Colloquium MathAlp

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Oscillating about coplanarity in the 4 body problem

For the Newtonian 4-body problem in space we prove that any zero angular momentum bounded solution suffers infinitely many coplanar instants, that is, times at which all 4 bodies lie in the same plane. This result generalizes a known result for collinear instants ("syzygies") in the zero angular momentum planar 3-body problem, and extends to the d+1 body problem in d-space. The proof, for d=3, starts by identifying the center-of-mass zero configuration space with real 3×3 matrices, the coplanar configurations with matrices whose determinant is zero, and the mass metric with the Frobenius (standard Euclidean) norm. Let S denote the signed distance from a matrix to the hypersurface of matrices with determinant zero. The proof hinges on establishing a harmonic oscillator type ODE for S along solutions. Bounds on inter-body distances then yield an explicit lower bound ω for the frequency of this oscillator, guaranteeing a degeneration within every time interval of length π/ω. The non-negativity of the curvature of oriented shape space (the quotient of configuration space by the rotation group) plays a crucial role in the proof.