Exo 31 $\quad x(t) = \cos^3 t - \cos t \qquad f(t) = (x(t), y(t))$

$\qquad y(t) = \sin^3 t + \sin t$

1) $f(t+2\pi) = f(t) \rightsquigarrow [-\pi, \pi]$ suffit

$f(-t) = ? \qquad x(-t) = x(t)$ symétrie Ox

$\qquad y(-t) = -y(t)$

$\rightsquigarrow [0, \pi]$ suffit

$f(\pi - t) = ?$

$$x(\pi-t) = (\cos(\pi-t))^3 - \cos(\pi-t)$$
$$= -(\cos t)^3 + \cos t = -x(t)$$
$$y(\pi-t) = (\sin(\pi-t))^3 + \sin(\pi-t)$$
$$= (\sin t)^3 + \sin t = y(t)$$

symétrie Oy $\rightsquigarrow [0, \frac{\pi}{2}]$ suffit.

2)

$$x(t) = \cos t(\cos^2 t - 1)$$
$$= \cos t\left(\frac{1+\cos 2t}{2} - 1\right)$$
$$= \frac{1}{2}\cos t(\cos 2t - 1)$$
$$y(t) = \sin t(\sin^2 t + 1) = \sin t\left(\frac{1-\cos 2t}{2} + 1\right)$$
$$= \frac{1}{2}\sin t(3 - \cos 2t)$$
$$x'(t) = -\frac{1}{2}\sin t(\cos 2t - 1) + \frac{1}{2}\cos t(-2\sin 2t)$$
$$= -\frac{1}{2}\sin t(\cos 2t - 1) - 2\sin t\cos^2 t$$
$$= -\sin t\left(\frac{1}{2}\cos 2t - \frac{1}{2} + 2\cos^2 t\right)$$
$$= -\sin t(3\cos^2 t - 1) = \sin t(1 - 3\cos^2 t)$$
$$y'(t) = \frac{1}{2}\cos t(3 - \cos 2t) + \sin t(\sin 2t)$$
$$= \frac{1}{2}\cos t(3 - \cos 2t) + 2\sin^2 t\cos t$$
$$= \cos t\left(\frac{3}{2} - \frac{1}{2} + \sin^2 t + 2\sin^2 t\right)$$
$$= \cos t(1 + 3\sin^2 t)$$

pour $t = k\pi$, $x'(t) = 0$ mais $y'(t) \neq 0$

$t = \pm\arccos\frac{1}{\sqrt{3}} + k\cdot 2\pi$: $y'(t) \neq 0$

$\rightarrow$ pas de pt sing.

t	0		$\arccos\frac{1}{\sqrt{3}}$		$\frac{\pi}{2}$
x'	0	$-$	0	$+$	$+$
y'	$+$	$+$	$+$	$+$	0
	$(0,0)$	$\nwarrow$	$\left(-\frac{2}{3\sqrt{3}}, \frac{5\sqrt{2}}{3\sqrt{3}}\right)$	$\nearrow$	$(0,2)$

4) $$\|f'(t)\|^2 = \sin^2 t(1-3\cos^2 t)^2 + \cos^2 t(1+3\sin^2 t)^2$$

$$= \sin^2 t(1+9\cos^4 t - 6\cos^2 t) + \cos^2 t(1+9\sin^4 t + 6\sin^2 t)$$

$$= (1-\cos^2 t)(1-6\cos^2 t + 9\cos^4 t) + \cos^2 t\,(1+6-6\cos^2 t + 9(1-2\cos^2 t+\cos^4 t))$$

$$= 1 - 6\cancel{\cos^2 t} + 9\cos^4 t - \cancel{\cos^2 t} + 6\cancel{\cos^4 t} - 9\cancel{\cos^6 t} + 7\cancel{\cos^2 t} - 6\cancel{\cos^4 t} + 9\cos^2 t - 18\cos^4 t + 9\cancel{\cos^6 t}$$

$$= 1 + 9\cos^2 t - 9\cos^4 t$$

$$\int_0^{\pi/2} \sqrt{1+9\cos^2 t - 9\cos^4 t}\,dt \approx 2.247$$

longueur totale ≈ 8.987

5) $$\vec{T}(t) = \frac{1}{\sqrt{1+9\cos^2 t - 9\cos^4 t}}\left(\sin t(1-3\cos^2 t), \cos t(1+3\sin^2 t)\right)$$

$$\vec{N}(t) = \frac{1}{\sqrt{}}\left(-\cos t(1+3\sin^2 t), \sin t(1-3\cos^2 t)\right)$$

$$f\left(\frac{\pi}{4}\right) = \left(-\frac{1}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)$$

$$f'\left(\frac{\pi}{4}\right) = \left(-\frac{1}{2\sqrt{2}}, \frac{5}{2\sqrt{2}}\right)$$

$$\left(f''\left(\frac{\pi}{4}\right) = \left(\frac{5}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)\right.$$

↳ orthogonaux!

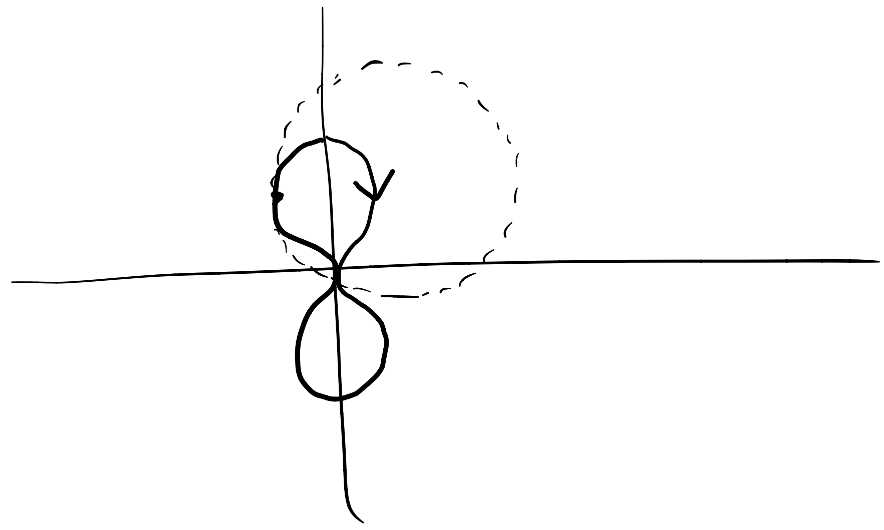
$$\vec{T}\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{13}}\left(-\frac{1}{2\sqrt{2}}, \frac{5}{2\sqrt{2}}\right)$$

$$\vec{N}\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{13}}\left(-\frac{5}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right)$$

$$f''\left(\frac{\pi}{4}\right) = -\frac{\sqrt{13}}{2}\vec{N}\left(\frac{\pi}{4}\right)$$

$$\frac{\sqrt{13}}{2} = \frac{\left(\frac{\sqrt{13}}{2}\right)^2}{R}$$

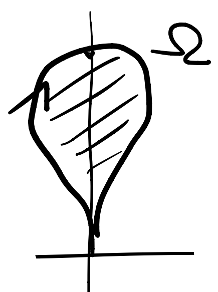
$$\boxed{R = \frac{\sqrt{13}}{2}}$$



$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

6)

$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$



$$1 - 3\cos^2 t = -\frac{1}{2} - \frac{1}{2}\cos 2t$$

$$\int_\Omega dx\wedge dy = -\int_\gamma x\,dy$$
$$d(x\,dy) = dx\wedge dy\ )$$
$$x(t) = \cos t(\cos^2 t - 1)$$
$$y'(t) = \cos t(1+3\sin^2 t)$$

$$-\int_0^\pi \cos^2 t\,(\cos^2 t - 1)(1+3\sin^2 t)\,dt$$
$$= -\int_0^\pi \frac{\cos(2t)+1}{2}\cdot\frac{\cos(2t)-1}{2}\cdot\left(\frac{5}{2}-\frac{1}{2}\cos 2t\right)dt$$
$$= \frac{1}{8}\int_0^\pi (\cos^2 2t - 1)(\cos 2t - 5)\,dt$$
$$= \frac{1}{8}\int_0^\pi \left(\frac{1+\cos 4t}{2} - 1\right)(\cos 2t - 5)\,dt$$
$$= \frac{1}{16}\int_0^\pi (\cos 4t - 1)(\cos 2t - 5)\,dt$$
$$= \frac{1}{16}\int_0^\pi (\cancel{\cos 4t\cos 2t} - 5\cancel{\cos 4t} - \cancel{\cos 2t} + 5)\,dt$$

$$\cos(6t) = \cos(4t+2t) = \cos 4t\cos 2t - \sin 4t\sin 2t$$
$$\cos(2t) = \cos(4t-2t) = \cos 4t\cos 2t + \sin 4t\sin 2t$$

$$\cos 4t\cos 2t = \frac{1}{2}(\cos 6t + \cos 2t)$$
intégrale nulle sur $[0,\pi]$

$$\dots = \frac{5\pi}{16} \approx 0.98$$