take a triangular piece of paper (equilateral).</td>
<td>

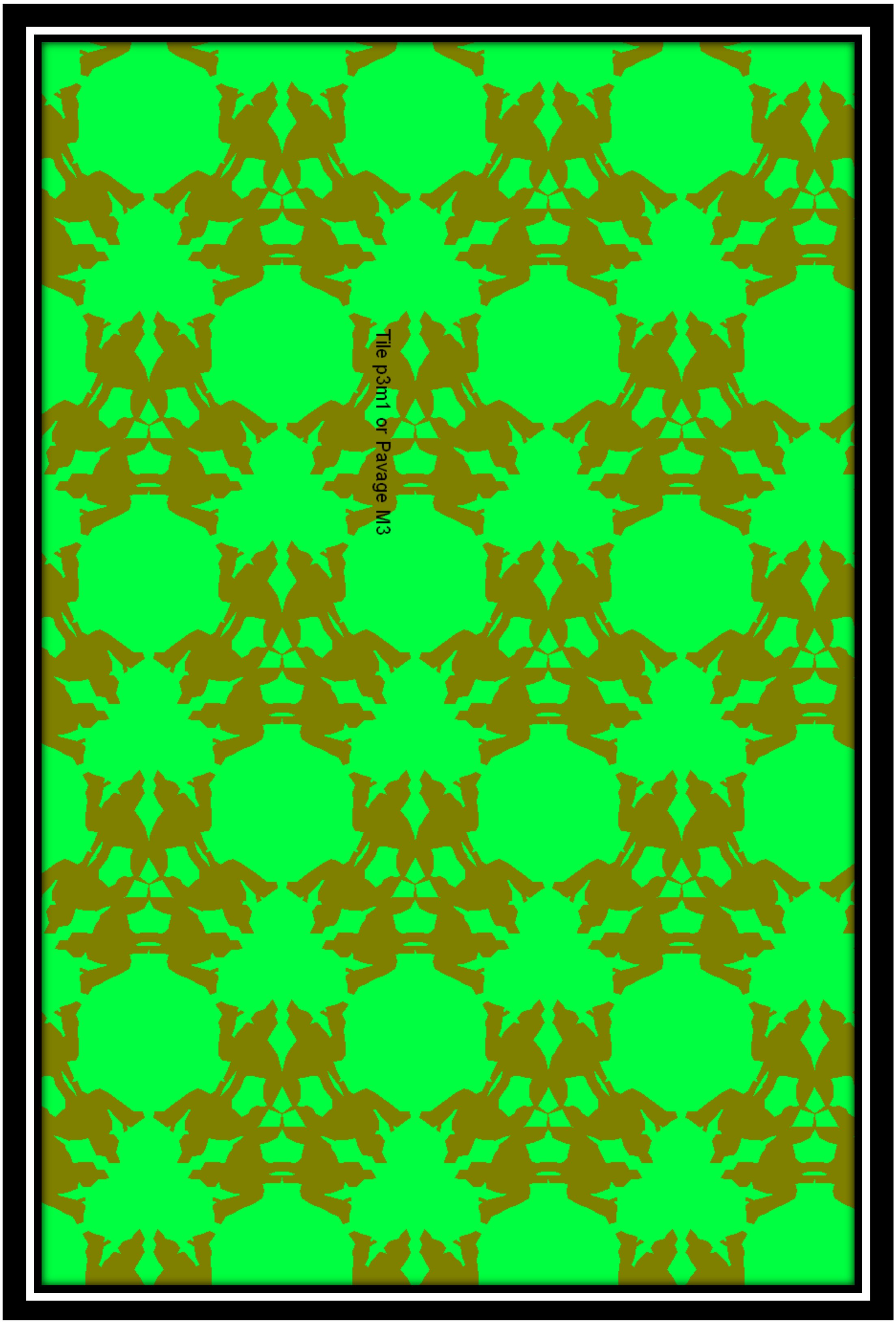
Draw a simple curve starting in 1 and going to 2.</td>
</tr>
<tr>
<td>

Draw a simple curve starting in 2 and going to 3.</td>
<td>

This is your pattern for this tiling group.</td>
</tr>
</table>

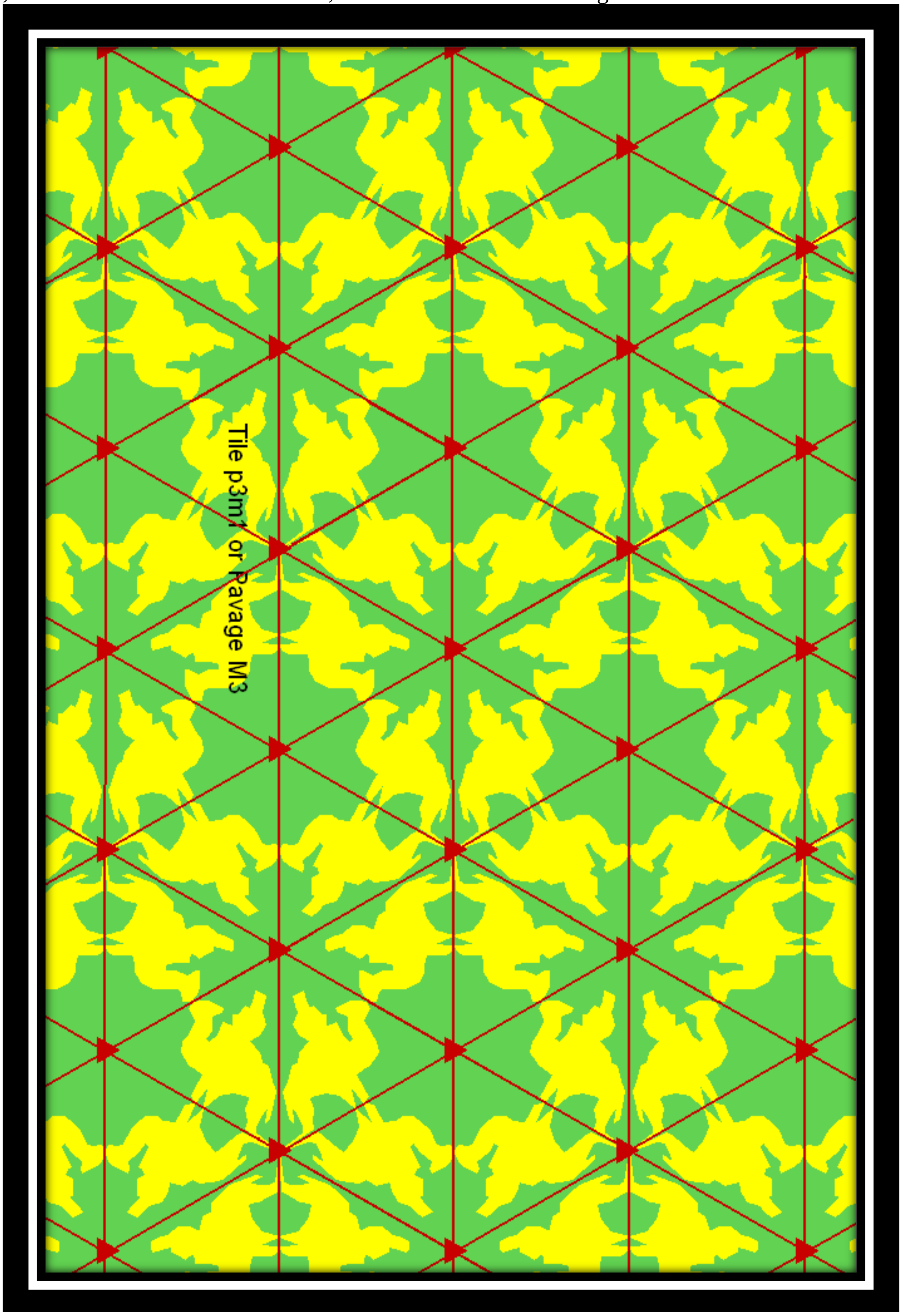
We fulfill the plane by using symmetries with axis the sides of the equilateral triangle:




Tile p3m1 or Pavage M3

Lattice and fundamental region for p3m1 (M3)

The following picture is the lattice associated to the Tiling p3m1 (M3), the fundamental region is an equilateral triangle, and as you can check there are translations following the sides of the triangles. The lattice is the same as the tiling p3 (R3), and we must add the mirror lines, that is the sides of the triangles.



Tiling p6m ou M6

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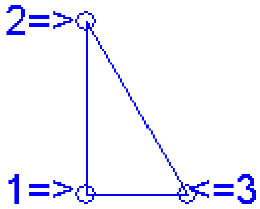
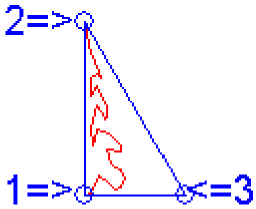
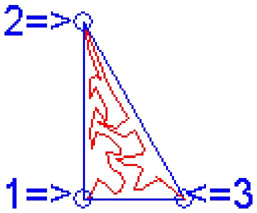
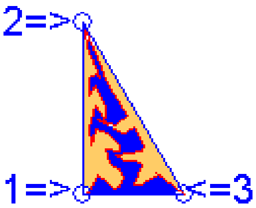
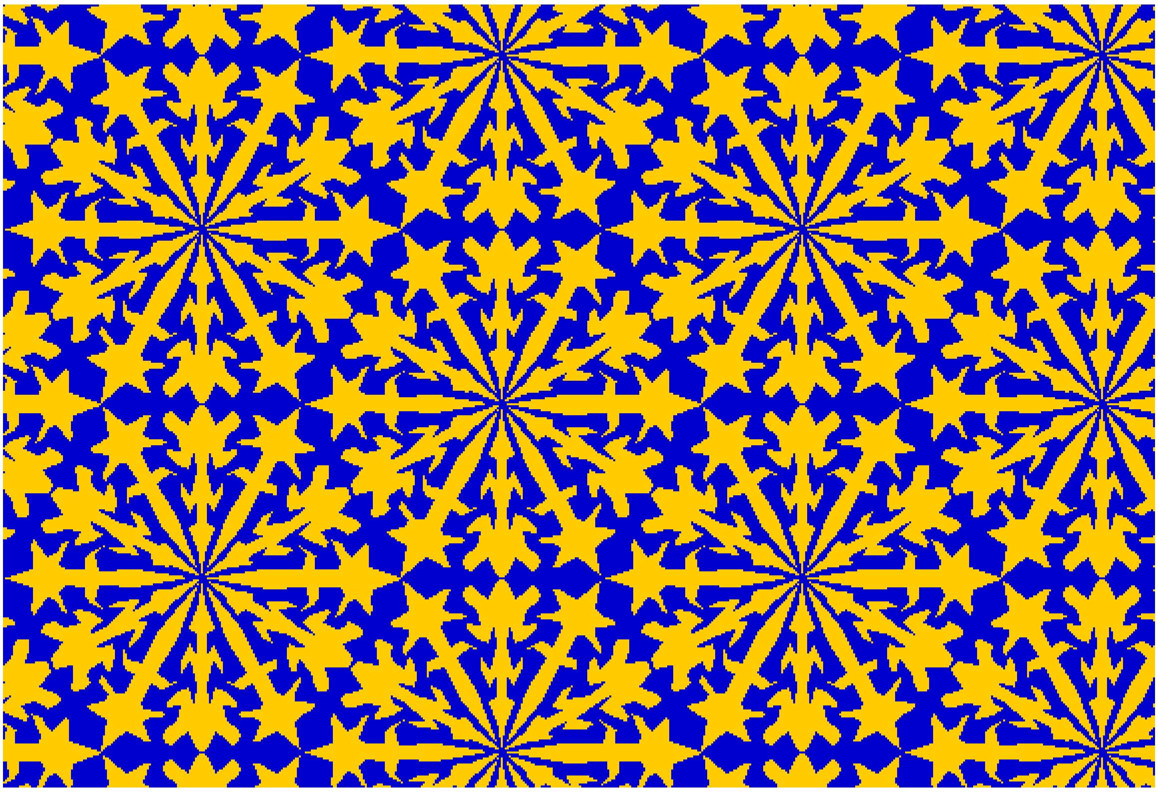
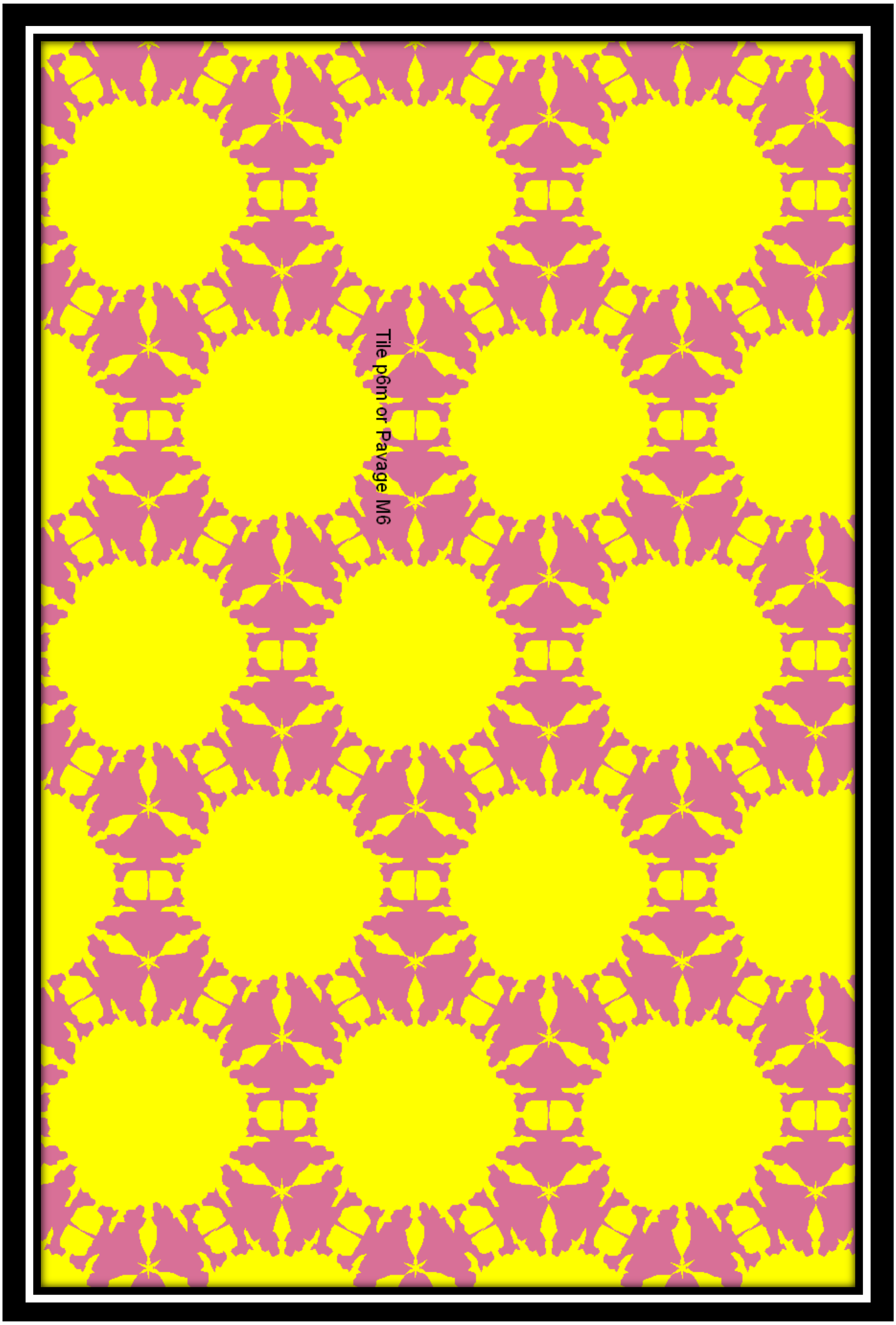
<td>
Take a triangular piece of paper. (The half of an equilateral triangle).</td>
<td>
Draw a simple curve starting in 1 and going to 2.</td>
</tr>
<tr>
<td>
Draw a simple curve starting in 2 and going to 3.</td>
<td>
This is your pattern for this tiling group.</td>
</tr>
</table>

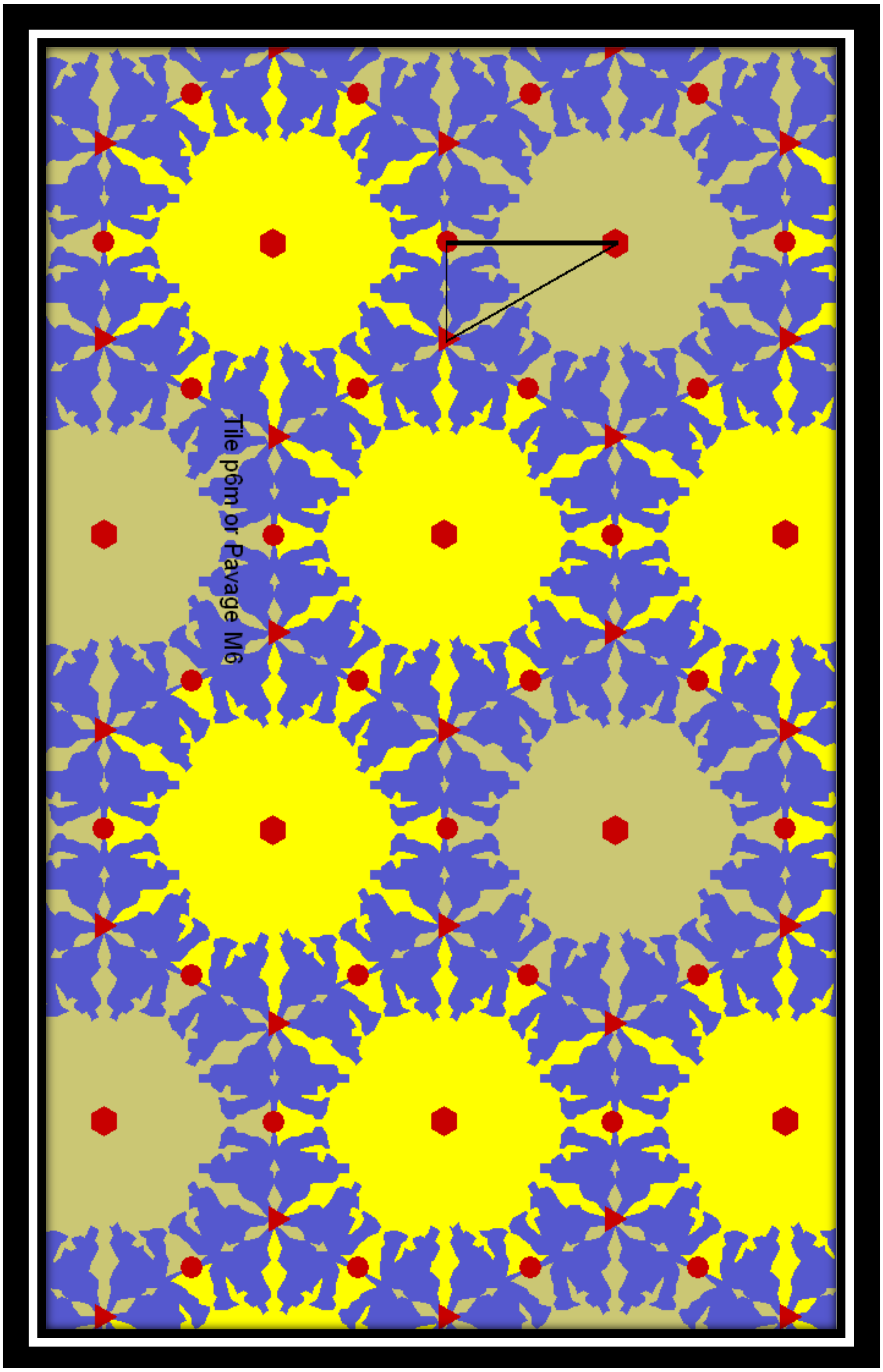
We fulfill the plane by using symmetries with axis the sides of the rectangle triangle:



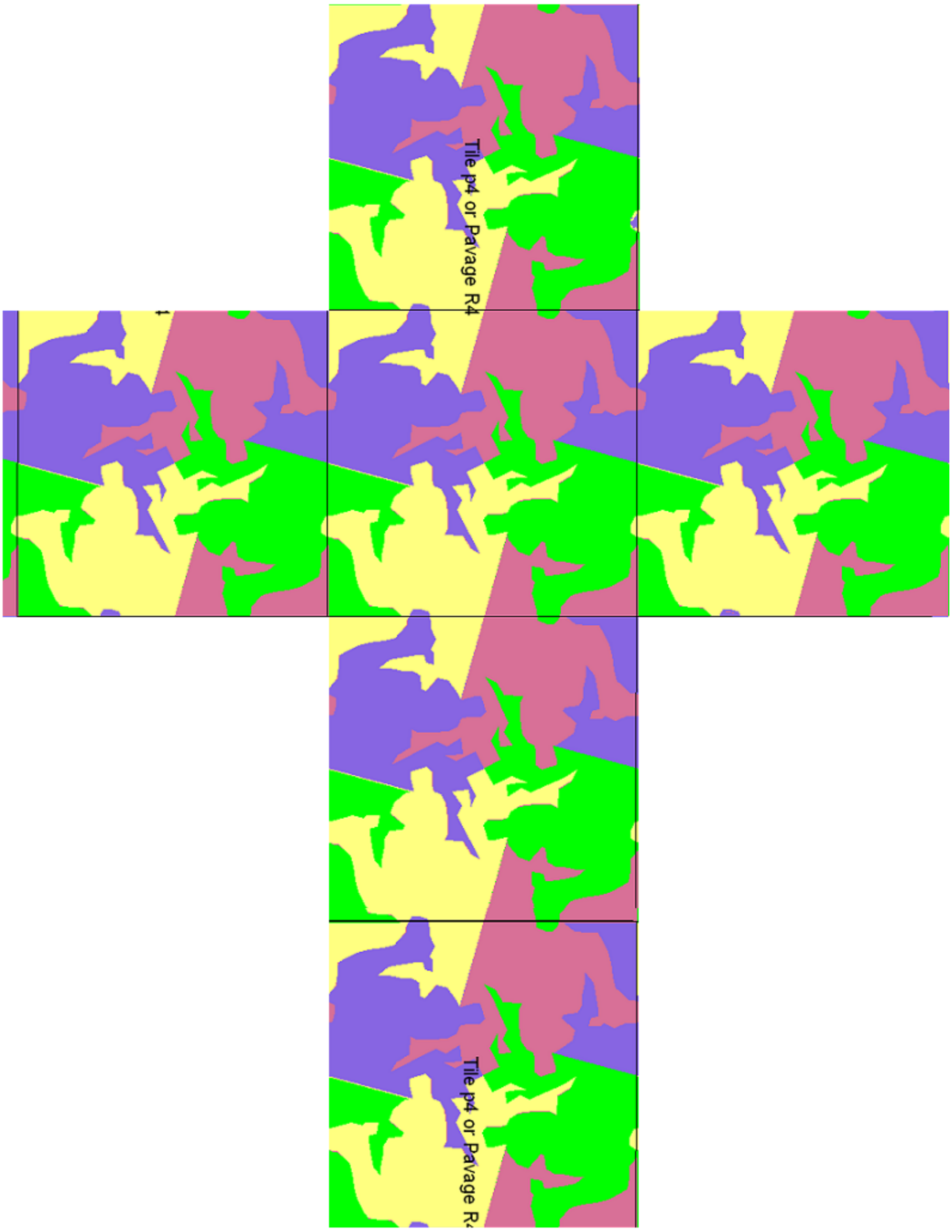

Tile p6m or Pavage M6

Lattice and fundamental region for p6m (M6)

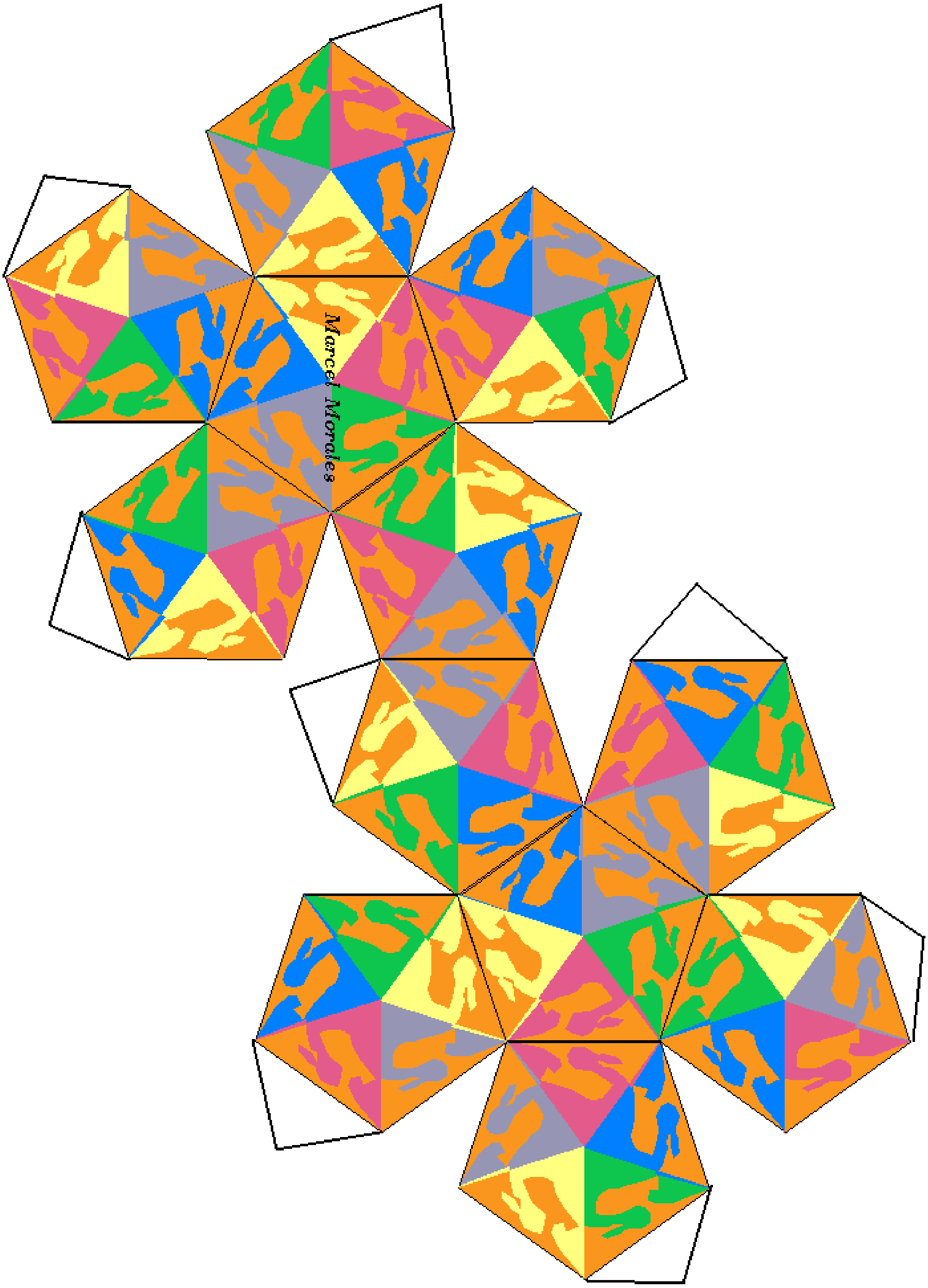
The following picture is the lattice associated to the Tiling p31m (M3R3), the fundamental region is equilateral triangle, and as you can check there are translations following the sides of the triangles. The lattice is the same as the tiling p3 (R3), and we must add the mirror lines, that is the sides of the triangles.

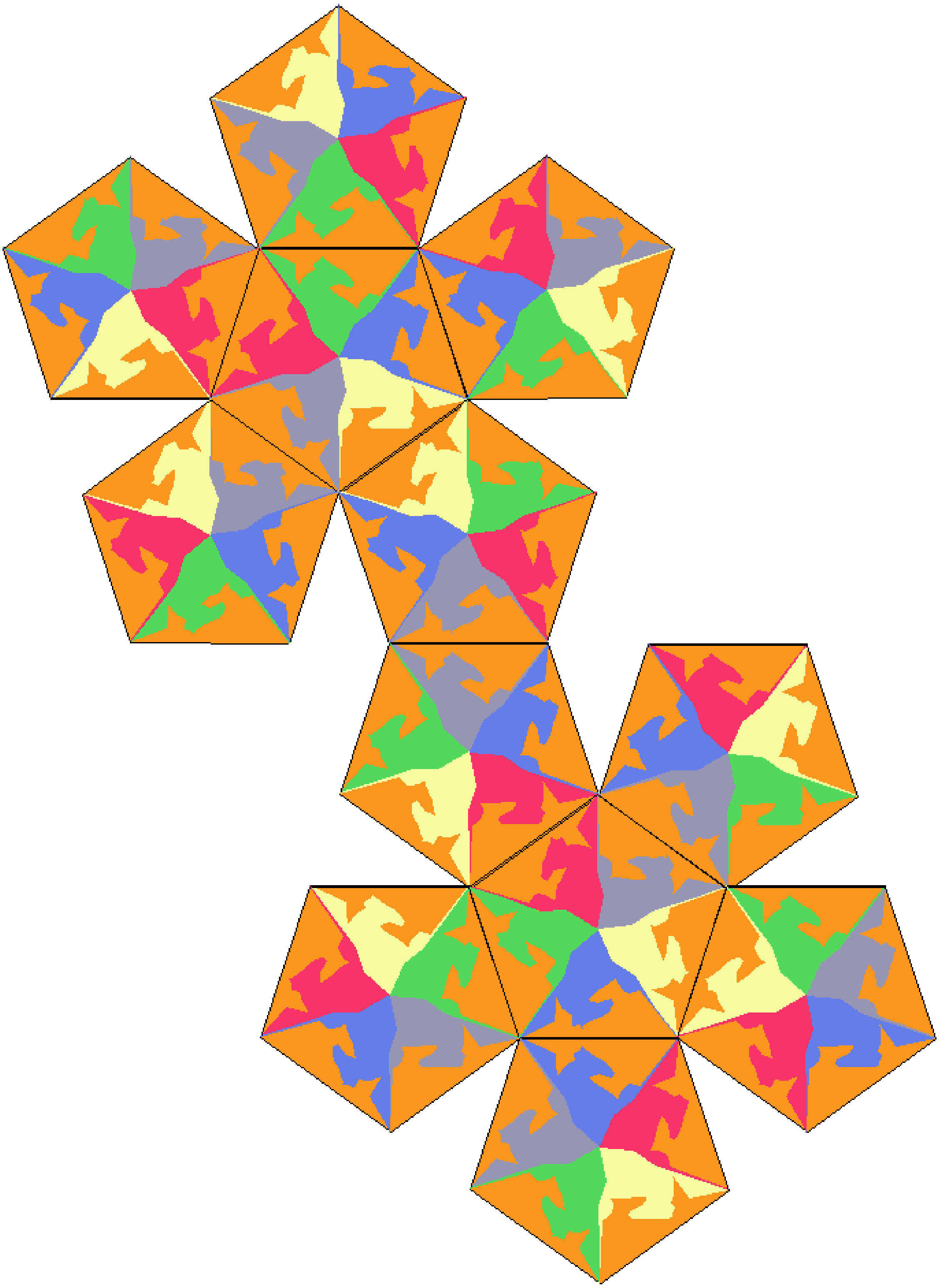


Pattern of a tiling of a cube

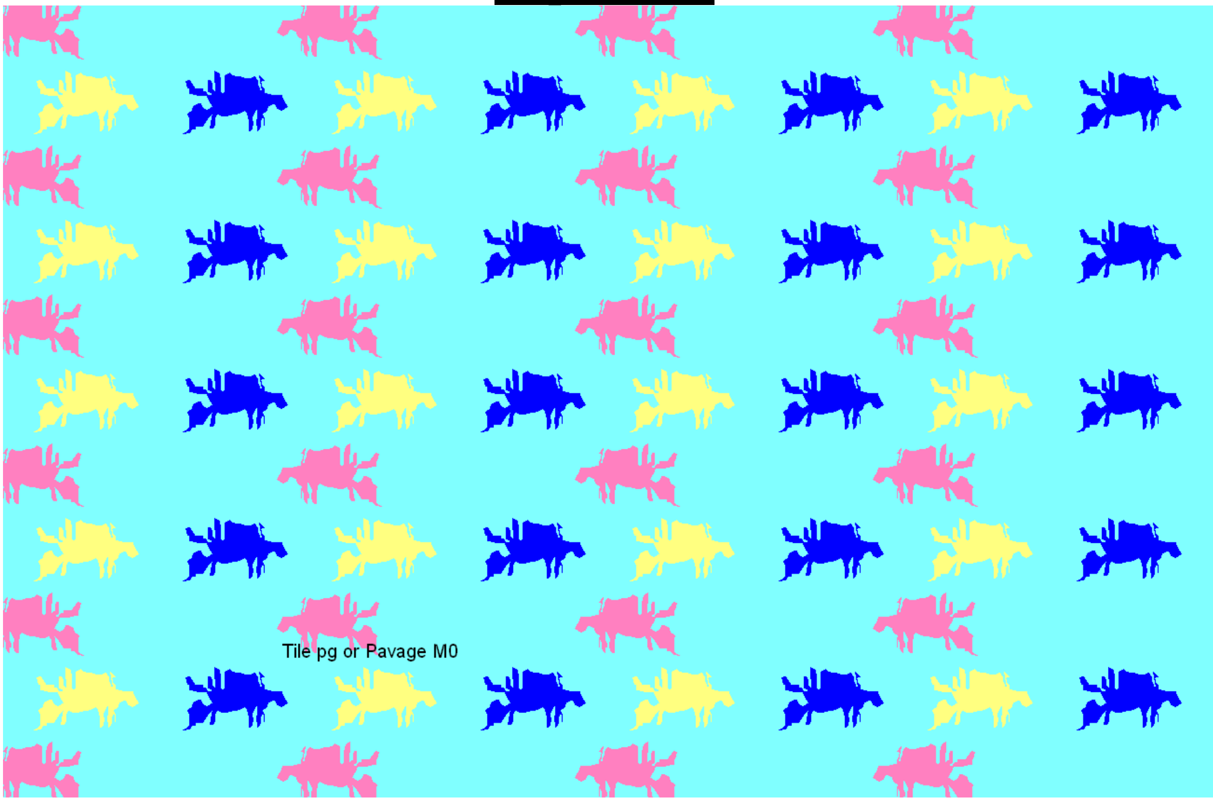


Pattern of a tiling of a dodecahedron


Marcel Morales



Paper wall



The hyperbolicTilings

We understand by hyperbolic geometry, the drawn of pictures and transformations by using the hyperbolic distance. The Hyperbolic plane is represented by a disk without the border, the border of the disk is the infinite; figures in the hyperbolic plane seem smaller when we approach to infinite. We give some examples of hyperbolic Tiling of the hyperbolic plane, drew by my software. My software is the first in the world to drawn automatically hyperbolic tiling and allows anybody to realize its own hyperbolic tiling. By contrast with Euclidian geometry there are infinitely many hyperbolic tiling groups.