France - Taiwan joint Conference on Nonlinear Partial Differential Equations

CIRM (Marseille, France), March 25 to 28, 2008

Organizing committee :

Thierry Gallay (Université Joseph Fourier Grenoble I)
Jong-Shenq Guo (National Taiwan Normal University)
François Hamel (Université Paul Cézanne Aix-Marseille III)
Jean-Michel Roquejoffre (Université Paul Sabatier Toulouse III)

Invited speakers :

From Taiwan :

Jung-Chao Ban	National Hualien University of Education
Chiao-Nien Chen	National Changhua University of Education
Kuo-Chang Chen	National Tsing Hua University
Jong-Shenq Guo	National Taiwan Normal University
John Meng-Kai Hong	National Central University
Sze-Bi Hsu	National Tsing Hua University
Chang-Shou Lin	National Taiwan University
Chun-Chi Lin	National Taiwan Normal University
Jenn-Nan Wang	National Taiwan University

From France :

Guy Barles	Université de Tours
Karine Beauchard	Ecole Normale Supérieure de Cachan
Henri Berestycki	EHESS Paris
Yann Brenier	CNRS et Université de Nice
Didier Bresch	CNRS et Université de Savoie
Anne de Bouard	CNRS et Ecole Polytechnique
Albert Fathi	Ecole Normale Supérieure de Lyon
Isabelle Gallagher	Université de Paris VII
Michel Ledoux	Université de Toulouse III
Emmanuel Russ	Université d'Aix-Marseille III
Laure Saint-Raymond	Université de Paris VI
Eric Séré	Université de Paris IX Dauphine

Plenary Talks

Entropy and Zeta Function in One-Dimensional Multilayer Cellular Neural Networks

Jung-Chao Ban, National Hualien University of Education

Abstract: In this talk, we study the one-dimensional MCNN (Multilayer Cellular Neural Networks). The parameter space for 1-D MCNN can be completely characterized. For a given region of parameter space, the local patterns with 1-layer input can be completely described; however, new concepts arise that do not have 1-D CNN analogues in computing the entropy and zeta functions. Thus, the "Sofic shift" is introduced to help us for understanding the complexity of such equations. Finally, the problem for 2-D case is also addressed here.

On the Generalized Dirichlet Problem for Viscous Hamilton-Jacobi Equations

Guy Barles, Université de Tours

Abstract: We consider the Dirichlet problem for a model viscous Hamilton-Jacobi Equation. Despite this equation seems to be uniformly elliptic, loss of boundary conditions may occur because of the strong nonlinearity of the first-order part and therefore the Dirichlet boundary condition has to be understood in the sense of viscosity solutions theory.

We describe the main characteristics of this equation, we explain why one has such loss of boundary data through a stochastic control interpretation and we show how to obtain the existence and the uniqueness of a continuous solution which is defined globally in time.

Controllability of Schrödinger equations

Karine Beauchard, Ecole Normale Supérieure de Cachan

Abstract: We consider a quantum particle, in an infinite square potential well, subjected to a control which is a time dependent uniform electric field. It is represented by a Schrödinger partial differential equation, on a bounded domain. We study the exact controllability of this bilinear control system, in different situations. In 1D, we can prove the local exact controllability around the eigenstates. Depending on the situation, the proof relies on a Nash-Moser implicit functions theorem, moment theory and the return method or expansions to the second order. Finally, we investigate the same question in 2D and 3D.

Travelling fronts with forced speed

Henri Berestycki, EHESS Paris

Abstract: This talk reports on joint work with Luca Rossi and is concerned with reaction-diffusion equations of the KPP type in inhomogeneous media. Specifically, the reaction term involves a bounded "favourable zone" which is travelling with a forced speed. Such a model is introduced to describe the effect of a climate change on a biological population. First, necessary and sufficient conditions are established to characterize the existence of travelling waves in cylinders or in all of space. This allows one to derive the asymptotic behavior of solutions of the evolution equation in various geometrical settings. Then, we describe problems which involve two different forcing speeds or periodic time dependence. Then, one is led to pulsating travelling fronts and the dynamics is related to principal periodic eigenvalues of parabolic operators.

Optimal Transport, Convection and Magnetic Relaxation

Yann Brenier, CNRS et Université de Nice

Abstract: We establish a connection between Optimal Transport Theory and classical Convection Theory for geophysical flows. Our starting point is the model designed few years ago by Angenent, Haker and Tannenbaum to solve some Optimal Transport problems by a gradient flow approach. We interpret this geometric equation as a generalization of the Darcy-Boussinesq equations for use in Convection Theory. This suggests that the Navier-Stokes-Boussinesq equations, the basic model in Convection Theory, provide a good framework for Optimal Transport related problems, such as Hoskins' Semigeostrophic equations and some fully nonlinear version of some Chemotaxis equations. In a different direction, we introduce a "stringy" generalization of the Angenent Haker Tannenbaum model. This model is closely related to the Magnetic Relaxation model investigated by Arnold and Moffatt. It is a gradient flow for which the equilibrium states are just the (variational) solutions of the Euler equations for incompressible flows. We also discuss a vectorial version of the Burgers equation, the 'cross-Burgers' equation.

Incompressible limit and non-isentropic fluids

Didier Bresch, CNRS et Université de Savoie

Abstract: The aim of this talk is to present mathematical difficulties and first partial answers on incompressible limit and non-isentropic fluids in the periodic case. After recalling briefly the isentropic case, we will present previous works in the whole space or exterior domain case and we will discuss problems coming from the fact that acoustic wave equations involve variable coefficients depending on space and times in the periodic case. This implies resonance phenomena but also crossing and multiple eigenvalues. We will present measure type estimates based on transversality properties and explain why

flows could avoid the resonant set for almost all initial data. This will help to conclude and show in fact that the limit dynamics, in the ill-prepared case, would be given by a similar system as the one studied by Bresch, Desjardins, Grenier, and Lin (2002) for almost all initial data. This is joint program with B. Desjardins and E. Grenier.

Stability criteria for reaction-diffusion systems with skew-gradient structure

Chiao-Nien Chen, National Changhua University of Education

Abstract: Reaction-diffusion systems with skew-gradient structure can be viewed as a sort of activator-inhibitor systems. In connection with calculus of variations, there is a close relation between the stability of a steady state and its relative Morse index. The stability criteria presented here were partially motivated by recent works of Yanagida.

Minimizing solutions of the planar n-body problem in certain topological classes

Kuo-Chang Chen, National Tsing Hua University

Abstract: The n-body problem concerns the motion of n celestial bodies moving in space in accordance with Newton's law of universal gravitation. In recent years variational methods have been successfully applied to the n-body problem to construct miscellaneous solutions with some equal masses. For most choices of mass, a special class of solutions called retrograde solutions can be also constructed by variational methods. In this talk I will briefly discuss a generalization of this result to the n-body problem with some topological constraints. In particular, I will outline a variational proof for the existence of some solutions for the four-body problem with distinct masses in a special topological class.

Random modulation and persistence of solitons for the stochastic Korteweg-de Vries equation

Anne de Bouard, CNRS et Ecole Polytechnique

Abstract: We consider the KdV equation perturbed by additive or multiplicative noise, white in time, and analyse the dynamical behaviour of the solutions which are close to solitons, in the limit where the amplitude of the noise goes to zero. We estimate the exit time of neighbouhoods of the soliton, with or without random modulations of the soliton parameters. In the additive case, the estimates we obtain are sharp, while in the multiplicative case, we obtain additional properties of the solution as e.g. "superdiffusion" of its order one part. We also estimate the exit time in the multisoliton case. The results have been obtained in collaborations with A. Debussche, E. Gautier and K. El Dika.

Denjoy-Schwartz and Hamilton-Jacobi

Albert Fathi, Ecole Normale Supérieure de Lyon

Abstract: Given a C^2 Hamiltonian $H(x,p)$, C^2-strictly convex in the moment variable, it has been shown by Patrick Bernard that one can always find C^1 strict subsolutions with locally Lipschitz derivative of the Hamilton-Jacobi equation. After explaining the general background for the nonspecialist, the talk will concentrate on the constraints imposed on smoother critical subsolutions by the implications of the classical Denjoy-Schwartz theory of Dynamical Systems on the circle or the 2-dimensional torus.

Some examples of global solutions associated with large initial data for the incompressible Navier-Stokes system

Isabelle Gallagher, Université de Paris VII

Abstract: It is well known that the incompressible Navier-Stokes system has a unique, global solution if the initial data is small enough (in scale invariant function spaces). In this talk we will give a notion of "best space" to solve the equations, and we will present examples of initial data, arbitrarily large in that space, which generate a unique, global solution. Those results will be related to blow-up theorems for "toy models" for the incompressible Navier-Stokes system

Those are joint works with J.-Y. Chemin, and with M. Paicu.

Motion by Curvature of Planar Curves with Two Free End Points

Jong-Shenq Guo, National Taiwan Normal University

Abstract: In this talk, we shall present some recent results on the motion of curvature of planar curves having two free end points. One motivation of this work is from the study of evolution of grain domains in polycrystals. We shall deal with a special case of three phases, in which the two triple junctions (or, two free end points) are moving along a straight line with fixed contact angles to this line. We first prove that the curve shrinks to a point in a finite time. Then we derive the existence of a unique self-similar solution. Finally, we show that the curve shrinks to a point in a self-similar manner, if initially the curve is a graph.

Global Existence of Weak Solutions to the Cauchy and Initial-Boundary Value Problems of Nonlinear Hyperbolic Balance Laws

John Meng-Kai Hong, National Central University

Abstract: In this talk we are interested in the global existence of weak solutions to the Cauchy and initial-boundary value problems of nonlinear balance laws. Applying

Lax's method and generalized Glimm's method, we construct the approximate solutions based on the corresponding perturbed Riemann solutions. The stability of generalized Glimm scheme is obtained by the wave interaction estimates. Then, the global existence of weak solutions can be carried out by showing the weak convergence of residuals for the source terms of equations. The results can be applied to establish the existence of globally Lipschitz continuous solutions for quasilinear wave equations.

On a nonlocal problem arising from Phytoplankton Blooms : Incomplete Mixing and Competition for Light

Sze-Bi Hsu, National Tsing Hua University

Abstract : In this talk we present a mathematical model of n species competing for light under an environment of abundant nutrients. The model takes the form of parabolic equations with nonlocal terms. First we present a complete result of steady state problem for the case of one species. Then we consider the case of parabolic PDE of two species competing for light. We obtain a partial result of extinction when the diffusion coefficients are equal. This is the joint work with Du Yihong, University of New England.

Measure concentration, functional inequalities and curvature of metric measure spaces

Michel Ledoux, Université de Toulouse III

Abstract : We present a circle of ideas between analysis, geometry and probability theory, around the concentration of measure phenomenon, geometric, information theoretic and functional inequalities, and evolution equations, that led recently to notions of curvature in metric measure spaces.

Uniqueness of solutions for mean field equations and application

Chang-Shou Lin, National Cheng Kung University

Abstract : In this talk, I will survey some of recent development of uniqueness of mean field equations in bounded domains in $\mathbf{R}^2$ or in a compact Riemann surface. As applications of uniqueness, I will give a sufficient and necessary condition for the existence of solutions at the critical value 8π. Another application is on the flat torus, the Green function has at most five critical points.

Unfolding elastic knots

Chun-Chi Lin, National Taiwan Normal University

Abstract : In this talk we introduce a method of unfolding or untangling a given smooth knot by the variational geometric flows. Namely, we associate an energy to knots, which consists of elastic energy and the so-called Moebius energy, and set up a gradient flow which decreases the energy. We show that, for a given smooth knot, it would remain smooth for all time during the gradient flow, and approach an equilibrium configuration asymptotically. Note that knot classes would be preserved during the gradient flows because self-penetration would not happen.

Elliptic and parabolic regularity for discrete elliptic differential operators

Emmanuel Russ, Université d'Aix-Marseille III

Abstract : Let Γ be a infinite graph and consider a second order elliptic differential operator L on Γ. Under suitable geometric assumptions on Γ (namely, conditions on the volume growth of balls and Poincaré type inequalities), we show that some elliptic and parabolic regularity properties on L are equivalent. Comparisons between L^p norms of $L^{1/2}f$ and ∇f are also given. Part of this work is joint with Nadine Badr.

About the influence of the wind on the oceanic motions

Laure Saint-Raymond, Université de Paris VI

Abstract : The goal of this presentation is to understand the influence of the wind on the oceanic circulation. More precisely, we are interested in obtaining a good theory for the general density and velocity structure of the ocean, including the non local effects of the wind.

We first describe the linear response of an homogeneous fluid to some wind forcing, which admits fast time oscillations and may be resonant with the Coriolis force. In addition to the usual Ekman layer, we exhibit another - much larger - boundary layer, and some global vertical profile. That means in particular that the wind effect is no longer localized in the vincinity of the surface.

However such a simple model do not capture the main features of the oceanic circulation since both the dependence of the Coriolis parameter with respect to the latitude and the stratification of the density are neglected. We will then turn to a more complex model to consider the important effect of the ventilation of the thermocline which occurs as oceanic density surfaces rise to intersect the oceanic mixed layer.

Traveling water waves : a global variational approach

Eric Séré, Université de Paris-Dauphine

Abstract : We consider the classical problem of periodic traveling water waves in presence of gravity and surface tension. We give a weak formulation of this free-boundary

problem. We find weak solutions of arbitrary momentum μ as minimizers of an energy functional. When μ is not too large, our weak solutions are classical solutions.

Identification of viscosity in an incompressible fluid

Jenn-Nan Wang, National Taiwan University

Abstract: In this talk I would like to discuss the inverse problem of determining the viscosity parameter in an incompressible fluid. I will consider the Stokes equations (linear) or the Navier-Stokes equations (nonlinear) as the governing system.