ERRATUM AND COMMENTS

Chapter II

p. 26, two lines before Lemma II.1.10, it says "The lemma below describe". The last word should be "describes".

Chapter III

p. 35, Remark II.1.12, it says "still hold". The last word should be "holds".

Chapter IV

p. 68, immediately after the first equation: read "where n is the number of antennas..." (instead of M)

Chapter V

- Exercise 5. (e) : the results of the previous questions allow only to prove $\delta_{min}(\mathcal{C}_{A,I}) \geq p$. The equality is actually true, but one needs deeper arguments to prove it.

Chapter VII

- p.177, before Remark VII.6.4 : the equality $\delta_{min}(\mathcal{C}_{A,\lambda,I}) = \dfrac{1}{p}$ is established, provided that we know $\delta_{min}(\mathcal{C}_{A,I}) = p$, which may be proved using Proposition VII.6.3., and the fact that I is principal (this last part is not obvious, and needs class field theory arguments to be proved).

- Exercise 4. (e) : $\alpha = -2 - i - \theta$ if $p = 29$

Chapter IX

- Lemma IX.5.25 is not true in full generality because Λ is not necessarily discrete. For example, take $n = 1$ and $k = \mathbb{Q}(\sqrt{2})$ and $n = 1$. There are also counterexamples if k is not real. One may take $n = 1$ and $k = \mathbb{Q}(i\sqrt{2\cos(\pi/9)})$. Then $z = \dfrac{1 + i\sqrt{2\cos(\pi/9)}}{1 - i\sqrt{2\cos(\pi/9)}}$ is a unit of $\mathcal{O}_k$ such that $z\overline{z} = 1$ which has infinite order. In fact the right statement is:

Lemma IX.5.25. Assume that $k/\mathbb{Q}$ is a quadratic imaginary extension. Let Λ be a subring of B which is also an $\mathcal{O}_k$-module of finite rank. Then $\mathbf{U}(B, \tau) \cap \Lambda^\times$ is finite.

Proof. Let $n = \deg(B)$. By Lemma IX.5.8, the injective morphism of k-algebras

$$\psi\colon \begin{array}{rcl} B & \longrightarrow & \mathrm{M}_n(\mathbb{C}) \\ b & \longmapsto & \mathbf{U}_b \end{array}$$

restricts to an injective group morphism $\psi : \mathbf{U}(B,\tau) \longrightarrow \mathbf{U}_n(\mathbb{C})$. Since ψ is injective, $\psi(\Lambda)$ is a free $\mathcal{O}_k$-module of finite rank. Since $k/\mathbb{Q}$ is quadratic imaginary, $\mathcal{O}_k$ is a full rank $\mathbb{Z}$-lattice of the $\mathbb{R}$-vector space $\mathbb{C} \simeq_{\mathbb{R}} \mathbb{R}^2$. Thus $\psi(\Lambda)$ is a $\mathbb{Z}$-lattice of $\mathrm{M}_n(\mathbb{C}) \simeq_{\mathbb{R}} \mathbb{R}^{2n^2}$. It follows $\mathbf{U}_n(\mathbb{C}) \cap \psi(\Lambda)$ is finite, since $\mathbf{U}_n(\mathbb{C})$ is compact. Consequently, $\psi(\mathbf{U}(B,\tau) \cap \Lambda^\times) \subset \mathbf{U}_n(\mathbb{C}) \cap \psi(\Lambda)$ is finite, and thus $\mathbf{U}(B,\tau) \cap \Lambda^\times$ is also finite. This concludes the proof.

- Remark IX.5.26 need to be modified accordingly. In fact, one can even assume that Λ is a free $\mathcal{O}_k$-module generated by a k-basis of B. Keeping the same notation, if we assume furthermore that $e_1 = 1_B$, then we may set

$$\Lambda = 1_B\mathcal{O}_k \oplus me_2\mathcal{O}_k \cdots \oplus me_{n^2}\mathcal{O}_k.$$