Errata to “Two questions on mapping class groups”

Louis Funar
Institut Fourier BP 74, UMR 5582
University of Grenoble I
38402 Saint-Martin-d'Hères cedex, France
e-mail: funar@fourier.ujf-grenoble.fr

August 2, 2011

The definition of property FA_n used in ([5], section 3) is the one given by Farb in [4]:

Definition 0.1. *Let $n \geq 1$. A group Γ has property FA_n if any isometric action on any n-dimensional CAT(0) cell complex X has a fixed point.*

However all complexes considered in [4] are complete and with finitely many isometry types of cells. It is convenient then to amend the terminology, as follows:

Definition 0.2. *Let $n \geq 1$. A group Γ has property FA_n if any isometric action on any n-dimensional complete CAT(0) cell complex X with finitely many isometry types of cells has a fixed point.*
A group Γ has property very strong FA_n if any isometric action on any n-dimensional complete CAT(0) cell complex X has a fixed point.

Recall also the following notion studied by Bridson:

Definition 0.3. *Let $n \geq 1$. A group Γ has property strong FA_n if any isometric action by semi-simple isometries on any complete CAT(0) cell complex X with the property that $\tilde{H}_n(Y) = 0$ for every open subset $Y \subset X$ has a fixed point.*

One is proved in [3] that any isometric action on a complete CAT(0) cell complex X with finitely many isometry types of cells is actually semi-simple so that strong FA_n implies FA_n.

Now the main result of ([5], section 3) should be restated with this terminology as follows:

Proposition 0.1. *If Γ is a finitely generated group with property very strong FA_{n^2-1} then the representations $\Gamma \to SL(n, \mathbb{C})$ have finite image.*

We claimed in [5] that Bridson proved in [1, 2] that M_g has property (very strong) FA_g in order to use the previous proposition to derive that representations $M_g \to SL([\sqrt{g+1}], \mathbb{C})$ have finite images.

However, the result of Bridson from ([2], see also [1]) is that M_g has strong FA_{g-1}. It is presently unknown whether M_g has the property very strong FA_{g-1}. Thus the proof given in [5] of the inequality $N_g \geq \sqrt{g}$ (Proposition 1.2 from [5]) is incomplete.

Nevertheless this inequality from Proposition 1.2 in [5] is true. A sharp estimate was recently obtained by Franks and Handel [6] and Korkmaz [7] independently, as follows:

Theorem 0.1. *The images of linear representations $M_g \to SL(2g-1, \mathbb{C})$ are finite, for all $g \geq 1$.*

Their result is sharp since the homomorphism $M_g \to Sp(2g, \mathbb{Z})$ is surjective. This also implies that $N_g \geq 2g-1$, which considerably improves the estimate from [5].

I am grateful to Martin Bridson for pointing out this error and for useful discussions on this subject and to Jaka Smrekar for pointing out that in [2] one proves that M_g has property FA_{g-1} and not – as we claimed in [5] – property FA_g.

References

[1] M.R.Bridson, *Semisimple actions of mapping class groups on CAT(0) spaces*, "The Geometry of Riemann Surfaces", Dedicated to Bill Harvey on his 65th birthday, 114, London Math. Soc. Lecture Note Ser., 368, Cambridge Univ. Press, Cambridge, 2010.

[2] M.R.Bridson, *On the dimension of CAT(0) spaces where mapping class groups act*, arXiv:0908.0690, Crelle's Journal, to appear.

[3] M.R.Bridson and A.Haefliger, Metric spaces of non-positive curvature, *Grund. Math. Wissenschaften*, Springer, 1999.

[4] B.Farb, *Group actions and Helly's theorem.* Advances Math. 222(2009), 1574-1588.

[5] L.Funar, *Two questions on mapping class groups*, Proc. Amer. Math. Soc. 139(2011), 375-382.

[6] J.Franks and M.Handel,*Triviality of some representations of* $MCG(S_g)$ *in* $GL(n,C), Diff(S^2)$ *and* $Homeo(T^2)$, arXiv:1102.4584.

[7] M.Korkmaz, *Low-dimensional linear representations of mapping class groups*, arXiv:1104.4816.