

I repeat two exercises from Sheet 2 because they were about some material I did not cover in my second lecture.

**Exercise 1.** Let  $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z)$  be a pointed Heegaard diagram,  $\mathbf{x}$  a Heegaard Floer generator,  $P$  a periodic domain and  $[P] \in H_2(M; \mathbb{Z})$  the homology class associated to it. Prove that

$$c_1(\mathfrak{s}_z(\mathbf{x})) = e(P) + 2n_{\mathbf{x}}(P).$$

**Exercise 2.** Let  $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z)$  be a pointed Heegaard diagram and let  $z'$  be another basepoint separated from  $z$  by the curve  $\alpha_i$ .

●  $z$



●  $z'$

Compute  $\mathfrak{s}_z(\mathbf{x}) - \mathfrak{s}_{z'}(\mathbf{x})$  for a Heegaard Floer generator  $\mathbf{x}$ .

**Exercise 3.** Let  $\mathbf{x}$  and  $\mathbf{y}$  be Heegaard Floer generators. prove that

$$\mathfrak{s}_z(\mathbf{y}) - \mathfrak{s}_z(\mathbf{x}) = \epsilon(\mathbf{x}, \mathbf{y}).$$

Be careful with the next exercise because I couldn't solve it, but it should be doable.

**Exercise 4.** Choose (arbitrary) orientations of the  $\alpha$ - and  $\beta$ - curves. Then to each intersection point  $x$  we assign  $\sigma(x) = \pm 1$  depending on the sign of the intersection (since the sign depends on the order in dimension two, we fix the convention that  $\alpha$  is first and  $\beta$  second). Given an a Heegaard Floer generator  $\mathbf{x} = \{x_1, \dots, x_n\}$  we define  $\sigma(\mathbf{x}) = \prod \sigma(x_i)$ .

Prove that, for every  $u \in \mathcal{M}(\mathbf{x}, \mathbf{y})$ , we have that  $I(\mathcal{D}(u)) \equiv 0 \pmod{2}$  if  $\sigma(\mathbf{x})\sigma(\mathbf{y}) = 1$  and  $I(\mathcal{D}(u)) \equiv 1 \pmod{2}$  if  $\sigma(\mathbf{x})\sigma(\mathbf{y}) = -1$ .

**Exercise 5.** Let  $(T^2, \alpha, \beta, \gamma, z)$  be the triple Heegaard diagram where, for some orientation of  $\alpha$ ,  $\beta$  and  $\gamma$ ,  $\alpha \cdot \beta = 1$  and  $\gamma = \alpha + n\beta$  for  $n \in \mathbb{Z}$ . The placement of  $z$  is up to you. Compute the maps  $F_{\alpha, \beta, \gamma}^-$ .