**Exercise 1.** Prove that $\tilde{HF}(M) = 0$ if and only if $HF^+(M) = 0$.

The next exercise is a slightly harder version of the previous one.

**Exercise 2.** Let $\tilde{F}: \tilde{HF}(M) \to \tilde{HF}(N)$ and $F^+: HF^+(M) \to HF^+(N)$ be linear maps such that the diagram

$$
\begin{array}{cccccccc}
\ldots & \to & \tilde{HF}(M) & \to & HF^+(M) & \to & \tilde{HF}(M) & \to & \ldots \\
 & & F & \downarrow & F^+ & \downarrow & F^+ & \downarrow & \\
\ldots & \to & \tilde{HF}(N) & \to & HF^+(N) & \to & \tilde{HF}(N) & \to & \ldots \\
\end{array}
$$

commutes. Then $\tilde{F}$ is an isomorphism if and only if $F^+$ is an isomorphism.

**Exercise 3.** Let $M$ be a three-manifold with boundary which has $g$ pairwise disjoint properly embedded discs $D_1, \ldots , D_g$ such that $M \setminus (D_1 \cup \ldots \cup D_g)$ is homeomorphic to a ball. Then $M$ is a genus $g$ handlebody.

**Exercise 4.** Let $S$ be a surface with boundary. Then $S \times [-1, 1]$ is homeomorphic to a handlebody. What is its genus?

**Exercise 5.** Let $\Sigma \subset S^3$ be a standardly embedded genus $g$ surface. Show that $\Sigma$ decomposes $S^3$ in two handlebodies and draw the corresponding Heegaard diagram.

**Exercise 6.** Which three-manifold is described by a genus $g$ Heegaard diagram $(\Sigma, \alpha, \beta)$ such that $\alpha_i$ is isotopic to $\beta_i$ for all $i = 1, \ldots , g$?

**Exercise 7.** Let $(\Sigma, \alpha, \beta, z)$ be a pointed Heegaard diagram for a three-manifold $M$. Show that the group of the periodic domains of $(\Sigma, \alpha, \beta, z)$ is isomorphic to $H_2(M; \mathbb{Z})$.

**Exercise 8.** Regard $S^3 = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = 1\}$. Given $1 \leq q < p$ we define the lens space $L(p, q)$ as the quotient of $S^3$ by the equivalence relation

$$(z_1, z_2) \sim (e^{\frac{2\pi i q}{p}} z_1, e^{\frac{2\pi i q}{p}} z_2).$$

Draw a genus one Heegaard diagram for $L(p, q)$.

**Exercise 9.** Prove that $S^3$, $S^2 \times S^1$ and the lens space $L(p, q)$ are the only three-manifolds with a genus one Heegaard splitting.