Exo 33

1) $P\,dx + Q\,dy$ fermée si

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$P = -y \,,\quad Q = x$$

$$\frac{\partial P}{\partial y} = -1 \,,\quad \frac{\partial Q}{\partial x} = 1 .$$

$\rightarrow$ pas fermée, donc pas exacte

2) $P = -y\,(x^2+y^2)^\alpha$

$Q = x\,(x^2+y^2)^\alpha$

$$\frac{\partial P}{\partial y} = -(x^2+y^2)^\alpha - y\cdot\alpha(x^2+y^2)^{\alpha-1}\cdot 2y$$

$$= -(x^2+y^2)^{\alpha-1}\left(x^2+y^2+2\alpha y^2\right)$$

$$= -(x^2+y^2)^{\alpha-1}\left(x^2+(1+2\alpha)y^2\right)$$

$$\frac{\partial Q}{\partial x} = (x^2+y^2)^\alpha + x\cdot\alpha(x^2+y^2)^{\alpha-1}\cdot 2x$$

$$= (x^2+y^2)^{\alpha-1}\left(x^2+y^2+2\alpha x^2\right)$$

$$= (x^2+y^2)^{\alpha-1}\left((1+2\alpha)x^2+y^2\right).$$

on veut $-x^2-(1+2\alpha)y^2 = (1+2\alpha)\,x^2+y^2$

ie $1+2\alpha = -1$, $2\alpha = -2$, $\alpha = -1$

3) $\int_0^{2\pi} (a\cos t\cdot b\cos t + b\sin t\cdot a\sin t)\,dt =$

$x = a\cos t \quad x' = -a\sin t$

$y = b\sin t \quad y' = b\cos t$

$ab\int_0^{2\pi} dt = 2\pi ab.$

4) aire $= \frac{1}{2}\cdot 2\pi ab = \pi ab.$

5) $x = \cos t$

$y = \sin t.$

$$\int_0^{2\pi} \frac{x y' - y x'}{x^2+y^2}\,dt =$$

$$\int_0^{2\pi} \frac{\cos^2 t+\sin^2 t}{\cos^2 t+\sin^2 t}\,dt = 2\pi$$

intégrale sur un chemin fermé $\neq 0$

$\rightarrow$ ne peut pas être exacte !

6) $\frac{\partial V}{\partial x} = \frac{-y}{x^2+y^2}$ $\quad\quad$ $\frac{\partial V}{\partial y} = \frac{x}{x^2+y^2}$

$$= \frac{-1/y}{1+\left(\frac{x}{y}\right)^2}$$

$$-\frac{1}{y}\int \frac{1}{1+\left(\frac{x}{y}\right)^2}\,dx = -\frac{1}{y}\int \frac{y\,du}{1+u^2}$$

$\frac{x}{y} = u$

$dx = y\,du$

$$= -\operatorname{atan}(u) + \phi(y)$$

$$= -\operatorname{atan}\left(\frac{x}{y}\right) + \phi(y).$$

$$V(x,y) = -\operatorname{atan}\left(\frac{x}{y}\right) + \phi(y)$$

$$\frac{\partial V}{\partial y} = -\frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\frac{-x}{y^2} + \phi'(y).$$

$$= \frac{x}{x^2+y^2} + \underbrace{\phi'(y).}_{\text{on peut prendre } \phi = 0.}$$

(potentiel marche pour $y \neq 0$).

7) $$\int_\gamma \omega = V(\gamma(b)) - V(\gamma(a)) = 0$$

$\gamma : [a,b] \to (\mathbb{R}^{+*})^2$

$\gamma(a) = \gamma(b).$