To integrate this effort with existing algorithms, we start with the framework of effective homology mentioned above, and we introduce an analogous definition of *polynomial-time homology*; see Sect. 2. In another paper [3], we show that various known constructions and operations on objects with effective homology have polynomial-time versions. With a repertoire of such operations, we also obtain a polynomial-time version of the algorithm of [2], as well as other algorithms, such as computing the higher homotopy group $\pi_k(X)$ in polynomial time for every figoritxed k, or computing the first k stages of a Postnikov system for X.

This Paper Here we make a significant step in this development. First we set up the framework of polynomial-time homology (modeled after effective homology mentioned above) and some tools of general applicability. Then, in the second part of the paper, we present our main technical result. The problem which we solve can will be briefly introduced in Sect. 2 below), a so-called *Kan simplicial set*. We use the standard Eilenberg–MacLane simplicial model for $K(G,k)$; see [8, Chap. III], [15

simplicial sets uses the language of category theory and is very elegant and concise; see, e.g., [13, Sect. I.1].

- $f : C_{\bullet} \to \tilde{C}_{\bullet}$ and $g : \tilde{C}_{\bullet} \to C_{\bullet}$ are chain maps;
- the composition $fg : \tilde{C}_{\bullet} \to \tilde{C}_{\bullet}$ is equal to the identity id

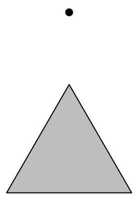

Fig. 1 A triangulation of the real projective plane with a discrete vector field (after Forman [10], Fig. 4.1). Pairs of vertices with the same label should be identified; thus, there are only one critical edge and one critical vertex

defined based on V, and they are locally effective assuming that X

To check that the iterations of Φ indeed stabilize after finitely many steps, it suffices to consider the case $c = 1 \cdot \sigma$, and then the stabilization follows easily from the above discussion of the action of Φ (and from the admissibility of the vector field V).

As the diagram suggests, we put $f := j\Phi^{\infty}$ and $g := i\Phi^{\infty}$. Then, since j and Φ^{∞} are mutually inverse and Φ

The first vector field will be denote by V_{bs} and called the

where $k_j \geq 2$ *is even, while all of the other*

The Anatomy of a Simplex Let $\sigma = [a_1|a$

i.e., we split the last component of σ into two equal halves.

Lemma 4.2 *This definition indeed yields a vector field, and the only critical sim-*

(B) *If* $i_j < i_{j+}$

11. P. Franek, S. Ratschan, P. Zgliczynski, Satisfiability of systems of equations of real analytic functions is quasi-decidable, in *Proc. 36th International Symposium on Mathematical Foundations of Computer ScienceS(MFCS)*